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PREFACE

This volume of QUARTERLY TRANSACTIONS is Part II of Volume 50, issued in June 1931 in accordance with the change of dates of issue explained in Part I of this Volume.

It contains the papers and discussions presented at the Winter Convention of the A. I. E. E. New York, January 26-30, 1931, and in addition, one paper, "Harmonic Generation by Means of Grid-Circuit Distortion," by F. E. Terman, D. E. Chambers, and E. H. Fisher, which was presented at the Pacific Coast Convention, September, 1930. It also contains the Report of the Board of Directors for the fiscal year ending April 30, 1931, and a financial report covering the same period.

A Table of Contents is printed on the front page, and an Index of Authors appears in the back of the volume. The complete subject and authors' annual indexes are printed in Part 4 of the QUARTERLY TRANSACTIONS each year.

The Newfoundland-Azores High-Speed Duplex Cable

BY J. W. MILNOR,* Fellow, A. I. E. E. and G. A. RANDALL,† Non-member

Synopsis.—A description is given of the cable laid by the Western Union Telegraph Company in 1928 between Newfoundland and the Azores. The plan of this cable is unique in that it consists of a combination of non-loaded and loaded cable, the latter being tapered throughout certain portions. By means of this arrangement it has been possible to secure duplex or two-way operation at a speed of 1,400 letters per minute in each direction. The general methods used in obtaining the duplex balance are described and the fundamental theory of the cable is given.

* * * * *

In September 1928 the final splice was made in a submarine telegraph cable of a new type connecting Bay Roberts, Newfoundland, and Horta, Azores. This cable adds an important link to the transatlantic cable system of the Western Union Telegraph Company. From Newfoundland it is extended to New York by land line connections, and at Horta connections are made with German and Italian cables.

This cable combines the advantage of high-speed operation characteristic of the new continuously loaded cable, with the facility of duplex or two-way operation inherent in the old non-loaded type of cable. Tests have shown that a signaling speed of 1,400 letters per minute cable code (42 cycles per second) simultaneously in both directions is now practicable, and the indications are that the ultimate duplex speed may be somewhat higher. If operated simplex, *i. e.*, in one direction only, the speed is somewhat over 2,500 letters per minute. The duplex speed is several times as high as any long cable has previously been duplexed, and provides the greatest message capacity of any existing transocean cable.

Prior to 1890, Heaviside had shown the important relation which exists between the inductance of a cable and its operating speed. However, it was not until recently that this knowledge could be applied to long telegraph cables, as a result of the development by the Bell Telephone Laboratories of a new magnetic alloy, permalloy, and of means for applying it to submarine cables.[1] The first cable of this type was laid by the Western Union Telegraph Company between New York and Azores Islands in 1924. The results obtained on that cable fully justified expectations, and since then a total of eight other long-loaded cables has been laid by various companies in the Atlantic and Pacific Oceans.

It has long been customary for the relatively slow speed non-loaded submarine cables to be operated simultaneously in each direction.[2] The electrical characteristics of loaded cables are such that they are much more difficult to duplex. For this reason all loaded cables laid between 1924 and 1927 were designed to be operated simplex only. Special devices were developed for quickly reversing the direction of transmission at regular intervals which may be as short as one minute. There are many situations, however, where duplex operation is decidedly advantageous, and it was inevitable that attempts should be made to develop a cable capable of high-speed duplex operation.

The desired result was obtained in the cable here described through the use of a combination of loaded and non-loaded cable, the latter being adjacent to the ends.

GENERAL THEORY OF DUPLEX CABLES

In order to duplex a cable it is necessary to provide at each terminal station a so-called artificial line which closely simulates the impedance of the cable as measured from its end. The highest practicable accuracy of balancing is desirable, as the cable efficiency is thereby increased. The two impedances should closely match throughout a range of frequencies from about half the fundamental signaling frequency up to about 1.6 times that frequency; a somewhat less accurate balance is necessary at frequencies outside of that range.

A loaded cable is more difficult to balance than non-loaded cable for three reasons:

1. There are irregularities in the impedance as measured at different points along a loaded cable, which are much more pronounced than if non-loaded. Difficulties in manufacture are such that variations from the mean values of inductance are appreciable and unavoidable.

2. The effect on balance of such irregularities is more

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[1] "Permalloy, An Alloy of Remarkable Magnetic Properties," by H. D. Arnold and G. W. Elmen, *Jour. Franklin Inst.*, V. 195, 1923, p. 621.

The Loaded Submarine Telegraph Cable, by O. E. Buckley, A. I. E. E. TRANS., 1925, p. 882.

A bibliography on loaded cable practise appears in an article by Foster, Ledger, and Rosen, *Jour. I. E. E.*, London, April 1929, p. 475.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

[2] Duplex operation was first introduced in 1871.

prominent in a loaded cable. The reason for this is that the velocity of wave propagation is slower and therefore the reflections from different parts of the cable tend to appear distinct, while in the non-loaded cable they are relatively merged. A more accurate balance is therefore necessary with loaded cable.

3. The impedance of a loaded cable is not constant. The inductance due to the loading material, as well as the effective resistance, vary with current as indicated in Fig. 2. The signaling current, of course, continually changes in magnitude, consequently the cable impedance undergoes frequent small variations.

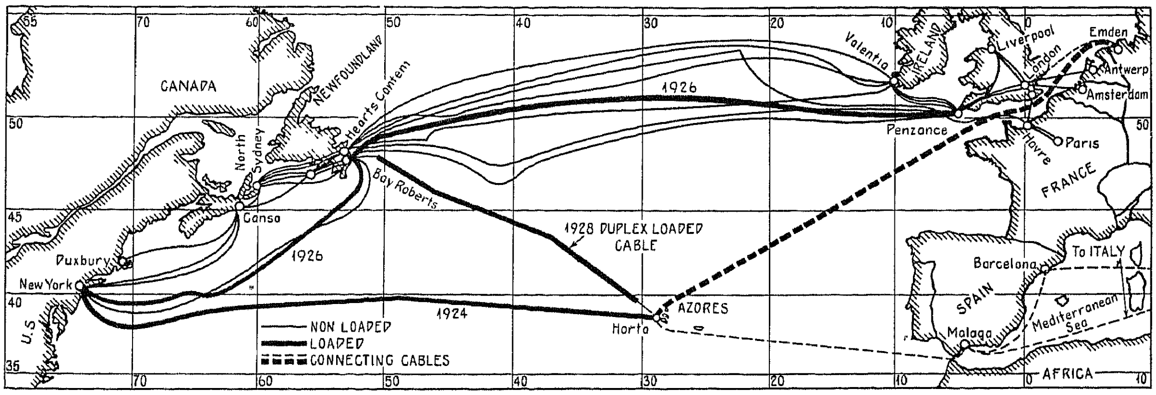

Fig. 1—Western Union Cable System in North Atlantic

Each of the above difficulties is reduced in importance by loading only the central portion of the cable, the end sections being non-loaded. For the non-loaded portion, a duplex balance is obtained having about the same order of accuracy as has been customary in the past in cables which are non-loaded throughout their length. In the part of the artificial line corresponding to the loaded portion, a duplex balance of a much lower degree of accuracy is sufficient, since it is distant from the terminals.[3]

The correctness of the above will be made evident in the Appendix, in which a more complete exposition is given of the influence of various factors. Some of these factors may be briefly summarized as follows:

The permissible attenuation from end to end of the cable as measured at the working frequencies, is determined by the degree of accuracy with which the artificial line as a whole can be adjusted to match the impedance of the cable. With higher accuracy a greater attenuation is permissible. The attenuation which is permitted from end to end of the *loaded* portion of the cable is determined by the accuracy with which the impedance of that portion of the cable can be matched in the corresponding portion of the artificial line.

In order that the most economical design of cable may be obtained, it is necessary that special consideration be given to methods for improving the stability of the impedance of the loaded portion of the cable, as affected by changes in the inductance of loading caused by varying current. The amount of change of inductance with current may be controlled to some extent through variations in the core design and in its treatment during manufacture. By the use of sea earths of proper length at the ends to hold the level of extraneous interference to as low a value as readily practicable, the use of a lower battery voltage at the opposite end of the cable is made possible. This results in lower current strength throughout the cable, with a corresponding gain in stability.

Description of 1928 Cable

A theoretical study completed in January 1928, had indicated the commercial practicability of this type of duplex cable. At that time telegraph traffic on the simplex loaded cable previously laid between New York and Azores, was growing so rapidly that some increase in the facilities was urgently needed. This growth could be met only by providing a second connection at an early date, and it was considered desirable to take

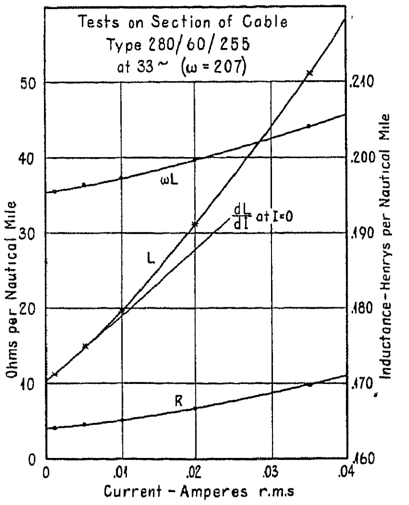

Fig. 2—Example of Variation of Loaded Cable Resistance and Inductance with Current

advantage of this new development in the cable about to be laid.

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[3] It should not be inferred from these statements that it would be impossible to duplex an all-loaded cable. If by proper design the attenuation from end to end were held to a reasonably low value, and other factors are correct, a balance should be practicable. The combination of loaded and non-loaded cable was used in the present case in order to obtain the desired results at a minimum expense.

Because of the nature of the new technical problems involved in the design of the cable and its balancing equipment, and the lack of time in which to make proper experimental confirmation of some of them, it was considered advisable that the cable be so constructed as to permit simplex operation, if for any reason the duplex balancing should not prove satisfactory. However, it was found that in order to provide for simplex operation at the higher speed, it was unnecessary to make any important change from the design which was found to be best for duplex working.

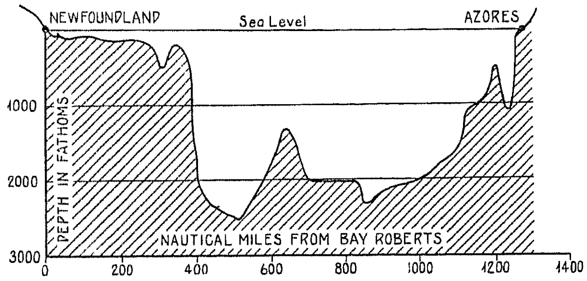

Fig. 3—Profile of Ocean Bottom Route of 1928 Cable

The formal order for the manufacture was placed in March, 1928, although some details of the design were not completed until much later. The cable was manufactured by the Telegraph, Construction and Maintenance Company, Ltd. of London, and laid by their cable ship "*Dominia*," the laying being completed on September 1st of that year.

The route followed by the cable is indicated in Fig. 1. A profile chart showing its depth is given in Fig. 3.

Plan of Cable. The total length of the cable is 1341.2 nautical miles. The non-loaded end portions are each 160 miles in length. The remainder of the cable is loaded, two different weights of conductor and loading being used. In Table I are given further details of the design, together with the results of electrical tests.

TABLE I
PHYSICAL AND ELECTRICAL CHARACTERISTICS OF EACH CORE TYPE

Factory values per nautical mile, at 75 deg. fahr.

	Ends	Intermediate	Center
Weight of copper—lb	508	329	284
Weight of loading—lb	non-loaded	51	60
Weight of gutta percha—lb	303	275	258
Resistance d-c.—ohms	2.34	3.60	4.18
Capacitance— μf	0.357	0.344	0.342
Dielectric resistance—megohms	746	919	1004
Section length—miles	2 x 160	2 x 70	881

Conductor. The conductor consists of a solid copper central wire closely surrounded by five or six thin copper tapes.

Loading. The loading throughout the central portions is Mumetal wire which was supplied by the Telegraph, Construction and Maintenance Company. This material is similar in general composition and characteristics to permalloy. The wire is wound directly on the copper conductor in a close continuous wrapping. The heaviest loading wire is about 0.008 in. in diameter. It is estimated that the total length of loading wire in this cable is about 50,000 miles.

An inductance range from 0.056 to 0.230 henrys per mile was obtained in different portions of the cable through controlled variations in the annealing of the loaded conductor. The average values of inductance in the various sections are indicated in Fig. 4. The permeability (corresponding to small magnetizing forces) ranges from 2,000 to 6,800. This maximum value materially exceeds any previously obtained in cable construction.

A special asphaltum flux was used to fill the interstices of the loaded conductor, for the purpose of equalizing pressure upon this strain-sensitive loading material.

Insulation and Armoring. The conductor, either loaded or non-loaded, was covered with a conservative thickness of a gutta percha mixture for purposes of insulation, in accordance with usual cable practise. This in turn was covered with wrappings of jute and a sheath of iron or steel wires to provide tensile strength and protection against injury. The portions near shore have much heavier sheathing than the main sections, and in addition have a wrapping of brass tape applied to the core as a precaution against injury by the teredo.

Sea Earths. The cable is single cored except near the ends. In order to reduce as far as possible the extraneous voltages picked up in shallow water, it is customary in cable practise to carry a second and sometimes a third core in the sections of cable near the ends. These additional cores are grounded at their outer ends to the cable sheath. At Bay Roberts the distance out to the end of the longer or receiving sea earth is 50.10 nautical miles. At Horta, Azores the corresponding distance is 2.39 nautical miles. The receiving sea earth connection at Bay Roberts differs from the other sea earth connection in the fact that a manganin resistance of 160 ohms is connected at sea in series with the sea earth core. This resistance assists in equalizing any

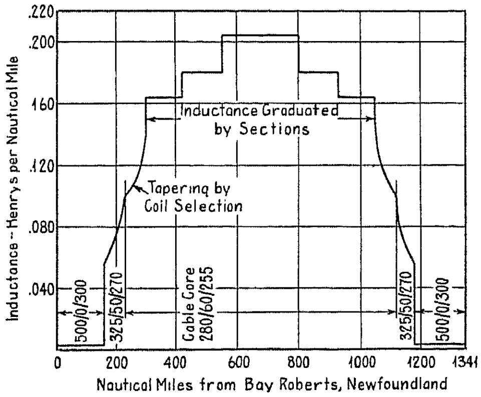

Fig. 4—Inductance of Cable

extraneous voltages picked up by cable and sea earth cores between that point and the receiving apparatus.

CABLE TESTS

Various tests were made during manufacture and upon the laid cable in order to ascertain its performance characteristics.

The results of the d-c. tests are as follows:

Total d-c. resistance of cable	4521	ohms
Total d-c. capacity of cable	463	μf.
Total dielectric resistance, mean of first 5 minutes	24.65	megohms

The attenuation of the cable was measured in each direction at various frequencies and with several sending voltages. The results of some of these tests are shown in Fig. 5. The curves show the attenuation of the fundamental frequency of a square-topped transmitted wave. The effect of changing the sending voltage was found to be small.

The time of propagation of signals over the cable is 0.235 seconds at 60 cycles. Based upon the measured values of time of propagation and attenuation at various frequencies, it has been estimated that during the process of laying, the strains placed on the cable caused an average loss in the inductance of about 13 per cent.

In Fig. 6 is shown a record of interference obtained at Newfoundland, using the 50-mile sea earth. At Azores where the cable reaches deep water within a few miles, the interference is much less.

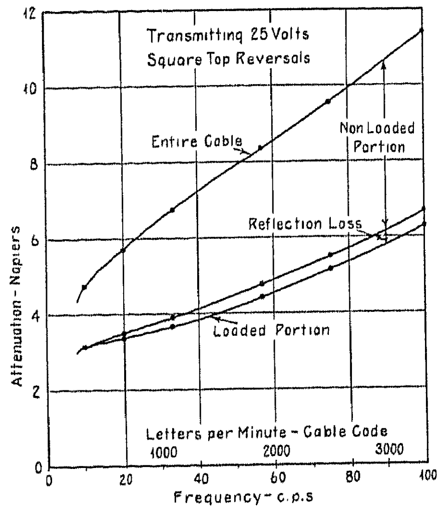

FIG. 5—ATTENUATION OF LAID CABLE

ARTIFICIAL LINE EQUIPMENT

An essential part of this cable project has been the provision at each cable station of an adequate artificial line for completing the Wheatstone Bridge arrangement which is necessary for duplex operation. This line is a network which is an electrical image of the cable, designed to match it mile for mile as accurately as is necessary for the purpose. A certain amount of work has been done by others[4] with a view to providing a network for general use which would contain less electrical capacity than the actual cable, and would therefore be relatively inexpensive. The resultant network has always been complicated in theory and possibly difficult to adjust. In the present work, no attempt was made to use such devices.

The present equipment is somewhat more complicated than in the case of the usual artificial line for low-speed cables. For a non-loaded cable the network usually consists of units of resistance and capacity only. In addition to these two elements, the present equipment includes means for simulating the inductance of both non-loaded and loaded portions, eddy current losses in the loading, sea return losses, and dielectric losses. A compensating arrangement is introduced to prevent filter oscillations between the condenser and inductance units of the artificial line, in order to avoid the use of an excessive number of coils. The artificial line is adjustable in every portion, the size of the steps at each point of the line depending upon the accuracy of impedance matching required at that point.

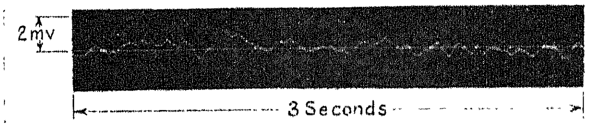

FIG. 6—RECORD OF INTERFERENCE IN CABLE, HIGH AMPLIFICATION

As originally installed, the artificial lines were set up on the basis of computed values, the initial error in matching being of the order of one per cent. Uncertainties in the exact distribution of the cable constants themselves prevented a closer initial balance. The final balances were obtained by a systematic adjustment of the various elements composing the line.

As in the case of the cable itself, the change of inductance with current in the coils of the artificial line must be given consideration in their design. In theory it might be practicable to provide artificial line coils whose inductance would vary with current in the same manner that the inductance varies in the corresponding portion of cable, thus permitting the use of relatively larger inductance variation in designing the cable. Such procedure would greatly increase the complexity of the artificial line. It was therefore considered desirable to hold the current-inductance coefficients (defined in the Appendix) of both the cable and artificial line to relatively small amounts, the values for the artificial line being kept smaller than those in the cable.

Because of the use of inductance coils, some other unusual precautions were necessary in this line. In the usual artificial line, the resistance elements are of wire having low temperature coefficient. This wire did not appear to be practicable in the inductance coils, and in order that the line should not be unusually susceptible to temperature changes, the copper resistance of the coils has been kept at a relatively low value. This necessitated larger coils than would otherwise have been used.

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[4] One of these methods is described by H. Salinger and H. Stahl, "The Simulation of Long Submarine Cables," *Elektrische Nachrichten Technik*, August, 1926.

Special precautions have been taken to prevent pick-up of extraneous disturbances by the coils. This is accomplished by a method of mounting and wiring the coils which effectively transposes them with respect to external fields.

Each artificial line occupies a space of approximately 210 cu. ft. These lines are mounted in heat-insulated cabinets. To correct for variations in the balance due to temperature changes along the cable and to a less extent in the artificial line, five adjustable resistances have been provided in the operating room. Observations over a period of more than a year indicate that these resistances need not be altered more than once daily to maintain a satisfactory balance.

The method used in making the final field adjustments to the artificial lines is not new, being similar in principle to that used for many years with non-loaded cables, except that a high-speed recording device was used to observe the balance. During the greater part of the work a single reversal at a time was transmitted into the cable, while a record was obtained of the resulting current caused by lack of perfect balance. In the final refinements of balance different combinations of telegraphic signals were sent at various speeds.

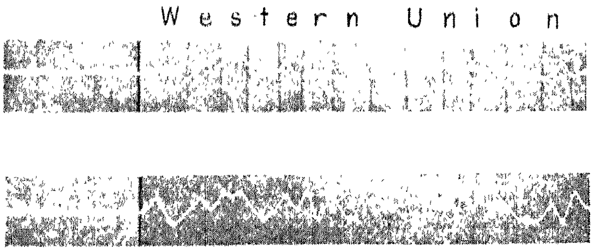

Fig. 7—Received Signals at Bay Roberts—Cable Code

Upper record—simplex, westward transmission
Lower record—duplex, eastward and westward transmission

A Western Electric Signal Shaping Amplifier[5] is used both during balancing and for commercial operation, the amplifier being standard except for a few minor alterations and additions.

In Fig. 7 are shown records of characters received, both simplex and duplex, over the cable. The speed here is 1,400 letters per minute cable code. A 12-volt battery was used for sending, and the strength of received signal with reversals at the above speed is about six millivolts. It has been estimated that the impedance of the artificial line matches that of the cable to an accuracy of roughly one part in 4,000, as determined when telegraph characters are transmitted. Such characters may be considered to be made up of many frequencies sent simultaneously. When determined with a sinusoidal voltage in the frequency range between 35 and 55 cycles, the estimated accuracy is better than one part in 7,000.

Relative Merits of Simplex and Duplex High-Speed Cables

In determining the kind of cable facilities best suited for handling telegraph traffic between two given areas, there are many factors which must be considered, and individual consideration for each project is therefore required.

The relative advantages and economics of simplex and duplex cables depend to a certain extent upon the length of the circuit and the desired message capacity. Consideration must be given to the relative amounts of urgent and deferred traffic; the difference if any in the load in the two directions; and the difference in time between the terminal areas. The nature of any existing cable facilities between the terminals must be taken into account in determining the method of increasing the facilities. It is also important to consider the manner in which the traffic would be handled in the event of a failure of one or more cables.

Where the load between two points is sufficient to justify only a single cable, and the difference in time between terminals is not excessive, the duplex type usually offers the more convenient method of handling it. An alternative arrangement is a simplex cable with automatic direction reversing equipment. The latter plan necessarily results in part of the traffic being somewhat delayed, a condition which in some cases is objectionable. In addition, much of the terminal equipment is idle half the time, and when working must be operated at relatively high speed. On the other hand, the duplex cable requires the somewhat complicated artificial line networks.

If the terminals of the cable circuit are so located that there is a considerable time difference between them, as for example between America and Japan, the simplex cable has the advantage, since the greater part of the traffic flows in one direction at a time.

Where the amount of traffic is enough to justify two or more cables, there are various possible solutions. The equivalent of a duplex circuit can, of course, be obtained through the use of two simplex cables worked in opposite directions. Ordinarily, however, the rate of traffic growth is not sufficient to justify laying two cables at the same time or within short periods. In some cases, a combination of simplex and duplex cables may be found to be the best ultimate solution when all factors are taken into account.

It appears probable that neither the simplex nor the duplex cable has as yet reached its limit of economical development, hence it is hardly practicable at this time to give a final answer to the above questions.

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[5] "Applications of Vacuum Tube Amplifiers to Submarine Telegraph Cables," by Austen M. Curtis, *Bell System Technical Jour.*, July, 1927.

ACKNOWLEDGMENTS

The fact that the application of non-loaded ends to a loaded cable would facilitate duplex working was discovered independently by Mr. J. J. Gilbert of the Bell Telephone Laboratories, and by the first named of the authors.

The cable under discussion and the duplexing equipment were developed, designed, and balanced by engineers of the Western Union Telegraph Company. Important contributions were made by Messrs. C. H. Cramer and W. D. Cannon. Most of the field adjustments of artificial lines were in charge of Mr. E. L. Newell. The route and the sheathing structure for the cable were worked out by Mr. V. H. Rowland.

Appendix

NOTES ON THEORY OF DUPLEX LOADED CABLE

Accuracy of Balance. The general theory of duplex balancing was published in a previous paper.[6] It was shown that the required degree of accuracy by which the impedance of the artificial line must match the impedance of a uniform cable, is proportional to the napierian base having as an exponent the attenuation. In the case of a composite cable of the type under discussion, the same theory is applicable providing that the proper value of attenuation is used.

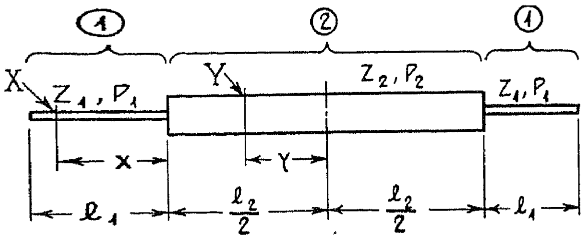

FIG. 8—NOMENCLATURE

Section 2 is long, *i. e.*, $\epsilon^{-2\alpha_2 l_2}$ is neglected

Z_1 and Z_2 are the characteristic impedances

P_1 and P_2 are the propagation constants per unit length

α_1 and α_2 are the attenuation constants per unit length

The full theory of a tapered loaded cable involves expressions which are somewhat unwieldy. The theory will be discussed here on the assumption that the loaded portion is uniform throughout, the cable being as indicated in Fig. 8. The manner of taking into account tapering of the inductance will be discussed later.

For determining the necessary accuracy of the duplex balance of the cable in Fig. 8, the following is the proper value of attenuation:

Total attenuation

$$= \log_\epsilon \left| \cosh P_1 l_1 + \frac{Z_1}{Z_2} \sinh P_1 l_1 \right| + \log_\epsilon \left| \cosh P_1 l_1 + \frac{Z_2}{Z_1} \sinh P_1 l_1 \right| + \alpha_2 l_2 \qquad (1)$$

This value is also found to be the natural logarithm of the ratio of sent to received voltage with the cable terminated in an impedance equal to the cable impedance as measured from that end. This expression, as well as those immediately following, have been developed by straightforward methods from the well known formulas for current and voltage of lines having distributed resistance, capacity, etc. In applying these expressions, they must in general be considered throughout a range of frequencies, preferably including all frequencies from zero to about twice the fundamental signaling frequency.

At points other than at the head of the line, the required accuracy of adjustment of the artificial line is less, since the effect of any irregularity is attenuated. It may readily be shown that at point X, Fig. 8, the required degree of accuracy is proportional to the napierian base with an exponent as given in equation (1), but with x written in place of l_1. At point Y, the required accuracy of adjustment is proportional to $\epsilon^{2\alpha_2 y}$.

Stability of Balance. The manner in which the inductance and resistance of loaded cable varies with current was indicated in Fig. 2. This tends to cause an instability of balance, which must be held to a low value as determined from the necessary accuracy of balance. An expression for the fractional rate at which the cable impedance as measured with alternating current changes with the current may be obtained by differentiation of the standard equation for characteristic impedance. For loaded cables the result is approximately:

$$\frac{d\,Z}{Z\,d\,I} = \frac{1}{2}\left(\frac{d\,L}{L\,d\,I} - \frac{j\,d\,R}{\omega\,L\,d\,I}\right) \qquad (2)$$

In general it is found that the effect on balance of the change of resistance is less important than the change of inductance.

It is convenient to introduce the expression, "current coefficient of inductance," as a measure of the fractional rate of change of inductance as measured with small values of alternating current; *i. e.*, the quantity $(d\,L/L\,d\,I)_0$. Its value depends upon the characterisics of the loading material and the diameter, in accordance with the following equation;

$$\left(\frac{d\,L}{L\,d\,I}\right)_0 = \frac{0.566}{d+t}\left(\frac{d\,\mu}{\mu\;dH_{max}}\right)_0 \qquad (3)$$

in which I is in amperes r. m. s.; d is the diameter of the copper conductor, and t is the thickness of loading material in centimeters. The quantity $(d\,\mu/\mu\,d\,H)_0$ is a property of the loading material. It might be expected to be constant for given material but in fact is found to vary to some extent with the absolute value of permeability, and to a lesser amount with the design and treatment of the core.

The necessary strength of received signal is determined from an estimate of the expected level of interference at each end of the cable, together with the expected amount of extraneous voltage due to the inaccuracy of balance. Knowing this value, the necessary signaling voltage at any point Y may be determined by the use of the following expression for attenuation:

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[6] *Submarine Cable Telegraphy*, by J. W. Milnor, A. I. E. E. TRANS., 1922, p. 30.

Attenuation from point Y to opposite end of cable

$$= \log_{\epsilon} \left| \cosh P_1 l_1 + \frac{Z_2}{Z_1} \sinh P_1 l_1 \right| + \frac{\alpha_2 l_2}{2} + \alpha_2 y \qquad (4)$$

The value of signal current at the point is readily determined from the signaling voltage.

Tapered Loading. There are two advantages of tapering (decreasing) the inductance near the ends of the loaded position: (1) tapering reduces the loss of signaling energy caused by reflections from the non-loaded portion; (2) tapering permits the use of lower permeability in the loading material in this portion, which results in a smaller current-inductance coefficient.

On the other hand, the use of lower inductance has the disadvantage of causing the attenuation constant per mile to be increased; and to avoid having the attenuation become too large it was advantageous to make use of a heavier copper conductor within 70 miles of each end of the loaded portion of the 1928 cable. This has the additional advantage that the current-inductance coefficient is also lowered when the size of conductor is increased, as indicated in equation (3).

A solution of the equations of signal propagation in a circuit having linearly tapered inductance and resistance, and uniform capacitance and leakance is as follows:[7]

The parameters of the circuit are assumed to be:
$R_0 + K_R x$, the resistance per unit length
$L_0 + K_L x$, the inductance per unit length
C and G, the capacitance and leakance per unit length
K_R and K_L, the rates of tapering
x = the distance from the head of the taper
Subscript zero always refers to the point $x = 0$

The fundamental differential equations of current and voltage at any point are:

$$-\frac{dE}{dx} = [(R_0 + K_R x) + j\,\omega\,(L_0 + K_L x)]\,I \qquad (5)$$

$$-\frac{dI}{dx} = [G + j\,\omega\,C]\,E \qquad (6)$$

Differentiating and substituting,

$$\frac{d^2 I}{d x^2} = (G + j\,\omega\,C)\,[(R_0 + j\,\omega\,L_0) + (K_R + j\,\omega\,K_L)\,x]\,I \qquad (7)$$

Substituting

$$\theta = x \sqrt{(G + j\,\omega\,C)\,(R_0 + j\,\omega\,L_0)} \qquad (8)$$

and

$$\alpha = \frac{K_R + j\,\omega\,K_L}{\sqrt{(R_0 + j\,\omega\,L_0)^3\,(G + j\,\omega\,C)}}$$

this becomes

$$\frac{d^2 I}{d\theta^2} = (1 + \alpha\,\theta)\,I \qquad (9)$$

This equation may be solved by infinite series with the result

$$I = k_1 S_1 + k_2 S_2 \qquad (10)$$

where

$$S_1 = 1 + \frac{\theta^2}{2\,!} + \frac{\theta^3\,\alpha}{3\,!} + \frac{\theta^4}{4\,!} + \frac{4\,\theta^5\,\alpha}{5\,!} + \frac{\theta^6\,(1+4\alpha^2)}{6\,!} + \frac{9\,\theta^7\alpha}{7\,!} + \frac{\theta^8(1+28\,\alpha^2)}{8\,!} + \frac{\theta^9(16\alpha + 28\alpha^3)}{9\,!} + \frac{\theta^{10}(1 + 100\alpha^2)}{10\,!} + \ldots \qquad (11)$$

$$S_2 = \theta + \frac{\theta^3}{3\,!} + \frac{2\,\theta^4\,\alpha}{4\,!} + \frac{\theta^5}{5\,!} + \frac{6\,\theta^6\,\alpha}{6\,!} + \frac{\theta^7(1 + 10\alpha^2)}{7\,!} + \frac{12\,\theta^8\,\alpha}{8\,!} + \frac{\theta^9(1 + 52\,\alpha^2)}{9\,!} + \frac{\theta^{10}(20\,\alpha + 80\,\alpha^3)}{10\,!} + \ldots$$

(NOTE: There are no simple expressions for the general terms of these series. Usually seven to nine terms are sufficient in practise.)

Combining (6) and (10), we have

$$E = -Z_0 \left[k_1 \frac{dS_1}{d\theta} + k_2 \frac{dS_2}{d\theta} \right] \qquad (12)$$

in which

$$Z_0 = \sqrt{\frac{R_0 + j\,\omega\,L_0}{G + j\,\omega\,C}} \qquad (13)$$

The constants k_1 and k_2 may be obtained from a consideration of the boundary conditions at $x = 0$, with the result,

$$I = I_0 S_1 - \frac{E_0}{Z_0} S_2$$

$$E = E_0 \frac{d\,S_2}{d\,\theta} - I_0 Z_0 \frac{d\,S_1}{d\,\theta} \qquad (14)$$

If Z_a is the impedance terminating the tapered section, the voltage and current at this point are defined by the equation:

$$E_a = I_a Z_a \qquad (15)$$

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[7] This solution was worked out by Mr. John W. Arnold, and was published in the *Proc.* I. R. E., February 1931, p. 304.

Substituting these values for I and E in equation (14), we may eliminate I_0 and finally obtain:

$$\frac{E_0}{E_a} S_1 \frac{dS_2}{d\theta} - S_1 = \frac{Z_0}{Z_a} \frac{dS_1}{d\theta} + \frac{E_0}{E_a} S_2 \frac{dS_1}{d\theta} \qquad (16)$$

A study of the functions S_1, S_2, $\frac{dS_1}{d\theta}$ and $\frac{dS_2}{d\theta}$ shows that

$$S_1 \frac{dS_2}{d\theta} = 1 + S_2 \frac{dS_1}{d\theta} \qquad (17)$$

Therefore:

$$\frac{E_a}{E_0} = \frac{1}{S_1 + \frac{Z_0}{Z_a} \frac{dS_1}{d\theta}} \qquad (18)$$

Similarly, it can be shown that

$$\frac{I_a}{I_0} = \frac{1}{\frac{Z_a}{Z_0} S_2 + \frac{dS_2}{d\theta}} \qquad (19)$$

When the rate of taper is zero, α becomes zero and

$$S_1 = \frac{dS_2}{d\theta} = \cosh\theta$$

$$S_2 = \frac{dS_1}{d\theta} = \sinh\theta \qquad (20)$$

that is, equations (14) reduce to the usual form of the equations of the current and voltage at any point in a uniform line.

Discussion

E. R. Shute: A study of the map given in Fig. 1 shows two loaded cables and seven non-loaded cables connecting Newfoundland with points east, while only one loaded cable and four non-loaded cables connect with the United States. The difference is made up by a system of land wires connecting the long cable sections with New York and other cities, thereby providing, by means of another route, additional assurance of uninterrupted service.

The Bay Roberts-Horta cable forms a diagonal of a quadrilateral. These triangular circuit layouts are greatly desired by the traffic forces because of the protection they afford in case of stoppage or failure of any one of the lengths.

The Commercial Cable Company has similar protection in its Canso-Horta cables.

The wisdom of these layouts was shown following the earthquake of November 18, 1929, when the loaded cable between New York and Horta was interrupted in deep sea.

There are many factors entering into the determination of whether a duplex or simplex cable shall be laid between two points. Generally speaking, where there is only one cable on the route, as in the case of the Bay Roberts-Horta cable, a duplex cable is indicated. A combination of two cables, one a high-speed simplex and the other a duplex cable, in most cases presents an ideal solution. Because of the time difference between terminals, the load generally is lopsided. With this combination you can turn the simplex cable in the direction of the larger file and thus have three times the capacity (assuming that the simplex cable works at twice the duplex cable speed) in one direction as in the other and have continuous communication in the opposite direction for fast services and replies.

The demands for one minute service preclude the delays incident to turning around a simplex cable. This demand is not a theoretical one, for without such service our brokers cannot take advantage of the difference in quotations on a stock on the exchanges on either side of the Atlantic.

Both the Western Union and the Commercial Cable Company support the New York-Horta-Emden route by the via London connections.

As mentioned before, where the permalloy parallels old type cables, as in the case of the New York-Bay Roberts-Penzance-London cable, shown in Fig. 1, the difference in time between the terminals tends to favor simplex operation. The New York-Bay Roberts-Penzance permalloy handles to a large extent only bulk traffic. It could be conveniently arranged to turn around every two minutes, but in practise it is not turned as often as every two hours. The New York-Bay Roberts-Horta cable in contrast is duplexed and handles traffic of all classes.

All of the loaded cables shown on the chart are operated by multiplex printers which are attended by experienced cable operators.

J. W. Milnor: Since this paper was prepared, further work has effected an improvement in the balancing with the artificial lines. The accuracy of balance is now about one part in 7,000 when telegraph characters are transmitted, and one part in 13,000 for any sine wave in the range between 35 and 60 cycles per second. The duplex speed has been increased also, and it is now practicable to transmit 1,600 letters per minute (cable code or multiplex characters) simultaneously in each direction, or a total of 3,200 letters per minute. By using multiplex equipment, five messages may be sent in each direction simultaneously over the cable.

Submarine Cable Telegraphy: Influence of Interference

BY J. W. MILNOR*

Fellow, A. I. E. E.

Synopsis.—*The design and operation of long submarine telegraph cables are affected to an important degree by extraneous electrical interference near the ends. The amount of interference in different cases is dependent upon a number of factors, some of which can only be determined by measurements made in the vicinity.*

The relation is first given between the amount of interference and the operating speed of the cable, assuming the interference to be sinusoidal. This is extended to include actual interference which is not sinusoidal. Since the computations with non-sinusoidal interference are too tedious for general field use, an experimental method is given which provides results which may be used directly in determinations of the effect upon cable operation.

Examples are given of the computation of the response of the receiving instrument under the influence of a transient extraneous voltage, the frequency characteristic of the receiving equipment being known. Conversely, examples are given of the computation of the frequency characteristic from the results of an experimental determination of the response of the receiving instrument to a transient impressed voltage.

* * * * *

IN the design of submarine telegraph cables most of the pertinent electrical properties are known or can be computed to reasonable accuracy. However, there is one factor, namely, extraneous interference near the ends, which is often highly uncertain yet which usually has a large influence both upon the optimum design and upon the limiting performance of the completed cable. In view of this, it is quite desirable that the effects of interference should be examined and that methods for measuring interference should be standardized.

In some cases, electrical interference may be caused by other cables, or by electrical railway or power systems in the vicinity. In general, however, the interference is chiefly of natural origin, akin to the atmospheric disturbances or static which affect radio systems.

Interference into cables from natural causes decreases with increasing depth of the cable.[1] In general it appears to be greatest at some moderate frequency, and decreases at higher and at lower frequencies. The interference depends to some extent upon the design of the sheathing of the cable, upon the electrical shielding if any, and upon the design of the loading, if a loaded cable. It appears generally to be greatest in low latitudes. Where justified, the interference may to a considerable extent be controlled by installing a sea earth, the proper length being dependent upon local and economical considerations.

In a previous paper,[2] the theory of telegraphy with particular reference to long cables was outlined, based upon a consideration of the manner in which sinusoidal alternating currents throughout a band of frequencies are transmitted through the system. With this knowledge at hand, the transient response of the receiving equipment to any form of applied voltage may be determined by performing one or two definite integrations as formulated in the paper referred to. For many purposes, however, it is unnecessary to compute the transient response, since the necessary transmission characteristics for sinusoidal currents may be determined once for all and subsequent computations may be restricted to the latter; in this respect this method of analysis affords important advantages for purposes of design. In the present paper the same method of analysis will be followed, and in order to avoid repetition, occasional references will be made to the previous paper.

The various curves in these papers are for the purpose of indicating the general principles as applied to a representative long cable, rather than giving results which may be accepted as final for use under all conditions. The characteristics of cable receiving equipment vary to some extent according to the design and speed of the cable with which the equipment is associated, and also because of differences of opinion of the cable operating attendants. The methods given for computing transients are of course not limited in usefulness to cable systems, but instead are applicable to various other classes of circuits.

RELATIONS BETWEEN AMOUNT OF INTERFERENCE AND CABLE SPEED

A standard expression for the useful single-frequency

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[1] "Extraneous Interference on Submarine Telegraph Cables," by J. J. Gilbert, *Bell System Technical Journal*, July 1926.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

[2] *Submarine Cable Telegraphy*, by J. W. Milnor, A. I. E. E. TRANS., February 1922, p. 20.

current received through a long non-loaded or loaded cable is the following:[3]

$$I_r = \frac{E_s}{Z_o + Z_s} \cdot \epsilon^{-Pl} \cdot \frac{2\,Z_o}{Z_o + Z_r} \qquad (1)$$

The corresponding deflection of the receiving instrument at any frequency is

$$D = \frac{E_s}{Z_o + Z_s} \cdot \epsilon^{-Pl} \cdot \frac{2\,Z_o\,M}{Z_o + Z_r} \qquad (2)$$

In studying the effects of interference, it is convenient to consider the disturbance to be introduced as a voltage E_d directly in series with the cable near the receiving end. For the present, the interference will be assumed to be sinusoidal; other transient forms of interference will be considered later.

The deflection of the receiving instrument due to interference is

$$D_d = E_d \cdot \frac{M}{Z_o + Z_r} \qquad (3)$$

The quantity $M/(Z_o + Z_r)$ is one form of frequency characteristic of the receiving equipment; it is a measure of the response both to signal voltages transmitted through the cable, and to interfering voltages. General information as to its character may be obtained by any one of three methods, as follows:

1. Sinusoidal voltages of various frequencies may be impressed in series between the cable and the receiving equipment, the deflection of the latter and (if desired) its phase relation being observed.
2. By analyzing the response of the receiving equipment to a pure impulse, as described in connection with Figs. 9 and 10.
3. By computation from the known electrical characteristics of the cable, the over-all arrival curve of the entire cable system having first been determined.

Thus, plots of each term of equation (2) were given in the original paper[4] for a representative cable (3,000 ohms, 800 μf., worked cable code at a dot frequency of 10 cycles per second), assuming fairly efficient receiving equipment. From these, a curve of $M/(Z_o + Z_r)$ against frequency may readily be computed. Such curves, shown here in Fig. 1, give the deflection of the receiving instrument per peak volt of sinusoidal disturbance as a function of frequency.[5]

In this figure, the two curves marked "Beat transmission" show the effect of curbing of transmitted signals, assuming that the shaping elements of the receiving equipment have been so altered that the resultant received signal is the same as with "block" signals. Such a change results in greater susceptiveness to low frequency interference, and less to higher frequency interference.

For satisfactory operation, a limiting value of D_d is probably one-fifth of the size of the deflection when reversals at dot frequency are transmitted through the cable. A safer value of D_d might be somewhat less. For cables operated duplex, the effect of any lack of perfect balance must also be included in determining the performance of the cable.

The above information may be made applicable to cables other than the one described by noting that the value of $M/Z_o + Z_r)$ which measures the susceptibility to interference, is proportional to $\epsilon^{\alpha l}$. The point of maxima tends to move to the right for cables operated at higher attenuation.

Where the interference originates some distance out from the receiving end of the cable, its effect is reduced. If a voltage E_x is induced along the cable due to an external magnetic field at a distance x from the receiving end of a long cable, its effect on receiving equipment is equal to that of a voltage $E_x\ \epsilon^{-Px}$ at the receiving end. Similarly, a current I_x induced electrostatically through the insulation due to a potential variation of the sheath, is readily shown to be equivalent to a voltage $I_x Z_o \epsilon^{-Pv}$ at the receiving end. The above value of E_d may therefore be considered to be an equivalent interference as measured at the receiving end, equal to

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[3] The nomenclature is as follows:

E = sending battery voltage.
l = cable length.
f = $p/2\pi$ = frequency.
F = dot or fundamental signaling frequency.
α = attenuation constant per unit length of cable.

The following are vector quantities,—functions of the frequency:

E_s = sending voltages as modified by curbing, if any, of transmitted signals.
P = propagation constant per unit length of cable.
Z_o = characteristic impedance of cable.
Z_s = impedance to outgoing current of equipment at sending end.
Z_r = impedance looking from cable, of complete receiving equipment.
I_r = peak current at junction of cable and receiving equipment.
D = peak deflection of receiving instrument at any frequency (see D', below).
M = magnification of complete receiving equipment at any frequency = D/I_r.

The following is a function of time t:

D' = instantaneous value of deflection of receiving relay or recorder. The maximum deflection (neglecting a momentary overthrow) is taken as unity, the deflection at any instant being expressed as a fraction of the maximum.

It is assumed throughout that all characteristics in the system are linear, with effects proportional to causes within reasonable limits. If the receiving relay is of the type whose armature moves between fixed stops, its motion is of course non-linear and the theory is inapplicable to its motion; however with proper allowances the quantities D and D' might represent some other factor, such as the armature torque or relay current.

The quantity M takes into account all equipment in the receiving-amplifying-shaping network, and is of course a complicated function of the frequency. The method of analysis makes it unnecessary to set up the solution of the components of the network.

[4] Milnor, loc. cit., Figs. 12 to 17 inclusive.

[5] Instead of giving the modulus and the angular lead as in Fig. 1, the frequency characteristic may be specified by giving the real and imaginary components, which will be designated respectively by u and v.

$$E_d = \int_0^l \epsilon^{-Pv} (E_x + I_x Z_o)\, d\, x + E_{SE} \qquad (4)$$

In this equation, E_x and I_x are the values per unit of length and E_{SE} is the potential variation, if any, of the

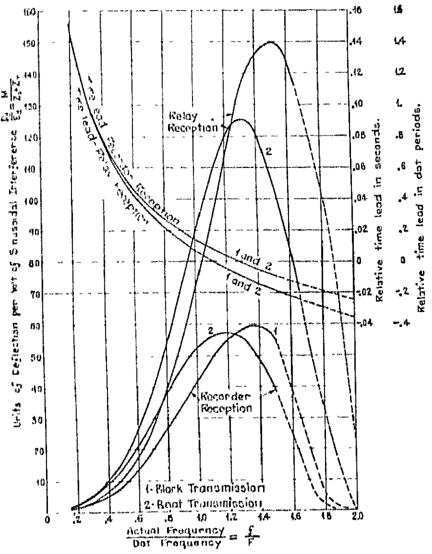

FIG. 1—FREQUENCY CHARACTERISTIC OF CABLE RECEIVING EQUIPMENT

earthing connection at the end of a short sea earth. If the sea earth is long the formula for E_d is somewhat modified.

ANALYSIS OF INTERFERENCE OTHER THAN SINUSOIDAL

In Fig. 2 is shown a typical record of interference on an actual cable, using a distortionless amplifier with high impedance input. This section will deal with the effect on signal reception of interference of this nature. The maximum peak in the record is 2.5 millivolts; however, it is found that its effect on reception is equivalent to that of a much smaller sinusoidal interference of the frequency at which the equipment is most sensitive, the effective value in different cases ranging from 5 to 60 per cent of the peak.

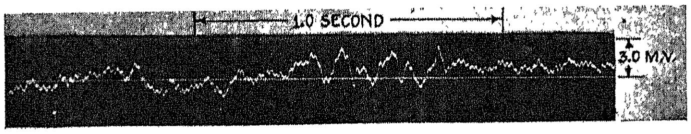

FIG. 2—RECORD OF INTERFERENCE ON CABLE. DISTORTIONLESS AMPLIFIER

In order to illustrate the manner in which interference affects operation, the curves in Fig. 3 have been computed showing the manner in which the receiving relay of the representative cable responds to various simple transient forms of interference. The method of computing is given below. An examination of these results indicates that an impulse of interference which is fairly short in time, causes a peak of response which is nearly proportional to the product of the voltage multiplied by the time in seconds. Where the interference is prolonged, the response is less than given by the above product. However, the response is greatest when two or more pulses occur in a manner to set up forced oscillation of the equipment.

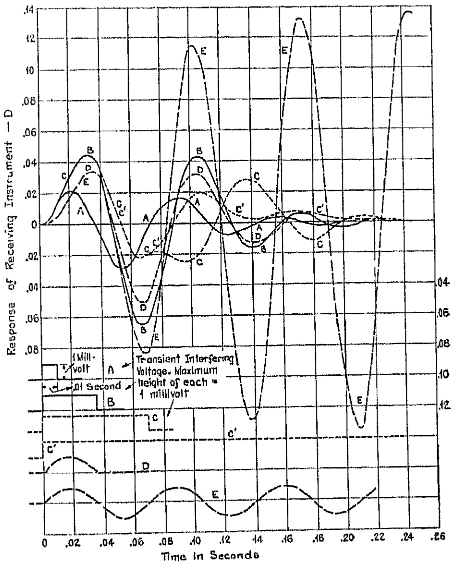

FIG. 3—RESPONSE OF RECEIVING INSTRUMENT TO INTERFERENCE IMPULSES

Curves such as in Fig. 3 might be computed directly when the frequency characteristic as in Fig. 1 is known.[6]

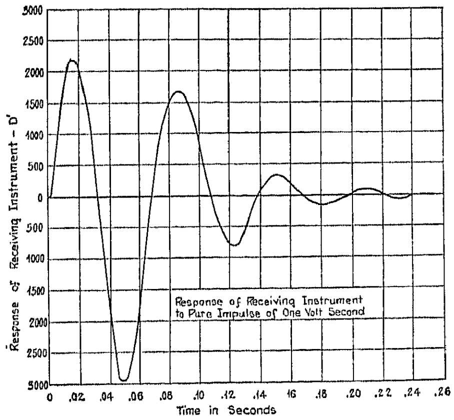

FIG. 4—IMPULSE CHARACTERISTIC

However, a more convenient method was first to compute an *impulse characteristic*, shown in Fig. 4, and then to build up the curves in Fig. 3 by means of the principle of superposition. The impulse characteristic as here used is defined as the transient response of the receiving instrument, when an instantaneous or pure impulse of voltage is impressed (across a small resistance) in series between the cable and the receiving equipment. The impulse characteristic is computed from the frequency characteristic by means of the equation[5,6]

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[6] Milnor, loc. cit., Table I and equations (**10**), (**12**) and (**13**) of Appendix. For a pure impulse of one volt second, $s = 0$ and $c = 1$. In the present case the integrations have been performed by breaking the curve representing u into several parts each of which was integrated analytically.

$$D' = \frac{E_i T_i}{\pi} \int_0^\infty (u \cos p t - v \sin p t) \, d p \qquad (5)$$

$E_i T_i$ being the strength of the pure impulse in volt-seconds. By taking advantage of the fact that D' is zero for negative values of t, this may be simplified into

$$D' = \frac{2 E_i T_i}{\pi} \int_0^\infty u \cos p t \, d p \qquad (6)$$

which is valid for positive values of t only.

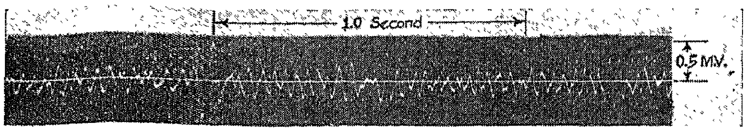

FIG. 5—RECORD OF CABLE INTERFERENCE. AMPLIFIER TUNED TO 30 CYCLES PER SECOND; DECREMENT = 0.8

It is, of course, entirely possible to determine the effect on signal reception by computation from a complete record such as shown in Fig. 2, by building up a curve of response of equipment by superposition using the impulse characteristic. The result would be similar in appearance to Fig. 5. However, such procedure would be rather tedious and impracticable in general studies of interference.

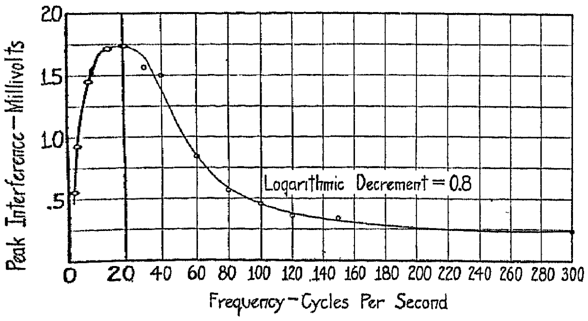

FIG. 6—EXAMPLE OF VARIATION OF MEASURED INTERFERENCE WITH FREQUENCY OF TUNED CIRCUIT

FIELD MEASUREMENTS OF INTERFERENCE

The most accurate field method for determining the practical effect of interference would be to use receiving equipment adjusted to give properly shaped signals on the given cable. This method has the objection that the shaping may not be known in advance of laying, and comparative studies of interference in different locations or in relation to the cable design would be difficult. The importance of interference in a given case is measured by the maximum or peak response that occurs during a suitable period of time.

Since the frequency characteristic of efficient receiving equipment is somewhat similar in form to that of a simple electrically tuned circuit, the method suggests itself of making use of such tuning during the obtaining of the field records. Such a method offers advantages over the other methods, and appears to be sufficiently accurate for the purpose. The point of tuning must correspond to the frequency at which the cable equipment is most sensitive, and the broadness of tuning should approximate that of the operating equipment.

A convenient arrangement of apparatus for the purpose is a vacuum tube amplifier working into an oscillograph, the amplifier having a symmetrically tuned circuit between two of the stages, and being otherwise distortionless. The tuning may be provided by means of a low-loss inductor and condenser in parallel, the combination constituting the external plate load of the preceding tube. In order to control the broadness of tuning, an adjustable resistor is placed in parallel with the inductor and the condenser. The broadness of tuning of a symmetrically tuned circuit is conveniently specified by stating the "logarithmic decrement" as used in radio parlance. A low value represents relatively sharp tuning.

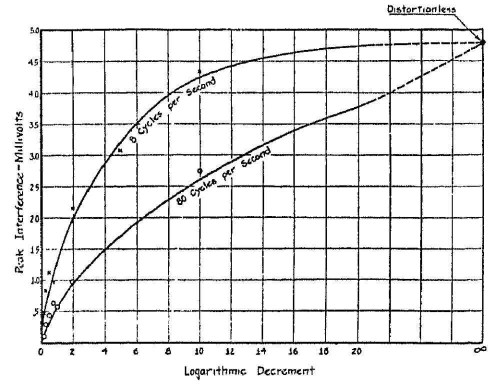

FIG. 7—EXAMPLE OF VARIATION OF MEASURED INTERFERENCE WITH DECREMENT OF TUNED CIRCUIT

A sample record obtained with such equipment is shown in Fig. 5. In Figs. 6 and 7 is shown the manner in which the peak interference varied at one location with different frequencies of tuning, and with varying logarithmic decrements. It is evident from the latter figure that the response of cable receiving equipment to interference is reduced with sharper tuning of the equipment, (that is, by adjusting it to be relatively less sensitive to the higher frequencies which are unnecessary for transmission). For efficient cable equipment, the broadness of tuning appears to be approximately represented in different cases by decrements from 0.7 to 1.2; for less efficient equipment the value is higher.

The measured interference is in general increased if the length of the record is increased, since the highest peak of interference on the record is recorded as the amount of interference under the given conditions. In Fig. 8 are shown three examples of the manner in which the peak interference varied with the length of record.

Under some conditions it is convenient to replace the oscillograph with an arrangement for automatically recording the peak voltages. There are different methods by which this may be accomplished, use being made of vacuum or gas filled tubes or of sensitive mechanical relays.

COMPUTATION OF FREQUENCY CHARACTERISTIC FROM IMPULSE RESPONSE CURVE

By means of a simple experiment, it is possible to determine the impulse characteristic of an actual cable circuit. The method is to insert between the cable and the receiving equipment a resistor which is small in value as compared with the characteristic impedance of the cable, and to discharge a condenser across the resistor. Under proper conditions such an impulse may for the purpose be regarded as an instantaneous or pure impulse, and its strength in volt-seconds is the product of the resistance, the capacitance and the voltage at which the condenser is charged. To avoid possible amplifier overloading, the deflection of the receiving instrument should be not greater than during regular operation. In the upper parts of Figs. 9 and 10 are shown examples of impulse characteristics obtained experimentally on two cables.

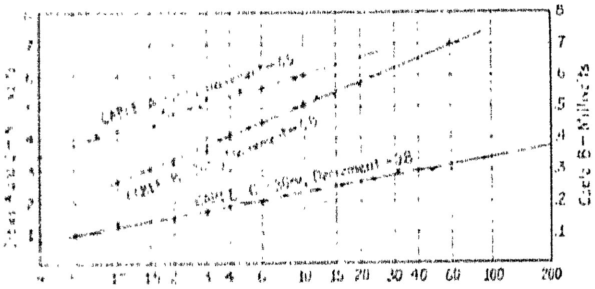

FIG. 8—EXAMPLES OF VARIATION OF MEASURED PEAK INTERFERENCE WITH LENGTH OF RECORD

A casual examination of such an impulse characteristic yields valuable information as to the approximate frequency at which the equipment is tuned, and also gives an indication of the sharpness of tuning and the sensitiveness.

The voltage frequency characteristic of the cable receiving equipment under investigation may be computed directly from the experimentally determined impulse characteristic. This operation is the inverse of that in equations (5) and (6). While such computations may not be justified in the average routine investigation, they are quite instructive as a check of the theory. The results of two such computations are shown in the lower parts of Figs. 9 and 10. Certain data regarding these cables are as follows:

For Fig. 9, non-loaded cable, resistance 4,740 ohms, capacity 806 μf., operated at a dot frequency of 5 cycles per second. Beat signal transmission, 0.8 contact. Unshunted receiving condenser. Heurtley magnifier working into syphon recorder, but signals shaped for relay operation.

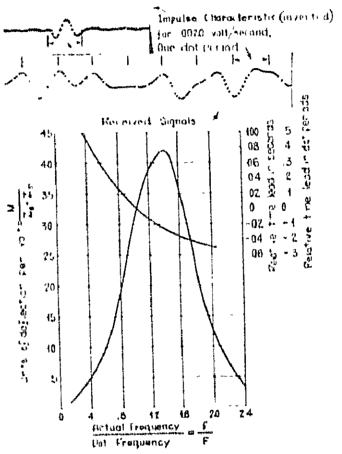

FIG. 9—FREQUENCY CHARACTERISTIC OF CABLE RECEIVING EQUIPMENT DETERMINED FROM IMPULSE CHARACTERISTIC

Fig. 10, duplex loaded cable with non-loaded end portions, operated at a dot frequency of 40 cycles per second. Block signal transmission. Western Electric vacuum tube amplifier working into oscillograph and relay.

These frequency characteristics are similar in general form to the characteristics for relay operation in Fig. 1, which were obtained by quite a different method. The differences between the curves are indicative of the variations which may occur in practise between cables operated at different efficiencies and with somewhat unlike signal shaping. The heights of the curves vary because the cables are operated with different strengths of received voltage.

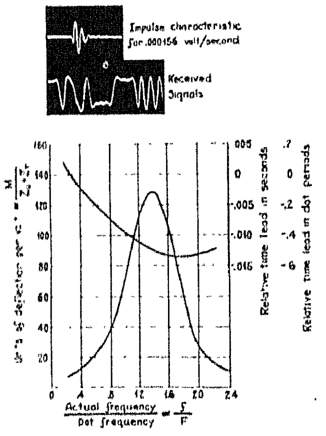

FIG. 10—FREQUENCY CHARACTERISTIC OF CABLE RECEIVING EQUIPMENT DETERMINED FROM IMPULSE CHARACTERISTIC

The method of determining the frequency characteristic from the impulse characteristic is to integrate through the transient in accordance with the formulas,[7]

$$\left.\begin{array}{ll} s = \int D' \sin p\,t\,d\,t & \text{between limits where } \cos p\,t = 0 \\ c = \int D' \cos p\,t\,d\,t & \text{between limits where } \sin p\,t = 0 \\ b = (D'_{-\infty} + D'_{+\infty})/2 & \end{array}\right\} \qquad (7)$$

The frequency characteristic of the pure impulse which was applied to the circuit is $R_i\,C_i\,E_i\,\underline{/0^\circ}$, in which the quantities are respectively the resistance, capacitance, and voltage used in obtaining the experimental impulse characteristic. The frequency characteristic of the receiving equipment is obtained by dividing the values obtained from equation (7) by the product just given, that is,

$$\frac{M}{Z_o + Z_r} = \frac{D_d}{E_d} = \frac{\sqrt{s^2 + c^2}}{R_i\,C_i\,E_i}\,\underline{/-\tan^{-1} s/c} \qquad (8)$$

Instead of the impulse characteristic, there is another characteristic which may readily be obtained experimentally, and which has similar uses. A small continued voltage may be suddenly impressed across a low resistance in series between the cable and the receiving equipment. The resulting response of the receiving instrument is similar in form to curve C' in Fig. 3

APPLICATION OF THEORY TO LOADED TELEGRAPH CABLES

The attenuation of a slow light-weight non-loaded cable increases nearly proportionately to the square root of the frequency. With heavy non-loaded cables, especially if operated at relatively high speed, the self inductance of the cable tends to reduce the attenuation, but this gain in transmission is partly offset by losses introduced by the sea return path. The usual result is that the attenuation is proportional to the frequency raised to a power slightly less than one-half.

In a loaded cable[8] there are additional losses due to hysteresis and eddy currents in the loading. The manner in which the attenuation changes is a much more complicated function of the frequency.

If a cable transmits a greater proportion of the higher frequencies, the receiving equipment will be required to provide less magnification at those frequencies, and vice versa. It has been the experience thus far that the appearance of the voltage frequency characteristic of the receiving equipment for loaded cable operation is generally similar in form to that in Fig. 1, with modifications in height to take account of the changed characteristics of the cable. The curve of time lead is more nearly flat, since currents at the different frequencies are retarded more nearly alike by the cable. The general method which is suggested for studying interference in the field is applicable to both non-loaded and loaded cables, provided that proper allowance is made for the sensitiveness and tuning of the equipment, in accordance with the design of the particular cable.

Acknowledgment is due to Mr. K. B. Eller for assistance especially in connection with the experimental studies.

Discussion

A. W. Breyfogel: The subject matter of the paper is of paramount importance to cable engineers for it is the existence and magnitude of extraneous electrical interference that places a definite limit on the speed of a cable. This statement is not to be construed to mean that if all electrical interference could be eliminated completely, that the speed of any cable could be increased indefinitely. The electrical characteristics of a cable are a measure of its signaling speed, but this speed is never quite reached in practise because of the detrimental influence of interference.

By utilizing the methods outlined in the paper, the deflection of the receiving instrument per peak volt of sinusoidal or transient forms of interference as a function of frequency may be predicted. In like manner, the deflection of the receiving instrument per volt of received signal may be predicted and from the results of these calculations a fairly accurate predetermination of the ultimate speed is obtained.

The author of the paper has presented the theoretical and practical means for determining the magnitude of the effect that such interference will have upon the receiving instrument but has refrained from presenting what practical means are available for minimizing the effect of such electrical disturbances.

Every cable is subject to a certain amount of electrical interference either from natural causes, from electrical railways or power systems in the vicinity or from other cables or from all of these sources at the same time. It is not always possible to eliminate such interference at its source and an effort must be made to minimize its effect upon the receiving instrument.

When dealing with non-loaded cables where the receiving equipment consists of a hot-wire Heurtley magnifier and a Brown drum relay or a stretched-wire relay the effect of extraneous electrical interference can be lessened to some degree by adjusting the free period of the moving parts of these instruments to a frequency near the dot frequency or a little higher than the dot frequency of the signal. The instrument, in this case, would approach a maximum deflection for the dot frequency of the signal and a deflection somewhat less than the maximum for higher and lower frequencies. The instruments' response to interference frequencies above or below the dot frequency would thus be reduced to a minimum.

Where interference is present, it is always of advantage to tune the receiving equipment to the lowest practical frequency consistent with reception of satisfactorily shaped signals. Such tuning may be somewhat harmful to the reception of the signal itself but generally it permits reception at a higher speed than might otherwise be obtained.

Mr. Milnor has shown that for satisfactory operation of non-loaded cables where the three-element cable code is used, the limiting deflection produced in the receiving instrument by interference cannot be greater than about 20 per cent of the deflection produced by the dot frequency. It is interesting to note that in practise this percentage is found to be about the maximum that can be tolerated successfully. A deflection of this magnitude will not carry the pointer of the drum relay beyond the boundaries of no-man's-land and will, therefore, not produce failures. Should the interference become of greater magnitude, the pointer will cross the borders of no-man's-land and cause failures.

The receiving networks can also be used to a more limited degree to minimize the effect of extraneous interference. The adjustments of the shaping networks associated with the Heurtley magnifier can be made more or less responsive to the signal, thus discriminating against interference.

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[7] Milnor, loc. cit., equation (8) of Appendix.

[8] "High-Speed Ocean Cable Telegraphy," O. E. Buckley, *Bell System Technical Journal*, April 1928.

When dealing with loaded cables where the prime receiving instrument is a vacuum tube shaping amplifier, the effect of interference can also be minimized to some extent by the adjustment of the shaping networks. In the amplifier, the tuning is done electrically and a certain amount of discrimination against extraneous interference is obtained.

During some recent tests on a loaded cable at speeds as high as 135 cycles, the effect of extraneous interference was successfully shaped out and excellent printing ranges were obtained with a resulting high degree of accuracy. If the interference could be completely eliminated, we believe that even higher speeds could be obtained with receiving apparatus that is now available.

Telegraph Transmission Testing Machine

BY F. B. BRAMHALL[1]

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Synopsis.—In the study of telegraph transmission efficiencies and the analysis of signal distortions a suitable signal recording device is a first requisite. This paper describes a machine developed for the purpose. This machine furnishes a graphical and permanent record of the received signals and shows their relation to sent signals, in a form which is most convenient for comparison and analysis. The unusual arrangement of the recorder is such as to adapt it especially to the study of both characteristic and fortuitous distortions and, in general, to the comparison and absolute measurement of short time intervals. Time intervals of one ten-thousandth of a second in the opening and closing of electrical circuits are graphically recorded without resort to photographic means.

INTRODUCTION

IT is the object of this paper to describe an improved method of measuring the quality of transmission obtained from telegraph or other signaling circuits or apparatus. A description of the transmission testing machine developed for this purpose is followed by a discussion of the typical results obtained in various classes of work and an explanation of the method of their analysis.

Although the machine was conceived and used several years ago by Mr. J. W. Milnor, Research Engineer of the Western Union Telegraph Company, the principle of the method has not previously been published. While the fundamental principle remains unchanged the machine has naturally gone through a considerable evolution as its many adaptations have become apparent. In its present form it is used very extensively in laboratory and field investigations of telegraph signal transmission, and in the testing and comparison of relays and repeating instruments.

QUANTITATIVE MEASUREMENTS OF SIGNAL DISTORTION

Any measurement of the quality of telegraph transmission is of necessity a measurement of the extent to which the received signaling pulses are distorted from the form in which they were originally transmitted. For the purpose of this paper, signal distortion will be defined as the deviation in length and time relationship of the received signal pulses from the absolute length and time relation of their transmission. This definition confines the measurement of distortion to the measurement of the time relationships which obtain between the signaling pulses as repeated by a relay or other repeating instrument.

The nature and effect of telegraph signal distortion as here defined was admirably explained in a paper called, *Measurement of Telegraph Transmission*, by Messrs. Nyquist, Shanck, and Cory, presented before the Winter Session of the Institute in 1927.

It should suffice here to point out only the distinction between the two general types of distortion designated as *fortuitous* and *characteristic*. Fortuitous distortion is that shortening or lengthening of the signal pulses which results from extraneous disturbing currents in the signaling circuit or from irregular and improper functioning of repeating or receiving instruments. From the very nature of its causes then this type of distortion may be expected to be quite irregular, fluctuating rapidly and perhaps very widely. Characteristic distortion may be defined as that shortening or lengthening of signal pulses which is governed by the sequence of the pulses themselves and by the electrical and magnetic characteristics of the signaling circuits and apparatus. Problems of the cause and elimination of characteristic signal distortion are most effectively attacked through an accurate knowledge of the current wave shapes which delay or hasten the time of operation of the repeating instrument. In the study of fortuitous distortions it is frequently desirable to supplement a knowledge of their effect on the signals with frequency analyses of the disturbing currents.

This paper will deal chiefly with the effects of signal distortion. The wave form distortions and irregularities will be dealt with only in so far as seems necessary in the explanation of the results obtained by the method of measurement described.

In all experimental studies of telegraph circuits and apparatus a method for measuring the fidelity with which the transmitted signals are reproduced in the receiving terminal is of prime importance. It is important to know not only the maximum signaling speed which can be obtained on a given circuit, but as well to know quantitatively what particular forms of signal distortion limit that speed. The most common method of determining the efficiency of transmission is to transmit into the circuit test signals of known characteristics while observing the integrity of their reception. The receiving element then, which constitutes the novel feature of the transmission testing machine, is the principal subject of this paper.

Methods of transmitting test signals which are substantially perfect are well known. In automatic signaling systems generally, the transmitting unit may usually be relied upon for substantially perfect transmission. The transmission testing machine here described, however, carries with it, in addition to the receiving element, a transmitting element which may be used for experimental testing work with both ends of the circuit available at a point.

Some of the common uses to which the machine is put are:

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

1. Absolute measurement of over-all transmission efficiency.
2. Detection and analysis of characteristic signal distortion.
3. Detection and analysis of signal distortion resulting from extraneous disturbances.
4. Study of repeating relay characteristics.

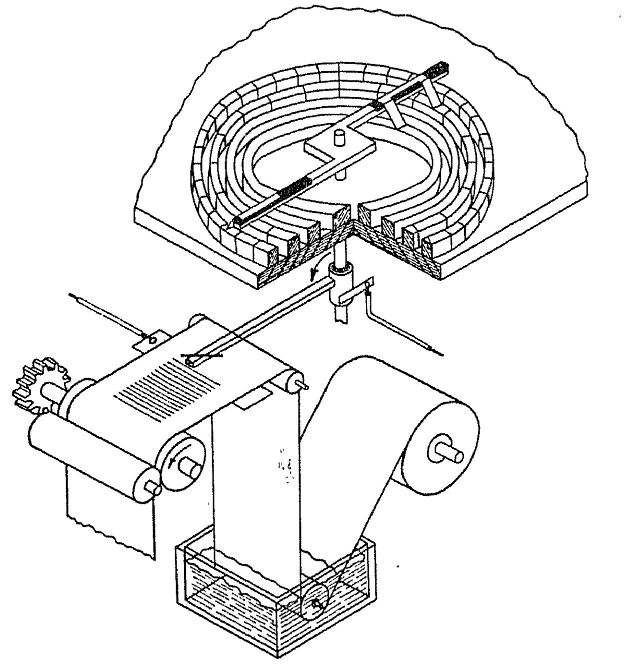


Fig. 1—Telegraph Transmission Testing Machine

Drawing of chemical recorder mechanism showing relation to transmitting commutator

5. Detection of hunting or irregularities in transmitting apparatus.
6. Study of duplex balance problems.

Transmission Testing Machine

Fundamental Principle. A conception of the principle of the machine can most readily be formed if it be considered first as arranged for testing circuits both ends of which are available at a point. Referring to Figs. 1 and 2, let the commutator transmit signals to the circuit under test. Consider the signals to operate a relay at the receiving end of the circuit. The contacts of the relay are made to control a potential applied between the metallic needle 13 and the platen 12. The arm which carries the metallic needle revolves on the shaft which carries the transmitting brushes. The metallic needle passes over a saturated paper tape 15 in light contact with its surface. The solution with which the paper tape is saturated and the material of the metallic marking needle are such that when the proper potential is applied to the needle a mark results from the electrochemical action. If in response to a transmitted signal pulse the receiving relay energizes the marking needle during the time of its transit across the paper, a mark is made continuously until the needle is again deenergized at the termination of that pulse. Thus, if the transmission over the circuit under test be perfect, the revolving needle will draw a line on the paper tape which is a true projection of the length of the transmitting segment.

The idea of transmission and reception being done on a common rotating member is, of course, no longer essential to a conception of the principle of this method of measuring. These two functions may obviously be performed by separate rotating members so long as proper synchronism of rotation is maintained. It is by this method that point to point testing is done.

Detailed Description. A detailed description of the recording mechanism is given with reference to Figs. 1 and 2. Fig. 1 shows the relation of the essential parts of the machine, Fig. 2 is a schematic wiring diagram.

Fig. 2 shows, developed, a transmitting commutator having a solid ring and a segmented ring. Each of the segments of the segmented ring is arranged for connection to either positive or negative transmitting potential, so that by the manipulation of switches any desired combinations of positive or negative pulses may be transmitted. The signaling currents pass from the solid ring through the circuit or apparatus under test to the coils of a receiving relay which in turn repeats the received signals to the recording device.

A suitable grade of paper tape is pulled by rollers over a contacting surface or platen after passing through a chemical bath. The marking needle attached to the rotating arm is carried on the shaft which drives the commutator brushes. When an electric current is passed through the marking circuit in such a direction that the marking needle is positive with respect to the platen a mark is recorded on the paper as a result of oxidation of the material of the needle.

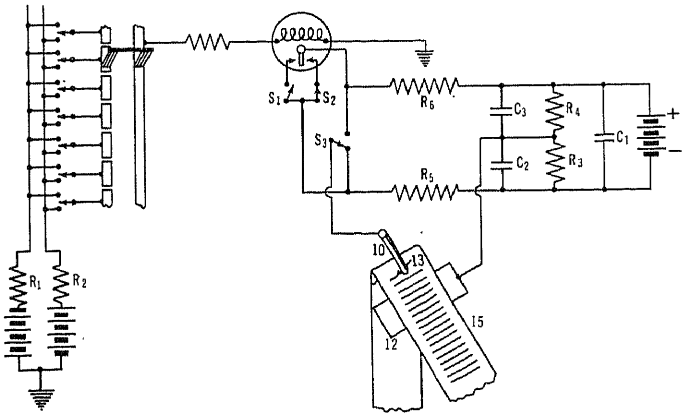


Fig. 2—Schematic Wiring of Test Circuit and Chemical Recorder

The principle of the chemical recording feature is well known. Telegraph systems which made use of the chemical tape recorder for signal reception have been used to some extent in commercial telegraphy.

The chemical recorder of this machine uses an iron needle marking on a paper tape which is saturated with a solution of potassium ferricyanide and ammonium chloride. The mark made with this solution is blue. Other solutions and needles of other materials produce different colored marks. The combination of the iron needle and potassium ferricyanide solution is perhaps the fastest acting of any that has been commonly used.

A common fault of chemical recording devices is the tendency of the metallic needle to mark lightly even when no potential is applied to it, and the tendency for the beginning and ending of the marks to be somewhat indistinct.

The marking circuit as arranged for this machine makes use of a capacity and resistance network to produce marks with a clear cut and sharp beginning and ending. With the switches of the marking circuit thrown as shown in Fig. 2, the needle marks when the marking contact of the receiving relay is closed. The resistances, R_3, R_4, R_5, and R_6, are so proportioned that when the circuit is open at the relay contacts current flows from positive battery through resistance R_4 then in part through the recorder and resistance R_5 to negative battery. The needle thus being negative with respect to the paper, no mark is made on the tape. When the circuit is closed at the relay contacts, however, current flows through resistance R_6, the relay contacts, through the recorder from needle to tape and through resistance R_3 to the negative pole. Under this condition a mark is made on the tape. The capacities C_1, C_2, and C_3 serve the purpose of accelerating the electrochemical action at the beginning of the mark and retarding the action at the ending of the mark. When the relay contacts close, a condenser discharges through the recording circuit in the direction of the flow of a marking current. When the contacts open, reversing the current through the recording circuit, the discharge of the condensers is in such a direction as to end the mark more abruptly. As a result the signals are clear cut and distinct. By causing a reversed current to flow during non-marking periods, the tendency of the recorder to mark even when the needle is deenergized is curbed.

By proper manipulation of the switches in the contact circuit of the receiving instrument, the recorder can be made to mark when either or both contacts of the receiving relay are closed. By reversing the switch S_3 and making suitable changes in the values of the resistances in the potentiometer, the recorder can be made to mark only when the relay contact circuit is open. The desirability of these arrangements will become evident as the methods of testing are explained.

The commutator and brush carriage of the machine are similar to those of the standard distributor used in multiplex telegraphy. They are, however, made somewhat larger to afford more perfect transmission. One segmented transmitting ring bears twice the number of segments of the other. One ring can, if desirable, be made movable like the receiving ring of the multiplex distributor. The inner ring has 20 segments. This ring is used for all testing work at normal telegraph signaling speeds. The outer ring is used for transmission at higher signaling speeds and for a variety of other purposes, such as maintaining synchronism with a distant transmitter and for range finding work. The two pairs of rings are each connected to standard multiconductor receptacles of the type used on multiplex apparatus and connection is made between the battery control switches and the commutator by means of the multi-pin plug. By proper orientation of this plug then, it is possible to observe the reception of any chosen element of the transmitted test signals. In loop testing work, where the test signals are transmitted from the machine, the characteristic lag of the circuit or apparatus may be accommodated by orientation of this plug.

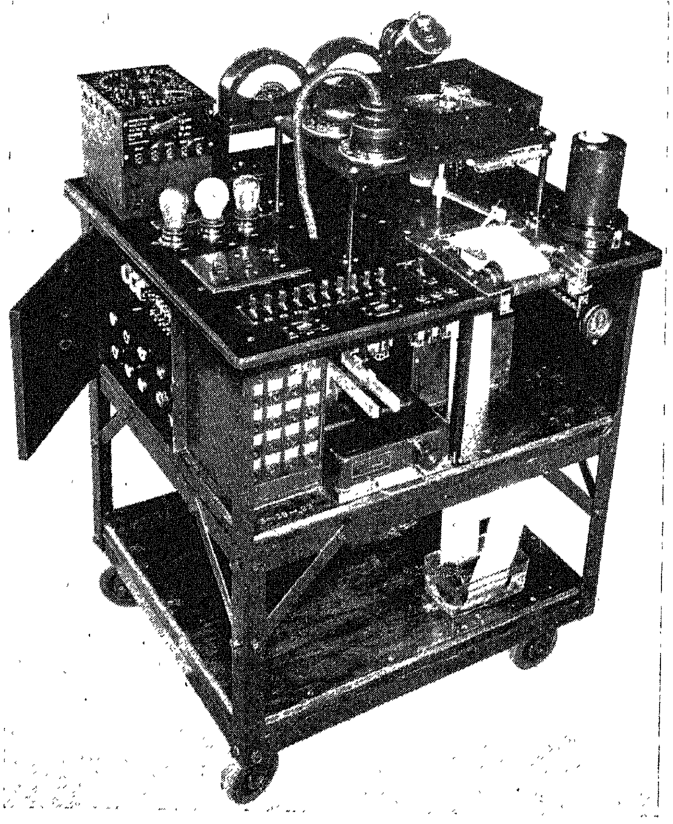


FIG. 3—TRANSMISSION TESTING MACHINE
One of the newer types, showing recorder and control apparatus

Numerous motor-drive arrangements have been used in transmission testing machines. An early machine was driven by a small shunt-wound d-c. motor. A train of gears and a friction disk afforded a wide range of speed adjustment in this machine. Such a drive proved satisfactory when the machine was used for relay testing and other work on which the time lag of the circuits under test was not appreciable.

The application of the machine to the over-all measurement of circuits of appreciable lag and to straightaway testing necessitated the development of a more constant drive. The drive used in the present machine to obtain uniform rotational speed is a synchronous motor drive. The driving unit is shown in Fig. 4. In this unit the rotating members of two driving motors are rigidly connected to the common shaft. One of these, a standard shunt-wound low-speed d-c. motor may be seen at the bottom of the unit. The synchronous motor is a standard phonic wheel commonly used in multiplex distributors. Above this motor will be seen the large mercury filled flywheel which is also commonly used with the multiplex distributor.

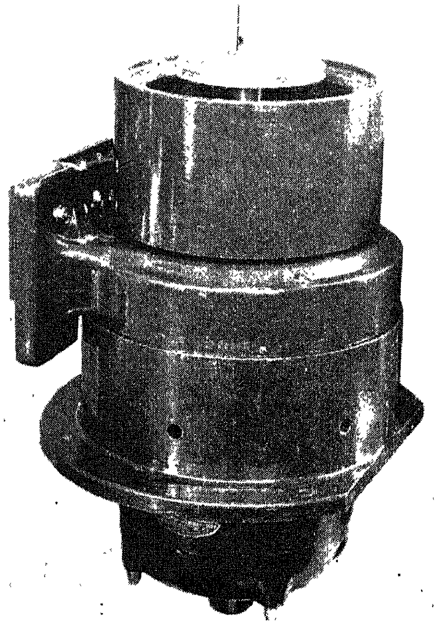


Fig. 4—Drive Unit, Assembled

Showing "phonic wheel" motor with flywheel above and direct-coupled d-c. starting motor beneath

A low resistance potentiometer is used to control the potential applied to the armature of the d-c. motor. The d-c. motor alone may be used when great constancy of driving speed is not required. It is also used as a starting motor for the phonic wheel drive or as an auxiliary drive to the latter.

The phonic wheel motor derives its power through the contacts of the tuning fork commonly used in connection with this motor in the multiplex automatic equipment. Synchronism of the phonic wheel motor with the driving fork is indicated both by a pilot lamp and by a meter in the phonic wheel circuit.

The driving fork is adjusted to vibrate at the desired frequency by means of removable weights on its tines. Small changes in its frequency are made by means of sliding weights. These may be moved without stopping the fork. A push-button switch is arranged to control a damping network so connected to the driving winding of the fork as to change its frequency of vibration slightly. By this means a predetermined change may be made in its frequency without disturbing the adjustment of the weights. When the machine is running synchronously with a transmitter located at a distant point, this control is very useful.

The circuits and controls of the transmitting commutator are not unusual. The telegraph transmitting potentials are brought through suitable protective resistances to a battery reversing switch. A rotary signal setting switch provides a convenient means of setting up on the commutator a number of predetermined combinations of testing signals. Ten different combinations of test signal pulses which have been found useful are shown by Fig. 6. These combinations are those used in the making of specimen records shown in this paper.

Local Testing Work

Polar Relay Testing. In order that the operation of the machine may be more readily understood the method of obtaining the test records in a very simple local test is outlined in some detail. Possibly the work in which the measurements are most simply made is that done in the study of polar relay operation. For the purpose of this description then, the case of a test of a polar relay in a local circuit will be considered.

In work of this class it is generally most convenient to transmit the test signals from the commutator of the machine. In the case of the polar relay test, bi-polar transmitting potentials are used. Suitable protective resistances in the transmitting potential leads are a part of the transmitting circuit arrangements of the machine. The solid ring on the transmitting commutator is connected to the windings of the relay through a suitable current controlling resistance. Or if the test is one intended to show the operation of the relay in a particu-

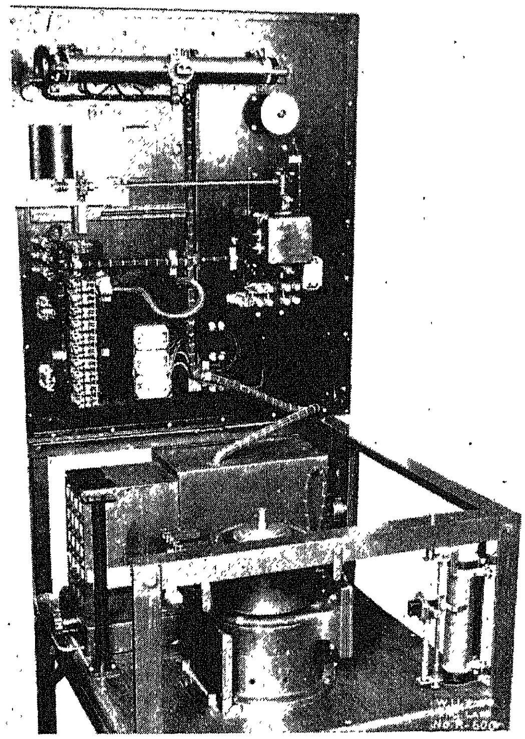
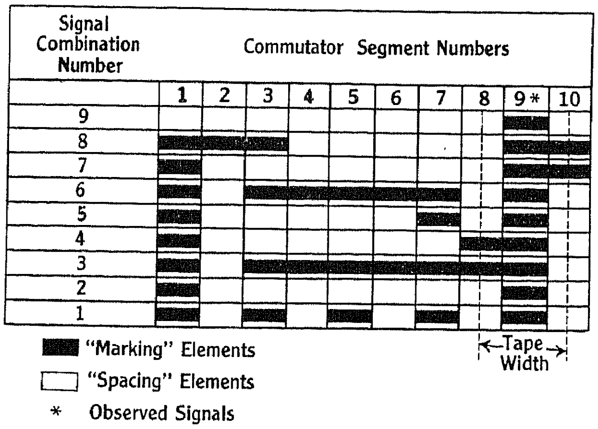


Fig. 5—Standard Semi-Portable Machine with Top Removed

lar network circuit, this network is used. The machine is driven at the desired operating speed for the test. Whether the test signals be alternate marking and spacing pulses or whatever combinations of signaling pulses may have been chosen, the operation of the relay armature against the contact stops is recorded on the chemical tape.

A typical record made in this class of work is that shown by Fig. 7. The signal combinations used in making this record were those shown by Fig. 6. When the record of Fig. 7 was made the marking needle of the chemical recording mechanism was so oriented with respect to the transmitting brushes that the signal pulse designated in Fig. 6 as No. 9 was recorded in the center of the tape. Since the tape moves as it does, away from the axis of the recording needle, the bottom, rather than the top, is actually the beginning of the record. The various combinations of test signal pulses used are, therefore, numbered from the bottom to the top. The numbers which appear on the record not shown in the illustration, are those of the signal combination chart of Fig. 6. If on this signal combination chart the segments blocked out in white are considered to be of such polarity as to cause the armature of the relay to move to its marking contact the test record is readily seen as a graph of the chart. In this test the machine was allowed to record the reception of a number of signals under each different signaling pulse combination.



Signal Combination Number	Commutator Segment Numbers									
	1	2	3	4	5	6	7	8	9*	10
9									■	
8	■	■	■						■	■
7	■								■	■
6	■		■	■	■	■	■		■	
5	■						■		■	
4	■							■	■	
3	■		■	■	■	■	■	■	■	
2	■								■	
1	■		■		■		■		■	

■ "Marking" Elements

□ "Spacing" Elements

* Observed Signals

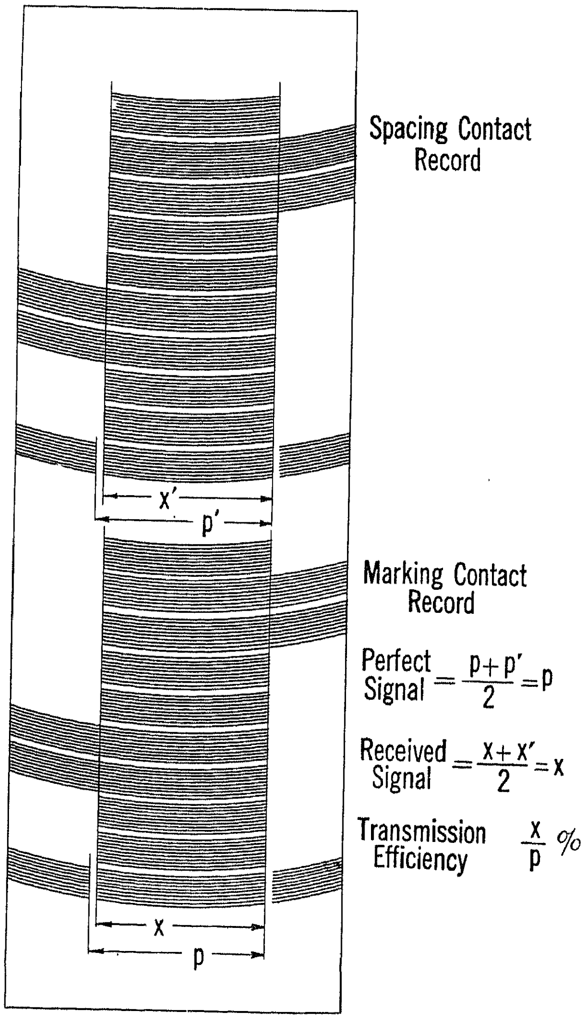
Fig. 6—Test Signals

Combination of test signal pulses used in the making of specimen records

Just as the lower half of the record of Fig. 7 records the contact made between the relay tongue and the marking contact stop, so the upper half records the contact of the tongue with the spacing stop. The upper half of the record is made by reversing the polarity of the transmitting potentials and connecting the spacing contact of the instrument, instead of the marking, to the recording circuit.

At the lower end of each half of the record it will be noted that the recorder made a nearly continuous mark across the tape. This portion of the record was made by arranging the recorder circuit to mark when the relay tongue was in contact with either contact stop. The only breaks in the mark recorded under this condition are those which represent the time during which the tongue was in transit from one stop to the other.

Had there been any appreciable difference in the magnitude of the potentials of the two transmitting batteries or had the receiving relay been biased, the marks recorded on the lower half of the record would have been of a different length than those on the upper half. This particular record shows the repeated signals to be comparatively free from bias. The method of measuring and interpreting a record of this type is very simple. It is largely explained by the dimension lines and lettering of the figure. The distances p and p' are those from which the basis of comparison, the length of the perfect signal, is taken. These distances are measured on the record of the operation of the relay on a-c. pulses. The distance p is proportional to the time elapsed between the departure of the receiving relay tongue from the spacing contact stop and its departure, at the next succeeding signal pulse of opposite polarity, from the marking contact stop. The distance p', measured on the other half of the record, is proportional



Spacing Contact
Record
x′
p′
Marking Contact
Record
Perfect Signal = (p+p′)/2 = p
Received Signal = (x+x′)/2 = x
Transmission Efficiency x/p %
x
p

Fig. 7—Specimen Record of Good Polar Relay Operation

to the elapsed time between the end of a repeated marking signal and the end of the succeeding repeated spacing signal. The length of the perfect signal is then taken as the average of the measured distances p and p'. As already mentioned the length of the perfect signal may be determined from the length of the transmitting segment and the radius of the path of the recording needle. However, since this radius may be altered slightly by adjustment of the needle a more accurate standard may be obtained from the measurements described. The repeated solid signal is taken as the average of the two measurements x and x'. The lines on which the x and x' measurements are made are drawn parallel to the edge of the tape and pass through the beginning and ending of those marks which record the signal latest to arrive and that earliest to be terminated. The average of the distances x and x' then, expressed in percentage of the length of the perfect signal, is taken as the transmission efficiency. In this particular case, the only appreciable difference between the repeated signal and the perfect signal is that which represents the transit time of the relay tongue.

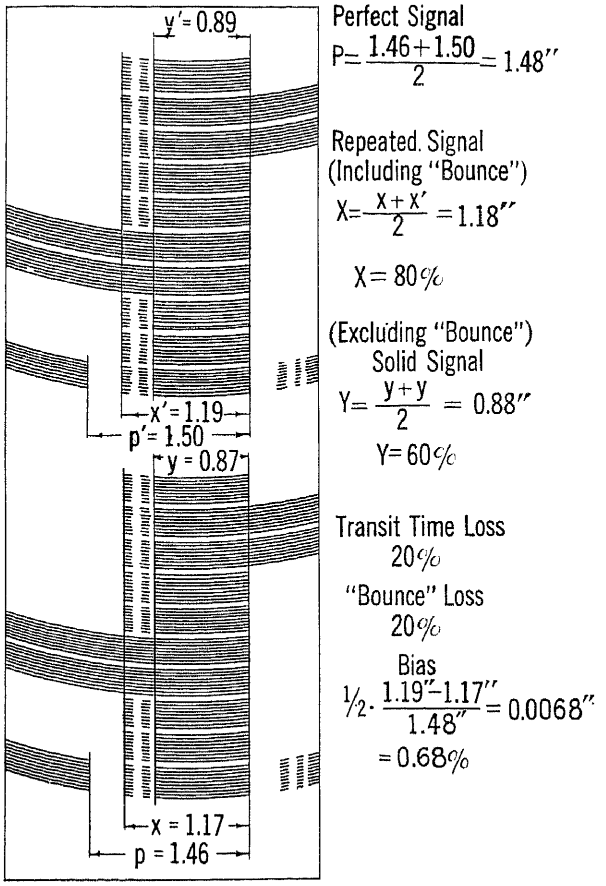


FIG. 8—SPECIMEN RECORD OF IMPROPER POLAR RELAY OPERATION

Quite obviously, if the recorder is sufficiently quick in its response, a mark will be made on the chemical tape even though the time duration of the electrical contact of the relay tongue with its stop be exceedingly short. If the characteristics of the relay be such that its tongue rebounds from the stop one or more times, the record, to be perfectly useful for this class of work, should permit a complete analysis of the repeated signal. Fig. 8 shows a record taken from a relay which repeated the signals less faithfully than did that used for the first record. In this case, the bouncing action of the relay tongue against its contact stop is very pronounced. The measurement of this record has been made in such a way as to determine not only the length of the solid portion of the signal but also the time elapsed between the initial contact and the final contact of the tongue after which it comes finally to rest firmly on its stop. For the sake of clearness the actual measurements made in analyzing this record are given. In this case the difference between the perfect signal measurement P and the total repeated signal X gives the transit time loss as before. A third measurement Y gives the length of the solid unbroken portion of the repeated signal. The difference between this and the X measurement is commonly called the "bounce" loss. It will be noted that the signals repeated from the marking contact are somewhat shorter than those repeated from the spacing contact. This bias is readily expressed in per cent of perfect signal.

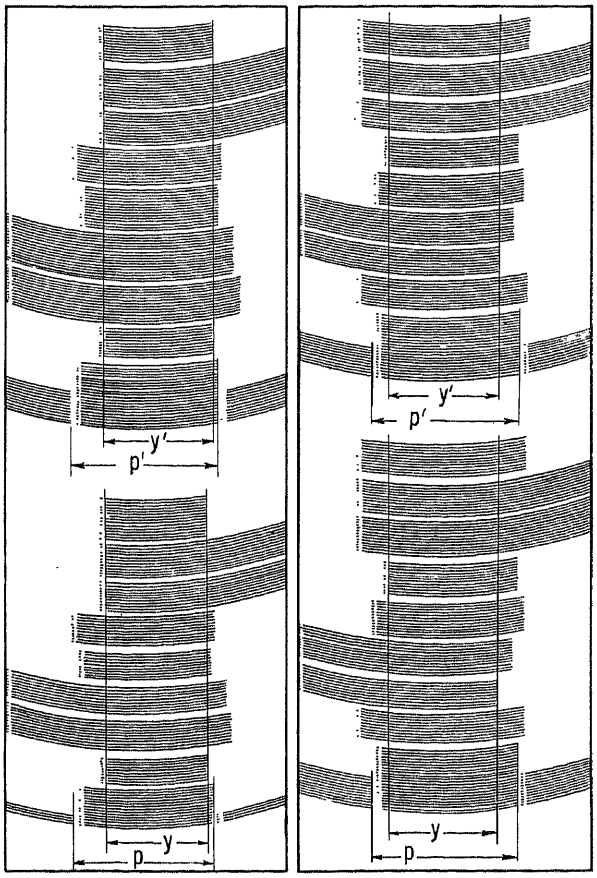


FIG. 9—TYPICAL CHARACTERISTIC DISTORTION

The time required for the relay tongue to move from one contact stop to the other and the time it consumes in bouncing are referred to as *loss*. The term *loss* is used in a limited sense. In the general case the transmission efficiency of circuits or repeaters is not impaired to anything like the extent that this *loss* would indicate. It is not the purpose of this paper to discuss the efficiency of operation of repeating instruments, but this point is brought out to avoid any misinterpretations.

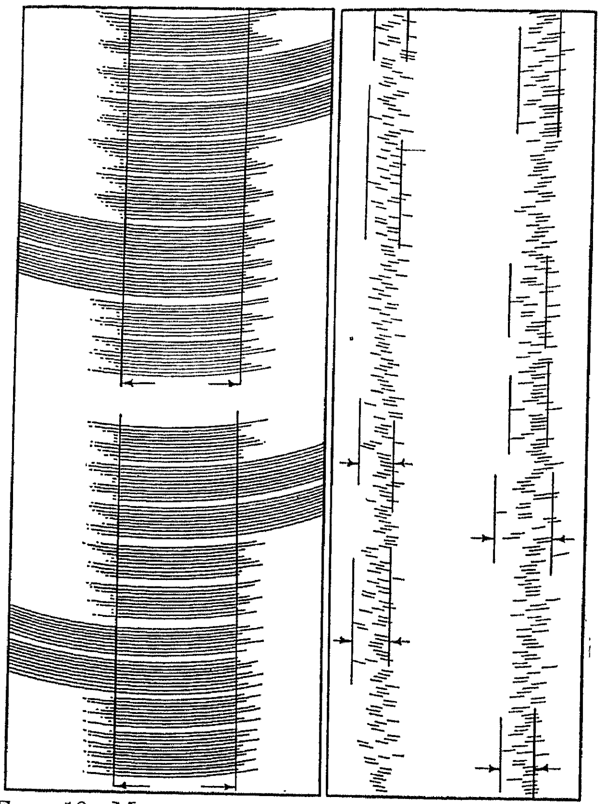


FIG. 10—MEASUREMENTS OF FORTUITOUS DISTORTION

Typical Characteristic Distortion. Should the relay under test or the circuit in which it is operating have such magnetic or electrical characteristics as to cause characteristic distortion of the repeated signal, the records may look somewhat like those of Fig. 9. An analysis of these records will show the particular field of usefulness which the machine has in the study of characteristic distortion. The left hand record of Fig. 9 shows characteristic distortion of the kind commonly designated as negative or underthrow. Distortion of this type is such as might be introduced by a relay designed for operation at a lower signaling speed than that at which the test was made or by sluggish signaling circuits or networks.

Distortion of the type shown by the right hand record of Fig. 9 is frequently the result of a highly damped current oscillation in a relay operating circuit. This distortion from the nature of its cause is called positive or overthrow distortion.

It is perhaps unnecessary to discuss the importance of an accurate knowledge of characteristic distortion in telegraph signaling circuits. However, the following consideration will be helpful in drawing an analogy between the transmission testing machine measurements and the familiar range finding operation commonly used to determine the quality of transmission over circuits operated by automatic means. In automatic signaling systems in which the transmitting and receiving apparatus run in synchronism it is, of course, of prime importance that all signal pulses be received so timed that they fall within the range of the receiving segments. In synchronous multiplex telegraphy the signals from the receiving relay are applied to a rotating brush which distributes them, through the segments of a receiving commutator, to the operating magnets of the printing machines. To be effective, therefore, a signal must be received during the time when the rotating brush is in contact with the proper segment of the commutator. The useful signal then is only that portion which arrives during the time when the brush is passing over the segment for which the particular signal pulse is destined. A general practise in handling automatic printing systems is to determine the "printing range," the angle through which the receiving segments may be oriented before failures occur in the printing machine. This range is a reliable measure of the transmission efficiency of the circuit and is entirely

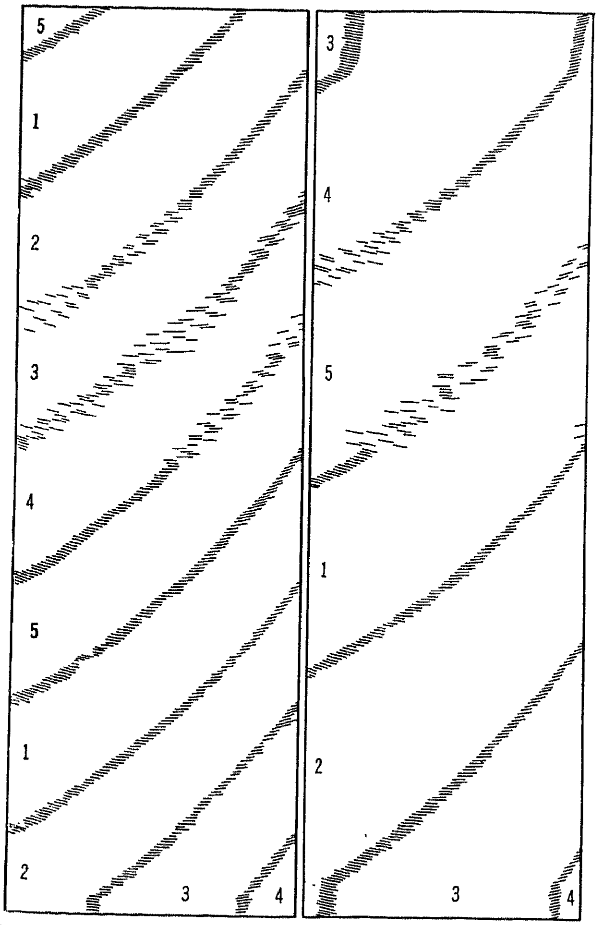


FIG. 11—STRAIGHTAWAY TESTING DRIFTING RECEIVING MACHINE

analogous to the operation of measuring the received signal from the transmission testing machine record. Making proper allowances for apparatus failures, relations are readily obtainable between "percent perfect signal" received and "printing range."

Fortuitous Distortion Measurement. It is, of course, not characteristic distortion alone that limits transmission efficiency and printing range. In land line telegraphy fortuitous distortions are generally far more serious. The effect of these distortions may be quantitatively determined from the transmission machine records. The left hand record of Fig. 10 shows the signals received over a telegraph line into which a periodic disturbance current of a considerable magnitude was induced. This record, like those of the preceding samples, was made on a circuit both ends of which were available at the test point. In general appearance the record is not unlike those already explained. The unevenness of the beginning and ending of the received signals is the result of extraneous currents in the line. The periodic nature of the disturbance is evident in the beats it produces in the beginning and ending of the signals.

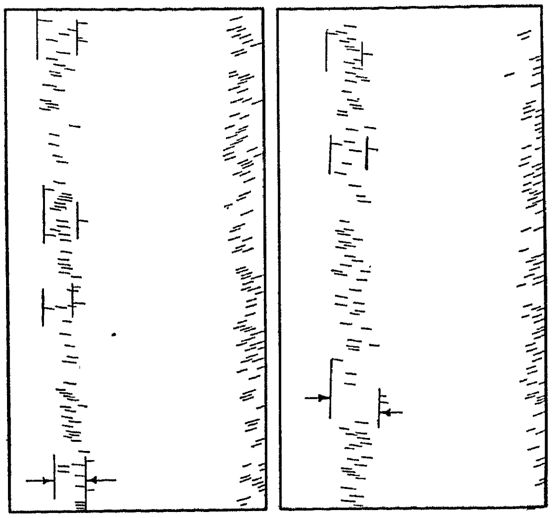


Fig. 12—Typical "Traffic" Signals

The right hand record of Fig. 10 presents a different appearance from any yet discussed. In this case the marks record the open circuit time of the relay contact circuit when both the spacing and marking contacts are tied together. In other words, the marks record the transit time of the relay armature. Fortuitous distortion is as readily measured from a *negative* record of this type as from a *positive*. For the isolation of fortuitous distortion the test signals are generally continuous a-c. pulses. This record may be considered as a *negative* in yet another sense. The measurements show not the percentage of signal repeated but the percentage made useless by the disturbing current.

The disturbing currents which cause fortuitous distortion are frequently, as in this case, of a very erratic nature, fluctuating widely from moment to moment.

Straightaway Testing

Obviously, any of the specimen records shown are as readily obtained when the signals are transmitted from a synchronous transmitting machine as when transmitted locally. Fig. 11 shows two records obtained in this manner. Both of these records were made with the receiving machine and the synchronous transmitter running at the same nominal rotational speed. At the bottom of each record the recorded signals show the two machines to be in very close synchronism. After the speeds had been thus approximately matched the receiver was caused to drift slowly with respect to the transmitter. The left hand record illustrates the signals from a standard four-channel multiplex distributor having 20 transmitting segments. The right hand record shows the signals from a three-channel, 15-segment, distributor.

The numbers on these records are arbitrary numbers assigned to the segments of the transmitting distributor. By drifting the receiving element with respect to the transmitting, all the signal pulses from the transmitting distributor are made to pass in review across the recording tape. When the transmitter and receiver are located at two distant points and the transmitter is sending a particular combination of positive and negative pulses, one of which it is desired to observe on the recording tape, it is convenient to drift the receiving machine in this manner until the desired pulse of the combination is located. The distorted appearance of the signals in the upper half of these tapes is the result of a rather severe hunting of the transmitting distributor. The hunting in this case was introduced intentionally.

Fig. 12 shows two specimen records of normal traffic signals on a multiplex automatic circuit. The transmitter in this case was the three-channel distributor of the multiplex apparatus, and the maximum dot frequency of the signals was approximately 45 cycles per second. The signal pulse combinations of traffic signals, as may well be imagined, cover every conceivable combination of positive and negative pulses. In the record of Fig. 11, with test signals which are continuous alternating current pulses, a mark is made by the chemical recorder for every pulse reversal. There is, therefore, a mark on the tape showing the beginning and ending of the particular pulse which is being observed for every revolution of the transmitting distributor. In the record of traffic signal pulses, however, many revolutions of the transmitter may be made with no marks recorded on the tape. From such a record it is not known whether the armature of the receiving relay is at rest on the marking contact or on the spacing contact. In the record of Fig. 12 the total losses of the circuit under test is recorded. The displacement of a particular mark on this record may be the result of any one of the various factors which introduce signal distortion, either characteristic or fortuitous, or more likely may be the result of a cumulative effect of all the distortions.

In tests of transmission made for the purpose of analyzing signal losses on a circuit it is frequently necessary to obtain a number of measurements. One may determine the characteristic distortion, another the fortuitous, and yet others may be required to analyze the latter and determine the magnitude of the components resulting from the various disturbing influences. A check test of the total losses is usually made by recording traffic signals. Such signals, because of the great variety of pulse combinations of which they are made, result in a maximum of characteristic distortion loss and are also subject to all the disturbances which produce fortuitous distortion.

If periodic disturbances, such as the currents induced by power distribution systems, are present in the circuit the losses resulting from these may not ordinarily be so readily separated from the normal telegraph "crossfire" loss. The transmission testing machine does permit, however, a separation of these two losses by a method which is rather unique. Measurements by this method depend for their success upon the condition that the frequency of the disturbing potential be fairly constant. If the frequency is sufficiently constant the testing machine may be driven at such speed that the signal displacement which results from this part of the disturbance is a constant amount. The effect of the power disturbance distortion as evidenced on the tape record is then a permanent shortening or lengthening of the observed pulse. It has the appearance of a uniform bias of the signals. That part of the distortion which results from the normal telegraph crossfire may then be measured by the method used in the measurement of the record of Fig. 12, without regard to the absolute length of the signal.

In general, measurements of over-all circuit losses are as readily made on a circuit which contains a number of repeaters as on a single repeater section. Obviously, however, the analysis of the losses may not be carried to the same length as it may in tests of a single section.

In this paper no mention has been made of the application of the machine to transmission measurement work on either ocean cable circuits, employing a three-element code, or on single Morse circuits. In work on circuits of the latter class, it is usually convenient to interpose between the receiving instrument and recording mechanism a polar relay which has two live contact stops from which the records may be made. It is difficult when recording from a single Morse relay of the conventional type to determine if the received signals are entirely free from bias. Frequently, in this class of work a special recording instrument is substituted. The terminal apparatus used for the three-element cable code presents no special problem. The records obtained from three-element signals are as conveniently interpreted as are those of the examples presented.

These are but a few of the more common measurements to which the machine is readily adapted. The field of work which the machine may be made to cover in telegraph transmission measurement, as may readily be appreciated, is very broad.

Discussion

H. H. Haglund: In his paper Mr. Bramhall has hewn very close to the line and described the machine only for the actual testing of transmission. As a matter of fact the machine has many other uses, one of which is described below.

For comparing the speed of oscillating systems of comparatively low frequency such as electrically driven tuning forks, the machine is particularly well adapted. In the past, five methods have been commonly employed for this purpose:

1. Comparing the rates of clock mechanisms driven by the two oscillators. This method is accurate only for comparing the average rate of the two systems.

2. Comparing the speeds with the aid of a stroboscope. This method is fairly accurate but very tedious if comparisons are to be made for periods longer than a few minutes at a time.

3. Setting up the two frequencies on a cathode-ray oscillograph in such a manner that Lissajous figures are produced and counting the interval between successive repetitions of the same figure. This method gives an indication of the erratic behavior of the oscillating circuits during short intervals, but does not give a good reading of the average rate unless long tedious observations are made.

4. Producing higher frequency harmonics in the two oscillators and comparing some given harmonic of the two for beats. This method can be made very accurate but also involves tedious continuous observation in order to obtain a comparison for both short and long intervals.

5. Setting up some form of counting system such as two multiplex distributors with the rates adjusted so that one distributor is corrected by a step-by-step mechanism whenever the speed of one gains or loses with respect to the other and counting the number of such steps that occur in a given interval of time.

The transmission testing machine may be used to make the comparison much more conveniently and usually with greater accuracy in the following manner:

Drive the motor on the testing machine from a relay controlled by the output of the oscillator which for the purpose of the test is considered as the standard. The motor makes one revolution for every ten cycles.

Run the second oscillator at the same speed or at a speed differing from the standard by multiples of one-half cycle. Operate the recording mechanism from a relay controlled by the second oscillator so that the needle of the machine marks during the relay travel time.

If the speeds are matched exactly, the short lines or traces on the paper will line up in two perfectly straight columns parallel to the edges of the paper. If the rate of one of the forks now changes, the lines of the traces will run off at an angle which can be readily measured.

The product of the tangent of this angle and the distance the paper feeds out per minute divided by the distance between the traces on the paper gives the change in rate in half cycles per minute. In practise the change of rate can often be obtained by observation. For example: If the traces begin to drift off at such an angle to the edges of the paper that in five minutes the right hand trace has moved over to the position previously occupied by the left hand trace, the change in rate is obviously one-half cycle in five minutes or one-tenth cycle per minute.

Since the machine will operate for several hours with practically no attention, a long run can be obtained during which a continuous record is produced. This involves no tedious counting and permits the variations to be carefully measured after the test has been run. Average rates, rates for successive intervals, maximum and minimum rates can all be obtained from the same record. Accuracies to one part in a million can easily be obtained except for short intervals.

R. B. Shanck: The device is a valuable addition to the tools which are available for the measurement of telegraph signal distortion. I believe that it is particularly well suited to the investigation of distortion which, with a regularly-recurring signal, oscillates through a definite cycle so as to form a regular pattern, as is the case with a-c. interference, for instance. The result of changes which affect distortion of this type is clearly evident immediately.

It is gratifying to note the tendency, in connection with the discussion and investigation of telegraph distortion, to employ the same approach and terminology throughout the world. Thus the separation of distortion into components, called bias and characteristic and fortuitous distortion, as now widely done in this country, has also been taken up abroad. Recently distortion was measured and reported in this way in connection with a voice-frequency telegraph investigation by a subcommittee of the International Consulting Committee on Telegraphy.

A Printing Telegraph Concentrator

BY WILLIAM B. BLANTON*
Non-member

Synopsis.—*The central telegraph office of a city is the focal point for the collection and delivery of commercial telegrams within that city and the surrounding territory. This central office is connected telegraphically to the public branch offices within this area as well as to the offices of those patrons who have sufficient telegraph business to warrant the telegraph company incurring the expense of providing such a connection.*

Printers employing the start-stop method of synchronism have been found exceedingly adaptable for use in circuits providing such telegraph connections, and the telegraph companies are using these printers in increasing numbers for this purpose.

The termination in a central office of a large number of these circuits, the majority of which is worked intermittently for the greater part of the time, requires the provision of some means of concentration in order that the traffic may be handled efficiently and economically.

This paper describes a multipled turret printing telegraph concentrator designed and used by one of the telegraph companies to meet this need.

* * * * *

INTRODUCTION

During recent years great improvements have been made in printing telegraph systems, particularly those using the start-stop method of synchronism.[1]

Some of the designations which are employed by the manufacturers and various users for telegraph printers adapted to start-stop systems are start-stop printer, simplex printer, teletype, telephone typewriter, typing telegraph, and teleprinter.

The telegraph companies are using in increasing numbers printers of this type on private tie lines which connect patrons' offices with a telegraph office, on public circuits which connect the public branch offices within a city to the central telegraph office, and in many cases on lightly loaded trunk circuits extending between telegraph offices in different cities.

For services of this kind, one of the telegraph companies has installed, since 1925, upwards of 20,000 of these printers. About 5,000 of this number were installed in patrons' offices. In fact, the greater portion of printer installations now being made are necessitated by the increasing number of printer tie lines which connect patrons' offices with a telegraph office.

The purpose of this paper is to describe a concentrator which was developed for use in central telegraph offices. Essentially, this concentrator provides for terminating a large number of start-stop printing telegraph circuits before a smaller number of operators so that any one of the circuits can be worked by any one of the operators.

Throughout the paper the terms "printer" and "printing telegraph circuit" should be understood to refer to equipment and line operated on the start-stop principle.

THE ADVANTAGES OF CONCENTRATION

The termination in a central telegraph office of a large number of printing telegraph circuits, the majority of which is worked intermittently for the greater part of the time, presents the problem of providing suitable facilities for efficiently and economically handling the traffic on these circuits.

Earlier methods included the termination of the heavily loaded circuits on operating tables individual to each circuit, and of the lightly loaded circuits in small concentrators each having four operating positions and a capacity of eight lines. While these arrangements are still satisfactory for small offices where few circuits are involved, they are unsuitable for handling the numerous circuits which are now terminated in the larger offices. For the latter conditions it has been found desirable to provide concentrators accommodating a large number of circuits and operating positions in order to obtain the advantages discussed in the following paragraphs.

The termination of a large group of intermittently worked circuits in a single concentrator permits the most efficient utilization of printer equipment and floor space due to the fact that any one of the concentrator positions can work any one of the circuits. Therefore, the number of operating positions provided for a given group of circuits need not exceed those required to handle the maximum number of these circuits which it is anticipated will be busy simultaneously during the peak load. The smaller floor area required to accommodate circuits when concentrated has been of a very distinct value in many offices, in that it has postponed for the time being the necessity of obtaining additional floor space to care for the increasing number of printer circuits.

The ability to work any one of many circuits from a concentrator operating position tends to utilize fully the time of each operator for useful purposes. Of course, this may be accomplished to some extent even with the termination of circuits on individual operating positions and in small eight-wire concentrators by shifting the operators from one operating position to another to meet the load changes. However, the impracticability of doing this to realize the maximum operating economies and at the same time not degrade the speed of service will become

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[1] T. E. Herbert, "Telegraphy," p. 632, Sir Isaac Pitman & Sons, Ltd.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

more apparent as the number of circuits is increased.

Besides the increased productiveness of each operator, several other operating economies accrue from the concentration of many printer circuits in one unit. The use of automatically operated supervisory arrangements, which give a continuous and accurate check on the number of operators required to handle the traffic load, permit a quick adjustment of the operating staff to meet the changing traffic load. The centralization of the operating force reduces the supervisory staff to a minimum. The bringing together of the operating force into a small group during the light load periods, particularly at night, permits further savings through the shutting down of lights, belt conveyors and other telegraph equipment in the unused portion of the office.

These economies are not obtained through any sacrifice in the speed of service. On the contrary, the speed of service obtained on each circuit in a properly staffed concentrator is consistent with that obtained on a circuit terminated on an individual operating table staffed continuously with an operator. Furthermore, better supervision of the movement of traffic is achieved due to the centralization of the operating force and the centralization of the various operating functions of each circuit.

The more intermittent the traffic load is on a circuit, the greater are the advantages in concentrating. Obviously, there would be no gain in providing concentration facilities for a heavily loaded circuit which requires the individual attention of an operator all of the time. However, on most heavily loaded circuits there occur periods, particularly at night, when the continuous services of an operator are not required. During such times it is desirable to work such circuits in a concentrator; and in order that advantage may be quickly taken of these periods, it is the present practise to terminate all start-stop printing telegraph circuits entering the larger offices in concentrators. This also has the advantage of uniformity of circuit arrangements in an office.

The concentration of heavily loaded circuits, the busy periods of which will probably occur simultaneously, does not effect any appreciable saving in operating positions. In fact, the addition of such a circuit to a concentrator practically requires the addition of an operating position. Thus, the value of a concentrator in which heavily loaded circuits will be terminated is greatly enhanced if the burden of concentration expense carried by each operating position is small.

METHODS OF CONCENTRATION

Before the equipment described in this paper was decided upon, three methods of concentration were considered; machine switching, manually attended switching board, and lines multipled before each operator. The following brief descriptions of these methods of concentration are not intended to apply to any particular existing concentrators but merely explain in very general terms the fundamental features of each method.

A machine switching concentrator would comprise essentially equipment which, in response to a calling signal from an outer office, would automatically connect the line of that office to the printer apparatus at an idle concentrator operating position, and also equipment which would enable a concentrator operator to initiate a call to any one of the outer offices. This latter equipment might consist of switches which would respond to electrical impulses, caused by a concentrator operator dialing the call numbers or letters of any one of the outer offices, so as to connect this operator's printer apparatus to the line of the desired office; or this equipment might be similar to that of a multipled turret concentrator, *i. e.*, the lines would be multipled before each sending operator thus enabling an operator to connect her printer apparatus to any desired line in order to initiate a call.

A manually attended switching board concentrator might comprise an arrangement somewhat similar to a manual switching telephone private branch exchange. Thus, the lines would be terminated on jacks located in the switchboard. The wires to the printer apparatus at the concentrator operating positions might be similarly terminated in jacks in this switchboard, or be terminated in cord circuits appurtenant to the switchboard. In either case, the switching operator or operators stationed at this switchboard would note the incoming calls, indicated by lamps or other means associated with the line jacks, and would connect these calling lines to printer apparatus at idle operating positions by inserting plugs, attached to cord circuits, into the proper jacks.

Outgoing calls might be handled in the same manner, the switching operator connecting the printer apparatus at the calling operating position to the desired line. Each cord circuit which the switching operator uses to connect a line to an operating position would be provided with a supervisory signal arrangement so as to permit a concentrator operator to give the switching operator a "disconnect signal" at the completion of the exchange of traffic with an outer office. Multiples of this switchboard would be necessary if the number of switching operators exceeded the number that can be stationed at one switchboard.

A concentrator in which the lines are multipled before each operator would also comprise means, generally lamps, which indicate incoming calls, and means at each operating position which enable an operator to connect her printer apparatus to any desired line. Thus, in answering or initiating calls, the concentrator operator would make the actual connection of her printer apparatus to the desired line.

A study of the economics and operating advantages of these three methods and their adaptability to the present methods of handling telegraph traffic led to the choice of the arrangement whereby the lines are multipled before each operator, or, in other words, a multipled turret concentrator.

Description of the Concentrator

The methods which have been generally followed in the design of multipled turret arrangements are to provide for each line a jack, calling lamp, and sometimes a busy lamp in each of the turrets, which are nothing more or less than small switchboards connected in multiple. Generally, one turret suffices for several operators, the number of turrets required being dependent on the maximum number of operators required at any one time to handle the traffic on the lines terminated in the turrets. Each operator is provided with one or more cord circuits, each terminating in a plug by means of which she connects an operating set to any desired line by inserting the plug in the proper line jack.

It can be readily appreciated that the cost of the turrets and the cabling required to connect the jacks and lamps in multiple comprises no small part of the total cost of a multipled turret arrangement. With the printer concentrator, which in some cases requires 70 operating positions for a unit terminating 100 lines, it becomes imperative that the cost per operating position of the turrets and cabling be held to a minimum. Two methods of accomplishing this suggest themselves: first, let each turret accommodate a large number of operating positions; second, simplify and reduce the cost of each turret and the cabling between the turrets.

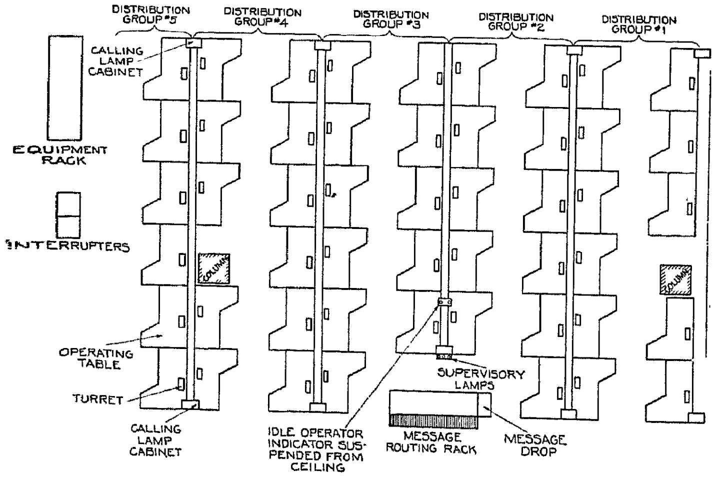

Fig. 1—Typical Concentrator Floor Plan

It is impracticable to follow the former because the distance between operators and the bulky equipment installed on the operating tables necessitate that the turrets be individual to each operating position if they are to be easily accessible. In order to simplify the turrets and reduce the number of conductors which will have to be multipled to each turret, several departures are made from earlier practises; no busy lamps are provided, the calling lamps are removed from the turrets and placed in lamp cabinets so located that a number of operators can observe one cabinet, and the concentrator line circuit is designed on a single conductor basis; in other words, each line appears in the turrets as one conductor without any associated local or private wires. Therefore, the cabling which is multipled from turret to turret consists of only one conductor for each line circuit which may be terminated in the concentrator, and the cabling required for the calling lamps is simplified since the number of calling lamp cabinets is considerably less than the number of operating positions.

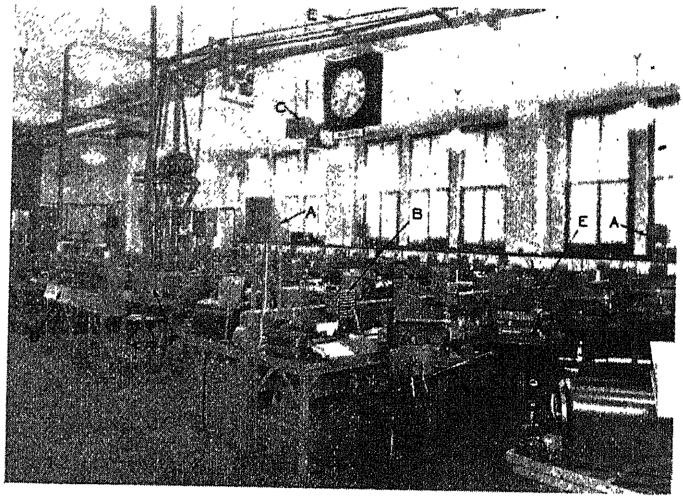

Fig. 2—General View of a Concentrator

A	Calling lamp cabinet
B	Turret
C	Idle operator indicator
D	Message routing rack
E	Message belt conveyor
F	Message drop

Layout of Concentrator. Fig. 1 shows in schematic form the relative location of the various items in a typical concentrator installation. Fig. 2 is an illustration of an actual concentrator installation having 50 operating positions and a capacity of 100 lines. The layout of this installation is identical with that of Fig. 1. Reference to these two figures when reading the following description will aid in correlating the various items.

Operating Table. The physical size of the printer and accessory operating equipment required at a central office operating position, even when it is not equipped as a concentrator operating position, requires considerable operating table area. The top of the standard table is 49½ in. (126 cm.) long, 21 in. (53 cm.) wide and on the right hand end of the table an extension projects forward. One of these tables equipped as a 100-line concentrator operating position is illustrated in Fig. 3.

The front edge of the printer is alined with the left hand edge of the extension which forms a 120-degree angle with the front edge of the table, while the gumming desk is alined with the front edge of the table. The printer is a "tape printer" as contrasted with a "page" printer. Each message, whether being transmitted or received, is printed on one side of a paper tape, the other side of which is coated with dry gum or glue. This tape issues from the left end of the printer. When transmitting, the operator allows this tape to go through a hole in the table top into a metal receptacle, whereas during the reception of messages, the operator by means of a thimble tape cutter and hand tape moistener pastes the tape on a telegram blank which is held on the gumming desk by means of a spring clip.

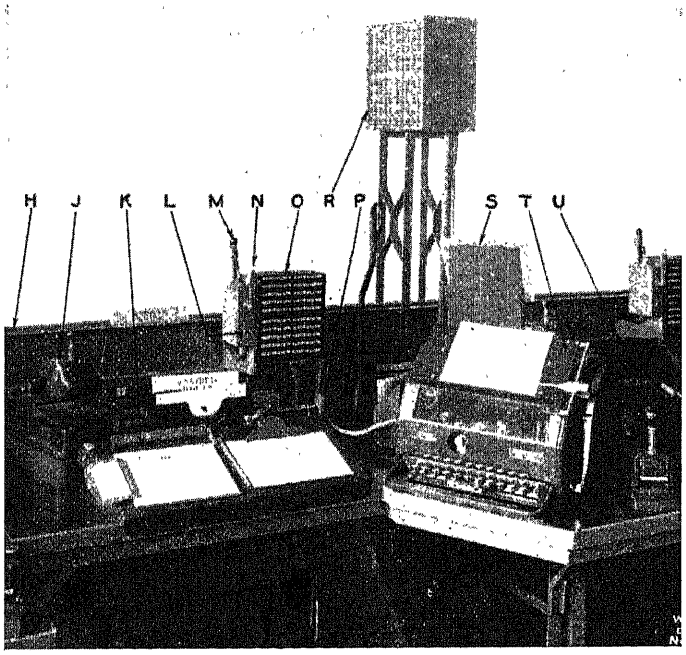

Fig. 3—Concentrator Operating Position

H	Belt conveyor
J	Time stamp
K	Gumming desk
L	Busy line lamp
M	Idle operator lamp
N	Answer-next lamp
O	Turret
P	Jack-plug
R	Calling lamp cabinet
S	Live message file
T	Printer
U	Number stamp

The other equipment on this table which is required at any standard central office operating position is as follows: the two-compartment, "V" shaped, live message retainer into which clerks called routing aides place messages awaiting transmission, full rate messages being placed in the bottom compartment and reduced rate messages in the top compartment; the number stamp used to number consecutively all messages transmitted from this operating position; the two compartment box located to the left of the printer into which the outgoing messages are placed after they have been transmitted; the three-compartment box directly behind the gumming desk which is used as a storage for message blanks; the time stamp which is used to stamp the time on all received messages; and the "goose neck" arrangement into which the operator inserts any message requiring the attention of the supervisor.

Along the rear edge of the table may be seen the belt conveyor on which all received messages are placed.

Turrets. To avoid using for the concentrator operating positions a table larger than the standard operating table, it is necessary that the turrets be of a small and compact design. Fig. 3 shows a turret, having a capacity of 100 lines, located near the rear of the table, midway between the sending and receiving portions of the operating table. It is not intended that the capacity of the concentrator be limited to 100 lines; in fact, installations have already been made where two of these turrets for 200-line concentrators have been mounted on the standard operating table. Moreover, it is quite practicable to install turrets for more than 200 lines on the same size table.

The use of a one-conductor multiple permitted a departure from standard telephone and telegraph practise in the design of the turret. Instead of jacks, the line conductors are terminated on solid plugs or studs molded in a bakelite panel.

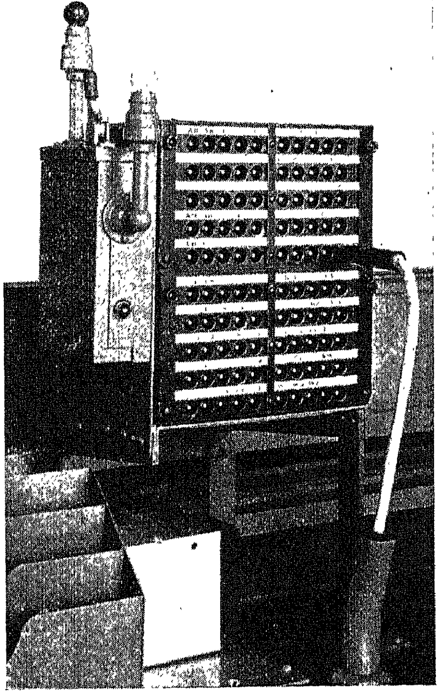

Fig. 4—Signal Assembly and 100-Line Turret

Fig. 4 gives a close-up face view of a turret equipped with two panels, each panel having a capacity of 50 lines. The line designations are printed on white cardboard and inserted in cellulose acetate tubing. By loosening either of the thumb nuts in the upper corners of each panel, the associated metal angle may be swung outward thus permitting the removal or replacement of the designation strips.

That portion of the face area of the panels which is used for the studs is approximately the same as it would be if telephone type jacks were used. However, this design of panel presents possibilities of greater face area reduction if that should become advisable in the future. For example, the horizontal spacing of the studs was governed by the desire to provide ample designation space for two or three letter designations.

The immediate gains obtained with this panel as compared with an equivalent arrangement of jacks and designation strips are: reduced cost of apparatus and installation, saving of space by use of a shallower turret, better insulation between adjacent circuits, and more robust construction. In connection with the last item, it will be noted that all springs and moving parts are included in the device, described later, which the operator uses to connect her printer to a line stud.

Cord Circuit. Associated with each operating position is an operator's cord circuit which includes the receiving and transmitting portions of the printer, the three lamps and lever type key contained in the signal assembly mounted on the left hand side of the turret, certain relays, jacks, resistance units, a vacuum tube, and other apparatus and equipment. This circuit connects to a three-conductor cord, the other end of which terminates in a "jack-plug," so termed because it has the features of both a jack and a plug as generally thought of in the communication art. It is by connecting this jack-plug to a line stud in the turret that the operator connects her printer to any desired line. When not in use, this jack-plug rests in a plug seat bushing fastened in the end of a short length of pipe located directly in front of the turret.

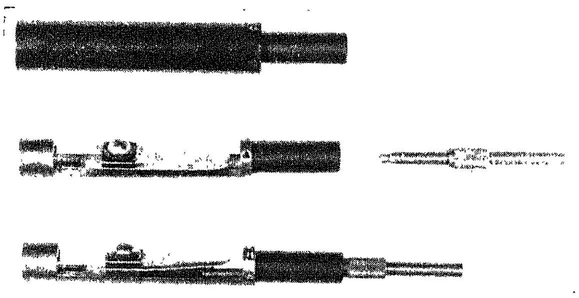

FIG. 5—SEVERAL VIEWS OF A JACK-PLUG

A metal cabinet is provided at each position for housing some of the equipment included in the operator's cord circuit and also other electrical equipment such as connecting blocks, fuse blocks, power receptacles, etc. The metal cabinet has a top plate somewhat larger than the cabinet proper. This enables the cabinet to be installed on the table by lowering it into a rectangular opening until the top plate rests on the top surface of the table. Access is gained to the equipment inside of the cabinet by means of doors which open underneath the table top.

Several views of the jack-plug are shown in Fig. 5. The top view shows a completely assembled jack-plug. The middle view is of the same arrangement except that the large bakelite sleeve is removed and also, a stud, of the type that is molded in the panels, is shown. The bottom view illustrates the operation of the jack-plug when it is plugged on a stud. The body of the jack-plug makes contact with the stud, and this stud operates a lever that extends up into the small barrel of the jack-plug so as to close the two springs.

Calling Lamps. The calling lamps serve to register an incoming call when an outer office initiates a call to the central office. As has been previously mentioned, lamp cabinets, each containing one lamp for each line terminated in the concentrator, are distributed among the operating positions so that each cabinet is visible to and used by a number of operators. It is the general practise to locate these cabinets so that no operator is more than four and one-half table lengths distant from a cabinet. Since the tables are arranged in double-sided rows, one cabinet may therefore serve as many as ten operators.

A view of one of these cabinets supported by a structure that is fastened to the belt conveyor guide angles may be obtained from Fig. 3.

There are no designations on the calling lamps, whereas, as has been noted, the line studs in the turrets are designated with the office letters. However, the calling lamps are in the same relative arrangement as the line studs with which they are associated. An operator, to answer an incoming call, notes the position of the lighted lamp and then attaches her jack-plug to the line stud having the same relative position. To aid the operators in performing this function, the 100 lamps and studs are arranged in the cabinets and turrets, respectively, in groups of 25 each. Thus the operator notes at a glance in which group the lighted lamp is located and its relative position in this group. Actual practise has demonstrated that the operators become exceedingly skillful in selecting the proper line in the turret by this method.

Busy Line Indication. The absence of busy lamps appurtenant to the line studs makes it essential that each operator's cord circuit be equipped with means that will give a "busy indication" when an operator plugs in on a busy circuit and, at the same time, not interfere with or mutilate signals which might at that moment be passing over the line circuit.

To obtain this feature with the one-conductor multiple, the concentrator circuit is designed so that positive battery[2] is applied to a line when it is idle and negative battery is applied to a line when a concentrator operator has her printer connected to it. Then by the use of a unidirectional current device, the operator's cord circuit is arranged so that when an operator plugs in on an idle line, the printer motor starts and the printer with its associated negative battery is automatically connected to the line. On the other hand, if the operator plugs in on a busy line, the unidirectional current device will function so as to keep the printer motor from starting and to prevent the printer with its associated negative battery from being connected to the line. Furthermore, the characteristics of the unidirectional current device are such that no interference is caused to the line signals which might then be passing over the line circuit.

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[2] In this paper, the word "battery" is used to denote any source of electrical energy which supplies current to telegraph circuits. The "batteries" referred to have one pole permanently grounded and are spoken of as "positive" or "negative" depending upon the polarity of the ungrounded pole.

When an operator plugs in on a busy line, the busy line lamp, having a red colored lamp cap and located in the lower portion of the front of the signal assembly (see Fig. 4), will light steadily until the operator removes her jack-plug from this busy line or until the line ceases to be busy. If the operator fails to see this visual signal and attempts to send on her printer, the printer will not function due to the fact that the printer motor is not running.

Answer-Next System. The answer-next system serves to expedite the answering of incoming calls in that it avoids delay that might result from the inattentiveness of the operators and confusion that might arise from several operators endeavoring to answer the same call. By virtue of the answer-next arrangement, the responsibility for answering an incoming call is placed upon the lowest numbered idle operator.

A white lamp, termed the answer-next lamp, is provided on the front of the signal assembly at each operating position as shown in Fig. 4. This lamp is always lighted on the lowest numbered idle operating position thus indicating to this operator that she is to answer the next incoming call. For example, if the first 10 operators are busy, the answer-next lamp on position 11 will be lighted. When the operator at this position answers a call, the lamp will be extinguished at position 11 and will light on position 12, provided of course that the operator at number 12 position is idle and that all lower numbered operators are still busy. If, on the other hand, a lower numbered operator than 11 becomes idle before operator 11 answers a call, then the answer-next lamp on position 11 will be extinguished and the lamp on the lower numbered idle position will light.

The answer-next lamp on the lowest numbered idle operating position glows steadily as long as there are no incoming calls, thus warning the operator that she should be in readiness to answer the next call. If there is a long interval of time before a call appears, it may be expected that the attention of the operator will wander and that she will not keep her eyes continuously on a calling lamp cabinet. Therefore, to attract the operator's immediate attention when a call does appear, the answer-next lamp begins to flash the instant a call is registered in the lamp cabinets.

By means of the lever type key located in the top of the signal assembly (Fig. 4) any of the operating positions which are not staffed, either because of light loads or of repairs being made to the table equipment, may be cut out of the answer-next system, or in other words, the positions may be "busied."

Operating Practises. Since many of the features of the concentrator about to be described are necessitated by the present practises followed in handling telegraph traffic in central telegraph offices, it is thought best to briefly outline these practises.

All messages received over printer circuits are placed on continuously moving belts which run along the rear edge of the operating tables and which carry the messages to the end of the table row. Other belts, located at the end of the table row, comprise a part of a mechanical conveyor system that carries all received messages (including those received by other facilities such as multiplex, telephone, pneumatic tube, and Morse) to the main routing center of the office. Here are located the requisite number of route clerks who ascertain the destination of these messages and dispatch them by means of other conveyors to "drops," points at which messages are discharged from the conveyor, located near the proper line wires. Where concentrators are not used, each outgoing message is removed from these drops by a clerk, called a routing aide, and carried to the operating position at which is terminated the line wire of the office to which the message is destined.

The handling of the received messages presents no problem to concentration. On the other hand, the present practises followed in handling the outgoing messages introduces many of the problems in the design and operation of large concentrators.

There are two requirements to be met in handling this outgoing traffic. First, each message must be expedited to its destination. To insure a satisfactory speed of service, there is specified a time limit within which each message must be retransmitted after it is received in the office. Second, full rate traffic must be given precedence over reduced rate traffic, such as day letters and night letters. In order to meet these requirements with a concentrator, messages for an office, the line of which is idle are taken to an idle operator for immediate transmission, while messages for an office the line of which is busy are taken to the operator then working that circuit.

At first thought it may seem that a more desirable method of designing and operating a large concentrator would be to divide the operating positions into sending and receiving positions and that all outgoing messages should be taken to sending operators who would transmit them to the proper offices at the first moment the lines to these offices became idle. However, it is apparent in considering the second requirement mentioned in the preceding paragraph, that this procedure would be open to the objection that a circuit might be busy at a receiving position then handling day letters, while at the same moment one or several full rate messages might be awaiting transmission at a sending position.

Sub-Routing Center. A sub-routing center, centrally located at one end of the rows of operating tables, is provided for each concentrator. All the messages received at the main routing center of the office which are destined for the circuits terminating in a particular concentrator, are dispatched by a mechanical conveyor to a drop located at the sub-routing center of this concentrator. This drop is located at one end of a message routing rack which has one compartment for each circuit terminated in the concentrator. The front view of a message routing rack provided with 100 compartments and with the message drop on the right hand end is shown in Fig. 6. Each of these compartments bears on both the front and rear of the rack the designation of the office with which it is associated.

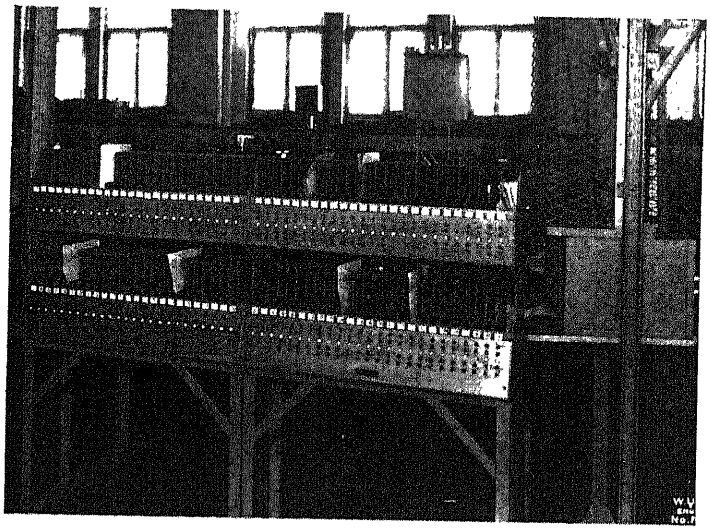

FIG. 6—FRONT VIEW OF MESSAGE ROUTING RACK

One or more routing clerks stationed at the rear of the rack remove the messages from the drop and sort them into the proper compartments according to the destination of the messages. These messages awaiting transmission may be thought of as falling into two classes, those for offices which are being worked at that moment by a concentrator operator and those for offices the line wires of which are idle.

Group Position Finder Facilities. To facilitate the distribution of messages destined for transmission on lines which are busy, the operating tables of a concentrator are divided into distribution groups. For example, the operating tables in the typical layout shown in Fig. 1 are arranged in five distribution groups. The number of tables included in a distribution group is not definite but depends on the layout and total number of tables in each particular concentration unit, the one requisite being that all the tables in any one distribution group shall face on the same aisle. In practise, one single-sided table row is designated as a distribution group if the table row is long, or two single-sided table rows facing on the same aisle, if the table rows are short.

On the front of the message routing rack a vertical row of lamps, one lamp for each distribution group in the concentrator, is provided underneath each compartment and its associated designation card. The rack illustrated in Fig. 6 has five lamps per compartment, one for each of the five distribution groups shown in Fig. 1.

When none of the lamps under a particular compartment is lighted, this indicates that the associated line is idle. The distribution of messages destined for transmission on idle lines is covered in the section below.

If one of the lamps under a particular compartment is lighted, this indicates that the associated line is being worked by a concentrator operator. The routing aide when removing a message from this compartment notes which one of the lamps is lighted. This serves to guide her to the proper distribution group. Then she views the turret on each busy operating position in this group and ascertains by the location of the jack-plug which operator is working the line in question.

An operator indicator system was devised which would denote under each compartment the exact location of the operator working the associated line. This arrangement was more intricate and costly than the group position finder arrangement. An analysis of the two showed that the individual position finder possessed very few advantages over the group position finder, and that these advantages were not sufficient to justify the increased cost of installation and maintenance.

Idle Operator Indicator. Messages for offices, the line wires of which are idle, are given to idle operators for immediate transmission. To facilitate this, an idle operator indicator, Fig. 7, is located within view of the routing aides when they are standing in front of the message routing rack.

The idle operator indicator contains one lamp for each operating position in the concentrator. The lighted lamps denote staffed operating positions where the operators are not working a line wire, whereas, the unlighted lamps denote unstaffed positions or positions where the operator is busy. The lamps are not numbered or otherwise designated. Instead, the layout of the lamps corresponds to the layout of the operating tables in the particular concentrator and the routing aides are required to note the relative location of the lighted lamps when they desire to know where the idle operators are located.

As a further aid to locating the idle operators, each operating position is equipped with an idle operator lamp mounted on the top and towards the rear of the signal assembly (see Fig. 4) and colored green to distinguish it from the white answer-next lamp. This lamp is lighted on each operating position where the operator is idle.

The idle operator indicator indicates to the routing aides the location of all of the idle operators. After the routing aides have left the message routing rack, the idle operator lamps on the positions guide them to the idle operators and also immediately indicate to the aides any change in status that may have taken place since they left the message routing rack.

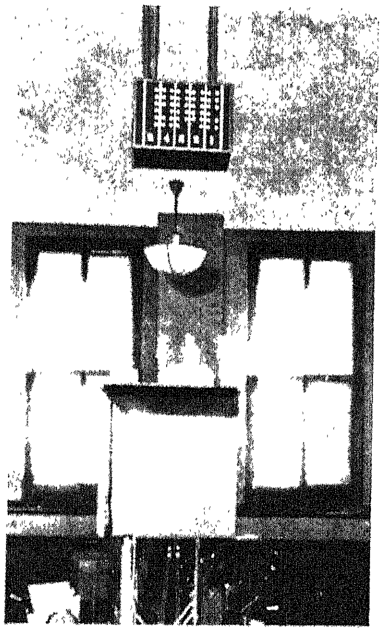

FIG. 7 IDLE OPERATOR INDICATOR AND SUPERVISORY LAMPS

Messages for idle lines are distributed to the highest numbered idle operators, the reverse order in which incoming calls are answered.

Supervisory Lamps. Three supervisory lamps are located at some point about the concentrator, generally on the supporting structure of one of the calling lamp cabinets as shown in Figs. 6 and 7, so that they are visible to the supervisor while he is performing his duties about the concentrator. These lamps serve to warn the supervisor immediately of conditions which require attention. The first lamp lights when there is one or more incoming calls. The second lamp lights when all the operators assigned to the concentrator are busy.

With the second lamp unlighted and the first lamp lighted for an appreciable interval of time, the supervisor is advised that the operator, on whose position the answer-next lamp is lighted, is not promptly answering the incoming call. Such a condition might exist through no fault of the operator if her answer-next lamp fails to light due to a burnt out filament. However, the supervisor is instantly made aware of that condition by the third supervisory lamp which lights at any time that an answer-next lamp fails because of an open filament.

When the second lamp lights denoting all operators are busy, the supervisor is warned that there is no "readiness to serve." A constant check must be kept on this feature in order that there will always be an operator available to answer an incoming call.

The idle operator indicator is utilized by the supervisor in determining the size of operating staff required throughout the day.

Equipment Rack. Fig. 8 shows a rack equipped with relays, resistances, fuses, and other apparatus required for a concentrator terminating 100-line circuits. Various alarm arrangements and testing facilities are also included on this rack.

Signal and Motor Control at Distant Office. When an operator at the concentrator plugs in on a line circuit, this automatically causes a visual signal to function and the printer motor to start at the distant office. The starting of the motor permits the concentrator operator immediately to supplement the visual signal with an audible signal by repeatedly depressing the proper key on her printer, thereby actuating the bell signal in the distant printer. The outer office operator acknowledges the call and proceeds to exchange traffic, having extinguished the visual signal by operating a conveniently located switch.

The outer office calls the concentrator by depressing any one of the printer keys, first starting the printer motor if it is not already in operation.

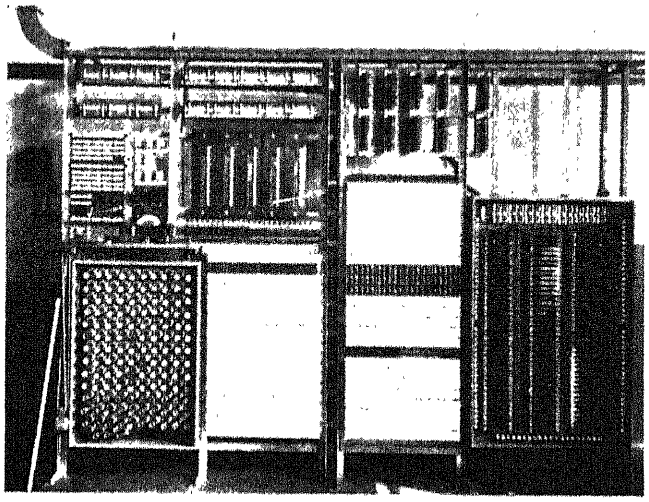

FIG. 8—EQUIPMENT RACK FOR A 100-LINE CONCENTRATOR

Circuit Operation. An explanation of several of the salient points of the concentrator circuit, shown in Fig. 9, may be of interest.

Positive battery, designated "signaling battery" in Fig. 9, of a potential of 110 volts, is applied to each idle line, resulting in a continuous current flow over the circuit. An outer office operator to initiate a call to the concentrator, throws her switch to the operating position, thereby starting her printer motor. Then by depressing any key of her printer the flow of line current is momentarily interrupted which causes the signal relay at the concentrator to light all of the calling lamps associated with this line. A concentrator operator to answer this call attaches her jack-plug to the proper line stud which applies the positive signaling battery to the plate of the vacuum tube in her cord circuit. A small plate current results which causes certain relays in her cord circuit to function so as to start her printer motor and connect her printer and its associated negative battery, designated "operating battery" in Fig. 9, to the line. The current flow which then results on the line circuit energizes a line relay that in turn energizes a cut-off relay which disconnects the signal relay and the signaling battery from the line.

It is essential that the cut-off relay remain continuously in the energized position during the entire time that the concentrator operator has her printer connected to the line. Since this must be accomplished on the same conductor over which are passing the line signals, consisting of intervals of current and no current, it is necessary to provide a slow-to-release arrangement for the cut-off relay which will not allow it to release except after a predetermined interval of no current on the line circuit.

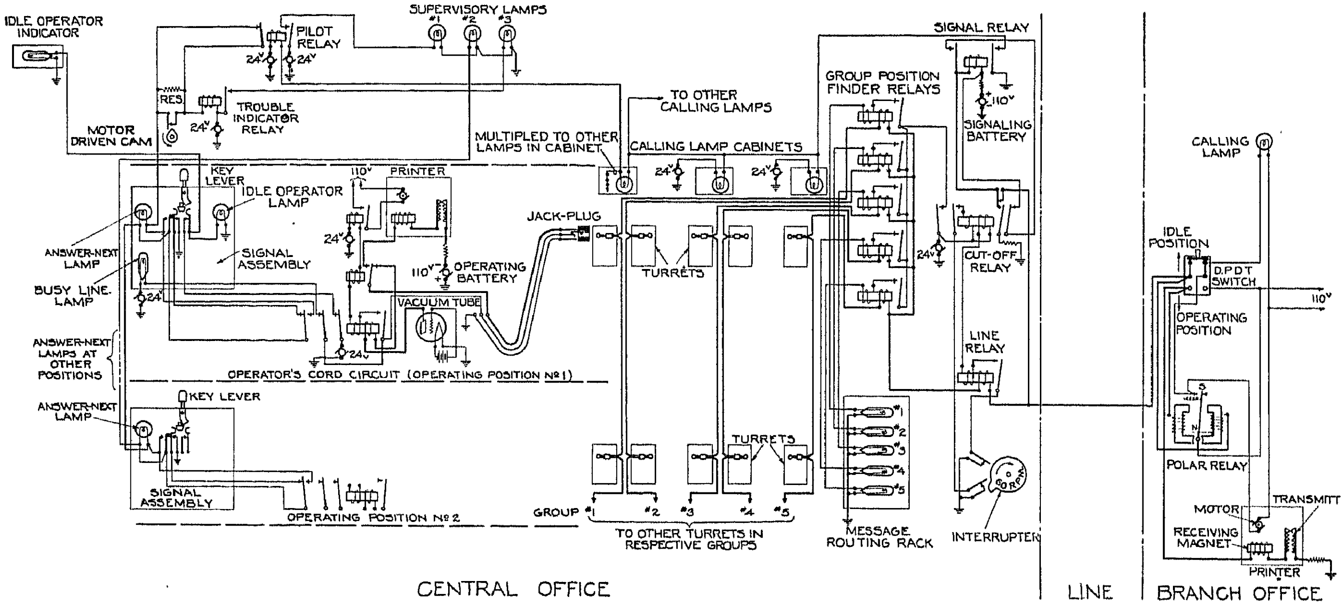

Fig. 9—Diagram of Concentrator Circuit

The start-stop printers shown in the illustrations of this paper employ a seven unit code transmitted at approximately 22 cycles per second, *i. e.*, each unit of the code takes 1/44 second. The first unit is always the start signal of no current and the seventh unit, the stop signal of current. The intermediate five units are in accordance with the Baudot code and consist of various combinations of current and no current depending on the character being transmitted. Since the seventh unit is always a current impulse regardless of what character is being transmitted, it is evident that the longest interval of no current that can occur when only one printer is transmitting is 6/44 seconds. If the second printer on a circuit is operated to "break" the first printer, longer intervals of no current may occur. However, experience has shown that even in such cases, at least one impulse of current is generally transmitted over the line every second.

The slow-to-release arrangement employed to maintain the cut-off relay continuously operated while an operator has her printer connected to a line consists of two locking circuits through the cut-off relay, either one or both of which when closed will maintain the cut-off relay in the operated position. A motor-driven interrupter is also included which momentarily opens at regular intervals and in a definite sequence these two locking circuits. The arrangement so functions that if an impulse of line current passes through the line relay at least once every second, these locking circuits will be restored after being opened by the interrupter in such a manner that one or the other will be continuously energizing the cut-off relay. When the concentrator operator disconnects, all battery is removed from the line and the cut-off relay will be deenergized shortly afterwards since there is no line current to restore the locking circuits after they are opened by the interrupter.

When the cut-off relay is deenergized, the signal relay with its signaling battery are connected to the line and are in readiness to receive another incoming call. It will be noted that the line current from the signaling battery does not pass through the line relay, thus it is evident that the cut-off relay remains in the deenergized condition during the idle period of the line circuit.

The signal and motor control device at the outer office is a polarized relay which lights the signal lamp and starts the printer motor when negative battery is applied to the line and extinguishes the lamp and stops the motor when positive battery is applied to the line.

The vacuum tube is the unidirectional current device employed to prevent a second operator from connecting her printer and operating battery to a line circuit which is busy at another concentrator operating position. If this second operator attaches her jack-plug to the stud of a busy line, the plate of the vacuum tube in her cord circuit encounters the negative operating battery (or no battery when spacing signals are being transmitted from the concentrator position) on this busy line and consequently no plate current will result, thus maintaining the relays in her cord circuit in the deenergized position. The impedance of the plate of the vacuum tube to ground is so high that no interference is caused to the line signals which might be passing over the line circuit.

Application of Concentrator

Numerous installations of the multiple turret concentrator have been made. Most of these concentrators were planned and installed on the basis of terminating 100 lines, while in a few cases concentrators accommodating 200 lines were installed. In some of the smaller offices having only a few circuits, concentrators were installed and equipped for less than 100 circuits, the installations being made, however, so that the capacity of the units can readily be increased to 100 or 200 hundred lines without unduly disturbing the initial installations.

Although this type of concentrator is adaptable to a larger number of lines, 100- and 200-line concentrators were selected as the standard sizes of concentrators because of the relative ease of finding the operator working any particular line, as well as the fact that these two sizes of units, particularly the former, meet the requirements of a large number of offices. The former size unit is essentially used for terminating heavily loaded circuits whereas the latter size unit is used for terminating comparatively lightly loaded circuits. In offices having one hundred or less circuits, no distinction is made between heavily and lightly loaded circuits as they are all terminated in one concentrator. At the present time most of the offices fall in this class which explains the greater usage of 100-line concentrators. In the larger offices, more than one concentrator is required. For example, ten 100-line and three 200-line concentrators have been installed in the New York City central office.

The traffic load on the circuits terminating in a concentrator determines the number of operating positions which is provided. The number of positions provided for the 100-line concentrators has varied from 20 to 70 positions and for the 200-line concentrators, from 30 to 40 positions.

The present practise is to terminate all start-stop printer circuits entering a central office in concentrators. Short city lines are connected directly to a concentrator multiple. Longer circuits, particularly trunk circuits, in which it is not practicable to operate the printers directly in the line circuit, are terminated in single line or duplex-half repeaters. The local sides of these repeaters connect to the concentrator multiples.

Duplex circuits which permit simultaneous transmission in both directions on one line wire are terminated in terminal duplex sets. From the local sides of these sets, a sending and receiving leg are each connected to a concentrator multiple. These two legs appear on adjacent studs in the turrets, the sending leg terminating on the left hand stud. In Fig. 4 will be seen adjacent studs having identical designations, for example the first two studs are designated *A B* and *A B*, respectively. In this particular case, this represents a duplex circuit.

During that part of the day when the exchange of business is heavy enough to keep both channels of a duplex circuit busy, the concentrator supervisor temporarily assigns two adjacent operating positions to this circuit. When the traffic falls to the point where it can be handled with one channel, certain switches on the terminal duplex set are operated so as to change the set for half duplex working, that is, transmission can be in only one direction at a time. In this condition, a concentrator operator can both send and receive on the left hand stud (the sending leg) in the same manner as she does on any single-channel circuit. When a duplex circuit is arranged for half duplex working, it is allowed to "float" in the concentrator, that is, it is not definitely assigned to any particular operating position.

Sometimes an outer office has such a heavy message load as to require the equivalent of a duplexed circuit but the economics of the situation dictate that a second single-channel circuit be supplied this office rather than install duplexing equipment with its attendant maintenance charges. Therefore, two adjacent studs having identical designations may also represent two separate circuits to an outer office; in fact, more than two circuits are sometimes required to public branch offices having a very heavy message load. Generally, however, such cases are taken care of with pneumatic tubes if the distance is not too great.

At night, the traffic decreases to such an extent that it is not sufficient to keep one channel continuously busy on some of the trunk circuits which are multiplexed during the day. It is then advantageous to convert such circuits to start-stop printer operation during these dull periods and to terminate the circuits in concentrators, utilizing concentrator multiples which connect to offices that are closed at night. This permits centralizing the night staff which results in operating economies due to employing the operators to better advantage.

It is the general practise to segregate the various types of circuits terminating in a concentrator. Thus the upper two rows in Fig. 4 contain public circuits and the lower eight rows, private tie lines; the circuits in each case being arranged in alphabetical order. Where more than one concentrator is installed, the private tie lines and public circuits are terminated in separate units. Segregating the private tie lines permits assigning operators for working these circuits who are best qualified and especially trained for dealing directly with the public.

Discussion

C. M. Brentlinger: The concentrator which Mr. Blanton has described was developed because the traffic department required a means of giving very fast service without incurring prohibitive operating expense. By fast service we mean the answering of incoming calls on private tie line or public branch office circuits within a very few seconds—as quick or quicker than a telephone is ordinarily answered. The unit as developed has enabled us to give this fast service at approximately the same operating expense as with the older type of equipment.

There are two general reasons why we do not put all circuits in a given office in one large unit, instead of splitting them into several 100- or 200-wire units, one reason having to do with equipment and the other with operation. The equipment on each operating table occupies so much space that it would be difficult to place a turret of the standard telephone exchange type before each operator; also the cabling and other expense involved in running several hundred wires in multiple to a large number of tables would be much higher than it is when the units are smaller and the lines limited to 100 or 200.

Another item where additional expense might be incurred in using one large instead of two smaller units, is in the provision of equipment to care for growth. Assume a 50-position 100-wire unit, which has become inadequate due to an increase in the number of circuits. We could add a few operating tables to this unit, put an additional turret on each of the existing operating tables, and thus accommodate the additional circuits by enlarging the unit; or, we could install a second unit of 50-wire capacity with say 10 tables. The latter would be the cheaper procedure on account of the excessive wiring expense involved in adding a turret to each of the 50 original operating tables, and extending the multipled circuits to them.

Aside from the equipment and installation expense, a large unit would be unwieldy to operate. For example, the handling of outgoing messages would be more difficult in a large unit. With our present layout, the circuit for which a given message is destined is either idle or is being worked in one of say 70 tables, and by means of the group-position finder is rather easily located. Multiply the number of tables and circuits by 3, 4, 5, or even 10, and the problem is not so easy, the group-position finding equipment would be very expensive, and the time required to carry the message from the message routing rack to the operating table would be increased. The use of one large unit in an office would require the distribution of all simplex traffic from one center, instead of the present convenient arrangement with a distributing point for each 100 or 200 circuits.

There is also an advantage in the smaller units with respect to incoming calls, for if the unit were very large, it would be impossible to establish the priority of calls and answer them in order. Obviously answering calls in sequence during peaks is absolutely necessary if the maximum answering time is to be kept down to a very few seconds. Of course equipment could be provided to establish the sequence of incoming calls, but this would be expensive.

There is no particular advantage to a large unit. A 70-position unit with 100 heavily loaded circuits requires a maximum of 4 supervisors and 70 operators. This is a sufficiently large force to permit efficient assignment, and no gain would result from having the whole simplex staff in one large group.

Another phase of the application of concentration units is the resulting saving of operating tables and floor space, as compared to separate tables or eight-wire telegraph type concentrators. In the new central office at 60 Hudson Street a striking comparison can be drawn between the old and new types of equipment. The new office requires only 608 operating tables for terminating 1,600 circuits. If old type equipment had been used nearly 1,000 operating tables would have been required for the same number of lines. These additional tables would have required 9,000 sq. ft. more floor space. The savings in operating tables and floor space can probably be better appreciated if we consider that they are the equivalent of the whole simplex departments in the Dallas, Jacksonville, Indianapolis, Milwaukee and Houston offices.

H. Mason: Apart from the mechanical and electrical details of this development, many interesting points in its application to telegraph service could be discussed. Fifty-nine installations have already been made in twenty-seven offices embracing perhaps 6,700 lines and 2,200 operating positions, some 25 per cent being allowed for growth. As yet the equipment is so young and the possible variations in practise are so numerous that we have not settled down to what we may safely consider the best operating routine for this equipment. As intimated in the paper, there may well be a difference in the proper technique for answering a customer's tie line and for answering a branch office wire where the distant operator is an employee of our own. It is not unusual to find operators who are highly successful on the branch wires but whose method with the customer leaves something to be desired. On the other hand, other operators get along excellently on the private tie lines but are less efficient when operating on the busier branch wires.

Evidently it is necessary for us to give an exceedingly prompt answer on customers' tie lines. On branch wires, when economy so dictates, we can impose a slower answer. In practise we average about a three-second answer on the former and a six-second answer on the latter. If, then, we establish two standards of speed of answer with lighter staffing on the slower group we must, on that account as well, segregate our tie lines and our branch wires into separate units.

The distribution of business outgoing from a concentrator presents a problem. This is a manual function with no automatic guide except an indication if the desired line is already being operated. Shall we endeavor to place outgoing business with the lowest numbered idle operator or shall we take it to the higher numbered positions where an idle operator may more likely be found? If but one operator is idle shall we give her an outgoing message, thus leaving no one available to quickly answer an incoming call? As yet our practises vary considerably. On a concentrator of tie lines it is sometimes required that two, or in extreme cases, three operators be left idle before outgoing business is distributed. On branch wires this is rarely done.

Another problem remaining to be solved is in connection with branch office operation where the branch is big enough to warrant two wires during the busy hours. When the branch opens up the second wire by calling on it the answering operator will probably be quite remote from the one working the first wire although the pins for the two wires, as stated in Mr. Blanton's paper, are adjacent to each other in the multiple. With this condition it might well be that on the second wire deferred business was being transmitted while on the first full rate business was awaiting its turn. At present nothing but close attention by an ambulatory supervisor can take care of this situation.

When twenty or thirty operators are working on one concentrator one or another operator is becoming free and idle with relative frequency. If, then, the "answer next" lamp appears before a high-numbered operator and an incoming call is received and said operator stalls along for but a few seconds before answering it may well be that the answer next light will jump to a lower-numbered position. We are considering changing the wiring so that when an answer next lamp once lights it will stay lighted until that operator does something.

In the New York office an interesting comparison may be made. Besides a considerable number of these concentrators there is also installed the modern equipment for the recording of telegrams telephoned in by patrons. Adopted from telephone company developments an automatic position finder is used so that an incoming telephone call is placed upon an idle operator. In this equipment the search for an idle operator does not always begin at the bottom but, in a manner of speaking, begins where it left off seeking on the last call. For this reason all assigned operators are likely to have been equally busy by the end of the day. This differs from the simplex concentrator scheme where evidently the low numbered operators are much more frequently occupied than those of higher numbers. We had thought that this would help in operator assignment by finding, when over-assigned, that the last operator would sit continuously idle hence clearly indicating the desirability of releasing her. This does not seem to work out in practise, probably in part due to our holding of several operators idle for the readiness to serve feature and also due to our method of distributing outgoing business. We are at present unable to say which method of automatic operator selection is preferable. They are both working very well.

That feature of the sub-distribution center which indicates if a desired line is already being worked so that the outgoing message can be taken to the operator thereon is of considerable benefit in concentrators handling branch office lines. Depending, of course, upon the load density of the wires, perhaps 85 per cent of the business carried from the distributing racks to the positions may have been subject to this feature. In concentrators full of tie lines, however, wires are in use such a small fraction of the day that the probability of an incoming message finding the line already being worked is quite small and in such case the distribution feature, while of course not a detriment, presents little or no advantage.

In actual practise these equipments have resulted in some extremely desirable savings in floor space. They have postponed major moves that the growth of simplex operation otherwise would have necessitated. They have not as yet resulted in payroll economies, partly due to their newness and our unfamiliarity with just the best procedures; also because they were installed during a period of declining loads and it is concededly difficult to cut operating costs in linear ratio to reduced files; but chiefly to the fact that we have elected to materially improve the speed of answer coincident with their installation. I have in mind a certain city, more or less typical, where the cost per handling is approximately the same now as it was before but where, prior to the installation of the concentrator, the average speed of service from branch to main was 4.1 minutes and six months later, after the installation, it was 2.7 minutes.

Most of us believe that along with its other advantages, labor savings as well will result when loads increase and when operating practises through trial and experience have crystallized into the most desirable forms.

Modern Practises in Private Wire Telegraph Service

BY R. E. PIERCE*

Member, A. I. E. E.

***Synopsis.**—Private wire telegraph service forms an important part of the communications service of the United States. This pape outlines briefly the practises which are followed by the Bell System in furnishing a million and a half miles of such service. Some of the practises discussed are: Central office arrangements, line transmission systems, and station apparatus. The paper closes with a description of a few typical applications for press associations, brokers, police and airways.*

TELEGRAPHY has the distinction of being the earliest application of electricity to the service of mankind and it still ranks as an important factor in our commercial, educational, and social activities. This paper is devoted to a statement of the present practises of the telephone companies comprising the Bell System in the furnishing of more than a million and a half miles of private wire telegraph service to press associations, brokers, financial houses, public service companies, and other organizations and individuals.

The term "private wire telegraph service" as used in this paper covers that type of service by which a complete telegraph communication system is furnished to the patron for his exclusive use. This type of service is sometimes also referred to as special contract telegraph service. The service is complete in that all the facilities including the station apparatus at the patron's premises are furnished and maintained. The circuit may be arranged for either the manual or machine method of operation to suit the requirements of the organization using the service. Throughout the paper the machine method will be referred to as the "printer method" since this is the term most universally used. Other terms which are used by various organizations to describe the same method are "typing telegraph," "telephone typewriter," "teletype," and "teleprinter."

It might be well to consider briefly some of the unique features which have influenced the telegraph engineering in connection with this private wire service and which have brought about the development of the particular methods and means as described in this paper. The commercial telegraph message business is essentially a point-to-point service and the telegraph facilities are designed and operated for the most part so as to furnish direct communication channels between two points. The private wire telegraph service, on the other hand, usually requires circuits with sending and receiving equipment at more than two points and often at many points and because of this fact the engineering problems involved are somewhat different than in the commercial telegraph field.

Another unique feature of the private wire telegraph service is that it is furnished very largely over plant which is used jointly for telephone and telegraph purposes. For many years the use of the telephone plant for telegraph was accomplished by the superposition of two direct current telegraph channels on a telephone pair below the voice range. There came a time, however, when this method did not provide a sufficient number of telegraph facilities to handle the rapidly growing private wire service over the major routes. It was at this time that the high-frequency carrier telegraph system[1] was developed to provide telegraph trunk channels above the voice range on open-wire lines. By this method it has been possible to obtain 10 two-way telegraph circuits on a pair of wires in the frequency range from 3,000 to 10,000 cycles.

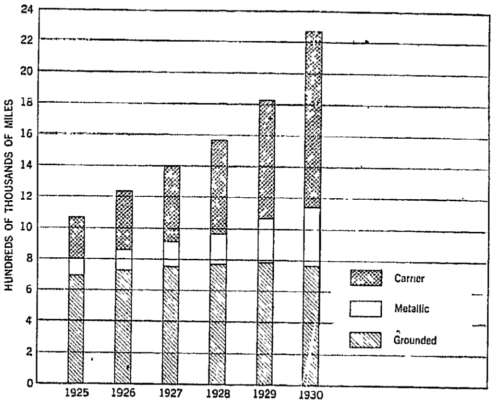

FIG. 1—MILES OF TELEGRAPH CIRCUIT IN PLANT

When it was decided to put the toll telephone plant in congested areas into cables, it was necessary to develop telegraph systems which would also be satisfactory in these long toll cables. To meet this need the metallic system and the voice-frequency system[2] were developed. The latter system provides for 12 two-way telegraph circuits on a standard four-wire telephone circuit but since it operates in the voice range it precludes the use of the circuit for telephone at the same time.

The chart shown in Fig. 1 illustrates mileage of the various types of telegraph facilities in the plant for several years past. The mileage given covers those

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[1] For references see Bibliography.

circuits in private wire service, the spare circuits maintained for protection of the plant in service and the circuits used by the telephone companies for their official business. The interesting point in connection with this chart is that a very large percentage of the growth has been accomplished with the carrier telegraph system. It will be noted that there are now more than a million miles of carrier telegraph trunk circuits in the plant. The development of the private wire telegraph service on the main routes is illustrated by the fact that on September 30, 1930 New York and Chicago were connected by more than 300 private wire telegraph circuits over the telephone wire plant. Of these 300 circuits, 120 were direct New York-Chicago voice-frequency carrier channels without intermediate drops and the growth of this direct business is such that another system of twelve channels is being added between these points about every six months. If these 300 telegraph circuits were each operated over a separate wire on a pole line it can be seen that more than seven complete 40-wire pole lines would be required to handle this service.

CENTRAL OFFICE TELEGRAPH ARRANGEMENTS

In the private wire telegraph services where all of the stations are within a small area, wires are assigned to the telegraph service from the telephone plant without any superposition. Since these networks are relatively simple from an engineering standpoint the longer services are chiefly considered here.

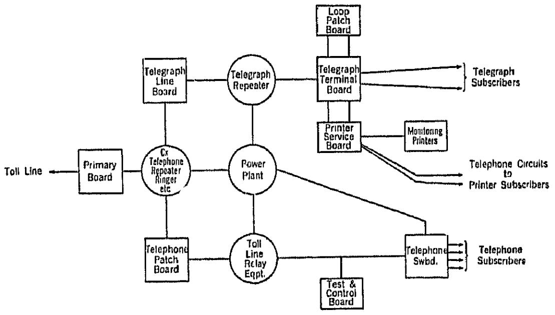


FIG. 2—CENTRAL OFFICE EQUIPMENT FOR TELEPHONE AND TELEGRAPH

The joint use of the outside plant for telephone and telegraph purposes requires that provision be made in the central offices, where the toll lines terminate, for testing and repeating equipment for the telegraph circuits as well as for the telephone circuits. The items of equipment which are generally involved in such a termination are shown in Fig. 2.

In addition to the items of equipment directly associated with the telegraph line circuit, there is, of course, a number of auxiliary facilities required in giving the private wire service. There is a complete network of test wires connecting all of the central offices where telegraph equipment is installed for use by the service maintenance people in routine tests and in clearing special trouble. There is also an intra-testroom communication system required in large offices so that men in different parts of the office may intercommunicate.

The printer service position, which is indicated in Fig. 2, is a comparatively recent development and provides a centralized location for terminating the printer subscribers' loops, the monitoring printers and the special telephone circuits for communication between the subscribers and the service people.

If the private wire subscriber having printer service wishes to communicate with the attendants who are in charge of the maintenance of his service, he has merely to call on the telephone provided for the purpose and usually mounted adjacent to the printer installation. Calling over this instrument connects him directly with the attendant at the printer service position in the nearest telegraph test room. If some difficulty on the circuit is reported, the telegraph attendant can immediately communicate with distant repeater attendants over test wires provided for the purpose, and can make measurements over the private wire network in question with the monitoring printers, transmission measuring apparatus and other testing means at his disposal. If the trouble cannot be located and remedied at once, spare facilities are patched into the network and service is resumed.

New Arrangements of Central Office Equipment. With the rapidly increasing number of telegraph circuits entering the repeater offices, it has been found desirable to completely redesign the central office equipment from the standpoint of mounting arrangements. At the present time, practically all of the central office telegraph apparatus being installed for use on the systems which have been mentioned above, is designed to mount on relay rack bays which are completely assembled and wired at the factory. A typical layout of such apparatus is indicated in Fig. 3 which shows a rack upon which are mounted four channel terminals of a voice frequency telegraph system together with a common monitoring set. This monitoring set can be plugged into any one of the four channel terminals or into the channel terminals on adjacent bays for supervision, testing or line-up purposes. This feature of a common monitoring set for several telegraph repeaters is a recent development which is now in quite general use on all of the telegraph systems being installed for giving the private wire telegraph service.

Methods and Means for Supervising, Testing, and Lining Up. In providing private wire telegraph service, it is necessary that the plant forces be equipped with efficient testing and measuring devices for determining the condition of the telegraph facilities at all times and for making the necessary adjustments and changes in cases of trouble. The extensive introduction of printer methods of operation has required the application of many new monitoring and testing facilities. Monitoring printers are provided in all of the principal telegraph offices with special switchboard arrangements for connecting these printers to the private wire telegraph facilities. In the Chicago office of the long lines department there are more than 60 of these monitoring printers which are used exclusively in the maintenance and supervision of the private wire telegraph service.

It will not be possible to go into detail concerning all of the interesting practises which are being followed in giving this private wire service, but a few of the more important items will be mentioned, first in connection with the line transmission systems and second in connection with the methods and means of sending and receiving.

LINE TRANSMISSION SYSTEMS

The telegraph circuits required for a private wire service may be divided into three general classes, which are illustrated in Fig. 4. First, the trunks or backbone circuits between telegraph repeater offices; second, side "legs" which connect the main telegraph offices with outlying offices, and third, subscribers' loops which run from the telephone offices to the station equipment on the subscribers' premises. One requirement which must be kept in mind in considering the facilities used in private wire networks is that all of the different systems used must be so engineered as to allow interconnection between them. It may be necessary to build up any one network out of a number of different systems. The references mentioned previously give descriptions of carrier and metallic systems and the methods of arranging the circuits of the terminal apparatus to allow this universal interconnection. These arrangements are typical of those used in all of the telegraph terminal apparatus.

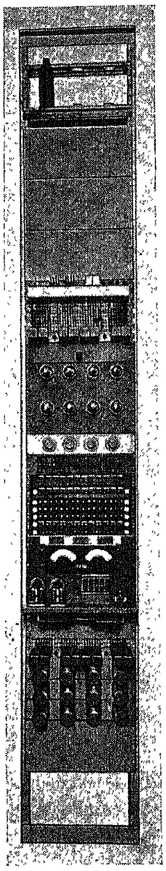


FIG. 3—VOICE-FREQUENCY TELEGRAPH SYSTEM

Four channel terminals with common monitoring set

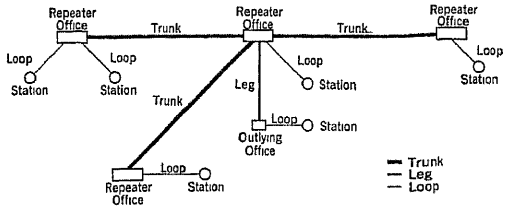


FIG. 4—TYPICAL LAYOUT OF A PRIVATE WIRE TELEGRAPH CIRCUIT

Carrier Telegraph Systems. Reference has been made to the rapid growth of carrier as shown in Fig. 1 and in this connection the following approximate data as of September 30, 1930, may be of interest.

Total mileage of carrier telegraph channels	962,000 mi.
Channel mileage of carrier on open wire lines	522,200 mi.
Channel mileage of voice frequency carrier in cables	439,800 mi.
The longest high frequency system is between Denver and Los Angeles	1,528 mi.
The shortest high frequency system is between Corpus Christi and Harlingen, Texas	134 mi.
The longest voice frequency system in cables is between New York and Chicago	1,001 mi.
The shortest voice frequency system in cables is between Cleveland and Pittsburgh	143 mi.

The New York office of the Long Lines Department has one ten-channel high-frequency system and 50 twelve-channel voice-frequency systems. These systems reach the following points.

One high-frequency system to Buffalo.

Voice-frequency systems as follows:

Albany	2	Cleveland	3	Rochester	1
Boston	6	Detroit	2	Syracuse	1
Buffalo	5	Indianapolis	1	Toledo	1
Charlotte	1	Pittsburgh	8	Toronto	1
Chicago	10	Portland, Me	1	Washington	5
Cincinnati	1	Richmond	1		

In addition to the carrier development mentioned above, there have been certain cases where it has been desirable to operate the voice frequency carrier telegraph systems on channels of carrier telephone systems.[3] The carrier telephone system is designed to operate on a pair of open wires and provides three two-way telephone circuits in the frequency range from 5,000 to 30,000 cycles. Fig. 5 illustrates the different items of equipment involved in the operation of the voice frequency telegraph systems on these channels. The hybrid coils which are usually used for terminating two one-way carrier telephone channels in single two-way telephone circuits have been removed and standard twelve-channel voice-frequency telegraph terminals have been connected.

One feature which has led to the use of carrier telephone channels for telegraph purposes is the possibility of operating voice-frequency telegraph for part of the distance over cables and part of the distance over open wires without telegraph equipment of any kind, intermediate between the two-voice frequency telegraph terminals. A good example of such an application was recently made by the Pacific Telephone and Telegraph Company. A toll telephone cable which was under construction between San Francisco and Los Angeles was completed from San Francisco to San Jose. The ultimate plan called for the operation of a large number of telegraph circuits direct from San Francisco to Los Angeles through this cable by means of the voice-frequency telegraph system. It was very desirable, however, to obtain additional telegraph facilities over this route in advance of the completion of the cable. The plan which was adopted to obtain these telegraph facilities was to install three standard voice-frequency telegraph system terminals in San Francisco and Los Angeles. These three systems were then put in service on the three telephone circuits of a standard carrier telephone system on open wire between Los Angeles and San Jose. Thirty-six telegraph circuits were then obtained from a single pair of wires in addition to the 1½ telephone circuits of the ordinary type. At San Jose these three carrier telephone channels were connected to three standard four-wire telephone cable circuits which terminated in San Francisco in the three-voice frequency telegraph terminals. This method of operation was continued until the cable was completed between Los Angeles and San Francisco when the same voice-frequency systems were put into service entirely on cable conductors. It is expected that this general method of superposing telegraph in the frequency range above the voice will have wide application in the future.

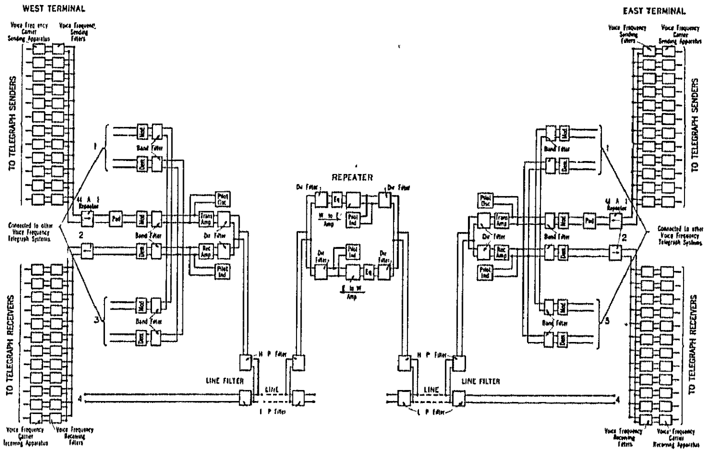


Fig. 5—Voice-Frequency Telegraph Systems Superposed on a Type C Carrier Telephone System

Two-Path Direct Current Systems. It has been the common practise in providing telegraph operation in both directions over a direct current system to use the well-known balance method. In many cases, it is found that a satisfactory balance can be maintained so that the speed at which full duplex operation can be carried on is only slightly less than the of speed transmission in one direction at a time. When the telegraph circuit is operated over line facilities which are liable to change in their electrical characteristics, it is found that frequent adjustment of the balancing networks is required if the circuit is to be operated full duplex at anything near its maximum efficiency, and in some cases considerable attention is required to maintain satisfactory transmission in one direction at a time. This instability requires attention from skilled attendants and at times there are delays or interruptions on a circuit. To overcome these difficulties, certain use is now being made of what is termed the "two-path method of operation." With this system, a separate telegraph channel is used for operation in each direction, these two channels being connected together and to the subscribers' loops at the terminals by appropriate apparatus. This, of course, is the application of a principle which is employed in long telephone circuits and in all of the carrier telegraph systems. With this arrangement, there is no balancing required, the full duplex speed is the same as the one-way speed and no skilled attendants are required on the circuit. Fig. 6 illustrates a terminal set arranged to operate on this principle. This unit is designed with two complete terminal sets on a relay rack mounting with complete power plant and is intended for installation in small outlying telephone offices where no skilled telegraph attendant is available. The only maintenance or lining-up required with this apparatus is to regulate the line current and adjust the relays, which can be done by the regular telephone central office personnel.

Regenerative Repeaters. When a private wire telegraph service involves a very complicated network it sometimes becomes desirable to divide the circuit from a transmission standpoint so that the signal distortion occurring in one part of the circuit will not be cumulative throughout the entire circuit. This result is accomplished in the case of printer service by introducing regenerative repeaters at regular repeater points where the circuit is to be divided. These repeaters receive signals from either direction on a distributor mechanism and then transmit them from another distributor. In this way the repeater will transmit perfect signals into one part of the network provided the signals it receives from the other part of the network are not distorted beyond the point where they could be recorded perfectly on a receiving printer.

Start-Stop Printer System. The multiplex and the start-stop[4] printer systems in common use differ in one important feature. In the multiplex system the signals are transmitted to the line and from the line to the receiving apparatus by distributing mechanisms which are operating continuously in approximate synchronism. In the start-stop system, the signals are transmitted to the line and from the line to the receiving apparatus by mechanisms which start from a given position, transmit one character and stop. In this latter arrangement, the maintenance of synchronism between the sending and receiving devices is necessary during the transmission of only one letter or character. The multiplex system is essentially a point-to-point system whereas the start-stop system is adapted to circuits involving a large number of stations, any one of which may send or receive. The advantage of the continuously synchronized system of transmission is the possibility of using a shorter code. It is only necessary to transmit the intelligence-bearing signals while in the start-stop system a signal is sent to start and another to stop the distant distributor mechanisms.

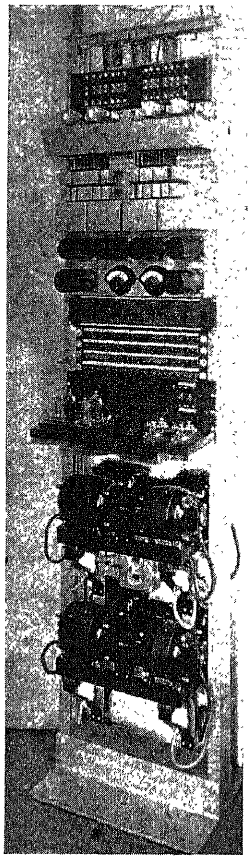


Fig. 6—Two-Path, D-C. Telegraph System
Two terminal sets with power plants

The fundamental advantage of start-stop over continuously synchronous systems is the fact that in the start-stop system the lag in the transmission of a signal from one point to another point in a circuit is automatically taken into account. With the continuously synchronized system and only two stations on a given circuit, an initial adjustment can be made to properly orient the receiving device with respect to the sending device which will hold as long as the lag remains fixed. With more than two points on a circuit any one of which may send to all the others or with transmission systems which are apt to be changed in a way that affects the lag, the adjustment to compensate for lag becomes difficult if not impracticable. This feature would considerably restrict the field of use for continuous synchronism in private wire telegraph service where such a large portion of the circuits require service between more than two points and where it is essential to maintain a flexible plant in order to maintain continuous service. This explanation will indicate why the multiplex system which is designed as a continuously synchronized system is used in the commercial telegraph message service while the start-stop system is used in the private wire telegraph service.

Station Apparatus

While there is quite a large mileage of telegraph circuits operated by expert telegraph operators with the key and sounder method, the newer applications of private wire telegraph service are, in most cases, being made with machine or printing telegraph methods of operation. The data in Fig. 7 indicate the change in the percentage of the private wire service mileage of the Bell System which is operated by the manual and the printer methods.

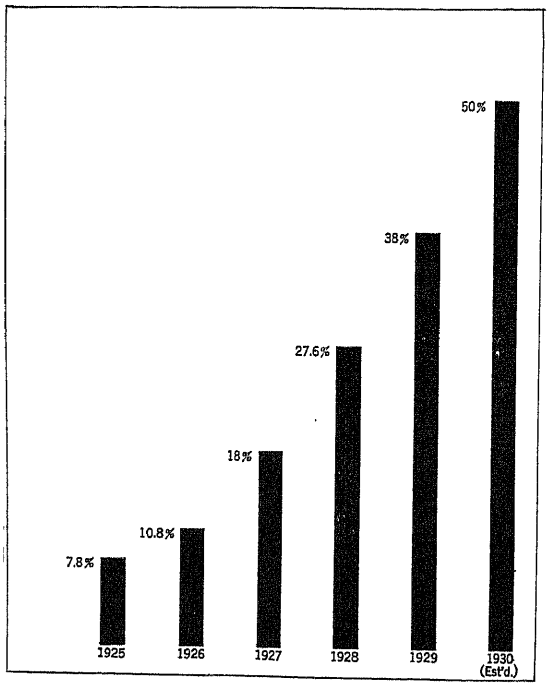


Fig. 7—Trend Toward Machine Operation in Private Wire Telegraph Service Bell System

In the private wire service which is furnished with printers, the machines for sending and receiving are set to operate at a speed which best suits the requirements of the subscriber. At the present time two standard speeds are provided. These are known as 40 speed and

60 speed which represent traffic handling capacities of 40 words per min. and 60 words per min., respectively. At the present time approximately 20 per cent of the printer private wire service is furnished at 40 speed and 80 per cent at 60 speed.

Recent Developments in Printers. In general two types of printers are used in telegraph service. One of these prints on a narrow tape while the other prints in page form. The commercial telegraph companies are almost universally using the tape printer. In private wire telegraph service, the brokers have adopted the tape printer, while the press and the commercial customers other than the brokers, in general, use the page printers. The comparative advantages of the tape and the page printers are quite generally understood, and need not be discussed in this paper.

The page printer which is shown in Fig. 8 has recently been developed to meet the exacting requirements of the press and commercial private wire subscribers. This machine is equipped with a stationary paper platen and a moving type basket. This feature allows the copy to be easily read while the machine is in operation and also reduces the possibility of trouble with the paper feeding mechanism.

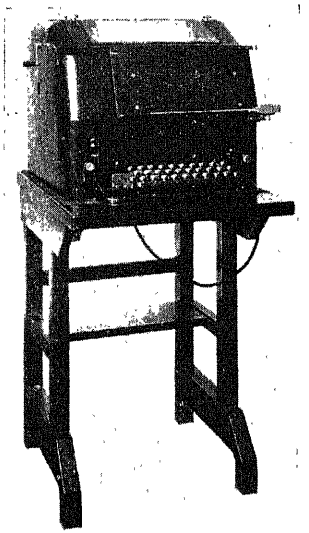
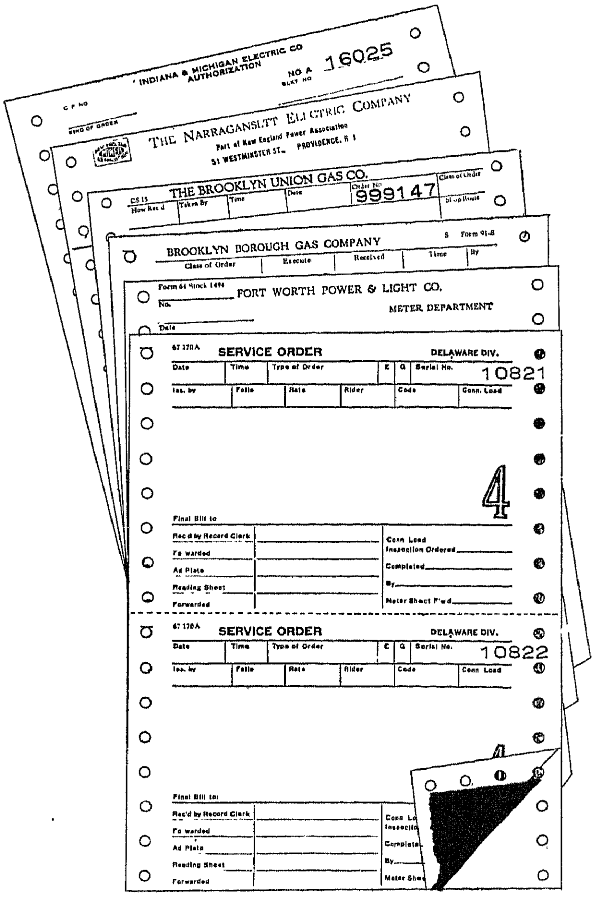


Fig. 8—Page Printer

Modern business methods call for the use of many printed forms in the routine correspondence of commercial concerns and to meet this requirement arrangements have been designed for handling organized printed forms in this new printing telegraph machine. Absolute registration is assured between different copies when carbons are used and between all the copies on different machines on the same circuit by feeding the paper with pins on the platen roller which engage perforated holes in the printed forms. Fig. 9 illustrates samples of forms which are used by different commercial houses in this type of machine.

This new page printer is also equipped with arrangements for starting and stopping the printer motors from a distance without any additional wire facilities. When this feature is used all printer motors on a circuit are normally idle. The motors on all machines start when a spacing signal is received over the line and stop when a particular combination of impulses is received from the sending keyboard.

Another practise which is expected to eliminate one of the most difficult problems in maintaining reliable printer service is the recent application of synchronous motors for driving the printers. It is of course understood that all machines on a circuit must operate at approximately the same speed so that the distributor mechanisms will remain in synchronism through one letter cycle. In the past where a-c. motors were used, they were usually of the ordinary series type with a contact governor. While these motors run at very constant speeds, the governor requires careful adjustment and occasional maintenance. Since the power systems of the country are becoming very closely regulated as to frequency, it has been found practicable to use synchronous motors of the self-starting type as the source of power in the printers. With this arrangement the speed regulation is in general better than with the carefully adjusted governor on a series motor and there is no adjusting or cleaning required to maintain accurate



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THE NARRAGANSETT ELECTRIC COMPANY
Part of New England Power Association
51 WESTMINSTER ST. PROVIDENCE, R I

THE BROOKLYN UNION GAS CO.
CS 15
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BROOKLYN BOROUGH GAS COMPANY
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Form 64 Stock 1494
No.
FORT WORTH POWER & LIGHT CO.
METER DEPARTMENT
Date

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Fig. 9—Samples of Forms for Use with Printer Equipped with Positive Paper Feeding Attachment

speed. It might also be mentioned that, in certain cases where the printers are operated in close proximity to high-gain radio receivers, the prevention of interference in the radio set from the printer operation has given an added incentive to the application of the synchronous motor.

In a great many printer installations, it is necessary to provide a local source of direct current power. In the past, (where only alternating current power was available) this has usually been accomplished by the use of a motor-generator but the present tendency is toward the use of rectifiers for this purpose. It is expected that the rectifier will be cheaper in first cost and will require less maintenance.

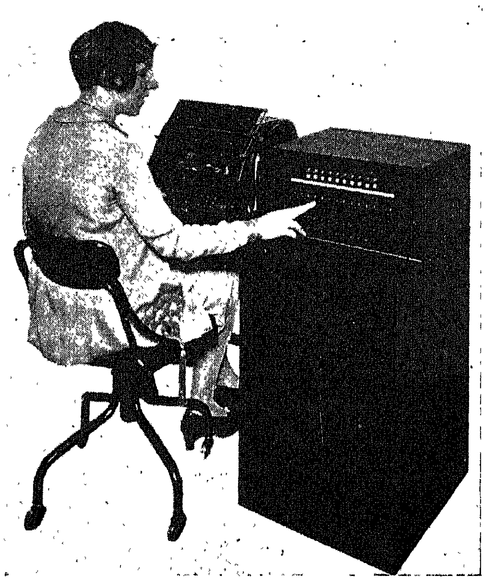

Fig. 10—A Ten-Line Switchboard with Tape Printer

Telegraph Switching Systems. Most of the private wire telegraph service is given on a fixed layout basis, that is, a given network is set up and operated by the subscriber for a given period each day. There have been certain types of service, however, where it was desirable to be able to interconnect stations on a network at the will of the subscriber. A typical application of this kind is found in the service for the Pennsylvania State Police. In this case, switchboards are provided which allow certain stations to be interconnected at the will of the operators at the switchboards and when a state broadcast is desired the operator at general headquarters may "seize" control of all the stations in the state so that more than one hundred police stations are connected to one sending machine and can be notified simultaneously.

Fig. 10 shows a typical private wire switchboard. This switchboard provides for ten lines which are connected to the operator's printer or together through a repeater by the operation of keys. Lights appear on the board to indicate a line is calling and the operator may monitor on a connection or disconnect the monitoring printer as desired.

For services where it is desirable to select particular stations and it is impracticable to provide a switchboard with individual circuits to each station, a selective calling arrangement has been developed. With this plan all stations are connected to the same circuit and selecting signals are transmitted from one station to another over the circuit used for passing the telegraph signals. The apparatus used to accomplish this consists of telephone dials, step-by-step switches and relays. Any point on the circuit may be called by dialing any other point on the circuit. A station may also dial by a single three-digit code all of the other stations or a group of stations. Busy signals appear at all stations when the circuit is in use. An audible or visible alarm is provided which will call the attention of the attendant to an incoming message. The motors of all printers are stopped when the station is idle but are automatically started when the station is called and automatically stopped when the calling station releases the circuit. Secrecy is obtained in this system because it is not possible for a station which has not been called to "listen in" when the circuit is in use by other stations.

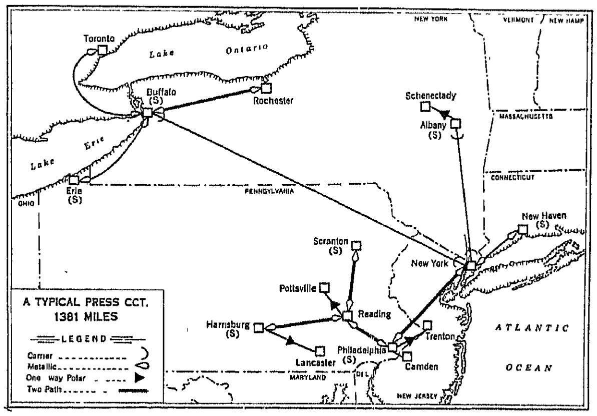

Fig. 11

TYPICAL APPLICATION OF PRIVATE WIRE TELEGRAPH SERVICE

Press. The press associations are engaged in the collection and the distribution of news. This service must be accomplished quickly, accurately and without interruption, and the private wire telegraph service is the means used for accomplishing this result. A typical telegraph network—one of the many circuits of one of the leading press associations in the United States—is illustrated in Fig. 11. It will be noted that this network contains approximately 1,400 miles of telegraph circuits and connects sixteen telegraph stations, five of which are arranged for both sending and receiving while eleven are for receiving only. The various types of telegraph systems involved in this particular service are indicated by the designations on the chart.

In a service of this kind most of the sending would be done from New York; it will be noted, however, that there are other points indicated on the network as sending stations. The circuit must be arranged with a "break" feature so that when desired any one of these other sending stations may take control of the entire network and transmit to all stations. To accomplish this the operator at the station transmitting the message operates the break key on his machine which transmits a break signal over the circuit locking up the machine which has been sending thereby indicating to the operator at that point that another station wishes to send.

The printers involved in the press work are, in most cases, of the page type using a continuous roll of paper. The sending machines are usually equipped for what is known as automatic transmission. The sending operator manipulates a keyboard which controls a perforator. This perforator prepares the message to be transmitted in the form of a perforated tape which is fed into a transmitter connected to the line. This transmitter is operated at a speed for which all the printers on the line are set. The perforator may be operated at any speed desired, and if the speed of the line is exceeded, as is usually the case, the message is stored up in the perforated tape and is transmitted to the line at a regular rate. With this method of transmission there is a continual flow of traffic over the line and any interruption in the operation of the keyboard does not interrupt the signals going to the line. This arrangement obtains the maximum efficiency from the line.

In certain cases a "bulletin" or "flash" of special importance must be transmitted. Arrangements are provided so that by throwing a key the transmission of signals from the perforated tape is stopped and the keyboard is connected directly to the line so that as each key is depressed the proper signals are transmitted. In this way the bulletin or flash may be sent directly to the line from the keyboard without any delay, and when it is complete the regular flow of traffic may be resumed by restoring the key previously mentioned. In order to attract the attention of the operators at the newspaper offices to such a flash or bulletin a bell is provided on the printers, which is caused to ring by the transmission of a particular combination of impulses from the sending keyboard.

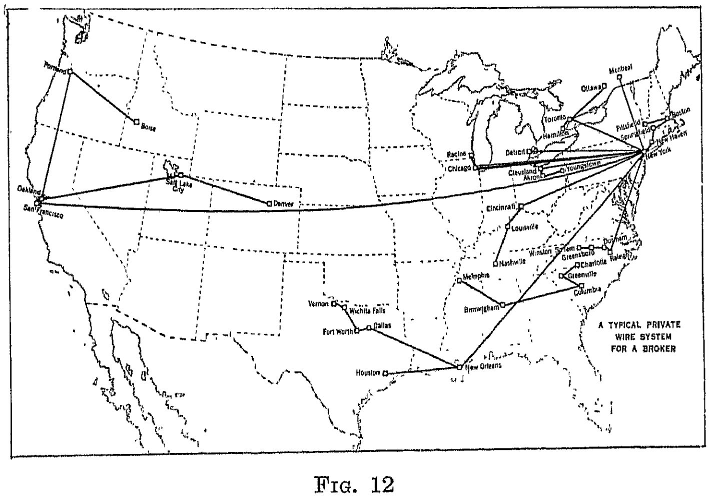

FIG. 12

Brokers' Service. A large percentage of the business involved in the trading on the stock and commodity exchanges is handled over the private wire systems of the various brokerage concerns. Fig. 12 shows a typical broker's service which comprises several separate circuits. This service is operated by the printer method.

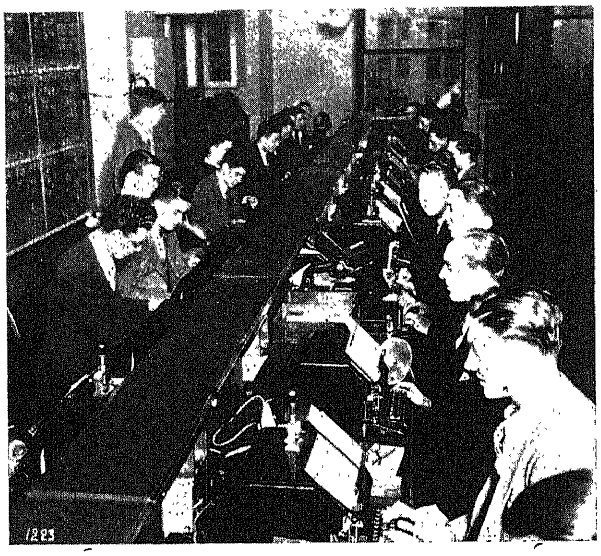

FIG. 13—WIRE ROOM OF A BROKER

Fig. 13 is a view in one of the wire rooms of this brokerage house. The wire network is used in the brokerage houses for passing information regarding orders which are placed before the market opens and for distributing certain market letters or market gossip. When the market opens the wires are kept busy with the passing of orders, confirmations and questions and answers concerning specific securities. It will be noted from the illustration that the tape printer is used and the cover of the machine is cut away in such a fashion as to allow the tape to be removed from the machine immediately after the last character of an order is printed. The orders are gummed to "buy" or "sell" order blanks and are then handled in the usual way. After the market closes in the afternoon the wires, in cleaning up the day's business, continue to handle a large volume of traffic. The records are checked and verified and all matters cleared before the opening of business on the following day. It is reported that during four days of the record month of October, 1929, one of the brokers using this service handled the remarkable total of 49,860 orders, confirmations and general messages over the firm's six private wire telegraph circuits.

Police Systems. The various police organizations throughout the country are finding the private wire telegraph service a reliable means of quickly disseminating police information. Fig. 14 illustrates the extent to which this service is being used in the cities and states along the Atlantic Seaboard. These police systems are essentially broadcasting networks for transmitting police information simultaneously to a large number of receiving stations from one or more central points. The information usually originates in a telephone call or a personal visit to a police station from which the information can be quickly transmitted to the central headquarters and from there sent out on the broadcasting network. The receiving machines make a record of the message regardless of whether or not the station is attended. In certain cases the receiving machines are equipped with an audible alarm so that a police officer at a distance may be advised that a message is being received. In certain cases it is desirable to provide arrangements whereby the headquarters from which the broadcast is transmitted will know whether the message is received at each point to which it was transmitted. In these cases an acknowledgment signal is arranged so that after each message is transmitted the ringing of a bell on the receiving printer indicates that the transmitting station requests an acknowledgment which can be sent back by the pushing of a button on the receiving machine. This acknowledgment lights a lamp on the switchboard at headquarters indicating that the message has been received.

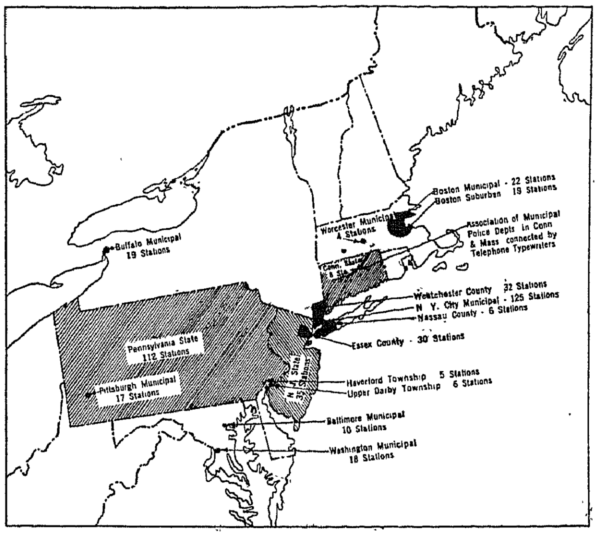
Boston Municipal - 22 Stations
Boston Suburban 19 Stations
Worcester Municipal 4 Stations
Association of Municipal Police Depts in Conn & Mass connected by Telephone Typewriters
Buffalo Municipal 19 Stations
Conn. State 8 Sta.
Westchester County 32 Stations
N. Y. City Municipal - 125 Stations
Nassau County - 6 Stations
Essex County - 30 Stations
Pennsylvania State 112 Stations
N. J. State 35 Stations
Pittsburgh Municipal 17 Stations
Haverford Township 5 Stations
Upper Darby Township 6 Stations
Baltimore Municipal 10 Stations
Washington Municipal 18 Stations

Fig. 14—Police Telephone Typewriter Systems Eastern States

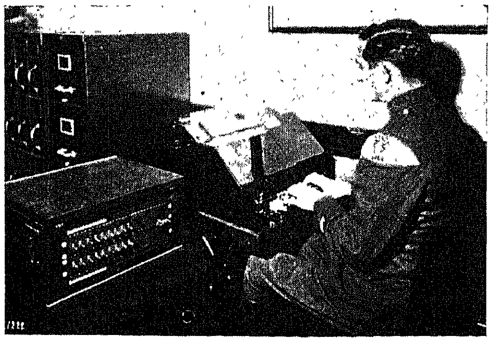

Fig. 15—A Police Switchboard and Sending Printer

Other police networks are set up with all stations connected to the same circuit so that any one may send and all of the others receive. Such a circuit connects seventeen of the principal municipalities in Connecticut. In this way police information originating in any of these towns can be broadcast simultaneously to all of the other towns on the network.

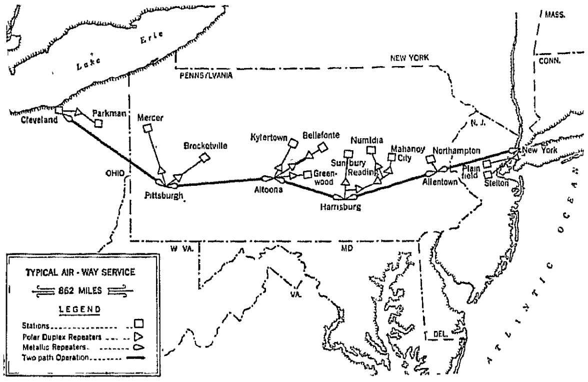

Fig. 16

Fig. 15 shows one of the switchboards and sending machine in a state police system.

Weather Reporting Systems for Airways. The Department of Commerce has contracted for quite a large network of private wire telegraph circuits for use in collecting and distributing weather information for use by aeroplane pilots. Weather information is collected on an hourly basis in sequence from the principal intermediate landing fields and critical points along an air route. It takes from four to five minutes to make such a weather collection over a typical air route between major landing fields. The weather information thus obtained is furnished to the air pilots before they leave the field or to pilots in the air by means of the radio telephone equipment of the Department of Commerce. Along the air route the printers are usually installed

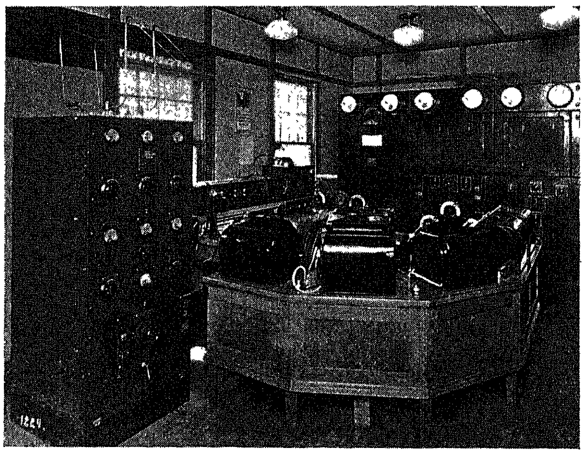

Fig. 17—An Air-Way Communication Office

adjacent to the field in what is known as a watch-house. In addition to the service mentioned in certain cases local private wire circuits are provided for the purpose of distributing weather information between the Weather Bureau, radio stations, airports, and air transport companies' offices.

In addition to the collection of weather information the private wire circuit is used in some cases for the reporting of the departure, arrival, and passing of planes. Fig. 16 shows a typical airway network and Fig. 17 gives a view in one of the Department of Commerce airway communication offices. It will be noted that the printers are of the tape type. The stations along the air route are usually connected in series so that each station receives the information sent from all other stations. In certain cases the bell signal on the printer is used to call the attention of a particular station with which it is desired to communicate.

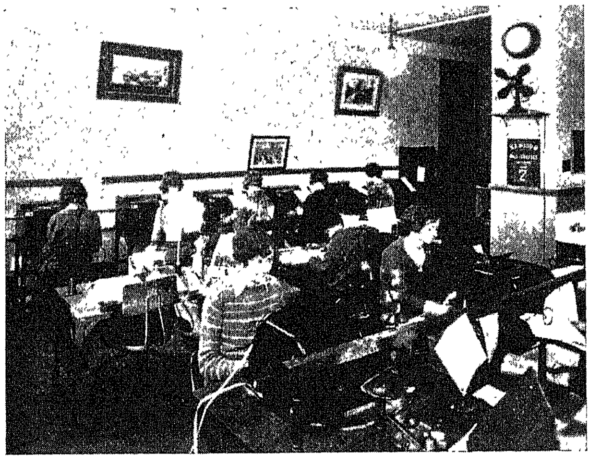

FIG. 18—AN INDUSTRIAL CONCERN COMMUNICATIONS OFFICE

Industrial Service. Factories are generally located with regard to the availability of raw materials and the ease of transportation. Executive and sales offices, on the other hand, are usually near the distributor, the exporter or the consumer. Private wire telegraph service is used extensively to provide direct communication channels between the widely separated units of modern industrial organizations. Fig. 18 shows a number of printers in the Chicago offices of an industrial company which are used for communication with several plants near Chicago and to units as far away as Indianapolis and Cincinnati. With the aid of this service the far-flung units of this organization are kept in constant touch with each other, thereby speeding up production and improving the service rendered to their customers.

In looking forward into the future, there seems to be every indication that private wire telegraph service will continue to grow and expand, and with the requirements continually changing it is to be expected that many new and interesting developments will be applied to this service.

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Discussion

J. O. Carr: It is apparent from the paper that the goal in this development is flexibility of the system and simplicity of the station apparatus.

One realizes the accomplishments better by comparing them with the earlier Morse system which was notably simple. A wire, a few batteries, a key, and relay and sounder at each station constituted the entire plant.

Mr. Pierce shows quite clearly the progress that has been made. In the Bell system various kinds of transmission channels have been developed which have far greater operating margins than the older types and suitable repeater apparatus has been devised so that all types of transmission channels may be interconnected quickly and interchangeably to produce any sort of a private wire system, completely flexible either with Morse or printer operation.

As Mr. Pierce has pointed out, when the use of printing telegraph passed beyond the stage of communication between two stations new problems arose. He has spoken of the matter of the method of synchronism. There were other problems which required deep study to solve. In a private wire system it may be necessary to permit transmission from a dozen stations and it is necessary that when the transmission shifts from one station to another that the signals be received correctly by all stations without the need for any change of adjustments. To meet this requirement it was necessary to greatly refine the transmitting apparatus to secure a high degree of uniformity in the character of the signals from each transmitter. The receiving mechanism also had to be refined in order that it would continue to print correctly through as great a range of variation in the received signals as possible.

That considerable progress has been made in improving the operating margins of both the transmission channels and apparatus is indicated by the statement that 80 per cent of the private wire service is at the 60-word per minute speed.

It is interesting to note from this paper, the use of the two-path d-c. method of operation. It was formerly considered necessary to utilize the telegraph channel to the fullest possible extent. Apparently it is proving economical to be somewhat wasteful of transmission channels if by so doing the continuity of the service can be increased and the need for expert attendance reduced.

After all the growth of the service is dependent upon the degree of satisfaction the subscriber secures from its use. The data on the growth of the service given in the paper indicate that it is giving satisfaction to the subscriber.

H. H. Nance: The discussion in Mr. Pierce's paper of the operation of voice-frequency carrier telegraph systems on channels of carrier telephone systems is of special interest. The Long Lines Department of the American Telephone and Telegraph Company has been operating a system between Chicago and San Francisco in this manner since September, 1929, and since that time other similar systems to the Pacific Coast have been established between Chicago and Seattle and between Denver and San Francisco. The Chicago-Seattle and Chicago-San Francisco systems are each over 2,300 miles in length and are the longest carrier telegraph systems we have in operation.

This method of operation is particularly valuable in times of line trouble because of the flexibility in rerouting the telegraph systems that is possible as a result of the extensive application of carrier telephone systems. In a recent storm in the middle west two of the Long Lines main transcontinental routes were severely affected. In order to maintain complete telegraph service to the Pacific Coast two Chicago-Los Angeles and two Chicago-San Francisco voice-frequency telegraph systems were temporarily set up using cable circuits between Chicago and St. Louis, carrier telephone systems St. Louis to San Bernardino, Cal., which systems are routed over the Long Lines southern transcontinental line through Texas, New Mexico, and Arizona and was not affected by the storm, and cable circuits from San Bernardino to Los Angeles and San Francisco. By this procedure complete service was maintained and in addition telegraph circuits were furnished to other wire using companies also affected by the storm.

Symposium on Coordination of Power and Telephone Plant*

Part I—Trends in Telephone and Power Practise as Affecting Coordination

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Member, A. I. E. E.

and

A. E. SILVER[2]
Fellow, A. I. E. E.

Synopsis.—*The general trends in telephone and electric power systems are outlined and the reactions of certain of these trends on coordination are described.*

In the telephone system, brief mention is made of the rapid growth of the dial system of operation, improvements in subscriber-station apparatus, rapid extension of new types of facilities for toll circuits and the growth of connections to foreign countries. Improvements in telephone service increase the importance of securing adequate coordination. The advantages of the use of cable facilities for toll circuits, of repeaters, and of carrier current systems as regards coordination of long distance and interurban telephone circuits are discussed. The benefits accruing from improved subscriber-station apparatus, central office equipment, abandonment of iron wire for the short tributary toll circuits and new methods of making sleeves at joints in open wire lines are outlined.

In the power system, brief mention is made of increasing use of larger generating units, and growing use of automatic devices to replace manual operation. Improvements in power service generally react favorably on coordination. The general trends toward higher voltages for transmission and distribution and the improved standards of construction accompanying these trends are described. The important matter of system stability and the practises as regards grounding of transmission circuit neutrals, lightning control and current limiting devices, and the reactions of these matters on coordination are outlined. Reference is also made to grounding of distribution system neutrals, service taps on transmission lines, general practises as regards transformer connections and improvements in wave shape in so far as these matters react on coordination.

In conclusion, it is pointed out that, while there have been influences working both favorably and unfavorably toward coordination, the preponderant trend is definitely toward an improvement. The benefits which have accrued from the activities of the Joint General Committee and the important function of the Joint Subcommittee on Development and Research are also mentioned.

GENERAL TRENDS

THE important benefits resulting from the cooperative handling of questions arising from the proximity of the physical plants of the telephone system and the electric power systems of the United States are emphasized when consideration is given to the extent and the rapid growth of these two industries. This growth is illustrated by Fig. 1 which shows that during the past decade, while the population of the country has increased 16 per cent annual telephone messages have increased 96 per cent and annual kilowatt hour usage of power 107 per cent. Another indication of the growth of these utilities is given by Fig. 2 which shows that during the past decade customers telephone stations have increased 88 per cent and customers of central stations 127 per cent. The leaders of both utilities confidently expect that, apart from temporary setbacks associated with recessions in general business, the recent rapid growth of these utilities will continue throughout the next decade.

Such a rapid growth of the two utilities both of which must supply the same customers with services essential to their comfort and prosperity, necessarily brings with it a large number of cases of physical proximity between the plants of the two utilities where, due to the widely different characteristics of the circuits involved, difficulties may arise. The necessity for active study of the coordination of the different systems and for the current handling of large numbers of individual situations will continue for a long time to come.

Associated with this rapid growth there has been another trend in these two utilities which has an important effect on coordination work. This trend is the steady improvement in the quality of service afforded to their customers.

In the telephone system the improvement in the standards of service, if considered by itself, tends to increase the noticeability and the reaction on service of inductive effects from outside sources. Such changes as the improvement in the characteristics of transmitted speech, including the extension of the band of frequencies efficiently transmitted, and the avoidance of cases in which interfering noises are produced from sources within the telephone plant, tend to increase the effect of moderate amounts of noise current induced in the telephone circuits from outside sources. Similarly increases in the extent of the service and in the speed of completing calls have led to increased reliance on prompt telephone communication which tends to increase the importance of avoiding interruptions. Five years ago the average interval of time between the placing of a long-distance toll call by a subscriber and the commencing of the conversation was 7½ minutes. At the present time it is a little less than 2½ minutes. Telephone users have now come to rely on the almost immediate establishment of telephone connections and are correspondingly more critical of interruptions or delays.

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*Joint work of the National Electric Light Association and Bell Telephone System.

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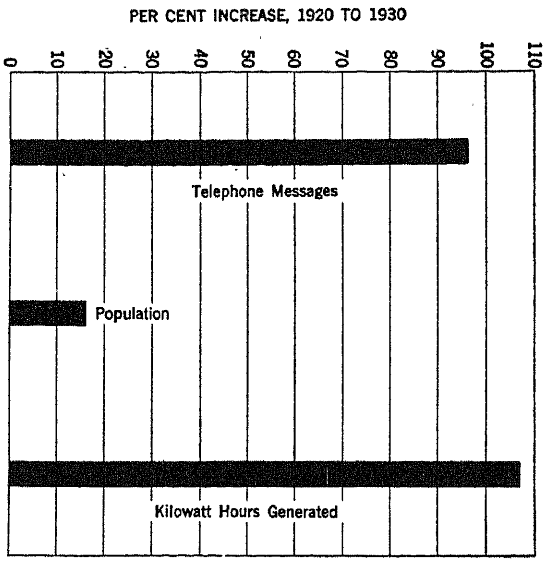
Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

The improvement of service has been associated with a particularly rapid growth of very long haul telephone business and a consequent increase in the average length of telephone circuits used for interurban and long distance work. This is illustrated by Fig. 3 which shows the growth in the last few years and the expected growth for the next few years of typical circuit groups of different lengths. In the period 1925 to 1929 while telephone toll business as a whole increased 59 per cent New York-Chicago business increased 170 per cent and the combined Chicago and New York business to Los Angeles and San Francisco 380 per cent. From the standpoint of coordination with other electric circuits the very long telephone circuit offers a more difficult problem than the circuit of moderate length because of the cumulative effect of exposures in different sections.



1930 Population	123,900,000
Telephone messages	21,600,000,000
Kw. hours generated	90 000,000,000

Fig. 1—Per Cent Increase, 1920 to 1930, in Population and in Telephone and Power Usage

Note: Values for 1930 are estimates based on best available data. Telephone data refer to Bell System

In the power industry one of the most important items in the improvement of service has been the steady decrease in the number of service interruptions. This has been brought about mainly by better standards of construction, including more systematic mechanical and electrical arrangement of circuits and apparatus, and increased numbers of circuits and sources of supply. The interconnection of power systems has figured largely in the last mentioned factor contributing to service reliability, by making available greater numbers of sources and by multiplying the routes over which power can be received at specific locations. While the increasing numbers of interconnecting and other types of lines bring new conditions for the coordination of power and telephone plants, improved construction and increased security of circuits and apparatus have a definitely beneficial effect upon matters of coordination by reducing the number of abnormal conditions of operation.

Other items in the improvement of the service given by the power industry are better voltage regulation and a great increase in the number of types of power consuming appliances and apparatus made available for the customer. Accompanying better voltage regulation are certain factors which definitely aid coordination, among these being better balance of currents in the separate phases of the circuits and more effective arrangements minimizing the tendency for currents to flow in the earth. The effect of increased numbers of types of utilization apparatus on coordination is problematical, though probably not of sufficient magnitude to be of practical importance.

Other trends which have a bearing on the improvement of power service are discussed in the section of this paper devoted to the power system.

While in some respects the general trends indicated above, namely, the extent and rapid growth of the two utilities, and the improvement of service standards, have by themselves tended to increase the importance and the difficulties of coordination work, these adverse tendencies have been offset by beneficial effects of improvements in plant design and construction and by the cooperative endeavor which has been carried on by the two utilities during recent years. It is a tribute to the effectiveness of this cooperative work that the degree of satisfactory coordination between the two systems is steadily improving. Fig. 4 shows that during the past 10 years the mileage of telephone toll circuits has increased 250 per cent and the mileage of power transmission lines over 100 per cent. The effect of such growth on the number of situations of proximity is illustrated by the fact that during the past three years the exposures of interest from a noise standpoint have increased from the equivalent of about 10 miles to about 14½ miles per 100 miles of open-wire telephone toll lead; while on the other hand the exposures not as yet adequately coordinated have in the same period de-

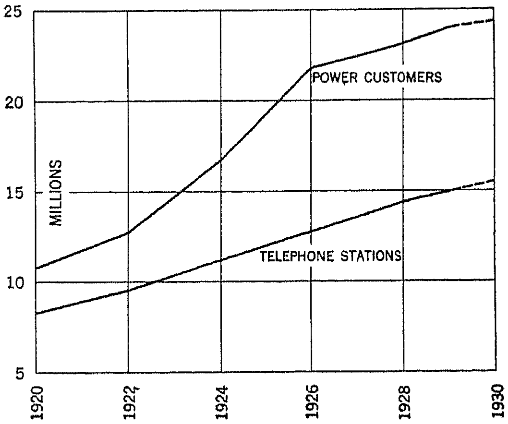


Fig. 2—Telephone Station and Power Customer Growth

Note: Values for 1930 are estimates based on best available data. Telephone data refer to Bell System creased from the equivalent of 2.6 miles to 1.5 miles per 100.

While the trends of practise in the design, construction and maintenance of the plants have necessarily been largely controlled by the fundamental requirements of service and economy in developing the two systems, and while the trends naturally have not all been in the same direction as regards their effect on the coordination problem, still the general trend of plant practise at the present time is in the direction to facilitate the coordination of the plants of the two utilities. In the following pages brief statements are made descriptive of the more important of these trends in the respective systems.

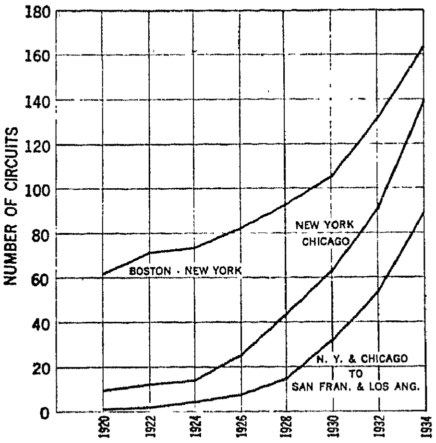


FIG. 3—LONG HAUL TELEPHONE CIRCUIT GROWTH TYPICAL CIRCUIT GROUPS

TRENDS IN TELEPHONE SYSTEM

The telephone plant is at the present time rapidly changing in its physical character through the application of important developments and changes in engineering and construction practise.

Probably the most fundamental and far reaching of these changes is the progress of conversion from manual to dial system operation. When present plans are completed this will result in the operation of approximately 80 per cent of the telephones of the Bell System on a dial basis, and a large part of the existing manual central office equipment will have been removed from service. With the application of the dial system there is a trend toward a greater concentration of central office equipment in one building, so that in the future as many as 100,000 telephones may be switched by the various central office units in a single building. While these trends are of the greatest and most fundamental importance from the standpoint of the development of the telephone business they do not affect the coordination problem in any material way and therefore need not be further discussed here.

An important trend in telephone practise has been the provision of apparatus designed for higher standards of service and greater convenience for use at the customers' station. This includes the hand set, new types of private branch exchanges and of auxiliary telephone station apparatus, and improvements of transmission characteristics. These changes in some respects affect the coordination problem and these effects are indicated below.

Another important fundamental change in the telephone plant and one of great importance from the coordination standpoint is the rapid extension of new types of facilities for toll circuits, that is, long distance and interurban circuits whose use involves what is called a toll charge. These changes and their effects on the coordination problem are discussed in this paper.

One of the most spectacular trends of development of the Bell System at the present time is the increase in the number of connections to foreign countries. Earlier connections to Canada and Cuba were supplemented in 1927 by service to Mexico and by transoceanic radio links providing service from New York to London through which connection is made to the principal European countries and in 1930 a similar radio link from New York to Buenos Aires through which connection is made to Montevideo, Uruguay, and Santiago, Chile. During the next few years it is expected that these foreign connections will increase to include generally all important points in South America, Australia, Japan, Honolulu and all other points which may offer an appreciable demand for service.

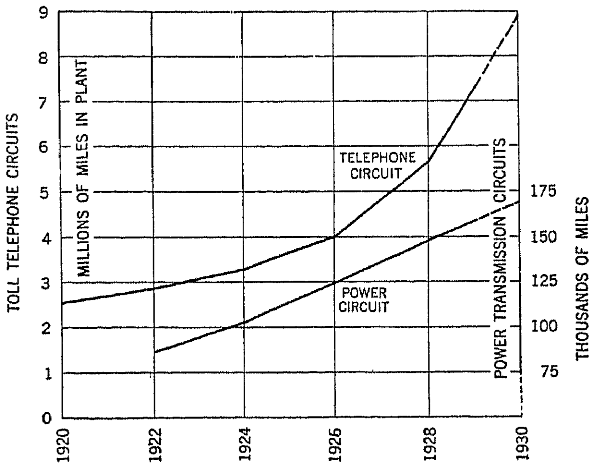


FIG. 4—TOLL TELEPHONE AND POWER TRANSMISSION CIRCUIT GROWTH

Note: Values for 1930 are estimates based on best available data. Telephone data refer to Bell System

These intercontinental circuits are not of such character and location as to be directly affected by the physical proximity of power circuits, but their efficiency is affected by the noise currents on connected circuits in the same way as other very long circuits are affected and this is discussed briefly below.

Toll Cable. The change in methods of designing and constructing toll circuits which is of greatest importance from the standpoint of general development of telephone plant is the great increase in use of cables for those circuits, including both the very long distance circuits and the shorter interurban circuits. This increase is shown by Fig. 5. A single cable may provide for from 250 to 500 telephone circuits and several hundred telegraph circuits, that is, as many circuits as would be provided by five to ten heavily loaded pole lines of aerial wire construction. This concentration of circuits in a single cable, a number of which can be placed on a single route, is in itself of great assistance in coordination problems by greatly reducing the number of routes for which coordination arrangement must be made. Furthermore the presence of the lead sheath together with the twisting of the cable conductors, the high degree of balance with respect to ground, and the mutual shielding effect of the many circuits in one cable practically prevent noise currents from being induced directly into the cable circuits from outside electrical sources. The shielding effect of the lead sheath when suitably grounded also provides substantial reductions in the voltages of fundamental frequency which may be induced along the cable conductors at times of trouble on neighboring power systems.

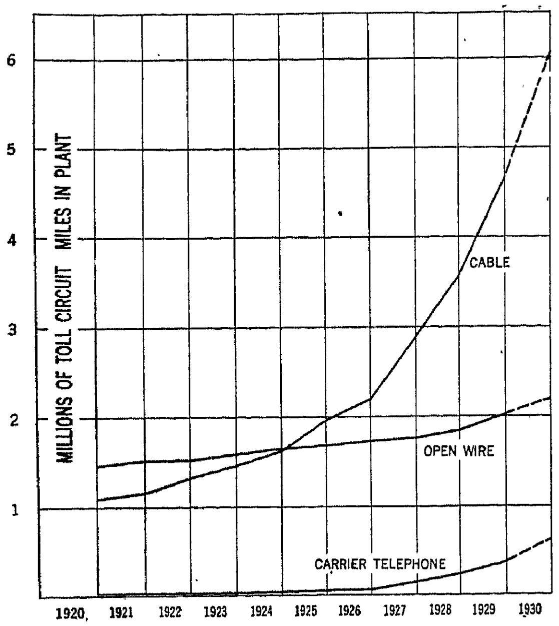


FIG. 5—TOLL TELEPHONE CIRCUIT GROWTH BY CLASSIFICATIONS

A telephone toll cable with its associated equipment costs about the same per mile as a twin circuit power transmission line of the 110-kv. class. This high cost has led to a large use of private right-of-way for new extensions of these cables, particularly for aerial cable construction. This, of course, has an added advantage from the coordination standpoint in tending to keep these important telephone routes off the highway, which are so much used for the distribution systems of both utilities. In the more rapidly growing cable routes underground conduit construction is employed and these in most cases are located along the highways. In these cases, however, the close proximity of several cables in the same conduit run offers a considerable amount of mutual shielding effect which reduces the susceptiveness of circuits in these cables to values approaching that obtainable by a single tape armored cable.

This tape armored cable, which recently has been placed in use in this country, is designed for burying directly in the ground, and has an increased degree of magnetic shielding. This is provided by two wrappings of steel tape outside the lead sheath which are necessary for the mechanical protection of the cable when ducts are not used. During the past year about 160 miles of this cable were installed and it is expected to have a considerable field of use in the future.

As indicated above, in all these types of cable construction the susceptiveness to noise induction is so greatly reduced that low frequency induction generally becomes the limiting factor relative to the permissible proximity of these cables to power circuits. The relative amounts of induced voltages with these different types of construction in comparison with open wire construction, while naturally varying with local conditions, are indicated in a general way in Table I.

TABLE I

Type of construction	Approximate relative volts on telephone circuits per ampere of inducing current at 60 cycles
Open wire	1.0
Single cable, aerial or underground—sheath well grounded	0.5
Buried tape armored cable—well grounded	0.2

Note: All values for cables assumed full size, *i. e.*, 2⅝-in. diameter.

The above figures are based on favorable conditions for obtaining low resistance ground connections on the cable sheaths. Such ground connections are necessary to provide the full shielding benefits, since the shielding is brought about by induced currents on the cable sheath flowing along the sheath and through ground. These sheath currents, because of the close coupling between the sheath and pairs, induce voltages into the pairs tending to neutralize the voltages induced into the pairs directly from the power system. The use of the tape armor, which is a magnetic material, increases the coupling between the sheath and pairs. The grounding conditions necessary for satisfactory shielding effects can usually be obtained, but situations sometimes arise in the case of aerial construction where it is difficult or impossible to obtain them.

While as noted above, the cable circuits are effectively protected from noise induction, the efficiency obtainable over the long circuits is limited in part by the noise currents occurring in the open-wire lines which may be switched to the long cable circuits. This is because the efficiency of the long cable circuits depends upon voice-operated switching devices which must not be operated by the noise currents. This is also true of the intercontinental circuits mentioned above. The extension of the circuits controlled by voice-operated devices tends therefore to increase the importance of good coordination of the entire plant.

Telephone Repeaters. Another important trend of practise is the extended use of telephone repeaters. The purpose of these devices is to amplify the voice currents and thus make possible higher efficiency and greater extension of long distance telephone circuits. Their use is essential to the great development of toll cable. Moreover, they are used widely on open-wire circuits. Without repeaters it was necessary on the long open-wire circuits to permit the power level of voice currents to sink to relatively low values. An extreme example of this is given by the New York-Denver circuit which, before repeaters were available for use on this circuit, had an over-all equivalent, using the highest grade of telephone construction which had been developed up to that time, of about 31 *db.*[3] With the application of repeaters to this circuit the level of voice currents could be kept relatively high throughout the circuit. This is illustrated in Fig. 6 giving level diagrams for the circuit as originally set up and later when provided with repeaters.

The use of repeaters contributes to reducing the susceptiveness of the telephone plant and thus aids coordination. On such a circuit as the original New York-Denver circuit just mentioned, a relatively small amount of noise current greatly impaired transmission because of the weak incoming voice currents. Although the repeaters naturally amplify the noise currents as well as the voice currents, the fact that the voice level is kept high throughout, results in great benefit which in this case, assuming similar exposure conditions in the various repeater sections, gives an improvement in the ratio of voice currents to noise currents of slightly over five.

Repeaters probably also have some effect in reducing certain of the effects of low frequency induction by the fact that they sectionalize cable lines at about 50-mile intervals and open-wire lines at intervals of 200 miles or less, and limit the power which can be transmitted from section to section. There is some evidence that this tends to limit acoustic shocks.

Carrier Telephone Systems. A third important trend in telephone practise is the extension in the use of carrier telephone systems for long circuits and the associated changes in aerial wire construction practises. The growth in use of this type of circuit is indicated in Fig. 5. The carrier systems are much less influenced by noise induction from power circuits because they occupy a range of frequencies (5,000 to 30,000 cycles) in which the harmonic power voltages or currents ordinarily are extremely small. Furthermore, in order to obtain economies inherent in the use of large numbers of carrier systems on the same telephone pole line it has been necessary to design systems of transpositions of much increased effectiveness and even to change the configuration of the wires in order to greatly reduce the inductive effects between the telephone circuits. These changes also result in reduced susceptiveness to outside inductive influences. The type of construction now recommended for new aerial wire lines in cases where the extensive use of carrier is anticipated is shown in Fig. 7. The two wires of each pair, except pole pairs, are spaced 8 in. apart compared with the previous standard of 12 in. Often transpositions are made as frequently as every second pole and are of an improved type giving better balance between circuits; also on the circuits on which carrier telephone is used the phantoms are abandoned. The relative susceptiveness to noise frequency induction of the various types of aerial wire construction has been tested for various typical conditions. The results of these investigations are summarized in Table II.

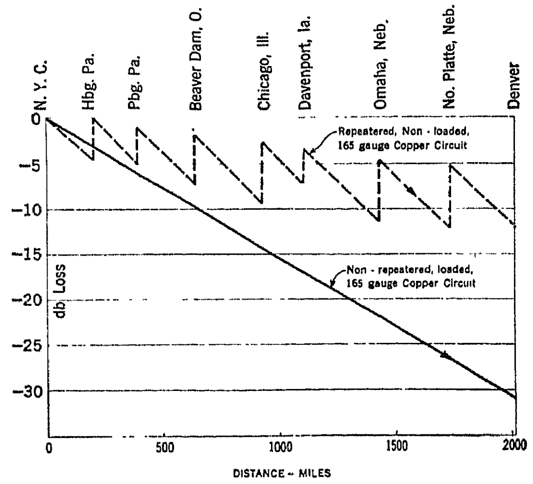


Fig. 6—New York—Denver Circuit Level Diagrams

TABLE II

Facility	Type of transposition*	Approximate relative susceptiveness†
12 in. phantom	Voice (brackets)	1.00
12 in. side	Voice (brackets)	0.50
8 in. pair	Carrier (break irons)	0.25 or less

*Voice circuits are not so frequently transposed as carrier circuits. Bracket type transpositions require two spans to complete the transposing whereas the break iron type completes the transposing on a single crossarm.

†Susceptiveness is used in the sense defined by the Joint General Committee, namely, "Those characteristics of a signal circuit with its associated apparatus which determine, so far as such characteristics can determine, the extent to which it is capable of being adversely affected in giving service, by a given inductive field."

Subscribers' Station Apparatus. To a large extent the trend of development in subscribers' station apparatus is toward new arrangements which provide greater convenience and more closely meet the needs of the users and which have no material effect upon the coordination problem. An important group of developments, however, centers about the improvement of the electrical performance of the station apparatus by removing impairments caused by the earlier types of apparatus. These changes, by improving the quality of speech as reproduced by the telephone system, tend to make more noticeable the impairments caused by the effects of currents induced from external sources.

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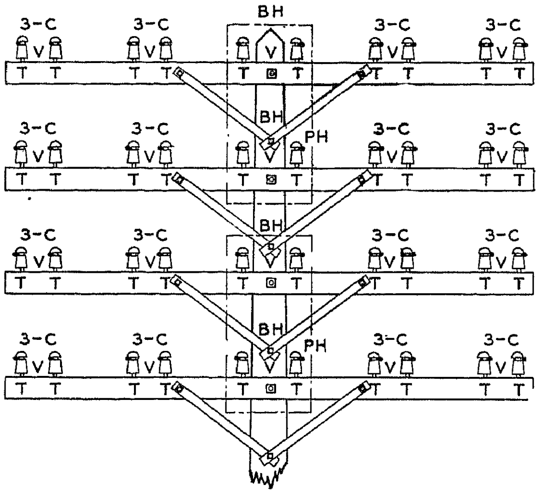
[3] This means that the ratio of output power to input power of this circuit is 0.0008.

The tendency toward an increase in the range of voice frequencies efficiently reproduced by the telephone system tends to increase the range of frequencies of induced currents which may cause noise interference as discussed in the introductory section. An extreme illustration of this is the circuits designed to transmit programs for radio broadcasting stations. The transmission characteristics of these circuits have been improved by including both higher and lower frequencies and in their most modern form these circuits efficiently transmit currents of frequencies in the range between 35 cycles and 8,000 cycles and are therefore capable of being affected by inductive noises over this wide range.



LEGEND

SYMBOL	FACILITY	TOTAL CIRCUITS	
V	VOICE FREQUENCY-PHYSICAL	20	
PH	VOICE FREQUENCY-PHANTOM	2	
3-C	CARRIER TELEPHONE	48	
T	D-C TELEGRAPH	40	
BH	CARRIER TELEGRAPH (10 CHANNELS)	40	
	TOTAL TELEPHONE		70
	TOTAL TELEGRAPH		80

FIG. 7—POLE LINE CONFIGURATION

Non-phantomed construction—8-inch spacing between wires of non-pole pairs

The room noise conditions at the subscribers' premises have an effect on telephone transmission. This noise besides acting directly on the ears of the telephone user is converted by the transmitter into electrical currents, a part of which actuates the receiver, thus producing noise. The present trend in telephone practise is very strongly toward a reduction of these effects. This will tend to bring into increasing prominence noise caused by induction in the telephone circuits which now in many cases is partially overshadowed by the reproduction of the noises in the room.

As partly offsetting this tendency steps have been taken to improve the degree of balance to ground of new station apparatus, particularly in the case of party lines. The new station apparatus with the improved transmission characteristics discussed above will be designed for reduced effect of noise currents entering from the line. Also, in extending the selective signaling features to rural areas, higher impedance ringers and a newly developed high impedance relay are being used in order to limit susceptiveness to noise from exposures between the rural open-wire extensions and rural power distribution circuits. Where central office equipment is being modified to permit of increased range of direct current signaling, or for some other reason, the reduction of susceptiveness is always a consideration. All of the newer repeating coils used for supplying talking battery to subscribers in common battery areas, which comprise the bulk of the local plant, possess a much higher degree of balance than the coils which were standard a few years ago.

Other Items. So far the changes which are associated directly with the major trends of development in the telephone plant have been described. The broad outlines of these developments depend on all of the factors affecting telephone service as well as coordination with power circuits. There are other features not directly associated with these main trends which, while introduced into the telephone plant largely because of the advantages to be gained in reducing susceptiveness to electrical influences, have also afforded other benefits. A few of the more interesting examples of these changes are given below.

Referring to the toll plant, there may be mentioned the recently adopted general practise of soldering aerial wire sleeve connections in order to insure a permanently high degree of series balance. Heretofore reliance had been placed on the contact between the wires and the twisted sleeve. The practise of soldering will be supplemented in the near future by a cold-rolled sleeve method, and it is confidently expected that these practises will result in material noise improvements. They will also probably reduce the maintenance required on open-wire toll circuits, particularly where exposures are involved.

Another item is the abandonment of the use of iron wire and substitution of copper for short tributary toll circuits. Coordination of the iron wire circuits is relatively difficult because of the development of resistance unbalances at the wire joints. The transmission efficiency is also improved by the reduced resistance afforded by the copper but this effect is generally of secondary importance in the short tributary circuits.

In toll offices improvements have been made in the balance of coils and condensers used for superposing telegraph on the telephone circuits. The use of repeating coils, commonly used for side-circuits, has been extended to phantom toll circuits. These coils act as insulating transformers to prevent noise voltages from the outside conductors being impressed upon the intricate cabling and equipment of the office.

Referring to the local plant, there are several noteworthy examples of modifications made principally for the purpose of reducing susceptiveness. Investigation such as that of the coordination between power and telephone distribution plants conducted at Minneapolis by the Joint General Committee, stimulated the development of means for reducing the susceptiveness of the telephone distribution plant. Present practises call for the interconnection of aerial and underground cable sheaths and the grounding of the aerial sheath in order that the benefits of the shielding action of the sheath currents as previously described, may be realized for noise induction. In cases where electrolysis conditions do not permit direct grounding, condensers of the electrolytic type are employed to prevent the flow of direct currents.

The telephone circuits have long been equipped with over-voltage protectors for the purpose of protecting apparatus and cables against lightning waves and against power frequency transients from the lower voltage distribution circuits, also with fuses for opening the lines in cases in which heavy currents flow. The trend in development of these devices has been principally toward more uniform operation and lower maintenance costs. With the rapid increase of voltage and capacity of power circuits generally, experimental studies have been undertaken of further means for maintaining the safety of persons working on or listening on the telephone circuits. At the present time, development work is being done on various devices for this purpose, some of which are fundamentally different in design and operation from those previously used. It is hoped that these devices, which are discussed in one of the following papers, will afford increased protection against overvoltages and improve coordination conditions.

TRENDS IN POWER SYSTEM

In the field of power generation marked attention has been paid, from the start, to methods of improving the efficiency of the generating process and reducing the investment per kilowatt of generating capacity. This has led to the development of larger and larger generating units. A single shaft unit of 160,000 kw. capacity and a triple-element unit of 208,000 kw. capacity are in operation. The latter consists of one high-pressure and two low-pressure turbines with their respective generators. Single shaft units of 200,000 kw. capacity are under construction and it seems probable that the trend in the future will be toward even larger units of both types. This trend toward larger units instead of the equivalent in small units has resulted in improved wave shape but otherwise does not directly affect coordination except in so far as it may reflect the general trend toward larger concentrations of power with the accompanying tendency to increased magnitude of system abnormals.

Another definite trend in the power industry, but one which is not of importance from the standpoint of coordination, is the increasing use of automatic devices to replace manual operation. Complete automatic operation is being practised to some extent in hydroelectric generating stations and is widely practised in substations of various types. The trend is definitely toward wider use of automatic devices and new types and applications of such devices are being constantly developed.

In view of the remarkable development and rapidly multiplying uses of thermionic tubes and related devices in other fields, and the theoretically potential applications in the power art, the question will doubtless be asked as to the trend of their application in the power field. However, other than application for current rectification, such as in railway work, it cannot be said that progress has advanced to the point of establishing a trend.

Those trends in power system development which are more directly concerned with matters of coordination are discussed in the following.

System Voltages. Referring to Table III, it is of

TABLE III

TOTAL CIRCUIT MILES OF TRANSMISSION LINES. BY VOLTAGES, YEARS 1926-1929 INCLUSIVE

Voltages	1926	1927	1928	1929	Per cent of total 1929	Average annual increase per cent 1926-1929
220,000	1,054	1,257	1,442	1,442	0.9	11.0
132,000	3,125	3,343	4,010	4,448	2.8	12.5
110,000	7,875	8,661	9,114	10,159	6.4	8.9
66,000	12,157	15,212	18,716	21,236	13.3	20.4
60,000	8,801	9,257	8,076	8,174	5.1	−2.4
44,000	7,517	8,492	8,732	8,761	5.5	5.2
33,000	23,831	24,706	27,451	28,523	17.9	6.2
22,000	10,130	10,429	11,545	12,583	7.9	7.5
13,200	19,496*	18,441*	19,551	21,340	13.4	3.1*
11,000	8,072	9,145	10,007	10,860	6.8	10.4
All other over 11,000	28,223	28,535	29,843	31,916	20.0	4.2
Total	130,281	137,478	148,487	159,442	100.0	7.0

*This apparent discrepancy is believed to be due to reclassification of these lines as between transmission and distribution facilities interest to note that the rate of increase of transmission line mileage, as a whole, is lagging behind the rate of growth of both installed generator capacity and electricity production. Furthermore, mileages of the higher transmission voltages, 220 kv., 132 kv., 110 kv. and particularly 66 kv., are growing at a faster rate than the group average. These comparisons reflect the increasing utilization of the higher voltages with the greater circuit capacities they provide. As power industry growth requires the handling of larger blocks of power and as greater distances between sources and markets are encountered, the development and use of circuits and apparatus to transmit at voltages higher than the 220 kv. initiated in 1923 must be expected as an economic necessity.

In the distribution field also, coincident with the development of rural service, there has been a movement to higher voltages in primary circuits, and indications point to the continuance of this trend in the future. Due to the distances involved, voltages from 6,600 to 13,200 (and even higher) have been used in rural work. In urban areas the high load densities encountered in some districts require the handling of large blocks of power in the primary circuits, and the lower primary voltages have often been replaced by higher voltages for such conditions. In addition to the greater capacities provided by the higher voltages, possibilities of system simplification by combining rural and urban systems and eliminating voltage transformations are of considerable economic importance.

While at first glance the pronounced trend to higher transmission and primary distribution voltages may appear to enhance the difficulties of coordinating communication and power lines, certain factors enter to offset this. As transmission voltages increase, line construction as a whole becomes more massive, greater clearances and wider rights-of-way become necessary and construction costs per mile rapidly rise. These greater space requirements weigh against the use of highway locations and, together with the higher construction costs, which make the shortest possible lengths desirable from an economic viewpoint, frequently influence the selection of direct cross-country private rights-of-way providing generally greater separation from communication circuits in the same territory.

The use of the higher voltage circuits, each transmitting many thousands of kilowatts, of itself tends to increase the problems of coordination. However, the greater separations obtained by the use of private rights-of way for these main transmission circuits in most cases eliminate the need for coordinative measures to control normal induction (manifested as noise in the telephone circuits) and, in case noise presents a specific problem, the greater separations simplify and render less extensive those specific coordinative measures which may be required. Induction due to power system abnormals too is mitigated or rendered easier of control.

In the case of distribution lines, the adoption of increasingly higher voltages is accompanied by more systematic grades of construction and greater clearances from communication circuits. The result, of course, is that fewer abnormal conditions of operation occur and the number of related disturbances in the communication circuits is correspondingly reduced. The possibility of contact between power and communication circuits is also reduced. This trend toward better grades of construction applies also to transmission lines and as noted previously to other parts of the power system.

System Stability. During recent years considerable attention has been paid to the development of methods for improving system electrical stability. One of the most important of these methods is the use of higher speed switching,—at present, faults can be cleared in 15 cycles, or less, of a 60-cycle wave. So far, high-speed switching has been applied mainly to transmission circuits. However, as development proceeds and cost of equipment required is reduced, the field of application of high-speed switching may naturally be extended to distribution systems. The result in the case of either transmission or distribution will be of course, to reduce the duration of transients. Akin to high-speed switching, the use of high-speed excitation of rotating equipment has been developed. This may tend to increase the maximum fault current values somewhat which would make coordination more difficult. However, the reduction in the severity of instability surges, in so far as such surges involve faults-to-ground, affords definite benefits from the coordination standpoint. It requires further study and observations to determine what, if any, inherent limitations or advantages it may possess with respect to coordination work.

The way has been paved for the development of high-speed switching by steady improvement in relaying practise. Selective operation of protective relays in power systems, during the early stages of relay development, was largely dependent upon an additive sequence of time intervals which might aggregate a considerable period in the case of the more remote units in the sequence. The development of relaying practise has included various methods of securing selectivity independently of time. This has accomplished large increases in the over-all speed of operation, at the same time improving selectivity. Coincident with these improvements there has also been a substantial gain through greater precision in design and workmanship and improved application of relays and related devices. These trends definitely aid coordination by reducing duration of transients, eliminating faulty relay operation, and steadily reducing the radius of influence of system abnormals.

With the growth in power systems and major interconnections, the use of bus or feeder current limiting reactors or other means of limiting the concentration of fault current flow is being given increasing application. Such practise acts to restrict the magnitude of inductive transients. In distribution systems the growing use of feeder reactors has a similar effect in matters of coordination.

For well-known reasons, among which are the avoidance of transient over-voltages resulting from arcing grounds and the economies made possible in apparatus insulation, it is predominant practise in America to ground the neutrals of transmission systems at important transforming centers, sometimes through resistors or reactors but usually solidly. In view of the prevalence of the latter method, a large proportion of higher voltage transformers now in service have been constructed with insulation between the neutral ends of the grounded windings and the core and tank, designed to support only the neutral potentials produced by fault currents regulating through the unavoidable impedance of grounding connections. The economies resulting from this method of construction become greater as rated operating voltages rise. The use of solidly grounded neutrals tends to make coordination more difficult in view of the possibilities for increased flow of earth currents.

On some large power networks with relatively great possible concentrations of short-circuit power and solidly grounded neutrals tendencies towards instability of operation have appeared. In some instances also oil circuit breaker characteristics, particularly as regards the older breakers in service, have become a source of concern. For these reasons, in these situations, increasing study and consideration are being given to the use of current limiting devices in the neutral where the characteristics of the apparatus and limitations of relaying will permit of such operation.

In some European countries, particularly in Germany, where grounding for the purpose of power system voltage stabilization is excluded by governmental regulation, dependence is extensively placed on the Petersen coil as a substitute. This device may be regarded as a special type of neutral impedance. The Petersen coil has been applied to but limited extent in this country although its possibilities for moderate voltage systems, especially for situations warranting only single circuit supply, are receiving consideration.

In this country, the increasing use of neutral impedance, as well as the use of other types of current limiting devices is an aid to coordination since it reduces the magnitude of abnormal induction.

Lightning Control. The major problem of the transmission art at the present time is the control of lightning in its effects on service. In those sections of the country in which lightning is prevalent, this natural hazard accounts for a large proportion of transmission circuit faults, approaching 100 per cent in the case of the heavier, higher class trunk transmission lines. The seriousness of this problem and the researches which some of the larger power utilities and apparatus manufacturers are conducting for its solution are being fully reported from time to time before the Institute and need not be discussed here. It is sufficient to say there is encouragement that methods for the solution of this problem, as it affects high-voltage trunk circuits, will be known in the not too distant future. Where adequate methods are found and applied the results of course, will be a decrease in the number of system disturbances which induce transients in communication circuits.

Present measures in power system practise, especially at the higher voltages, directed toward the control of service interruptions caused by lightning include: improved application of overhead ground wires, improved grounding connections at the supporting structures, the improved use of wood for lightning insulation, and the use in shunt with line insulators of fused gaps or other valve devices to "spill" the surge without dynamic current follow up. There is also under consideration the application on grounded neutral systems of single-phase switching. All of these measures, with the exception of the last, are helpful from the coordination viewpoint since their effect is to avoid or reduce system faults or at least to decrease the magnitude of earth fault currents and hence of the accompanying voltages induced in nearby communication circuits.

Single-phase switching involves the use of individually controlled and operated single-phase circuit breakers. Upon the occurrence of a single-phase fault-to-ground, the breakers on the faulty phase only would open, leaving the other two-phase conductors in circuit to maintain connection momentarily between source and load. In a short interval the breakers controlling the faulty phase would be reclosed automatically.

Single-phase switching has not progressed beyond a preliminary consideration of its possibilities. If applied in situations of proximity, the residual voltages and load currents while one phase of a three-phase grounded neutral system is momentarily open circuited may constitute a problem in coordination.

Underground Construction. The use of underground construction in distribution systems is seldom economical but is increasing in high-load density districts and in some residential areas primarily due to requirements for civic improvements and the relieving of surface congestion. The reduced influence on communication circuits of such underground circuits as compared to overhead construction, is too well known to need repeating here. Coincident with the more recent developments in underground distribution certain special situations have brought about the development of underground cable suitable for use in high-voltage transmission circuits, inclusive of 132 kv. Underground installations involving these transmission voltages are highly special, comparatively few in number and small in extent. However, they have a definitely favorable effect upon coordination problems within the territories surrounding them.

Aerial cable construction for both distribution and transmission circuits has been used to a limited extent and has a definitely beneficial effect upon coordination matters. Whether this type of construction will be extended in the future is not evident.

Grounding of Distribution System Neutrals. One of the difficult tasks encountered in distribution systems is that of obtaining adequate grounding of primary and secondary circuits. Because of this difficulty the establishment of neutral networks grounded at many points has become a practise. In most cases in the past two separate neutral networks have been provided, one for the primary and one for the secondary system. However, in several localities these two separate neutrals have been combined into a common-neutral arrangement providing in this way an increased multiplicity of ground connections to both the primary and secondary neutral conductors. Further extension of the use of this system is probable. This arrangement introduces features of interest from the coordination standpoint, because of the increased opportunities for the flow of currents through the ground. Experience and investigations so far, however, indicate that with adequate attention to coordination this arrangement is comparable in its effect on neighboring communication circuits, to other types of distribution systems.

Service Taps on Transmission Lines. In some rural situations, it has been found economically impracticable to initiate distribution lines due to distances involved. However, in many such situations immediate electric service is urgently required and in some of these cases, transmission lines may be located relatively close to the point where service is desired. In such cases the only alternative to a long distribution line is to tap the high-tension transmission line when this can be done by some less expensive method. Such methods have been developed and applied to a limited extent. More study and field experience are needed to determine the effects of these installations on inductive coordination should they become extensively employed.

Transformer Connections. In distribution practise, the trend toward higher primary voltages has been accompanied by the use of the Y connection of the primaries of transformers as a step in the transition from one voltage class to another. Thus 2300-volt delta systems have become 2,300/4,000-volt Y connected systems, 6,600-volt delta systems have become 11,000-volt Y systems, and the 7,620/13,200-volt Y connection is being used. The use of the Y connection of the primary of distribution transformer banks is sometimes necessarily accompanied by a similar connection of the secondary. Such YY connections are usually in urban situations. Also, these banks usually represent only a small portion of the total transformer capacity on the circuits.

On large transformer banks and in the higher voltages delta-Y connections have long been the prevailing practise. However, where the YY connection is used for purposes of grounding, especial attention has been given to controlling the effects of this connection in situations of coordination, and for the absorption of triple harmonic currents it is common practise to use delta-connected tertiary windings in such installations. This subject is discussed more fully in another paper in this symposium.

Wave Shape. The connection of primary circuits directly to generating station buses results in service and economic advantages by eliminating transformations thereby improving voltage regulation and aiding system simplification. This practise, however, tends to make coordination more difficult as those harmonics which may be present in the generated voltage can flow directly out over these circuits. However, the important bearing of the wave shape of generators and apparatus of various kinds on the coordination problem has long been realized and is receiving increasing attention. Even before the formation of the Joint General Committee the general problem of apparatus wave shape was being studied both as to the amounts of various harmonic components which were present in apparatus wave shape and as to the relative effect of these components when appearing in communication circuits by induction from power circuits. As a result of this study an instrument was developed for measuring "telephone interference factor" of a voltage wave. With this instrument as an aid a better understanding of the bearing of wave shape has been gained by the apparatus manufacturers and there has resulted a gradual improvement in the wave shape of new apparatus.

It is recognized that there is a median line beyond which general improvement in the inherent wave shape of apparatus would not justify the attendant increased difficulties of design and increased manufacturing costs, —to avoid the alternative of applying in specific cases, available and less expensive methods of externally correcting wave shape. Work is now in progress cooperatively between the manufacturers and users looking toward the establishing of a measure of wave shape in apparatus design which will strike an economic balance between benefits and burdens.

The increasing use of rectifiers for conversion from alternating to direct current has an influence on inductive coordination. Considerable study has been devoted to this matter as a result of which methods for control of the distortion of the d-c. voltage wave caused by the rectifiers have been applied in several instances and a solution of this part of the problem appears to be in hand. More study and experience are needed as regards the specific conditions under which the wave shape distortion of the alternating current supply would require consideration.

With the progress being accomplished in the design and application of apparatus and the better understanding of the influence of circuit and transformer connections on inductive relations, problems concerned with wave shape can be expected to steadily decrease. The status of the cooperative study of this subject is described in this symposium.

CONCLUSION

A brief outline has been given here of the general trends in plant development and operating practise in telephone and power systems with special regard to those trends which affect the problem of coordination. While naturally there have been influences working favorably and others working unfavorably toward the problem it is clear that the preponderant effect of the development now being applied in the two industries is reducing the proportion of new situations in which specific coordinative measures are necessary. While to a considerable extent, as indicated in the body of the paper, this is due to the natural trends of plant design associated with new developments within each of the industries, it is also true that the extent of the progress made is due in no small measure to the careful study of all phases of the problem being conducted by the Joint Committee of the National Electric Light Association and the Bell Telephone System.

Under the guidance of this Committee and soon after its formation, the types of situations of physical proximity were classified and certain broad principles of cooperation were recommended. Soon thereafter more complete principles and detailed practises were formulated. These principles and practises were printed and widely distributed to companies and individuals directly interested in the problem of coordination.

The principles and practises thus set up were largely qualitative and the need for an organized program of research to establish quantitative data and to develop improved physical facilities for coordination was early recognized. Accordingly, the Joint Sub-Committee on Development and Research was organized, and assigned the work of determining both experimentally and by field experience quantitative data covering the various aspects of coordination problems, and the development of detailed methods of effecting physical coordination. Under this Sub-committee a very large volume of research work has been undertaken. Results of some of this work have been published and a considerable amount is now in progress. The three papers to follow in the symposium discuss much more fully three of the most important aspects of coordination work at the present time and tell of the work being done in these fields by the Joint Subcommittee on Development and Research and by the other branches of the Joint General Committee's organization.

In reviewing this subject one is impressed by the number of ways in which the coordination problem touches both the telephone and power fields, and with the very large amount of cooperative work which has already been done. This work, as has been indicated, has resulted in great progress in the satisfactory handling of coordination matters of all types. This matter concerns two industries both of which are in a period of rapid development and change, both as regards their size and as regards the physical arrangements which constitute their plants. Many new developments in each plant require consideration from the standpoint of coordination. It is evident, therefore, that if the ground already gained is to be held and further progress made the channels of cooperation between the two industries must be kept in operation both for the consideration of new problems arising with new developments in the industries, as well as for the further perfection of the cooperative methods of handling specific problems. These papers in other words do not constitute in any sense a final report. They are intended to show the present status of two very active and rapidly changing arts and to indicate the highly satisfactory results which have followed from a number of years of sincere cooperative effort between the telephone and power industries.

Part II—Status of Joint Development and Research on Noise Frequency Induction

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Associate, A. I. E. E. Fellow, A. I. E. E.

FOREWORD

THE Joint Subcommittee on Development and Research is the agency through which the National Electric Light Association and the Bell Telephone System carry out technical work on problems of physical relations which vitally affected their respective growth and operating practises. In the present paper and companion papers the status of this joint development and research work is described.

The present paper, Part II of the Symposium, is concerned with problems of induction in telephone circuits under normal operating conditions of power systems which result in noise. Part III of the Symposium treats of induction at the power-system fundamental frequency, principally that occurring at the time of grounds, short circuits or other abnormal conditions of power systems. Part IV of the Symposium treats of the physical relations and of the special noise-frequency and low-frequency problems brought about by the close proximity of the two types of service when occupying the same poles.

The Joint Subcommittee on Development and Research has subdivided its work among eleven project committees and assigned to each the actual carrying on of specific research work. Certain of the project committees are engaged on the problems described in this paper, while the remainder are concerned with the development and research problems of the companion papers, Parts III and IV of the Symposium. The names of these project committees, together with a statement of the phase of the problem considered by each, is given in Volume I of "Engineering Reports of the Joint Subcommittee on Development and Research."[1]

Naturally the first steps taken by the Joint Subcommittee were the review and appraisal of existing information and the exchange of data between the two interests represented. This paper includes a statement of the problem, with some review of the factors involved, the results accomplished by the subcommittee and the work projected in connection with each factor.

CLASSIFICATION OF FACTORS CONTRIBUTING TO INDUCTION

There are certain characteristics of a power circuit with its associated apparatus that determine the character and intensity of the electric or magnetic field which is set up in the surrounding medium. These characteristics are termed "influence factors."[2]

Likewise, there are certain characteristics of a communication circuit with its associated apparatus which determine its responsiveness to external electric or magnetic fields. These characteristics are termed its "susceptiveness factors."[2]

There is a third group of factors which refer to the interrelation of neighboring power and communication lines by electric or magnetic induction or both. These are termed "coupling factors."[2]

Inductive interference is thus the manifestation in the telephone circuit of a combination of influence, susceptiveness and coupling; and inductive coordination consists in the control of factors in all three of these classes to the degree required for satisfactory operation of both services.

METHODS OF CONTROL

Physical Separation. The first method which comes to mind for the control of inductive effects is that of physical separation obtained by placing the power and telephone lines on separate routes. A separation between lines of a few hundred feet practically eliminates the noise-frequency problem whereas the low-frequency problem may exist with much greater separations. Since the same customers desire both communication and power services, the two kinds of distribution lines are necessarily often located on the same streets and highways. Power transmission lines and toll telephone lines do not, in general, have to be placed on particular routes and, therefore, separation can often be employed where such lines are involved. Cooperative advance planning on the part of the utilities in laying out their plants makes it possible to employ separation where it is readily feasible and economical.

Frequency Separation. Another method of fundamental importance is the use of frequency separation. By this method, circuits to be coordinated are arranged so as to be responsive to different frequencies or bands of frequencies, and comparatively unresponsive to the frequency or band of frequencies employed for the other circuits. It is thus possible to make many different uses of electricity involving transmission in the same

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1. For references see Bibliography.

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medium. This solution is familiar to us in the coordination of radio services.

Fig. 1 shows a diagram of the various uses of the frequency spectrum for electrical transmission and the manner in which power and communication services are coordinated by means of frequency selectivity.

The first commercial electrical energy available was in the form of direct current. Shortly thereafter, alternating current was used for the transmission of power. The nominal frequencies of the current used for this service in the earlier days range from 16⅔ cycles to 133 cycles. In American practise the frequencies used for power purposes have practically settled down to either 25 or 60 cycles. There is one extensive 50-cycle system and a few odd frequency systems. These latter of 30, 33, and 40 cycles, and perhaps others, are being rapidly eliminated, due to the importance of interconnecting them with 60-cycle systems. At the present time, there is some tendency for the use of higher frequencies in special machine shop applications. This use, at present, is principally at 180 cycles and need not concern us here as its extent is usually confined within a factory building.

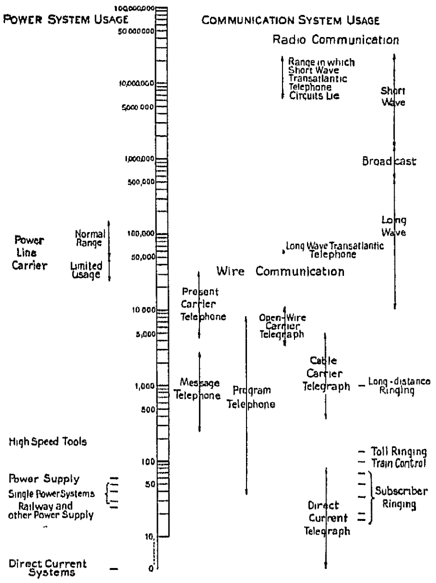

Fig. 1—Frequencies Used for Electrical Transmission

In message telephone transmission, the prime consideration is the transmission of intelligible speech. While the range of response of the human ear is from about 16 cycles to 15,000 cycles per second, human speech occupies a narrower range and a still narrower band is adequate for intelligibility. The present voice-frequency telephone circuits, especially the longer ones, operate within a frequency band of about 250 to 2,750 cycles per second. The frequency selectivity at the edges of the band is not sharp, however, so that extraneous currents at frequencies outside of this band may also give rise to noise. This is particularly true at the lower end with some of the local exchange circuits. High quality telephone circuits for program transmission cover a wider range. This may be on certain circuits as much as from about 35 to 8,000 cycles per second, thus overlapping the fundamental frequencies used for power transmission.

Control of Power Levels. Coordination by frequency separation becomes inadequate when the power levels of the various classes of services differ greatly as with power and telephone services. Thus, although incidental powers at harmonics of the power circuit fundamental frequency are negligible in comparison to the power at the fundamental frequency, they are large compared to the power employed in the telephone circuits and fall directly within the frequency range of the telephone circuits.

While the powers involved in telephone transmission are small as compared to those on power lines, they are in turn large as compared to the acoustical power received from the talker or delivered to the listener. The ordinary telephone transmitter is an amplifier, delivering to the line several hundred times the voice power which actuates the diaphram. On the other hand, the receiver requires an electrical power a hundred or more times that which it delivers as sound to the listener's ear.

It is obvious that the relative levels of harmonic frequency power in the power circuits and voice-frequency power in the telephone circuits are of major importance in inductive coordination. These considerations have had large influence in the power field in the control of wave shape of rotating machinery and transformers, and in the telephone field in fixing limitations on such factors as wire sizes, spacings of repeaters and instrument efficiencies.

Balance. Among the most important methods of coordinating power and communication circuits is the control of their respective balances to ground and to each other. A power circuit with absolutely balanced voltages and currents impressed, and with the various conductors arranged in such a way that they would not establish external electric or magnetic fields, would not have any effect on any type of neighboring communication line.

Likewise, a telephone circuit in which there were no unbalances and in which the conductors were arranged in such a way that in the presence of an electric or magnetic field they would not have any voltages induced between them would not become noisy from any neighboring power circuits. Such an ideal state is impossible, but much has been accomplished by care in the design of the lines and equipment and by the transpositions of the conductors.

Shielding. It is possible to materially reduce electric fields by interposing between disturbing and disturbed conductors grounded conductor surfaces known as shields. Magnetic fields can likewise be reduced by interposing conducting paths which circulate current to set up counter magnetic fields. The power and telephone cables in use are probably the simplest examples of shielding. A cable sheath is almost 100 per cent effective as a shield for electric induction, either on a power cable or on a telephone cable. The sheath is less effective as a shield for magnetic induction, because of its finite conductivity. It does not seem feasible at this time to obtain anywhere near perfect magnetic shielding.

FACTORS CONTRIBUTING TO NOISE FREQUENCY INDUCTION

Influence Factors

There are two characteristics of a power system which are of primary importance in determining its inductive influence upon neighboring telephone systems, *i. e.*, its wave shape and its balance. The wave shape is determined by characteristics of apparatus associated with the system. The balance is determined by the degree of symmetry of the supply voltages, load impedances, and of the series impedances and shunt admittances of the lines. While it is not practicable to design rotating machinery or other apparatus using magnetic cores entirely free from harmonics, or to realize ideally balanced three-phase systems, it is practicable to control both these factors within limits which, in conjunction with a similar degree of control on the coupling and in the susceptiveness of the communication circuits, permit satisfactory operation of both services without unduly burdening either.

The work on influence factors which has been conducted by the Joint Subcommittee on Development and Research has, therefore, been directed for the most part toward the study of the wave-shape characteristics of power systems and apparatus and methods for their improvement and the investigation of factors affecting the balance of the power systems and methods for their control.

Wave Shape. In initiating its work on influence factors, the Joint Subcommittee found little information available as to wave shape which might be expected on operating power systems equipped with various types of apparatus. In order to obtain a broad picture of wave-shape conditions as they exist in the field, the Subcommittee conducted an extensive survey of wave-shape conditions on 34 operating power systems in the eastern half of the country. The program was arranged to obtain information as to the average and range of magnitudes of harmonics present in various types of transmission and distribution systems under normal operating conditions, to observe the relation between the wave shape of generating machinery under open-circuit conditions and under load, to study the effects of various transformer connections on wave shape, and to observe the effects on wave shape of various types and magnitudes of load.

The measurements made included analyses of the phase-to-neutral and phase-to-phase voltages and phase currents on a large number of generators, transmission lines and distribution feeders. Wherever practicable, data were obtained as to the balance of the operating systems by measurements of residual voltages and residual currents. Measurements were also made of the telephone interference factors[3] of the voltages and currents. Where telephone circuit exposures suitable for test purposes existed, noise measurements were made on the communication lines to aid in determining the relation between power system wave shape and balance and telephone circuit noise. The actual measurements were for the most part conducted by the operating companies with the cooperation, during the first part of the testing program, of representatives of the Joint Subcommittee.

The mass of data accumulated during this survey is being summarized in several technical reports which it is anticipated will yield much valuable information pertaining to the wave-shape problem. An important practical application of these data will be in connection with the prediction of wave-shape conditions on new lines which are to be involved in exposures with communication systems and on which noise estimates are desired.

In general, the survey data indicate that the magnitudes of the harmonics present in voltage and current diminish with increasing frequency, with the exception that a pronounced dip occurs in the region from 800 to 1,500 cycles. This is, no doubt, a result of the efforts of the machine designers to closely control the harmonics in this important region. Frequencies above 2,000 cycles become extremely small except where these may be introduced on the power circuits by superposed carrier communication or signaling services. In general, the frequencies used for such services have been in the range from 50 to 200 kc., which is above the range employed for carrier communication on telephone lines.

In the general survey of wave shape, no efforts were made to select feeders involved in cases of inductive interference. Aside from the survey work, however, representatives of the Subcommittee have participated in a number of investigations of such cases in which power system wave shape was an important factor. Much valuable data as to wave-shape conditions under which coordination difficulties are experienced have been obtained from these studies, while in obtaining these data the subcommittee representatives have been of service to the local companies in the solution of the particular problems.

A limited amount of theoretical work has been carried on having to do with the effects of load on the harmonics observed in the open-circuit voltage of rotating machines. This work which was based on Blondel's two-reaction theory was supplemented by laboratory tests on several small machines. It was found that the reactions which take place within the rotating machines, particularly when two or more are operating in parallel, are so complicated as to practically preclude accurate computations of the effects. However, the data obtained from this investigation have been valuable in connection with later studies.

Balance. A balanced power circuit is one in which the voltages between the various phase conductors and ground are equal in magnitude and sum up vectorially to zero and in which the phase currents are also equal in magnitude and sum up vectorially to zero. In a three-phase system where the currents or voltages are not equal but do sum up to zero, the currents or voltages can be resolved into two balanced three-phase systems, one of positive phase sequence and one of negative phase sequence. In cases where the currents or voltages do not sum up to zero they contain a single-phase component which is usually termed residual or zero-phase sequence component. Any three-phase system can be resolved into its balanced and residual components and each treated separately. The coupling for the residual components is usually much larger than for the balanced components and is therefore frequently of major importance in coordination problems. Differences in the magnitudes or departures from phase symmetry of the three impressed phase-to-neutral voltages, load or line unbalances, give rise to residual currents or voltages.

Experience has indicated that the outstanding factor in the unbalance of power systems is the existence of triple-harmonic voltages and currents which may arise either in rotating machinery or in transformers which are connected in star with grounded neutral. Since the triple-harmonic voltages in the three phase-to-neutral legs are in phase, they act in a path consisting of the phase conductors and an external return as, for instance, a metallic neutral or ground.

A large measure of control may be exercised on the magnitudes of the triple-harmonic residual voltages and currents by the use of certain transformer connections and by not operating the transformers at high flux densities.

The magnitudes of triple-harmonic residual currents in grounded-neutral systems may be minimized by the use of star-delta connected transformers, in which case nearly all the required triple-harmonic current circulates in the delta. The opposite extreme occurs with star-star connections in which case the full triple-harmonic magnetizing current flows in the two systems which the transformer interconnects, the relative magnitudes in each depending on their relative impedances. Where a star-star bank is connected at one terminal of a line, with a star-delta at the other, the neutrals at each end being grounded, practically the entire third harmonic required by the star-star bank may be expected to circulate in the line connecting the two.

An effective method of control for cases in which star-star connections are required due to phase relations is the provision of a third set of windings or tertiaries in the transformers, the impedance of the tertiaries with respect to the other windings being sufficiently low to furnish an adequate path for the triple-harmonic magnetizing current. An alternate method of control, which also provides like phasing on the two sides of the bank, is the use of zig-zag connected transformers.

In four-wire multi-grounded neutral distribution systems, it has been found helpful in controlling the residual triple-harmonic currents from the single-phase load transformers to provide star-delta connected banks at various points in the network with neutrals connected to the system neutral. In some cases, these have been three-phase load banks, in others, special banks installed as a method of control.

The subcommittee is continuing its work on wave shape and balance through a laboratory study of transformer harmonics and transformer connections. These tests are being made on small model transformers, typical of the designs which are used for large sizes on transmission systems. It is planned to develop the theory applicable to harmonics from transformers on three-phase systems from the work on these laboratory models. It is planned to supplement the work by tests on large transformers in the manufacturer's shops and in the field.

A number of severe noise situations has been created during the past few years when star-connected generators, operating with grounded neutral,[4,5] have been connected directly or through star-star transformer banks to transmission or distribution systems. The interference in these cases resulted from triple-harmonic residual components impressed on the system by the particular generator operating with the grounded neutral. The magnitudes of these currents depend on the triple-harmonic components in the generator phase-to-neutral voltage and the impedance to ground of the system. The methods of control which have been successfully applied in these cases include the following:

1. Isolating the generator neutral and supplying the system ground through a suitably designed transformer bank.
2. Grounding the neutral of only those generators designed to be free from triple harmonics in their phase-to-neutral voltage.
3. The use of selective devices such as reactors or anti-resonant circuits commonly called "wave traps" in the generator neutral for suppressing the disturbing triple-harmonic components.

Non-triple harmonic residual voltages and currents may exist from differences of phase-to-neutral load impedances and from differences in the capacitances to ground of the three-phase wires.

In multi-grounded neutral four-wire systems differences in the single-phase loads connected between the individual phases and the neutral may be important sources of residual current. A considerable measure of control may be exercised by restricting the size of single-phase areas and balancing the load on the different phases.

Capacitance unbalance to ground may be due to single-phase branches on three-phase distribution systems. Usually, the more important effect is on the single-phase branch where the residual voltage is practically equal to the phase-to-neutral voltage. The unbalancing effect on the three-phase system may be minimized by equalizing the lengths of the branches connected to the several phases. The residual voltage on the single-phase branch can, where necessary, be eliminated by the use of isolating transformers or by converting to a three-phase branch.

Capacitance unbalance may also be due to dissymmetry in the arrangement of the wires of the circuit to each other and to ground. These unbalances are lowest in triangular configurations of the wires and largest when all the wires are in the same vertical or horizontal plane. With multi-circuit lines, a considerable measure of control may be obtained by suitable phase interconnection of the circuits. Transpositions are also effective in controlling these unbalances.

Coupling Factors. The coupling between power and communication circuits is, of course, determined by the degree of their proximity, but it may be greatly modified by the balance of the two classes of circuits to each other and with respect to ground. While the most direct and certain method for reducing coupling is to avoid proximity, means are available for minimizing the coupling where necessary.

The work on coupling of the Joint Subcommittee on Development and Research, in the voice and carrier-frequency range, has been directed toward two objectives: (1) development of improved methods for predetermining the coupling to be expected in new cases of exposure, and (2) development of improved methods for reducing coupling for given degrees of proximity.

Several years ago the California Joint Committee on Inductive Interference[6] completed an extensive series of computations on coefficients of induction which was expressed in the form of curves for various physical relationships of power and telephone lines. These coefficients indicate the voltages induced in short, isolated, untransposed telephone circuits by unit voltage and current on similarly untransposed power circuits. They do not include the small separations involved with jointly used poles.

These curves and others based on them have been used for many years in determining relative coupling, when comparing different exposures, different routes involving various degrees of exposures, different configurations of power and telephone circuits and for other comparisons where all factors were substantially equal in the situations being compared, except those involved in determining the coefficient of induction. For these purposes they have been very useful. Methods have not, however, been available whereby these coefficients could be used for computing noise where transposed circuits were involved and where many telephone wires were on the line, which exert an important shielding effect on each other.

The Joint Subcommittee on Development and Research has been conducting experimental studies both for highway and wider separations, and those occurring with jointly used poles, so that the effects of transpositions and of mutual shielding of the many wires involved might be properly taken into account in determining the noise currents in the metallic circuits.

In determining the coupling between power and telephone circuits, it is desirable to differentiate between the effects of the balanced and residual components of the voltages or currents of the power circuit, between the effects of voltages and those of currents, and on the telephone line between induced voltage which acts directly in the metallic circuit, termed "metallic-circuit induction," and that which acts in the circuit composed of the wires with ground return, termed "longitudinal-circuit induction."

Since the residual components act in a circuit having ground as one side with the wires in parallel for the other, while the balanced components are confined to the wires of the system, the coupling for the residual components is much greater than for the balanced components. The coupling for the balanced components may be reduced by the use of power circuit transpositions, while such transpositions have no effect on coupling for the residual components.

The distance between the power and telephone wires is usually large as compared to the spacing of the wires of the telephone circuit, so that the longitudinal induced voltages are large as compared to the metallic-circuit voltages. The effect of the telephone transpositions being merely to equalize the relations of the two sides of the telephone circuit to the power circuit, such transpositions do not change the magnitude of the longitudinal voltages, but do reduce the metallic circuit voltages.

The relative magnitudes of inducing voltages and currents differ widely among various power circuits, and may vary greatly with time on any given circuit. They will also differ considerably at a given time and on a given circuit among the various frequencies involved. For this reason it is necessary to consider separately the coupling arising through the electric and magnetic fields.

Voltages induced in metallic circuits for the separations between lines usually encountered are practically proportional to the spacing of the wires of the telephone circuit. Voltages induced in 8-inch spaced pairs are thus approximately two-thirds of those induced in 12-inch pairs, while those induced in phantoms on 12-inch spaced side circuits are twice those induced in the sides. The longitudinal voltages are, however, practically independent of the wire spacing so that the contributions which these voltages make to noise in the metallic circuit are unchanged except as the change in spacing may affect the balance to ground.

Spacing of the wires on the power circuit and their configuration also have an important effect on the coupling for the balanced voltages and currents, the coupling, in general, increasing as the spacing increases. Coupling for the residual components is, however, affected only to a minor degree by the spacing and configuration. Much information bearing on these matters is included in the material on coefficients of induction published by the California Commission referred to above.

Measurements of coupling have been made by the Subcommittee in a number of situations. These have included cases of (1) exposure of overhead transmission lines and open-wire toll telephone circuits at highway separations, (2) overhead distribution lines and subscribers' telephone cables in joint use and at street separations and (3) overhead distribution lines and subscribers' open-wire circuits in joint use. Information was obtained on coupling both for voltages and currents and for the balanced and residual components. The results of the work on overhead distribution lines and subscribers' telephone cables have already been published.[7] The other data are to be published as soon as they are prepared in suitable form.

The work on overhead distribution lines and subscribers' circuits is relatively complete, covering a wide range of conditions typical of those encountered in the field. Various arrangements of primary and secondary conductors covering single-phase and three-phase, three-wire and three-phase, four-wire systems were investigated. The shielding effect of the telephone cable was determined and, with the open-wire subscribers' telephone circuits, the shielding effect of the various telephone wires on each other.

For telephone cable circuits when the sheath is grounded at either one or both ends, the inductive effect of the power circuit voltages on the wires enclosed is negligible as compared to that of the power circuit currents. Furthermore, because of the close association of the wires of a pair in the cable and the frequent twist, the metallic-circuit induced voltages are neligibly small as compared to the longitudinal voltages so that, in general, only the magnetic longitudinal coupling factors are of importance in these situations.

The work further indicates that, for most practical problems involving overhead distribution lines of the multi-grounded type and subscribers' cable circuits, a knowledge of the coupling for the residual or unbalanced currents is sufficient, the effect of the balanced currents being relatively unimportant. However, in cases where the line currents are particularly heavy or contain exceptionally large harmonic components, the balanced currents become important.

In the range of frequencies used for telephone transmission the ratio of open-circuit voltage induced on a telephone line through electric induction to inducing voltage on the power circuit is substantially independent of frequency. When the exposed section of line is connected to the remaining section of the telephone line or to terminal apparatus, a current is set up which is approximately proportional to the frequency of the induced voltage. The circuit will perform as if there were a small condenser connected between the power circuit and telephone circuit and the induced current experienced will be proportional to the frequency and the magnitude of the inducing voltage on the power circuit.

The coupling between power and telephone circuits for currents is in the nature of a mutual inductance, so that the voltage induced in the telephone circuit is proportional to the magnitude of the inducing current in the power circuit and its frequency.

This statement applies strictly only to induction from the balanced current components. Induction from residual current in the power circuit is complicated by the effect of the finite conductivity of the earth. With increase in frequency the earth currents tend to be closer to the surface and the coupling with the telephone circuit tends to increase less rapidly than would follow from proportionality with frequency. The departure from linearity is not large in the frequency range from 250-2750 for highway separations and for joint use.

Transpositions afford one of the most powerful means available for controlling coupling of power and open-wire telephone circuits in given situations of proximity. Transpositions operate by neutralizing, in one section, inductive effects which arise in a closely adjacent section. It is evident that, in order for transpositions to be fully effective, conditions must be substantially alike among the various sections to be neutralized as regards relations of the power and telephone circuits to each other, to ground, and among the various circuits on each line. This latter condition more often applies to the telephone lines, as they usually comprise many circuits.

These conditions require that balanced and coordinated systems of transpositions be provided between each point of discontinuity in the exposure. By "discontinuity" is meant any point at which an important change takes place in the physical or electrical conditions of the circuits, such as loads, branch circuits, series impedances, etc.; any change in configuration, in the separation of the two classes of circuit or in their position relative to ground or to some other circuits which may be associated with either power or telephone circuits closely enough to appreciably modify the induction.

In addition to meeting these conditions, the telephone transpositions must also satisfy the requirements for minimizing crosstalk among the various telephone circuits. This, in general, requires telephone transposition arrangements of considerable complexity. For this purpose standard transposition arrangements are available,[8] adapted for different lengths depending upon the distances between the successive discontinuities.

In most cases unavoidable irregularities occur in the spacing of poles, in distances between power and telephone circuits, in presence of shielding objects, such as trees, and in height of poles, which it is not possible to treat as discontinuities and take into account in the transposition design. In cases where these irregularities are large, the effectiveness of the transposition arrangements is greatly impaired. The extent to which the effectiveness of such arrangements is impaired due to these non-uniform conditions is a problem not easily susceptible to mathematical analysis and reliable information is not now available. The subcommittee is planning to investigate this problem experimentally by tests on a number of situations involving operating circuits.

Susceptiveness Factors. The degree to which telephone transmission is adversely affected by noise-frequency induction depends not only upon the magnitudes of the induced voltages as determined by influence and coupling factors, but also upon the susceptiveness factors of the telephone system. These include the manner in which the induced voltages and currents are propagated to the circuit terminals together with the reactions of the circuit unbalances, thus relating the current in the terminal apparatus to the induced voltages, the sensitivity of the receiving apparatus and the operating power level of the telephone circuits.

Propagation Effects and Balance. Important differences exist with respect to propagation effects and balance between open-wire and cable circuits and between toll and exchange systems.

As pointed out in the discussion of coupling, only the magnetically induced longitudinal voltages and currents affect telephone cable circuits. Because of the absence of electric induction and direct metallic-circuit induction and because of the important shielding effects exerted by the cable sheath and the various telephone circuits on each other, telephone cable circuits are much less susceptive than open-wire circuits.

In open-wire telephone systems consideration must be given both to electric and magnetic induction and to voltages directly induced in the metallic circuit as well as to those induced in the longitudinal circuit. In a line composed of a number of circuits, the currents set up in any one circuit depend, not only upon the voltage induced in that circuit and its impedance, but also upon the currents and voltages which are set up in the rest of the telephone circuits on the line. It is not possible, therefore, to calculate the induced currents merely from a knowledge of the magnitudes of the currents and voltages on the power circuits and the coupling between the power circuits and isolated pairs of wires on the telephone line, considered independently.

These mutual effects among the various telephone circuits exist both within and without the exposed sections. Thus, the propagation of the induced voltages and currents along any one circuit is influenced both by the electrical conditions of this circuit and also by the conditions of all other wires on the line. Additional complexities arise in the propagation of the induced voltages and currents, because of non-uniformity in impedances to ground at terminals, points where circuits join or leave the line, and where lengths of cable may be used at terminals or at intermediate points. The impedances to longitudinal induced voltages and currents vary over a wide range depending on the number of wires on the line, the relative position of the exposure and the circuit terminals and the occurrence of sections of cable. Due to reflection effects from these irregularities, peaks of current and voltage may exist along the circuits which are large as compared to the corresponding magnitudes at the exposure terminals. If circuit unbalances happen to exist at these maximum points, metallic-circuit voltages and currents thereby introduced are increased.

While the distribution of longitudinal voltages and currents among the various wires upon the telephone line depends upon the nature of the inducing field in which it is placed, the experiments of the committee have shown that a satisfactory degree of approximation for studying propagation effects can be obtained by energizing all wires on the line simultaneously at the same potential from a common source. An extensive experimental study has been made in this way by the committee in which the magnitudes of the longitudinal voltages and currents at various points along the line have been measured as well as metallic-circuit currents set up through the unbalances at the sending and receiving ends of the line.

By making measurements of this sort on a considerable number of lines of different types of construction and different transposition arrangements, it is hoped to obtain statistical data whereby the metallic-circuit voltages and currents at the circuit terminals may be determined from the magnitudes of longitudinal voltages and currents as measured at exposure terminals.

Unbalances in toll circuits are the result of commercial variation from the balanced condition, since the circuits are designed to be symmetrical. These unbalances may consist of resistances in joints, capacitance or inductance unbalances due to irregularities in transposition spacing or to omitted or unspecified transpositions, or differences in the impedances of apparatus connected in series with the wires or between them and ground. These unbalances are fortuitous both as regards their magnitudes and location along the toll circuits. Some increase in importance with frequency and others decrease. These, combined with the irregularities in the propagation of the longitudinal voltages and currents, cause the resulting metallic-circuit currents in individual circuits to vary in an erratic fashion with frequency. The general trend is one of proportionality, independent of frequency within the important range, between the longitudinal currents and voltages at the exposure and current in the metallic circuit at the terminals. Taking into consideration the effects on coupling, the currents at the terminals increase approximately in direct proportion to the frequency of the inducing voltage or current on the power circuits.

Because of the lower susceptiveness of cable circuits together with the high degree of balance of the terminal apparatus and because of the more general use of private rights-of-way, cases of noise-frequency induction into toll cable circuits have been comparatively infrequent. For this reason the attention of the subcommittee as far as toll systems are concerned, has been directed toward open-wire circuits.

In exchange circuits certain inherent unbalances exist due to the arrangements employed for supervisory signaling, for selective ringing, and for coin-box service. The supervisory system utilizes a low impedance relay connected in series with one side of the central-office interconnecting circuit. The selective ringing scheme involves connecting the ringer windings from one side of the line to ground at the station set. For the coin-box service, a coin-collect relay winding is connected between one side of the station set and ground. These unbalances have been investigated in detail by the committee and the results have been published[7] as described later.

The unbalance of party lines due to the ringer ground is usually much more important than that of the central-office interconnecting circuit due to the supervisory relay. Both are, in general, more important than the cable unbalances. Coordination difficulties between telephone exchange systems and power distribution systems thus usually involve the party line circuits before the individual line circuits are affected.

The controlling unbalance in the exchange plant when in cable being in the nature of an inductance between one side of the line and ground, its importance decreases with increasing frequency of the induced longitudinal voltage. This effect largely counter-balances the increase in coupling with frequency. Thus, in most situations involving joint use of poles by distribution circuits and exchange cable telephone circuits, induced currents of the third and fifth harmonics of the power circuit fundamental frequency assume chief importance.

Exceptions are cases where outstanding harmonics in the range between 800 and 1,500 cycles are present on the power circuits. In these cases, particularly where the exposures are long, the central-office apparatus unbalances may be more important than those of the party-line station apparatus.

The method which has been found most generally applicable for reducing the susceptiveness of exchange cable circuits is the grounding of the cable sheaths. This reduces through shielding the magnitudes of the longitudinal voltages and currents. Special station sets having lower susceptiveness have been used in specific cases where their use appeared to be the best method.

Power Level and Sensitivity. The magnitudes of the induced currents in the telephone system having been determined by the influence factors, the coupling, and the susceptiveness of the telephone circuits, the degree to which they impair telephone service depends upon their intensity as compared to the intensity of the telephone currents.

Consideration has been given by the subcommittee to the possibility of increases in power levels (a) on local exchange circuits and (b) on toll circuits. Little promise has been found in the proposal to raise voice power levels in the local exchange plant as a means of reducing the effects of noise. As previously pointed out, present telephone transmitters materially amplify the power received from the voice so that the electrical power on the telephone line is some hundreds of times greater than the acoustic power applied. In development work on telephone transmitters, telephone engineers are proceeding on the basis that more is to be gained by improving the frequency response of the transmitter than can be gained by mere increase of power. This line of development has, of course, the effect of raising power levels at frequencies where they have been relatively low.

Two proposals for application to the toll telephone plant were studied. One would involve changing the repeaters now in use at terminals and at intermediate points to a more powerful type and equipping all toll circuits with terminal repeaters of this same type. This would permit raising the power levels without altering the relative levels of the various telephone circuits and thus would not change the crosstalk. Another would involve such changes only on certain long toll circuits, leaving the remainder of the circuits at their present levels. As the result of a trial installation, it was found that to realize any appreciable change in level on these circuits, very extensive changes would be required to avoid crosstalk from the higher level circuits to the remaining ones which were not changed.

The levels employed in carrier telephone circuits, while somewhat lower than those used on voice-frequency open-wire telephone circuits at the receiving end, are higher at the sending ends than the corresponding voice-frequency levels. Since the power-system harmonics in the carrier-frequency range normally are small as compared to those in the voice-frequency range, carrier-frequency open-wire systems experience considerably less noise from power systems.

Effects of Noise. The actual voice power level on telephone circuits varies over a wide range, depending upon the particular user, his distance from the telephone central office, and by the transmission loss in the connection between the two subscribers. Impairment caused by a given amount of line noise on the circuit may also vary over a considerable range, depending upon the voice power level and the noise in the room where the telephone is being used. The method in use by the Bell System for an engineering basis in considering the effects of noise on telephone conversation is to substitute for the noise increases in the transmission loss of the circuit. Thus, the circuit with its actual loss and noise is represented by a circuit of lower noise and increased transmission loss. These added losses are known as Noise Transmission Impairments and are abbreviated N. T. I. The N. T. I.'s were determined from articulation tests and judgment tests made on noisy and quiet circuits, and were set up on the basis of typical talker volumes, transmission equivalents, and amounts of room noise at the station terminals. Additional transmission loss was added to the quiet circuits so that noisy and quiet circuits gave equal articulation or were judged by the observers to be equivalent in their transmission performance. Thus, the N. T. I.'s are used to indicate an additional transmission loss or impairment which is occasioned by the presence of the noise.

The articulation and other tests on which these N. T. I. ratings were based are now being supplemented by tests conducted under the direction of the subcommittee. Measurements are being made of the effects of various magnitudes and sorts of line noise in the presence of typical amounts of room noise, as determined from a room noise survey made by the subcommittee, and for representative toll connections and talker volumes. The line noises being employed are those found typical from an extensive survey made by the subcommittee on open-wire toll circuits throughout the country. These tests will afford a basis for comparing various methods of measuring line noise, including ear comparison methods now in general use and new visual meter methods now under development. Thus, this work should lead to a mutually acceptable method for measuring noise and a basis upon which agreement may be reached as to the impairment in telephone transmission caused by noise.

Published Results. As various phases of the technical work being done are completed, they are published in the form of Engineering Reports which are released by the Engineering Subcommittee of National Electric Light Association and Bell Telephone System. Eight reports, of which five refer to matters concerning noise-frequency induction, have already been issued. Other reports dealing with this subject have been recently approved by the Engineering Subcommittee and will soon be issued. Certain other technical results which have come from the Subcommittee's work have been presented by various individuals connected with the work in papers before the A. I. E. E. Still other results have been published in brief articles in the N. E. L. A. *Bulletin.*

One of the problems, of which the technical work of the committee has been completed and published, is that of inductive coordination of primary distribution systems and exchange telephone circuits in cable. The results of this work are given in detail in a report[7] entitled "Minneapolis Joint Use Investigation." This report includes information on influence factors applying to various types of power distribution systems, including three-phase, three-wire and three-phase, four-wire systems with various arrangements of neutral grounding, data on coupling between various typical arrangements of these systems and telephone cable circuits, and information on susceptiveness characteristics of telephone systems, including unbalances of lines and apparatus. To facilitate the use of this information in the day-by-day coordination problems handled by the operating companies, a summarizing report[9] entitled "Short-Cut Methods for Calculating Noise in Local Telephone Subscribers' Circuits in Cable Due to Exposures to Power Distribution Circuits" has been prepared. This report presents empirical formulas for estimating noise-frequency induction and includes a brief discussion of the technical factors involved and the approximations underlying the formulas. Means are described for reducing influence by the control of triple-harmonic exciting currents and load unbalances of power distribution circuits, and for reducing susceptiveness by grounding telephone cable sheaths and by controlling the unbalances of the telephone station equipment. The information should be useful to engineers of the operating companies in the cooperative planning to avoid induction troubles.

While a large part of the experimental work connected with the problem of joint use of local open-wire subscribers' circuits and power distribution circuits has been completed, the detailed technical reports have not yet been completed for publication. However, a summarizing[10] report which it is believed will largely fill the needs of the engineers of operating power and telephone companies has been completed. This is entitled "Short-Cut Methods for Calculating Noise in Open-Wire Subscribers' Circuits Due to Joint Use Exposures to Power Distribution Circuits."

Three reports have been issued dealing with the problem of coordination of open-wire toll circuits and overhead transmission and distribution lines. The first[11] discusses the "Termination of Isolated Exposure Sections to Obtain Normal Metallic-Circuit Currents," which affords a means of taking into account the shielding effects present when the line is in normal operating condition. The second report[12] describes "A Method of Measuring the Balance of Open-Wire Telephone Circuits with Respect to Longitudinal-Circuit Induction," which should be useful to the field in the making of special tests and in supplying statistical data of value for estimating noise effects on open-wire line circuits. The third report,[13] dealing with "Methods of Measuring Noise on Open-Wire Toll Circuits," is a detailed presentation of the various types of tests for studying noise problems on toll lines, and includes a discussion of the method of analyzing the test data.

Another report[5] deals with "The Effects on Inductive Coordination of Generators Feeding Directly on the Line and Operating with Grounded Neutrals." This report includes a detailed discussion of the factors involved and describes methods which have been developed for control of the triple-harmonic residual currents and voltages which occur with this method of operation.

The results of the work done by the subcommittee on a survey of room noise in telephone locations was described in a recent paper.[14] While this was an incidental phase of the general study on effects of noise on telephone transmission, it was felt to be of timely value, particularly in respect to the methods of measurement employed. Using the results of the data obtained in surveys of wave shape on operating power systems and analyses of noise current on telephone circuits, a paper[15] was prepared on the frequency response characteristics of telephone transmitters and receivers. This paper indicated that there appeared to be no advantage, in reducing effects of noise, in shifting the resonance points of telephone transmitters and receivers from their present region, as the frequency distribution of the noise currents was such as to give a minimum in this resonance region.

At the time that the joint work was started the need arose for considerable special apparatus to make the measurements which were required. Some of the important pieces of apparatus for the work in the voice-frequency range were sensitive single-frequency voltmeters and ammeters. These needs were taken care of by the development of sensitive analyzers whereby single-frequency voltages or currents could be selected from complex wave shapes on either power or telephone circuits. One form of this apparatus has been described in a paper before the Institute[16] and another in a serial report[17] of the National Electric Light Association.

In connection with the survey of room noise, a room noise meter was developed. This was described in the paper[14] previously referred to which presented the results of this survey.

FURTHER WORK OF THE SUBCOMMITTEE

When the subcommittee started its work there was before it an accumulation of technical problems which had arisen as the arts developed without such close cooperation as now exists. The statements given above regarding various phases of the subcommittee's work on noise-frequency induction indicate the substantial progress which has been made in the solution of these accumulated problems. They convey also a general picture of the work which the subcommittee has immediately before it.

It must not be thought, however, that when these accumulated problems have been solved the work of the subcommittee will be completed and its efforts discontinued. This cooperative work must always bear a relation to the total development efforts of both the power and communication fields. As has already been pointed out, this work is concerned with two electrical arts which have been particularly noteworthy for their success in constantly developing their technical methods and expanding their services. These developments will surely continue and constant consideration of the physical problems of coordination is needed to insure that such developments act to steadily improve rather than to make more difficult the coordination of power and communication circuits.

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Part III—Status of Joint Development and Research on Low-Frequency Induction

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INDUCTION at power system fundamental frequency, commonly called "low-frequency" induction, has different characteristics and produces quite different effects from induction at the noise frequencies discussed in the paper by Messrs. Blackwell and Wills. Since very little has been published on low-frequency induction, it seems desirable, in order to make clear what the Joint Subcommittee on Development and Research is doing on this subject, to explain the problem in some detail.

The disturbances in communication circuits due to low-frequency induction are in general discrete occurrences, coincident with accidental grounds or other faults on neighboring power lines, rather than being continuous and due to normal power line operation.

Three-phase power circuits, when operating normally, are so nearly balanced with respect to earth at their fundamental frequency, and telephone circuits of the ordinary type are relatively so insensitive at frequencies of 60 or 25 cycles, that induction at these low frequencies under normal power line conditions is rarely a practical problem. But when abnormal conditions, particularly faults to ground, occur on power lines, large unbalanced voltages and currents at fundamental frequency exist temporarily and at such times there may be induced in neighboring telephone circuits voltages which are hundreds of times as great as under normal operating conditions. The induced voltages under abnormal conditions may reach values sufficient to cause hazard to telephone employees or interruption to service. Although such abnormal conditions occur infrequently and usually last for only the very short period required to interrupt or clear the power circuit, the effects which may be produced are so serious that protection against this type of induction is an outstanding problem in the coordination of power and telephone systems. A large part of the subcommittee's work has for its object the development of means for controlling and minimizing such induced voltages and their effects.

While low-frequency induction is not usually severe except under abnormal conditions, power circuits which operate at any time on an unbalanced basis, or which are closely coupled to grounded wires capable of carrying large currents may, even under normal operating conditions, create a problem of low-frequency induction in paralleling telephone circuits in addition to setting up high-frequency disturbances as explained in the Blackwell-Wills paper. This is particularly true where the exposed telephone circuits are used for special services such as the transmission of radio broadcasting programs. Grounded types of telegraph and other signal circuits also are sensitive to low-frequency induction.

CLASSIFICATION OF FACTORS RESPONSIBLE FOR INDUCTIVE EFFECTS

The same three classes of factors which combine to underlie the noise-frequency problem appear also in the low-frequency problem. As they appear in the latter, these are:

1. "Influence factors" in the power system, which are concerned with the magnitude, duration, and frequency of occurrence, of unbalanced voltages and currents.
2. "Susceptiveness factors" in the communication system, which are concerned with the nature and seriousness of the effects produced by the induced voltages.
3. "Coupling factors" which determine the magnitude of the voltages induced in the communication system, per unit unbalanced voltage or current of the power system.

In the low-frequency induction problem, the coupling factors are largely dependent upon the characteristics of the earth and the relations of power and telephone systems to the earth. If the earth were an insulator instead of a conductor there could, of course, be no such thing as fault current in the earth and the coupling between power and communication circuits would be much less. It would not then be hazardous for a lineman when in contact with earth to touch a charged wire. Or if the lines were not in proximity to the earth, there would be no chance for a lineman working on the wires to get in contact with earth. Neither in power systems nor in telephone systems is it actually necessary that the earth be used as part of an operating circuit but, as the earth is a conductor, and power and telephone lines and apparatus are located on its surface, it is essential in both systems that the earth be taken into account in circuit problems and that paths to earth for protective purposes be established at certain points.

It must not be assumed, however, that the earth is a perfect conductor. For the most part the materials of which the earth's crust is composed are of relatively

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low conductivity. From numerous measurements in various places the average conductivity over considerable volumes of earth has been found to range from 10^{-11} to 10^{-15} abmho per cm. cube. The resistance of an earth path is therefore not zero but may be many ohms or even in some cases many hundreds of ohms. Most of this resistance is in the immediate vicinity of the electrodes and can be reduced by increasing the surface area of contact between electrode and ground.

In discussing the low-frequency induction problem, it is convenient to consider the factors controlling:

(1) The magnitude of induced voltages, (2) the frequency of occurrence of induced voltages, (3) the duration of induced voltages, and (4) the effects produced by induced voltages.

FACTORS CONTROLLING THE MAGNITUDE OF INDUCED VOLTAGES

The magnitude of voltages induced in telephone systems in specific cases depends chiefly on the magnitude of residual currents and voltages resulting from power circuit faults to ground and on the exposure conditions.

Residual Currents and Voltages. A balanced power circuit is one in which the voltages from the various phase conductors to ground are equal and sum up vectorially to zero and in which the phase currents also are equal and sum up to zero. Under this condition all the currents in the circuit are *balanced* currents and all the voltages are *balanced* voltages. If, however, one phase develops a fault to ground, this relation becomes disturbed, the voltages to ground of the phases become unequal, and their vector sum, which is the *residual* voltage (3 times the so-called uniphase or zero-phase sequence voltage) of the power circuit, is no longer zero. The currents in the three phases likewise become unequal and when added vectorially their sum, which is the *residual* current (uniphase or zero-phase sequence current) of the power circuit, is no longer zero. In most low-frequency induction problems residual current is far more important than residual voltage.

Residual voltages and currents are equivalent to single-phase voltages and currents applied to a circuit consisting of the three line conductors in parallel as one side, and the earth as the other side. Their large inductive effects are due to the great dimension of the loop formed by this earth return circuit, much of the return current being effectively so deep in the earth that its neutralizing action is small. In the case of the balanced components, the inductive effect due to the voltage or current of one conductor is largely neutralized by the voltages or currents of the other two conductors.

The chief characteristics which determine the magnitude of the residual voltages and currents are (1) the power circuit voltage, (2) the impedances of the neutral ground connections, (3) the line and apparatus impedances, (4) the fault and earth impedances, (5) the sources of power supply, (6) the character of ground wires if used, and (7) the circuit configuration including ground wires.

When a fault occurs between a phase conductor and earth on a power system having neutral ground connections, these neutral connections, together with the fault, line conductors and earth, form a closed circuit for the residual current. Unless the neutral impedance is very high, *e. g.*, approaching that of an isolated system, the shunting effect of the capacitance to ground of the line conductors may for most purposes be neglected and practically the same value of residual current exists at all points along the line between the fault and the neutral connection to ground. For simplicity, a system with a single line and single neutral ground connection may be assumed. With this picture in mind, it is clear that the value of the neutral impedance may be an important factor in determining the magnitude of the residual current. If the fault occurs near the point where the neutral is grounded, the line and apparatus impedances being low, a small impedance in the neutral may control the current. On the other hand, for faults occurring at points remote from where the neutral is grounded, the impedance in the neutral connection may have to be relatively large to materially reduce the residual current.

As one limit there is the solidly grounded neutral, *i. e.*, no impedance is inserted and as good a ground as practicable obtained. This obviously permits maximum residual current when ground faults occur. Unless the grounding impedance is very high the residual current, and not the residual voltage, is the controlling factor in grounded-neutral systems.

As the other limit there is the isolated neutral, *i. e.*, the impedance from neutral to earth is infinite. In this case no residual current passes through the neutral. At the ends of the line the residual current is zero, gradually increasing to a maximum at the point of fault. The circuit for residuals is through the capacitance of the line to ground, the magnitude of this capacitance controlling the magnitude of the residual current, which is much less than with grounded neutral systems except in cases of double faults when it may be very large. With a single fault the residual voltage may be a more important factor in respect to induction than residual current.

The impedance of the fault itself depends upon a number of things, including the type of line construction and the earth conditions. The subcommittee has under way investigations to gather data on the range of fault impedances under different conditions. To determine the maximum residual current, the fault impedance may be taken as zero. In many instances, this approximation gives sufficiently close results, particularly if the fault is remote from the grounded neutral so that line, neutral and apparatus impedances are controlling.

In case of conductors falling upon the ground, local earth conditions largely determine the fault impedance. On a steel tower line an insulator breakdown results in a relatively short arcing path to grounded metal, whereas, in wood pole construction, the pole itself introduces considerable impedance unless nullified by guys or other metal.

The foregoing discussion of residual current has been confined practically to the situation brought about by single faults to ground. Double faults at separate locations sometimes occur and these are equivalent to a phase-to-phase short circuit through the earth, giving a large residual current in the intervening section of line. If the two faults in such a case are on opposite sides of an exposure, very severe induction may result. Experience shows that double faults at separate locations constitute only a few per cent of the total faults occurring on grounded-neutral power systems but are a much larger percentage of the total faults on systems normally isolated from ground.

The presence of ground wires on a line may have considerable influence on fault impedance. Being connected to ground at frequent intervals, such wires decrease the impedance to ground where a breakdown occurs between a phase conductor and a ground wire or any metal in contact with a ground wire. A ground wire tends to increase the total residual current but on the other hand its controlling function, from the induction standpoint, is that of a shielding conductor tending to decrease the induced voltage.

Circuit configuration does not have a large influence on unbalances due to abnormal conditions, but it has an important effect upon any unbalance of a power circuit under normal operating conditions. To be balanced, the phases of the power circuit must be symmetrical with respect to each other and to earth. To the extent that the capacitances and inductances of the several phase conductors differ, residual voltages and currents will result. Transpositions afford a means for compensating for these circuit unbalances.

In cases for which protective measures are being considered, it is important to be able to estimate the magnitude of the residual current when faults occur at different points on the power system. Apart from inductive effects, this is a question of importance to power companies, since forecast of currents under different fault conditions is essential in the design and setting of protective relays. Much work has therefore been done by different investigators on methods of predetermining these currents. Helpful mathematical methods have been developed, though sometimes the results obtained by their use are open to question due to lack of accurate values of some of the important impedances. Proper allowance for fault impedance and the effect of ground wires is sometimes difficult to determine and in cases of complicated networks approximations usually have to be made. To facilitate the numerical computations, calculating boards of varying degrees of elaborateness have been developed. The subcommittee is investigating this matter and by experimental work is checking the results of estimates and acquiring further knowledge of the range of the variable factors. Through this work, it is hoped to increase the convenience and accuracy of these important computations.

Exposure Conditions. The relationship between power and telephone lines with respect to the exposure conditions is defined by the "coupling coefficient" or "coefficient of induction," a factor which, when multiplied by the value of current (or voltage) in the power line, gives the resulting voltage set up in the telephone line. A power line and a neighboring telephone line have several different coupling coefficients corresponding to different conditions, such as, whether the induced voltages are due to power current or power voltage, to balanced or residual components, and whether they are voltages induced along the conductors (or to ground) or are induced directly in the metallic circuit. Low-frequency induction is predominantly magnetic in character and the coupling which is most significant is that between the power conductors and the telephone conductors, both considered with earth return. The induced voltages are due principally to "longitudinal circuit induction."

A number of dimensional factors affects the magnitude of this coupling, such as the length of the exposure, the separation between lines, and the locations of ground connections on the two systems. Local conditions as to earth conductivity and the arrangement of geological strata for some distance below the earth's surface, constitute other important factors. An accurate mathematical evaluation of the coupling between earth return circuits is difficult. Formulas have been developed under simplifying assumptions as to symmetry and homogeneity, which aid in explaining and interpreting experimental results and in predicting approximate values of coupling in cases where experimental measurements are not available.

Assuming uniformity of exposure conditions the coupling varies directly with length of parallelism, except for end effects or interactions between ground connections of the two lines. Increase in separation of the lines diminishes the coupling but the exact relationship depends upon the distribution of current in the earth which in turn depends upon the frequency and the earth conductivity. In many cases different strata of different conductivities are involved in the path of the earth current, which adds to the difficulty of correlating experimental and theoretical results. The effect of earth conductivity on coupling is accentuated as the lines are more widely separated. At roadway separation, large differences in earth conductivity affect the coupling only moderately; but at separations of one-half mile to one mile, coupling values may differ by 20 to 1 or more, due to the range in value of earth conductivity. Irregularities in exposure conditions such as changes in direction of one or both lines, crossovers, and angular exposures of varying separation, are complications which frequently occur in practise.

The voltages set up in neighboring communication circuits by power currents are due usually to inductive coupling but in some cases are due partly or wholly to resistive coupling. It is seldom necessary in practical studies to try to segregate these two components of voltage, since their effects in the telephone system are not a function of the phase relationships of these components to the power line current which produces them. It is not unusual to speak of inductive coupling as including both inductive and resistive coupling.

Any grounded circuit in proximity to power and telephone lines within an exposure brings about a certain amount of shielding through the reaction of the currents induced in this conductor upon the primary magnetic field set up by the residual current in the power circuit. In this respect a shield wire acts like a short-circuited turn on a transformer. The effectiveness of the shielding depends upon the conductance of the shield wire, the manner and effectiveness of its grounding, and its position with respect to the power and telephone wires. Such a wire affords maximum shielding when closely coupled to either the power wires or the telephone wires, when its conductance is high, and when its ground connections are of low resistance.

The variation of coupling with separation and with earth conditions is of great practical importance in the coordinated location of lines. Most of the subcommittee's study of coupling, therefore, involves field investigations of the variation of coupling with separation under different earth conditions, and is furthermore directed toward devising convenient and accurate methods of predetermining coupling in practical cases. Also, by studying and correlating experimental data derived under different conditions and from widely separated parts of the country, the subcommittee hopes to arrive at a better empirical basis for estimating coupling. Some of the work on this subject has already been presented.[1]

FACTORS CONTROLLING THE FREQUENCY OF OCCURRENCE OF INDUCED VOLTAGES

The frequency of occurrence of induced voltages in paralleling communication lines, while chiefly dependent on the frequency of occurrence of faults on the power line, is also somewhat affected by the location of the exposure with respect to the location of neutral grounding points. For example, if there is only one neutral ground, faults occurring between it and the exposure will produce relatively little induced voltage.

The frequency with which faults occur is usually traceable to features of electrical and mechanical design, the character and amount of insulation, and the location of the lines. Specifically, the factors which appear to be responsible for the majority of faults on power lines are: poor configuration, inadequate spacing and clearances, inferior insulation, lightning, fog, smoke and dirt, birds and animals, proximity of lines to external objects apt to interfere with operation mechanically or electrically, and certain mechanical features of design affecting the strength of construction, such as ineffective anchors, guys, or conductor and ground wire supports, particularly at angles and deadends, and insufficient bearing areas of subsurface structures.

FACTORS CONTROLLING THE DURATION OF INDUCED VOLTAGES

The length of time faults are permitted to remain on a power system is controlled by the kind of protective relaying employed and by the type and condition of the circuit breakers and other terminal equipment. The type of relay system, the degree of sectionalization obtained, the adequacy of the circuit breaker as to speed and rupturing capacity, and the maintenance of the equipment are the most important factors.

Generally, the type of fault has little effect on the duration if there is sufficient current to operate the relays. Conditions have been noted, however, where the fault is of such high impedance that the current is not adequate for the operation of the relays. Such high impedance faults usually occur on wood pole lines and may result in burning of pins, crossarms and poles. They may also occur on steel tower lines as the result of branches of trees getting in contact with conductors.

EFFECTS PRODUCED BY INDUCED VOLTAGES

Low-frequency and transient voltages induced on telephone circuits may produce a variety of effects depending upon their magnitude and duration. These effects include service interruption, false signals, telegraph signal distortion, damage to plant, electric shock, and acoustic shock.

Telephone circuits are very low energy circuits, the voltage for talking purposes rarely exceeding one or two volts, with maximum current measured in milliamperes. For signaling purposes a maximum of 165 volts peak, is used with currents limited to about 0.10 ampere. For telegraph service the voltages are limited to 135 volts between wire and ground, while the current is limited to less than 0.10 ampere. By contrast, the voltages due to induction, in some cases of exposure, may be a thousand volts or more.

Service Interruption. When the telephone protectors are operated by induced voltage the behavior of the protector discharge gaps depends upon the magnitude of the voltage and current and the length of time the discharge lasts. In cases where the discharge is not promptly extinguished or where the current is very high, the discharge gaps may become permanently grounded. This causes interruption to service until the affected protectors can be replaced, the time necessary for such replacement depending, of course, upon the protector locations.

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[1] For references see Bibliography.

False Signals. False switchboard signals are likely to be coincident with protector operation They produce a bad service reaction due to operators answering false calling signals and cutting off connections because of false disconnect indications.

Distortion of Telegraph Signals. The induced voltages appear in just the same paths over the wires as the operating voltages of grounded telegraph. The effect of such induced voltages depends on their magnitude, character, and duration. Voltages much lower than those sufficient to operate the protectors may cause detrimental effects ranging from a slowing down of speed to complete failure. Where the duration is short, the effect may be limited to distortion of signals, or, if the voltages are high enough, to momentary interruptions.

Damage to Central Office or Other Telephone Plant. The dielectric strength of the telephone plant is adequate for the voltages used in communication service, with appropriate factors of safety, but higher voltages may sometimes, notwithstanding the protective devices, cause dielectric failure, thus damaging the plant, particularly cables and wiring or apparatus in telephone offices.

Electric Shock. Telephone linemen in the course of their work upon wires at relatively close spacing, cannot avoid getting in contact with the wires and if the wires were subject to sufficient induced voltage, the men would be liable to receive electric shocks. On severely exposed lines such voltages are liable to occur at any time, suddenly and without warning. Electric shock might either injure a linemen directly or startle him and cause him to lose his hold and fall from the pole. Voltage to ground due to induction appears not only within the exposed section of line but considerably beyond. A similar, and in some respects worse, condition may exist with respect to employees working on cable circuits which are either exposed or directly connected to exposed circuits. In cables the wires on which the foreign voltage appears are very close to the grounded metal sheath and usually also to other wires at approximately earth potential, as well as to the earth itself. This problem has become more difficult with the rapid growth of the telephone and electric power systems and is engaging the subcommittee's serious attention.

Acoustic Shock. Acoustic shocks are liable to occur with the breakdown of telephone protector discharge gaps, which temporarily unbalances the circuit and causes a sudden and abnormally large current in the receivers. This current gives rise to sudden and severe flexures of the receiver diaphram, which produce loud sharp noises in the ear of a person using the receiver. Telephone operators, due to the nature of their work, are particularly liable to acoustic shocks, the effects of which range from minor reactions to severe general disturbances of the nervous system which may be painful and of long duration. In addition, if danger of severe shocks exists, the operating force may become fearful and the impaired morale seriously affect the service.

TYPES OF PROTECTIVE MEASURES

The foregoing effects of induction from paralleling power lines may be reduced by: (1) measures in the power system to limit the influence, (2) measures in the communication system to limit the susceptiveness and (3) coordinated location of lines or other means to reduce the coupling. As a solution in a specific situation, one measure may be sufficient or two or more measures may be required, depending on the conditions. The solution should afford the necessary protection without hampering the development or operation of either system. Where there are two or more alternative solutions, the one which is best from the engineering standpoint, including both the technical and economic aspects, should of course be applied.

Cooperative planning in advance of construction is especially important in situations involving low-frequency induction, because of the wide ranges in magnitude both of coupling factors and of residual currents. By advance notifications of construction it is possible to bring up for analysis the low-frequency effects which the proposed construction would bring about and, if necessary, to agree upon changes in the plans to prevent or reduce these effects.

As to the physical dimensions and relations of power and telephone lines which constitute an exposure there are no blanket rules for guidance; each case requires specific consideration. Due to differences in geological conditions and other variable factors, a given length of parallelism at a given separation might give satisfactory results in one location, whereas an exactly similar physical relationship of lines in another location might result in the communication system being rendered inoperative at times of power system fault. This fact emphasizes the necessity of advance planning and cooperative study of situations as they arise. Such cooperation may easily lead to a satisfactory solution of situations which at first seem very difficult. On the other hand situations which at first appear devoid of any possibilities of trouble may on careful study be found to require protective measures.

Protective Measures for Power Systems. It will be evident from the foregoing discussion that protective measures to reduce the inductive influence of power systems should be directed to limiting the magnitudes of unbalanced currents and voltages, particularly under abnormal conditions, and to reducing the duration and frequency of occurrence of abnormal conditions. Of such protective measures some are concerned with fundamental questions of line and system design and must be incorporated in the construction plans, while other measures are of such a character that they may either be incorporated in the original construction or added later if found necessary as a result of subsequent experience or developments in either the power or telephone system.

Fault-Resistive Design and Construction. As mentioned in the paper by Messrs. Harrison and Silver the methods employed in reducing the frequency of occurrence of faults are primarily involved in the design and construction of the power line, *i. e.*, adequate insulation, clearances, and spacings, and so arranging the component parts of the structure that the line will in effect be fault-resistive. Increasing demands for better service by the public combine with considerations of inductive coordination to justify greater attention to fault-resistive line construction.

Faults may result from improper guying of poles, *i. e.*, guys so located that the spacing between guys and conductors is inadequate, or the path from insulator to crossarm brace and thence to the guy is insufficient to withstand the voltages imposed. The conductor spacing may be inadequate or the configuration of the circuits may be such that the sudden unloading of conductors coated with sleet will result in their whipping together, or, if a ground wire is used, it may be so located that the unloading of sleet will cause the conductors to whip into the ground wire, or the design of the line, either steel tower or wood pole, may be such that inadequate strength is provided for the mechanical loads incurred.

Attention is being given to the location of lines as a material factor in limiting the number of outages resulting from external sources, such as lightning, broken trees, blasting, and automobiles. For example, lines built in valleys are less subject to failures due to lightning and wind storms than lines built over hills.

There is little need to call attention to the grade of insulation employed on power lines as recent lightning studies and papers have emphasized the importance of rationalization of insulation throughout the plant. By this method it is hoped that preferential points of failure would be established, thus permitting prompt restoration of service without damage to expensive equipment since most of the faults would be confined to the line.

The amount of insulation to be employed on lines is affected by topographical and climatic conditions. Lines in areas relatively free from lightning or shielded from lightning disturbances may, of course, employ less insulation without increasing the number of faults. On the other hand, lines built in areas where lightning is prevalent may justify not only higher insulation but also, on steel tower lines, the use of ground wires as an additional protection. Areas where salt fog, smoke, or chemical fumes are prevalent require special treatment as to the form of insulation used.

Laboratory tests and limited field experience indicate that a proper utilization of the inherent insulating properties of wood in structures may result in considerable improvement in line operation. The subcommittee is investigating the service performance of wood pole lines of differing designs with a view to determining how much may be accomplished in reducing the number and severity of faults by suitable arrangements of metal braces, fittings, guys, etc., to avoid so far as possible shunting out the insulation of the wood.

To the experienced designer the protective measures to be employed on lines subject to frequent faults are obvious, namely, the rearrangement and reconstruction of the tower or pole top to obtain greater spacing between conductors or greater clearance between conductors and other metal parts. In some cases spacing and clearances would be materially improved by utilizing a triangular configuration so that the conductors are not likely to come in contact with each other or the ground wire when sleet or other conditions cause whipping or dancing of the conductors. In other cases, merely a relocation of the point of attachment of guys would improve conditions without materially decreasing the strength of the structure.

Fault-Current Limiting Measures. Resistors, or reactors, in the neutral ground connection of a power system provide a means of directly limiting the magnitude of the residual currents, except in cases of double faults. In cases where the residual currents can be so far reduced as not to set up induced voltages of high values in the communication system without reacting unfavorably on power system operation, this method alone may afford a satisfactory solution. In such cases it has the further advantage of reducing the stresses to the power system due to the fault current. Where it is impracticable to clear up a situation by residual current limitation alone, this method may be effectively used in combination with other protective measures.

The reduction in residual current which will be brought about by adding a given amount of impedance in a neutral ground connection can be estimated with reasonable precision. It is not so much this question therefore, that requires study by the subcommittee as it is the question of the limitations and costs of this protective measure, and its reaction upon the power system. Included in this work is a study of the relative advantages of inductance as compared with resistance for accomplishing such current limitation. The subcommittee has under observation a number of installations of current-limiting devices and is engaged in experimental and theoretical studies and in field observations by means of recording instruments to determine the possibilities of this type of protection.

In non-grounded power systems a single fault on a phase conductor results in the charging current of the system flowing to earth through the fault. The other phases, rising to full line voltage above the grounded phase, create a system unbalance which may manifest itself by induction in paralleling communication lines. In such cases the problem is one of electric induction except for the magnetic induction set up by the charging current.

When double faults occur on either grounded or nongrounded systems, severe magnetic induction is liable to result and under these conditions it is difficult to limit the residual current.

Shielding. Ground wires on a power line, while tending to increase the total residual current, serve the purpose of shielding by reducing the strength of the external electric and magnetic fields set up by the residual voltages and currents. The net effect of ground wires from the low-frequency standpoint is to reduce the voltages induced in paralleling communication circuits under abnormal power circuit conditions. The effectiveness of such shielding depends on the impedance of the shielding conductor and its ground connections. Under favorable conditions the induced voltage at 60 cycles in paralleling communication circuits may be reduced about 40 per cent by this method. Such ground wires, if used on wood pole lines, have a disadvantage in that they impair to some extent the insulating property of the poles.

High-Speed Circuit Breakers and Relays. Very sensitive high-speed relay systems have been developed which, together with high-speed types of circuit breakers reduce the time duration of a power line fault to approximately 1/10 second, as compared with one-half second to three seconds required by the older forms of relays and circuit breakers, thus tending to minimize the effects of induction. On the other hand inadequate relaying, or the omission of automatic circuit breakers, may extend the duration of faults to a point where the hazards to power apparatus are serious. High-speed breakers and relays are expensive and it is difficult to justify them solely as a remedial measure for induction, particularly as the speeds of operation now available for relays and breakers on power systems, have not reached values which make them a complete solution of coordination problems. However, with the increasing size and interconnection of power systems, high-speed relays and circuit breakers are playing an increasingly important part in promoting power system stability.

Periodic testing of relays and circuit breakers accompanied by complete overhauling at regular intervals, will do much to reduce the duration of faults and to prevent improper functioning of the equipment.

The subcommittee is following the developments in high-speed breakers and relays with much interest. If such devices should come into general use for all classes of service it is expected that they would materially improve the whole inductive situation.

Improvement in Balance. As mentioned above, low-frequency induction between power and communication lines is sometimes experienced under normal operating conditions. On grounded telegraph and signal lines the trouble usually manifests itself by a chattering of telegraph instruments or by false signals. Improvement in balance of the power line by transpositions will in some cases correct the difficulty.

Protective Measures for Communication Systems. In general, measures applicable to the communication system to prevent or reduce the effects of induced voltages take the form of arrangements or devices for removing or counteracting the voltages to ground or the currents in the telephone circuits which might be produced by the induced voltages.

Bell System Standard Protectors. It is Bell System standard practise to equip all telephone circuits which are exposed to the liability of foreign voltages, with electrical protective devices. These devices are made in various forms and combinations for different plant and exposure conditions. The protector used at central offices and at subscribers' stations includes a discharge gap which operates at approximately 350 volts and a fuse which opens the circuit at about 10 amperes. Such devices are intended to offer a measure of protection against lightning discharges and against the voltages and currents resulting from accidental contacts with foreign wires or from low-frequency induction.

In order to protect telephone linemen or others working on open-wire lines against electric shock from induced voltages, it is necessary that the voltages between line wires, and between each line wire and ground, be kept low. The use of protectors at central offices does not so protect the linemen as the impedance drop on the line wires permits high voltages between wires and ground at other points, such as the terminals of the exposed section.

It appeared however, that protectors of the Bell standard type might be used on open-wire lines at locations immediately adjacent to exposures to limit induced voltages to ground. A number of installations of this kind has been made but observations over a period of time show that they introduce serious troubles as the protectors, being subjected to heavy discharges, often become permanently grounded thus interrupting service. It also sometimes happens, as all the line wires are not always equally exposed, that some of the protectors operate and others do not, resulting in objectionable voltages between line wires.

Relay Protectors. In view of the inadequacy of existing forms of protectors for such use, the subcommittee is experimenting with a "relay protector." This device includes Bell standard protectors in combination with a relay which operates to short-circuit them upon the occurrence of a discharge, thus relieving the protectors of the duty of carrying the large discharge current and greatly reducing their tendency to become permanently grounded. In more recent types all the relays at a protector point are electrically interlocked, so that when any relay operates all line wires are grounded within a few cycles.

Several trial installations of relay protectors have been made and are under observation. To guard against voltages to ground within the exposure these protectors have to be placed within, as well as at the ends of, the exposed section of line. Where the longitudinal induced voltage is large, protectors are required at a number of points within the exposed section.

The effective application of such protectors requires grounds of the order of one or two ohms and an important feature of the investigation is to devise methods of constructing and maintaining such grounds at remote points along the line.

The subcommittee is investigating in the field and in the laboratory the effectiveness, cost, reaction on service, and other practical questions relating to the installation and maintenance of this method of protection.

Acoustic Shock Reducers. Since acoustic shock due to induced voltages involves dissymmetrical discharges across the two sides of the protector, efforts have been made to devise a protector which would break down and discharge symmetrically, *i. e.*, provide two reliable low-impedance paths for heavy discharges, which would at all times have very closely the same arcing impedance. Thus far the subcommittee has not been successful in developing a practicable protector of this kind.

For the purpose of equalizing the voltages on the protector during the discharge period, an accessory device termed a "discharge balance coil" is under investigation. It consists of two equal windings on a common core, each in series with the discharge gap of one side of the line, and so arranged that the fluxes set up by the circuits in the two windings are in opposition. The "booster" action of this coil tends to equalize the discharge currents. This reduces acoustic shock from induced voltages, provided all protectors are so equipped and the line itself has no large unbalances. When however, voltage is impressed on one wire only of a telephone circuit, as by accidental contact, these coils have a detrimental effect on the action of the protector in reducing voltage to ground, as they introduce impedance in the protector discharge path.

Development work is also being conducted on other types of acoustic shock reducing measures which do not attempt to prevent unbalanced current but merely to shunt it out of the telephone receiving circuit. Obviously a device acting on this principle to be successful must be practically instantaneous in operation. One of the most promising of such devices consists of a high ratio step-up transformer with its primary connected directly across the receiver to be protected. The secondary is connected to a low-voltage discharge gap. Any abnormal voltage across the primary operates the discharge gap and the transformer becomes a low-impedance shunt. A number of field trials of these reducers applied to operator's receivers has been made. While not affording the full degree of protection desired they have been found to reduce substantially the severity of acoustic shocks and it is believed that they will be of considerable benefit in cases where some form of protection against acoustic shocks to operators is urgently required.

Another device based on the shunting principle consists of opposingly poled copper oxide rectifiers connected across the receiver. These have the property of greatly diminishing impedance with increasing voltage. The problem is to obtain a sufficiently sharp change in impedance with voltage, while avoiding a normal impedance so low as to cause serious transmission losses. As an aid to this end, biasing batteries are under investigation.

The committee has also investigated the saturating characteristics of a vacuum tube for acoustic shock reduction. The properties of a vacuum tube are such that the output current cannot be increased substantially beyond a definite value regardless of the input voltage. This feature can be made use of to limit shocks by a design which will pass currents substantially without distortion up to approximately the highest value of signal current used, thus cutting down the shock voltages which exceed the normal signals. While quite effective, this method involves apparatus which is more bulky and expensive than the transformer and spark-gap type reducer. Telephone repeaters accomplish this result to some extent and are being investigated by the subcommittee, to determine the quantitative reduction of acoustic shock by this means under practical conditions.

In cases where toll or trunk lines are exposed, an acoustic shock reducing device which could be placed at the ends of the lines would have the advantage of protecting subscribers as well as operators. Development work to obviate certain difficulties in using such a device is under way.

An effort is being made to develop a telephone receiver which will saturate between the values of current required for effective speech transmission and values of current which produce acoustic shock. This requires a sharp bend in the saturation curve of the iron employed in the receiver magnetic circuit. Until the development of permalloy, this feature was not approachable, but experimental permalloy receivers have now been developed, and, while it has not yet been possible to achieve the end sought without serious sacrifice in transmission, work along this line is continuing.

Improved Insulation. A slight reduction of susceptiveness to interference by low-frequency induction could be secured by providing increased dielectric strength to ground in communication circuits and their associated apparatus. Another method would be to insulate or isolate all conducting parts of the communication system so as to prevent contact by employees or others with wires or apparatus which may carry a dangerous voltage. Neither of these appears practicable at this time.

Drainage. Drainage is a method for controlling the parts of the circuit in which the induced voltages appear and causing these voltages to be consumed in those parts where they are least harmful. This is accomplished by connecting the telephone conductors to ground, preferably through balanced impedance coils, at certain points throughout the exposure. Assuming low resistance grounds at the drainage points, the resulting voltage to ground at such a point after drainage is established is limited to a value corresponding to the voltage drop over the impedance of the coil and ground connection. If this impedance is small compared to the other impedances in the drainage section, the voltage to ground at the drainage point is a small part of the total voltage induced in that section.

Under present conditions, the application of drainage is limited to special situations where interference with circuit testing and maintenance is of relatively minor importance and where superposed d-c. telegraph and carrier telephone are not used.

Neutralizing Transformers. The neutralizing transformer is a device for introducing into an exposed communication wire a voltage in opposition to the voltage induced by the disturbing circuit, thereby to a certain extent neutralizing the latter. The neutralization is effected by means of transformer action, the primary coils of the neutralizing transformer being connected to conductors which are grounded at the terminals of the exposure (or section of exposure), so that the voltage induced in these conductors will send currents through the transformer primaries. These primary currents induce in the secondaries of the transformers voltages substantially in opposite phase to the voltages induced in the telephone wires by the power circuit. The secondaries being connected in series with the exposed communication wires, the neutralizing action is obtained.

On account of introducing crosstalk and adversely affecting telephone transmission and carrier, application of neutralizing transformers has been confined chiefly to telegraph circuits. No applications of these devices to power line exposures have been made. They are, however, being studied by the subcommittee to see whether the objections mentioned above can be overcome and to determine their possible field of application.

Shielding. Shielding on a telephone line may be effected by special grounded conductors, by working conductors, or by cable sheaths. Miscellaneous structures such as pipe lines or rails in the immediate vicinity of an exposure also introduce more or less shielding. The employment on a telephone line of a high conductance shield wire, well grounded at the ends of the exposure and at intermediate points, may reduce the induced voltage by as much as 40 per cent at a frequency of 60 cycles. As bearing on the prevention of electric shock from induced voltages on telephone lines, shielding has a disadvantage in that it may, depending somewhat on the method of construction, add to the chance of a lineman making contact with grounded metal.

Use of Cable. A metallic sheath enclosing the conductors of a cable is a type of shielding. The lead sheath of a 2⅝ in. diameter aerial telephone cable, if effectively grounded at the ends, as when directly connected to an underground cable sheath, reduces the voltages induced in the conductors within the cable by about 50 per cent at 60 cycles. The additional shielding brought about by the surrounding earth when such a cable is placed underground is negligible at low frequencies, although underground construction has an advantage in affording a low-resistance ground for the sheath. The large number of conductors in a cable affords mutual shielding which varies from a negligible to a considerable amount depending upon many factors, important among which is the extent of the cable beyond the ends of the exposure. If two or more cables are close to one another through an exposure, each benefits by the shielding action of the others, so that the shielding increases with the number of cables.

If the lead sheath of the cable is surrounded by magnetic material as by armoring or placing cable in iron pipe, the shielding may be largely increased. With the form of iron tape armored cable referred to in the Harrison-Silver paper, which is now in trial use, shielding at 60 cycles is about 80 per cent, assuming effective grounding. Armoring a cable increases its cost substantially but has an advantage apart from shielding in that the cable being protected by the armor against mechanical injury may be buried directly in the earth without conduit. The armor is protected by impregnated wrappings but its life has yet to be determined. The shielding afforded by this type of cable has been studied experimentally under practical field conditions. Other installations and studies have been made abroad. It is probable that there may be a field of use for this type of cable in situations for which it is best adapted.

Coordinated Location of Lines. Since the magnitude of induced voltages for given power line conditions depends upon the inductive coupling of the two classes of lines, which in turn is dependent upon their relative location, particularly their separation and length of parallelism, it is possible by advance cooperative planning of new power and telephone line locations to minimize and in some cases to forestall inductive effects in the telephone system. If the cost of remedial measures which inductive exposures would render necessary can be avoided, additional expense in locating lines to avoid such exposures may be justified and where a complete solution is obtained in this way both parties secure greater freedom in the construction and operation of their lines. However, with the rapid expansion of both services, the possibilities of complete solution by separation of lines alone are becoming more and more rare, particularly for lines along highways.

Coordination of Grounding Practises. The occurrence of a fault on a power system usually results in raising the ground potential at the points of grounding as well as at the point of fault, but if steps are taken to coordinate the grounding of the power system and the telephone system serving the power company, particularly at transformer and generating stations, the effects in the telephone system of the earth potential gradient caused by a power fault may be minimized. For example, if in a switching station the same ground should be used for the power system neutral and for the telephone system, a power fault might cause the switching station ground to rise many volts above the distant telephone exchange ground, and result in operating the telephone protectors and possibly interrupting service. If, however, independent grounds sufficiently separated are used at the switching station, or an insulating transformer is placed in the telephone circuit, the power neutral ground may rise in potential without unduly affecting the telephone system.

Comprehensive consideration of the low-frequency coordination problem involves a study of the reactions between the grounding practises employed by power companies and those employed in telephone and telegraph systems. There is considerable diversity in practise with respect to methods of grounding. Some power transmission lines and primary distribution lines are not provided with any designed grounds, although most such lines have grounded neutrals and a few lines are grounded in such a way that operating current flows through the earth. In built-up communities there are underground pipes, cables, and other structures along which current in the earth will flow to a greater or less extent. These structures have varying degrees of conductivity and some of them have, either by design or by accident, high-resistance joints. Consequently the paths of earth currents are exceedingly complex. The conditions as to earth currents and earth potentials necessary to be known in order to work out any coordinated scheme of grounding would usually have to be determined by tests.

The different kinds of grounds to be considered include those on: power transmission circuit neutrals, lightning arresters, power distribution primary neutrals, power distribution secondaries, railway systems, building conduits, telephone protectors, batteries, ringers, telegraph circuits, lightning rods, electrolysis protection systems, various types of signal circuits such as fire and police alarm systems, and so on. The grounding practises for all these different systems should be carefully studied and coordinated in order to prevent so far as possible harmful reactions among them. Such a study of course goes considerably beyond the scope of this subcommittee.

Comparison of Different Protective Measures. The ideal protective measure would be one which furnished adequate protection and had no unfavorable reaction from an economic or service standpoint on the system to which it is applied. However, the work thus far has not disclosed any measure which fully meets this ideal.

The relative advantage of different measures resolves itself into a question of the best technical results which can be obtained at the least over-all cost. The solution of problems consists of finding measures which afford the highest degree of protection which is practicable and reasonable under the circumstances. In the investigation of a specific case it may be found that certain protective measures can be combined with other work in such manner that the cost is not wholly chargeable to coordination for the reason that other results of value are secured. For example, shielding may be obtained at small cost if improvement of performance of a transmission line justifies the installation of ground wires; or, the benefits of shorter duration of induced voltage by the use of high-speed circuit breakers and high-speed relays may be secured in connection with a program for improving the stability of power systems.

No other measure affords such complete protection against all effects of induction as adequate separation. However, measures applied to power systems such as fault current limitation which strike directly at the source of low-frequency induction are of a basic character and permit a closer association of the two classes of lines, a very important consideration in congested areas. Measures which affect only the frequency of occurrence of faults, or their duration, while very helpful, are not as effective from a protection standpoint as measures which limit the magnitude of residual currents and voltages.

As to measures which would allow telephone circuits to operate through a strong inductive field, the use of lead-sheathed cable surrounded by magnetic material seems to offer the physical possibility of affording the most effective protection. Precautions would be required, however, to prevent the shielding structure itself from rising to a dangerous potential with respect to earth. On open-wire lines where the occurrence of high induced voltages cannot be prevented, some form of protector for limiting the magnitude of voltage to ground seems to be a logical line of development.

Devices such as acoustic shock reducers, which protect only against a single effect of induced voltages, do not afford a solution of most specific situations, but have to be used in combination with other protective measures. In many situations, no single protective measure is adequate and if the exposure is severe several may be required.

In considering the effects which a new exposure may produce, all the relevant factors are capable of advance determination except frequency of occurrence of induced voltages, which has to be estimated on the basis of experience or judgment and a statistical analysis of line failures.

Selection of measures to be employed in specific cases should be made with the above considerations in mind to the end that the best engineering solution may be obtained irrespective of whether the protective measures are applied to the telephone system, to the power system, or to both.

Reaction of Physical Exposures and Lightning on Low-

Frequency Induction Problem. As telephone circuits which are exposed to induced voltages may also be exposed to possible contact with power circuits and to lightning, any comprehensive scheme of protection must take into consideration the high currents resulting from contact and the high voltage due to lightning. In this connection there are some points of difference in the reactions on the protection scheme of induction, contact, and lightning.

Contacts between power and telephone wires may occur at crossings or conflicts or they may occur on joint pole construction as described in the paper by Messrs. Huber and Martin. In any event such contacts can occur only where the two lines are in close proximity, whereas in cases of inductive exposure, a fault outside as well as inside the exposure may produce disturbances in the telephone circuits. Moreover, in cases of contact, wire or structure failures are generally involved while faults may cause induction which do not involve falling wires. Contacts impose on the telephone line the full voltage to ground of the power conductor at that point, whereas induced voltage is usually only a fraction of the power circuit voltage. This does not mean that the imposed voltages due to contact are always higher than those due to induction, because the majority of exposures to contact do not involve the higher voltage circuits while the opposite is true regarding inductive exposures. In cases of contact only part of the wires of the telephone line are usually involved whereas in the majority of induction cases substantially the same voltage is induced on all the wires. The voltages imposed on a telephone line by contact as well as those by induction may extend over the full length of the conductors involved.

In addition to the effects of contact between wires of the two systems, there is a distinct class of hazard to linemen of both utilities introduced by situations of insufficient clearance due to improper construction or inadequate maintenance on the part of one or both utilities.

Voltages on telephone lines by lightning produce effects somewhat similar to the effects produced by power lines but lightning voltages differ from the other voltages in that their duration is much shorter. Lightning makes necessary protector discharge gaps of very high speed of operation in order to prevent serious overvoltages on the telephone system, whereas contacts with power circuits make necessary a protector of high current-carrying capacity.

COMMITTEE'S PROGRAM OF WORK

The program of work on low-frequency induction undertaken by the Joint Subcommittee on Development and Research through its project committees is laid out to develop the essential facts bearing on the problem of telephone protection in a broad sense, including causes, effects, and remedial measures. The program covers not only the technical but also the economic aspects of the problem. The problems of lightning and physical contact under conditions of conflict or joint use are also included, as the measures finally adopted must protect against voltages from these sources as well as voltages induced by power systems.

Extensive field trials of all promising protective measures, are under way in order to determine their practicability under operating conditions. As the work progresses, it is expected to issue from time to time reports covering the applicability, efficacy, limitations, and conditions of use, of various measures. This should result in a better understanding of the problem and more effective and economical solutions of specific situations as they arise.

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Part IV—Status of Cooperative Work on Joint Use of Poles

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Synopsis.—*Because of the necessity of reaching the same customers, electric supply and telephone lines commonly use the same streets and highways. In urban communities, the joint use of poles for these two services has been very widely adopted and practises for joint use construction have been established from experience gained in past years. In rural communities, joint use is not always practicable or economical. Joint use involves many engineering and economic problems which have received the careful consideration of the Joint General Committee of the National Electric Light Association and Bell Telephone System.*

This paper describes some of the problems which have been encountered in joint use, and briefly outlines the work which is being conducted by the Joint General Committee in connection therewith.

It is concluded that in specific cases proposed for joint use all factors should be studied cooperatively by the companies concerned and that everything practicable should be done to facilitate joint use construction and extend its usefulness.

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TELEPHONE and electric light and power services are supplied in the same areas and to customers who are to a large extent common to both utilities. It is therefore necessary that both types of service be carried along the same streets and highways.

Experience has shown that safer and more satisfactory conditions can often be secured if the power and telephone circuits are carried on the same poles. This is due in part to the fact that clearances and climbing space can be more readily maintained where both classes of circuit are carried on the same poles rather than on separate poles on the same side of the street. Where separate lines are placed on opposite sides of the streets and alleys, it is difficult to secure and maintain proper clearances for service wires to buildings where these cross the line of the other utility.

Joint use of poles by the power and telephone companies results in the use of fewer poles on streets and highways and better appearance of aerial lines. It is, therefore, more desirable from the public point of view. It conserves pole timber and in many cases is more economical to both classes of utility than separate lines.

Because of the above mentioned advantages, joint use of poles by power and telephone companies has been widely adopted. No complete data are available as to the extent of such joint use at the present time, but it is estimated that there are at least five million poles jointly used by the power and telephone companies in the United States.

Both of these classes of utility have been growing rapidly in the past twenty-five years and the development, design, and construction of the physical plant of each has kept pace with the growth in territory and number of customers served.

While earlier types of distribution plant were such that the possibility of contacts between wires of the two utilities and other hazards could be satisfactorily met by proper construction methods, protective devices, etc., later developments have increased the use of types of power distribution circuits regarding which questions frequently arise as to how service can be properly maintained and extended on jointly used poles.

These questions have received and are receiving careful consideration by the Joint General Committee of the National Electric Light Association and Bell Telephone System. This committee has recommended certain principles and practises for the joint use of wood poles which are intended for use as a basis on which electric supply companies and communication companies should work out their mutual problems and has undertaken important research work in connection with these matters through its Joint Subcommittee on Development and Research.

The principles and practises mentioned were presented in a report of the Joint General Committee under date of February 15, 1926, and while it is beyond the scope of this paper to consider these principles in detail, the following recommendations are of interest in that they indicate the way in which this matter is generally being approached:

Each party should:

(a) Be the judge of the quality and requirements of its own service, including the character and design of its own facilities, both now and in the future.

(b) Determine the character of its own circuits and structures to be placed or continued in joint use, and determine the character of the circuits and structures of others with which it will enter into or continue in joint use.

(c) Cooperate with the other party so that in carrying out the foregoing duties, proper consideration will be given to the mutual problems which may arise and so that the parties can jointly determine the best engineering solution in situations where the facilities of both are involved.

It will be observed that while each party retains full responsibility for facing and meeting its own problems, it is recommended that both parties cooperate in working out mutual problems involving the joint use of poles and in finding the best over-all engineering solution in

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each situation. These are among the most important of the principles which have been recommended and are the basis upon which practically all cooperative work is being carried forward.

It is the purpose of the following paragraphs to describe what has been done and what is being done by the Joint Subcommittee on Development and Research in connection with the engineering and economic problems which have been encountered in joint use work.

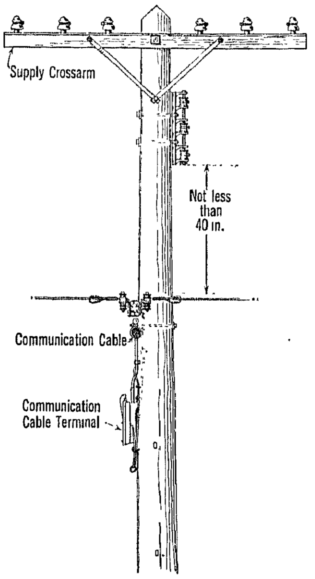

Fig. 1—Typical Jointly Used Pole

Construction Practises

Joint use construction practises have undergone almost continual change and improvement from the time joint use was first adopted and continued development is to be expected in the future. However, many of the fundamental requirements for securing satisfactory conditions on jointly used poles were recognized at an early date and form the basis for present day practise.

In the matter of relative levels it has been recognized that power wires should as far as practicable be carried in the upper position. In general, they are larger and stronger than the telephone wires. This is inherently so because of the current carrying capacity required. Placing power wires in the upper position on jointly used poles avoids the necessity of telephone linemen climbing through power circuits, the exact nature and characteristics of which they are not always familiar with.

Clearances must be provided which give sufficient space below the power wires so that power linemen will not have to come in contact with telephone wires while they are working on power wires. This neutral space must also provide sufficient clearances above the telephone equipment so that telephone linemen may work on the telephone plant without danger of coming in contact with power equipment. Clear climbing space must also be provided so that linemen may climb poles without having to be extremely careful to avoid falls or contacts with circuits from which they may receive physical injuries.

Fig. 1 shows one method for securing satisfactory conditions on a jointly used pole carrying circuits which both the power and telephone groups have recognized as being suitable for joint use.

In the matter of mechanical strength, joint use follows the practise in the construction of separate lines. That is, strength of construction should be provided such as to stand, with reasonable factors of safety, storm conditions which experience indicates are likely to occur from time to time in any particular area.

With regard to the matter of insulation and electrical strength, practises as to the size and type of power insulators have followed developments in the general field of power construction. Wires to street lights and underground connections to aerial plant require vertically run wires on jointly used poles. The location, insulation and mechanical protection of these have received special consideration to eliminate hazards to workmen.

Sufficient clearances between vertically run circuits of one type and the equipment of another utility on jointly used poles have also been found to be very important from the standpoint of avoiding interruption to power and telephone services.

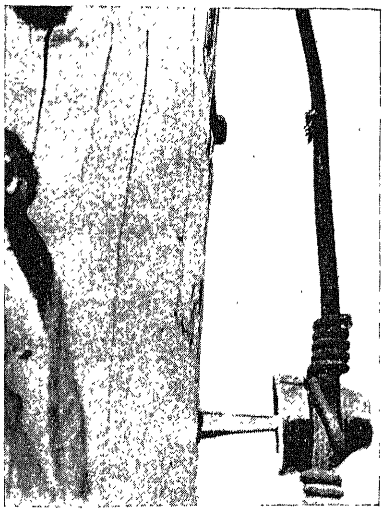

Fig. 2—Frayed Insulation Showing Breakdown of Insulation Between Power and Telephone Plant*

In the course of electrical storms, lightning may induce high voltages on either supply or communication wires. If the separation between the supply and communication facilities is not adequate at any point, these induced voltages may break down the insulation and arc between the two as illustrated in Fig. 2. Damaged plant may, of course, result from lightning alone. However, when lightning has established an arc between the power and communication circuits the normal voltage of the supply circuit may maintain the arc. This results in the transfer of power into the telephone plant at voltages which may be well above that for which it is insulated and may cause trouble on both the power and telephone system. This sort of abnormal belongs to the general class that includes insulator flashovers, short circuits on cables, tree grounds and similar power system occurrences that always carry the probability of damage to the power system or service.

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*In order to obtain a satisfactory photograph of the points of arc the vertical drop has been straightened out so that the clearance shown is much greater than existed at the time of the arc.

While vertically run attachments with improper clearances have played a large part in causing such occurrences, any situation where insufficient clearance between power and telephone facilities is provided may result in similar trouble.

Emphasis has, therefore, been placed in present day standards on the necessity of maintaining proper clearances as well as strength of construction to prevent this kind of abnormal. Experience has shown that where these clearances are adhered to this type of abnormal is kept to a reasonable minimum.

The Joint General Committee is giving careful consideration to the matter of construction standards on jointly used poles. Pending the development of complete specifications covering recommended practises under various conditions, they have recommended the National Electrical Safety Code to be used as a guide to practise.

PROTECTIVE DEVICES

Both telephone and supply circuits are equipped with protective devices which are fundamentally the same in principle. They may be divided into two general classes:

1. Those which provide protection from abnormal voltages consisting of protector blocks in the telephone plant and lightning arresters in the supply system.
2. Those which provide protection from abnormal currents consisting of heat coils and fuses in the telephone plant and fuses and circuit breakers in the supply systems.

These protective devices are a secondary defense against abnormal conditions which it is impracticable to avoid either by design or through adherence to construction standards.

Even when all practicable precautions with regard to clearances, strength of construction and insulation have been taken, accidental breaks occur in both power and telephone wires. In some cases there are direct contacts between such wires. Higher than normal potentials are also introduced into the telephone and power circuits by lightning and other causes.

It is because of the limitations of protective devices and other protective measures that joint use with certain types of circuits has been in question. Considerable differences of opinion exist between engineers as to the degree of hazard involved in joint use between telephone plant and power circuits of various types, voltages, and connected power. The problem has increased in importance as the use of higher distribution voltages and greater generating capacity have been employed.

This matter is under investigation by the Joint Subcommittee on Development and Research. Studies are now in progress in one rural area and in one suburban area to determine the over-all advantages and disadvantages of the use of higher distribution voltages and of joint use with these voltages under present conditions.

The first experimental work done by the Joint Subcommittee in connection with these problems was a detailed study of the characteristics of various types of fuses. This study covered all of the well-known commercial types of telephone fuses and a number of experimental models. The operating characteristics of these fuses were obtained at voltages of 2,300, 4,000, 7,500 and 13,200. The current range was from 16 to 1,000 amperes. These tests were carried on in a laboratory where 20,000 kva. of generating capacity was available. The tests showed the dependability that could be placed upon the various fuses for interrupting voltages of the range from 2,300 to 13,200 volts. They showed under what conditions the fuses could be depended upon and the ranges where the available type of fuses could not be depended upon for safe operation.

A number of the experimental models showed considerable promise of improvement over existing models, and this work will be carried further to determine what improvements can be made in the operating characteristics of fuses.

The next phase of the problem taken up included a study of the operating characteristics of various types of overvoltage protectors suitable for use on communication circuits. The experimental work covered breakdown with direct current, 60-cycle alternating current and a complete study with a cathode-ray oscillograph of the behavior under steep wave fronts for carbon block protectors, neon and vacuum tubes.

These tests showed that the carbon block protector has a breakdown point with all types of applied wave fronts which is sufficiently fast and low to protect the insulation that is now used in the communication plant, as shown by similar tests on condenser and cable paper. The shortcomings of these blocks lie in their tendency to permanently ground the circuit when carrying current for any appreciable length of time.

The tests with steep wave fronts were carried to a rate of rise of 36,500 volts per microsecond, and it was determined by tests of propagation of steep wave-front voltages through telephone cable that it was practically impossible to subject the plant to voltages with any faster rate of rise than those used in the protector tests.

The problem of adequate protection of the telephone plant in joint use, obviously, cannot be solved by the development of the telephone protective devices alone. The protective devices in the power system are equally important.

One of the important functions of the power system protective devices is that of clearing power system faults in a reasonable time interval. Obviously, telephone protective equipment cannot be expected to prevent damage to telephone plant in case of contact between the wire circuits of the two utilities when power system protective devices fail to operate and the physical contact of the circuits is maintained over an indefinite period of time.

One problem in the development and research work is the fixing of the part that the protective devices on each system must play in abnormal conditions. It is necessary that the over-all protective equipment be adequate and that the burden of overcoming weaknesses in the protective equipment of one system be not thrown on the protective equipment of the other. There are inherent limitations in both classes of protective equipment that must be defined.

Therefore, the next step in this investigation is a determination of the over-all characteristics of power circuit and telephone circuit protection under typical conditions of contact between the two plants.

While protective devices are an important element in connection with joint use involving certain types of power circuits employing the higher distribution voltages, there are also other important considerations. The general insulation of the telephone plant must also be considered, especially in connection with drop loops attached to and entering subscribers' premises. These matters are also being studied by the Joint Subcommittee.

All of these problems, as is the case of others being studied by the Joint General Committee, are being approached on the basis of determining the best over-all engineering solution such that both systems can provide their services in the most convenient and economical manner.

INDUCTIVE COORDINATION

In the early history of joint use, noise induction problems involving street lighting circuits appeared. Other interesting problems were encountered such, for example, as the accidental grounding of one corner of an isolated delta power system with its resulting unbalanced voltage inductive effects on open-wire telephone circuits, which type of telephone construction then predominated.

As these problems arose they received careful study and with the development and extended use of telephone cables and the use of improved operating methods in power and telephone distribution generally, inductive coordination of power and telephone distribution systems in the urban communities became less troublesome and did not for a time receive any large amount of consideration.

However, during recent years the introduction and extended use of various types of multi-grounded distribution systems described in the paper by Messrs. Harrison and Silver and the existence of certain types of signaling on local telephone circuits, have contributed toward making important the consideration of noise inductive effects in connection with joint use. This matter is discussed more fully in the paper by Messrs. Harrison and Silver.

The technical factors involved in inductive coordination problems under joint use conditions are complicated. The details regarding these factors and the results of the extensive studies of these matters by the Joint Subcommittee on Development and Research are described in the paper by Messrs. Wills and Blackwell.

The various operating problems which have arisen almost since the birth of the power and telephone industries and the investigations conducted by the Joint Subcommittee on Development and Research indicate the importance of giving careful consideration to the inductive coordination features of joint use and of including this factor in studies of the relative advantages and disadvantages of joint use as compared with separate lines. This factor should, of course, be considered from both its technical and economic aspects.

Much can be accomplished in the inductive coordination of the two distributing systems by cooperative advance planning. In urban areas where the telephone circuits are largely in cable, there is about a two to one ratio in the inductive effects between a joint line and separate lines across the street. In rural areas where the telephone circuits are largely open wire, the ratio of the inductive effects on joint lines as compared with separate lines across the highway, is much greater, other things being equal.

In urban areas the power and telephone companies can through cooperative planning frequently arrange to establish important power feeders and telephone circuits on separate streets and thereby avoid large inductive effects and permit more extensive joint use of branch lines. A careful review of the equipment used on the power and telephone circuits and the introduction of operating practises designed to limit the inductive susceptiveness of the telephone circuits and the inductive influence of power circuits, form an important part of advance planning and cooperation.

As described in the paper by Messrs. Wills and Blackwell, these latter factors include such items as limitation of the odd triple frequency series arising in

Y-connected generators feeding directly on the line and in single-phase service transformers. Suitable limitations of the unbalances existing among the loads connected between the three-phase conductors and the neutral, limit the ground return components.

Grounding of aerial telephone cable sheaths to provide for increased shielding and the use of central office and station equipment providing a higher degree of balance with respect to ground are helpful.

The matter of joint use may involve both rural and urban communities. It is more generally associated with the latter because of the severe limitations in physical space available for utility use. In the case of rural lines where the telephone circuits are largely in open wire and the exposures between particular circuits are likely to be long, joint use is not always practicable. In these cases locations for separate lines are usually available.

Furthermore, joint use in rural areas is not always economical from a purely construction standpoint due to the fact that relatively longer spans can often be used on the power lines and both utilites are able in many instances to use shorter and lighter poles than would be practicable in joint use. Joint use with telephone toll circuits or power transmission lines has not, in general, been found desirable. Types of construction vary so widely and service requirements and inductive effects are such that it becomes uneconomical to carry out such construction.

CONCLUSIONS

Joint use of poles by power and telephone companies has many advantages, both from the standpoint of the public and from the standpoint of the wire using companies. This is especially true in built-up communities.

Important problems brought about by developments in practises, particularly in the use of high-voltage distribution, remain to be solved.

Careful adherence to generally accepted practises with regard to clearances, strength of construction, insulation and inductive coordination is necessary in order that the advantages of joint use can be secured.

In considering specific cases proposed for joint use, it is advisable that all of these factors be studied cooperatively by the companies concerned, to the end that good service, safety and economy by both classes of utility may be promoted.

It is important that everything practicable should be done to facilitate joint use construction and extend its usefulness. The Joint General Committee of the National Electric Light Association and Bell Telephone System is continuing its efforts in this connection.

Discussion

J. J. Smith: The paper by Messrs. Harrison and Silver is very interesting in showing how in spite of the approximate two-to-one growth in both the telephone and power systems in the last decade which necessarily brings with it a large number of cases of physical proximity, the difficulties of coordination have not correspondingly increased in this interval. Sometime ago I had occasion to review some estimates that were made in 1920 as to what the situation in 1930 would be. Looking backward it is now easy to see that these estimates were altogether too pessimistic and for this I believe the joint work of the N. E. L. A. and Bell System can take some of the credit.

Considerable discussion is devoted to conditions on the power system which do not seem to be representative of the more usual type of practise on power systems. I have in mind more particularly their references to the use of transformers with graded insulation and Y-Y transformers. The consensus of opinion of engineers with whom I have discussed the matter seems to be that if 220-kv. transformers alone are considered, a large proportion of the transformers are constructed as described in the paper. If, however, we consider systems 66 kv. and above the majority of the transformers have sufficient insulation on the neutral end of the winding to permit considerable impedance to be inserted between line and ground. With reference to the section on transformer connections I feel that by far the more common practise is the use of the delta-Y connection and that the Y-Y connection referred to is the exception rather than the rule.

From Messrs. Wills and Blackwell's paper it would appear that with reasonable balance on both power and telephone systems the problem of noise is not normally a serious one. The difficulties with regard to noise lie mainly in special situations involving unbalance on one or the other of the systems. Within the next decade it seems, at the present time, that we shall witness the introduction of vacuum tube devices on a fairly large scale. As Messrs. Harrison and Silver have pointed out, their uses up to the present time have not been sufficient to establish a definite trend. Experience so far would tend to show that they will present new types of problems, but there is reason to believe that with a continuation of the cooperative work described in this symposium these will be solved satisfactorily.

Attention may be called to the fact that some of the work being done by these committees is of interest in other fields. For instance, work on methods of measurement of noise is being carried on by other committees and societies in connection with machinery, street, and other types of noise. There is sufficient correlation among the various bodies working on this subject to ensure that they are all working along somewhat similar lines and also to avoid undue duplication of effort. The relation between the effects of lightning on transmission lines in causing outages and the problem of low-frequency induction is another example.

The subject of coupling between ground return circuits has always been a very difficult one and a large amount of theoretical work has been done on it, all directed towards estimating induced voltages by calculation. In some cases these calculations checked reasonably well with test and in others there was no agreement. As is pointed out by Messrs. Conwell and Warren for separations of one-half mile to one mile the coupling coefficients may vary as much as 20 to 1 due to the range of earth conductivity. I believe that at the present time in order to make a reasonably accurate estimate it is necessary to conduct tests in a given location. I should like very much to hear what hopes the sub-committee have of establishing an empirical basis such as they refer to in their paper for estimating coupling.

In the foreign literature on the subject suggestions have been made at various times as to the amount of energy required to produce acoustic shock. I wonder if the committee's work has shown any correlation or a complete lack of correlation between these quantities.

If one unfamiliar with the subject were to attempt to estimate the interest in the coordination of the power and telephone plant by reference to papers presented in the last ten years before the Institute or published in electrical journals, he would be led to conclude that little attention was being given to the subject, since the amount published has been comparatively small. This conclusion, however, would be entirely incorrect as is apparent to anyone familiar with the large expenditure of time and money on the program being carried out by the Joint Committee of the N. E. L. A. and Bell System. It is to be hoped that this symposium is only the start and that further papers giving more detailed accounts of the studies will be presented before the Institute in due course.

M. D. Hooven: There are apparent several outstanding points of interest to power company engineers who in the field must cope with these problems of coordination in their infinite form and variety. As Chairman of the Foreign Systems Coordination Committee, National Electric Light Association, I may, to some extent at least, present the general viewpoint of these engineers.

Most important is the educational value of the material presented. Nowhere is there available in such concise and understandable form any review of the whole problem as here presented, such a review filling an often expressed need. With particular reference to the problem of low-frequency induction, it has seemed evident during the past few years that this problem is becoming of increasing importance; not all of its ramifications are yet understood. The Conwell-Warren paper supplies a needed source-book of information. It is the intent of the Foreign Systems Coordination Committee to bring all the material of this symposium to the attention of every engineer working on coordination problems.

The last three papers of the symposium each stressed the importance of cooperative advance planning and the Harrison-Silver paper showed how interwoven were the advances of the art in each of the two systems. The Development and Research work will continue to bring forward facts and findings, but much of the good of these results will be lost if the two systems are not planned cooperatively. The Foreign Systems Coordination Committee, in disseminating the results of this work, is fostering the adoption of routines of cooperative advance planning in all the geographic divisions of the Association.

L. T. Robinson: When the experimental work of the Joint Development and Research Subcommittee was started in 1923 there was very little information available which would allow an attempt to be made to proceed in a logical manner from cause to effect in dealing with the problem of coordination. The California Railroad Commission Report of 1919 deals almost wholly with conditions on the power circuit and with coupling between the power and telephone circuits. There is practically nothing in it to allow a determination of the effects on the telephone system of voltage induced in it by external means. It was necessary therefore to develop apparatus for use in such work and this resulted in the various forms of wave analyzers, etc. referred to in these papers.

As a result of the more accurate measuring devices it was possible to accumulate data from different sources which might be correlated among themselves. The principal sources of such data were:

1. Tests made on both power and telephone apparatus under carefully controlled conditions to determine their characteristics when considered alone.
2. Tests made in the field, such as those given in Engineering Report No. 6 on the Minneapolis tests where the relation between the systems were studied.
3. Tests of a statistical nature such as the survey of wave shape on 34 systems and the noise measurements on paralleling telephone lines.
4. Investigation of field cases which arose from time to time in which the Committee cooperated in obtaining data.

As a result of this work I believe it is now safe to say that for a given situation, if the various influence factors are known, a reasonably close estimate of the conditions to be expected can be made. In order to obtain the influence factors for a given location certain data may have to be obtained by test. For instance, due to the variation of harmonics in different power systems it would be desirable to make a direct determination in any given case. Variations in ground conductivity may make individual tests necessary in a given location, also the conditions of parallelism between the power and communication system must be obtained from observation, maps or records.

For these reasons it is evident that it would be extremely expensive to study in detail every contemplated extension of both power and communication systems *a priori*. The more logical way to proceed is to set up reasonable standards for both power apparatus and telephone apparatus which will be such that, if they are met, satisfactory conditions will obtain in the average situation. Special situations will of course arise needing more detailed study but these can be handled individually. The setting up of such standards should be based upon the most economical scheme of supplying power and communication to the public. Advantages have come from general acceptance of such an idea without the fixing of limits for either kind of plant equipment or apparatus. These may, perhaps, advantageously follow in due course. Using the means and methods which have been developed during the work of the Development and Research Committee there has been, as pointed out in the papers, marked progress made.

Reference to the paper by Messrs. Harrison and Silver shows that the number of telephone and power customers has about doubled within this 10-year period. Both telephone service and power service has been improved. It cannot of course be claimed that this improved situation, in spite of the increased congestion in many areas, is due entirely to the work of the Committee but this work has played an important part in the results.

It now appears that we are faced with a similar problem in the relation of the use of radio to other electrical industries. There is some tendency at the present time to expect that the whole burden of reducing troubles with radio reception should be borne by other industries such as by adding corrective equipment or redesigning the apparatus. However, we again find the three factors—"influence factor," "susceptiveness factor," and "coupling factor"—have the same importance and that an economic solution may involve not only changes in the electrical distribution system but also in the radio broadcast and receiver system as well.

Steps looking toward a cooperative study of this situation are now being taken and it is felt that the results will be as successful as the work described in connection with the coordination of power and telephone plants. The degree of success will depend in my opinion upon the extent to which general principles so successfully applied to the problems discussed in these papers are followed.

H. S. Phelps: Frequently one of the most difficult tasks confronting those responsible for the administration of coordination studies is that of acquainting their executives with the fundamentals surrounding the problem. The paper by Messrs. Conwell and Warren should be most helpful to one called upon to outline the essential features of low-frequency induction.

Speaking of several of the factors affecting the problem of low-frequency induction, I am glad to see that attention has been called to the importance of the frequency of occurrence of the induced voltages as well as their duration.

In studying specific problems these factors must be given serious consideration. It is entirely unreasonable to presume that the same coordination measures should be expected or desired in situations involving the most sturdy types of construction, built for maximum service reliability, that might be justified where less reliable transmission systems are involved.

In determining the importance of the effects produced by induced voltages due consideration must be given to a number of factors. Among these are included:

Character of communication service normally rendered over the lines involved in an exposure situation and the degree to which it might be affected adversely by abnormal potentials.

The likelihood of damage to physical plant as the result of abnormal potentials.

The physical hazard that might be imposed on employees at times of abnormal potentials.

Outlining the chief characteristics that determine the magnitude of residual currents and voltages should be particularly helpful. Making specific reference to such items as fault and earth return impedance and also the character and arrangement of ground wires immediately makes it evident that they are important factors.

Previous to the work that this symposium is reporting, information concerning fault and earth return impedances was extremely meager. As a result, there has been much controversy during many coordination studies resulting from attempts to name a mutually satisfactory value of fault impedance for substitution in the classic equations for determining "fault current." There is little wonder that such has been the case when values have been recommended in good faith that varied in value all the way from zero ohms to fifty ohms.

In pointing out some of the factors that have a material influence upon the inductive coupling between transmission circuits and communication circuits attention has been called to the important part played by earth conductivity. Also, there is a rapidly growing recognition of the widely varying earth conductivity conditions. That computed and observed values of inductive coupling differ by no more than a ratio of two to one is truly a matter of surprise when it is realized that earth conductivity conditions vary by as much as fifty to one within a distance of a few miles in some localities. On the other hand the earth conductivity conditions may remain practically constant throughout the area served by an extended power transmission system.

As the result of all these things marked changes have taken place in the procedure for calculating inductive coupling. Whereas it was formerly the practise to substitute more or less arbitrary values for earth conductivity generally based on personal opinion for the coefficients in the equations for calculating coupling, recent developments include a preliminary field study to determine earth conductivity values that apply in particular situations. The new method makes possible much more reliable data, from which conclusions may be drawn concerning the probability of experiencing trouble as the result of projected exposures.

Now that the benefits of ground wires are being given more serious consideration, as the result of the careful studies of the lightning protection problem, the benefits that may be derived from them in the coordination problem should also be taken advantage of wherever possible. While ground wires do increase the magnitude of fault currents, still at the same time they provide, in general, a decided net benefit through their shielding effects. The benefits to be derived through the use of ground wires warrants additional study to determine more fully the influence on their effectiveness of such factors as conductivity, resistance of their earth connections, location with respect to the power conductors and their influence on the coordination of systems in situations where currents of large magnitude are being transmitted.

Very serious consideration should be given the recommendation that advantage be taken of cooperative advance planning. Such studies invariably result in better appreciation by all of the parties at interest of the problems with which the individual parties are confronted. Also, such cooperative efforts develop ways and means for solving the problem in a manner that is mutually satisfactory.

Livingston P. Ferris: Among the factors "blamed" for the inductive effects of a power system on a communication line, we find the earth itself, particularly in the case of low-frequency induction. Messrs. Conwell and Warren first blame it for being a conductor at all, as they indicate that the problem would vanish if the earth were an insulator. Furthermore, they point out that inductive effects may differ by 20 to 1 due to differences in earth conductivity between different locations, other conditions being the same. Of course, we have to take the earth as we find it, and it would be as well to consider how it may affect our practical problem.

Assume, in a region of high and homogeneous conductivity, that we have power and telephone lines parallel to one another at a separation of 100 ft., and for just enough length so that with a ground on the power line the voltage induced in the telephone line reaches a value, beyond which we agree that some remedial measures should be taken to control it. Adhering to this same value of induced voltage and length of exposure, what then is the effect of diminishing earth conductivity on the permissible closeness of these lines, without resorting to any remedial measures? This is shown graphically by the curve on Fig. 1, using scale A. If we start off at a wider separation, say 1,000 ft., giving, of course, a lesser induced voltage per unit of length, and correspondingly, a longer exposure, the effect of diminishing earth conductivity is shown by the same curve, using scale B. The range of conductivities on this chart corresponds to conductivities which have been met in practise. Thus, to compensate for differences of earth conductivity within the practical range, our lines may have to be separated 3,000 ft., instead of 100, surely of great practical moment. If it were possible to superimpose on this chart a cumulative plot of the number of telephone lines within different increasing distances from power lines throughout the country, this would be even more forcibly evident.

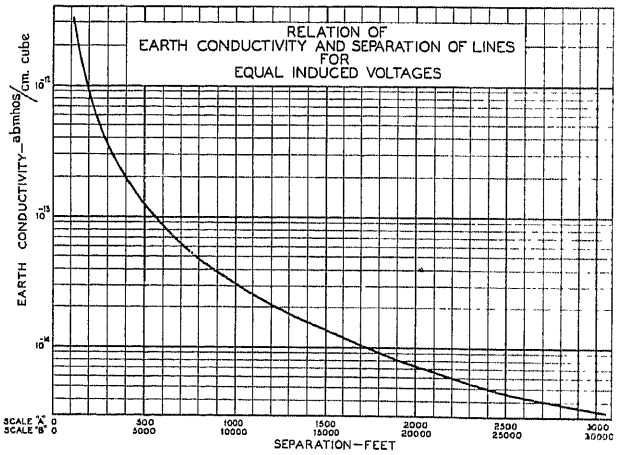

Fig. 1

In many practical cases, therefore, the answer to the question—"To be, or not to be—an exposure?" lies in the bosom of the earth, which we are powerless to change. But we can provide more information concerning earth conductivity, its variation with conditions and localities, and convenient means for predetermining its effective value in specific cases. With an appreciation of the need and importance of this information for practical purposes, I am sure that both power and telephone engineers will further lend their aid and support to the committee in its researches on this problem.

H. M. Trueblood: In the presentations and discussions of the papers, several allusions have been made to the wide range of subjects which the Joint Subcommittee on Development and Research has had to cover in its studies. The division of the work is such that some of the project committees are engaged upon questions relating very largely to telephone systems, while others are giving their attention to questions which concern power systems almost exclusively. Each project committee is made up of members representing both industries, so that the natural and desirable result has been that the power and telephone engineers who carry on the detailed technical investigations are getting well acquainted with each other's problems.

A good example of this is to be found in the work of the project committee assigned to study the possibilities of neutral impedance as a measure of coordination. It is true that one or two points affecting the value of this device in reducing induction require some further study. For example, there is the question of the relative importance of induction from unbalanced voltage and charging current, both of which tend to increase as neutral impedance is increased, and of induction from short-circuit current through the neutral, which decreases with increasing neutral impedance.

There is also the matter of the relative advantages of different types of neutral impedance from the standpoint of inductive coordination in the voice-frequency range. But, as the paper by Messrs. Conwell and Warren points out, the direct effects of neutral impedance in reducing induced voltages are not primarily the objects of our study, since to a large extent they can be calculated with all the precision necessary in practical work. On the contrary, the principal questions involved in the use of neutral impedance are those arising from its reactions on power system operation, such as the possibilities of overvoltages, of relaying difficulties, of improvements in system stability, and of reductions in stresses on equipment or in damage due to short-circuit currents.

While it is obvious that some of these reactions upon the operation of the power system will, in turn, react upon the inductive coordination problem, their larger aspect is in their relation to the functioning of the power system as such, and it is here that the subcommittee's work in connection with neutral impedance is now mainly concentrated.

These questions, of course, are of great interest to the power industry, apart from their bearing on inductive coordination, and the subcommittee has received much valuable assistance from power operating and manufacturing companies and from other committees working on similar questions. The continued support and cooperation of the industry is necessary in this work, since to a large extent practical results can be arrived at only through observations and tests on operating systems.

D. H. Gage: In the telegraph field of communication we are particularly concerned with the problem of low-frequency induction. We find in practise that our telegraph and printer circuits, where exposed to paralleling power lines, are not only interrupted during abnormal conditions on the power circuits, but are subject to interference from what might be termed normal induction.

These results are sometimes very puzzling and it seems that there is a field for investigation where unexpected interference results from an exposure of a communication line to an apparently well-balanced power circuit with very little current in the neutral.

Then again there may be a number of comparatively short exposures at different places on the same line, none of which alone would cause serious interference, but the results of the different exposures may combine in such a way as to materially affect the operation of our circuits.

We find in the papers that double faults occur with greater frequency on systems isolated from the ground than do double faults on systems with grounded neutrals. In case of a double fault on a non-grounded system do the faults occur simultaneously or does one follow the other and is the second one a result of the first, if so, why?

It is generally conceded that the joint use of poles by power and communication companies has advantages.

In the consideration of this problem the committee has apparently given a great deal of attention to the engineering and economic problems involved, but there is one phase of this joint use problem that is apparently not considered an engineering feature. Perhaps it has no place in an engineering study, but it is an important part of the problem and there should be no one better qualified to pass on it in some respects than the engineer. It is the problem of joint line agreements.

There is usually little difficulty in reaching an agreement on the questions of construction specifications, clearances, or division of costs. The character of circuits on jointly used poles is one for discussion. In the present state of the art it is generally conceded that joint use with certain types of circuits is undesirable, but the art is changing all the time and who can say what the future will develop? Furthermore, while the characteristics of the power circuits may be acceptable to the communication companies for joint use today, future developments may require the power companies to change the character of its circuits, what then shall be the procedure?

Again, take the question of liability, what is a fair provision in case of injury to person or damage to property resulting from joint use of poles? These are questions that should be covered in a joint use agreement and while they may be outside the scope of an engineering study, is it not reasonable that the engineers should give these questions careful consideration.

A. E. Silver: Mr. J. J. Smith, in discussing the sections dealing with Y-Y connections of transformers and graded insulation of transformers and apparatus, has expressed a preference for the delta-Y connection, and questioned the extent of the use of graded insulation in equipment now in service.

It was not intended by the authors to bring into question the theoretical preference of the delta-Y connection as compared to the Y-Y connection. However, in making the selection in any situation, due weight must be given to the nature of the apparatus and other existing power system features with which new equipment must coordinate. As indicated in the paper, the trend in the larger units, particularly those associated with transmission lines, has been toward the use of the delta-Y connection, or its equivalent, the provision of the tertiary winding. Nevertheless, in the distribution field, this trend is less marked and there is extensive and satisfactory application of the Y-Y connection, as for instance the widely used 2,300/4,000-volt system, in which single-phase lighting units distributed between the different phases of the primary constitutes the equivalent of the Y-Y connection.

As to the extent of the presence of graded insulation in various types of existing apparatus, I am unable to give a definite figure. However, graded insulation is present in many systems and in those cases, constitutes a limitation on the use of the neutral impedance.

Notwithstanding steady growth in capacity and in spread and in intimate intermeshing of the plants of the communication and power industries, the better understanding of the problem on all sides that is coming from the cooperative treatment in general and the Development and Research work in particular, will continue the trends firmly in the direction of more simple and effective coordination.

H. L. Wills and O. B. Blackwell: The third paragraph of Mr. J. J. Smith's discussion which refers to the Wills-Blackwell paper, is interesting in two ways. The first part of it might lead to the impression, while undoubtedly not intended by its author, that little remained to be done on the subject of noise induction and that the remaining difficulty was principally a matter of balance. While the unbalance difficulties stand out particularly we should not lose sight of the other factors and their relation to unbalance. For example, if the wave-shape distortion increases, the balance must, in general, be correspondingly improved to obtain an equivalent degree of coordination. In power distribution systems, particularly, this may introduce a considerable handicap on account of difficulties with single-phase extensions and branches. Furthermore, wave shape needs careful consideration in connection with new developments in the art. The reaction of large mercury arc rectifiers on the wave shape of power circuits from which they are supplied is a case in point. In certain situations the effects of these loads have created difficulties on telephone lines exposed to single-phase branches as supplied from the same power system.

Mr. Smith mentions in the latter part of this same paragraph another interesting development in the possible introduction of vacuum tube devices in power systems on a fairly large scale. We can, however, all agree with him in taking a hopeful view of the situation based on the continuation of the close cooperative work in solving the new problems as they arise.

H. S. Warren: From a technical standpoint the most interesting feature of the low-frequency induction problem is undoubtedly the great distance to which the inductive influence of power transmission lines extends. It might be supposed that if a telephone line is separated from a paralleling power line by 1,000 ft., or in any event, by 2,000 ft., it would not be disturbed by the power line. However, both experience and careful experimental tests have shown that such a supposition is not correct and that substantial induced voltages are experienced in some cases where the separation between lines is a mile or more. It also is of great technical interest that the coefficients of induction between the parallel power and communication lines have widely different magnitudes in different places depending on the geological structure of the earth at the different locations. One of the most important problems in low-frequency induction therefore is the development of simple means for determining coefficients of induction.

In reply to Mr. J. J. Smith's question as to whether the Committee is able to hold out hope of being able to predict coefficients of coupling in new situations, I may say that where the lines are parallel at roadway separations, the coefficient of coupling may now be estimated, by assuming an average earth conductivity, with a precision not greatly inferior to the precision with which we may estimate short-circuit currents. However, the dependability of our estimates of coupling falls off rapidly with increasing separation of lines, and for separations of several hundred feet or more, there are at the present time no means of making dependable predictions without some kind of actual exploration or electrical survey of the local conditions. Of course average values can always be assumed but in view of the wide variations of these coefficients I think it probable that some method of determining them locally will always be advisable in important cases. Of course, as time goes on, tests made in connection with specific cases, or for other reasons, are adding to our knowledge of the characteristics of the earth in different localities with respect to coefficients of induction, and this tends gradually to reduce the amount of specific testing work required in connection with new situations.

The Committee's problem is largely one of devising methods of making such determinations, which will be simple to carry out and the results of which will be easy to apply. The Committee is also interested in gathering data on the variation of effective earth conductivity with geographical location and geological conditions, in order that the correlation, if any, of these factors may be established. Such a known correlation would, of course, be of practical value in estimating coefficients of coupling. The above may serve also to answer the question asked by Mr. L. T. Robinson as to the possibility of setting up standards so as to avoid the necessity of studying each specific situation.

In reply to Mr. Gage's question, double faults on power transmission lines with isolated neutrals occur in sequential order, a ground on one phase raising the voltage to ground of the other phases, so that the insulation fails at some point where it is weaker than elsewhere. For practical purposes, however, particularly from the inductive coordination standpoint, the two faults in most cases may be regarded as simultaneous. The last sentence in the paragraph to which this refers would stand as written. The Committee expects to make tests with a view to determining more exactly the characteristics and properties of arcing grounds and other faults.

It appears to be a fact that the seriousness of voltages imposed on communication wires by induction is less generally recognized than in the case of voltages impressed on communication wires by actual contact with high-voltage wires. If a power wire breaks or otherwise makes contact with a telephone wire and thus imposes on it hundreds of volts, the seriousness of the condition is apparent to everyone, but that voltages of the same order of magnitude and capable of producing equally injurious effects, may be set up in communication circuits by induction, is not always recognized. This may be due in part to the fact that the introduction of induced voltage into a communication circuit is not accompanied by any visible effects such as the arcing and burning which often occur at points of contact. It is hoped that our paper will aid in bringing about a better understanding of the low-frequency induction problem and also that it may indicate to operating companies, at least in a general way, methods of protection against the effects of low-frequency induction.

Field Tests on Thyrite Lightning Arresters

Using Artificial Lightning of 1,500,000 Volts

BY K. B. McEACHRON* and E. J. WADE*
Member, A. I. E. E. Associate, A. I. E. E

Synopsis.—*The impulse characteristics of thyrite station type arresters of various ratings have been studied when connected to a 45-mile transmission line. The surges were supplied by a 1,500,000-volt impulse generator.*

The volt-time curves for arresters of various ratings were obtained under the conditions corresponding to the arrester on each incoming circuit, on the bus with two incoming circuits and on the bus with one incoming and one outgoing circuit. The oscillograms were found to check the calculations closely.

With the arrester 90 ft. from a nearby tower the effect of tying arrester and tower grounds together was studied, and also the effect on flashover at nearby towers.

The effect of flashover at a tower 1,660 ft. away on the operation of the arrester was determined.

Oscillograms are given comparing the protective characteristic of a single-stack arrester with that of a two-stack arrester of the same rating. There is no discernable difference between the two characteristics.

It has been proposed that line insulation for a distance of one-half mile from the station should be limited. Oscillograms are given for two arrester ratings, showing that, for the waves used, no protection is offered by the arrester at a distance of 2,180 ft. and a large part is lost at 500 ft. It is concluded that the insulation should be limited at the arrester if the arrester is to protect it safely.

The effect of the length of arrester connections is given, showing that on the 69-kv. arrester a loop 30 ft. in length reduces the potential 35 kv. The conclusion is that a long arrester circuit allows reflection of original wave at the transformer, while a long transformer arrester circuit allows the reflection of the arrester potential. Both the arrester circuit and the arrester transformer circuit should be kept as short as possible, but the former is more important than the latter.

* * * * *

THE study of the protection of lines and apparatus against the effects of lightning naturally divides into two parts. One deals with natural lightning itself, while the other is concerned with the use of artificial surges made to simulate natural lightning. The purpose of the study of natural lightning is to disclose the form of wave which may appear on the line, whether lightning strikes the line or some other point nearby. To study the effects of natural lightning on the operation of apparatus is very unsatisfactory and frequently will give rise to erroneous results. It is necessary to be able to apply the same impulse over and over again to determine how apparatus behaves and the effect of lines or apparatus on the waves themselves. The studies with natural lightning will determine the character and form of waves which may appear in service, while the investigation with artificial lightning will disclose the performance characteristics of lines and apparatus.

TESTING EQUIPMENT

This investigation is a continuation of the one made last year[1,2] in cooperation with the Consumers Power Company on their S-19 line in western Michigan, extending from Grand Rapids to Croton Dam, a distance of about 45 miles. In all of the tests to be described, which deal only with problems relating to lightning arresters, the impulse generator was located at the Croton end of the line.

The impulse generator consisted of 60 banks of oil capacitors charged in parallel to 25 kv. and discharged in series, according to the Marx circuit. Each bank had a capacitance of 0.50 μf., giving a total capacitance of 0.00833 μf. for the entire generator. The open circuit potential of the impulse generator was 1,500,000 volts. In order that the generator might be used in wet weather a canopy was constructed over the generator, as seen in Fig. 1.

A portable cathode ray oscillograph designed by Mr. E. J. Wade was found to be of great value when taking oscillograms at different points along the line.

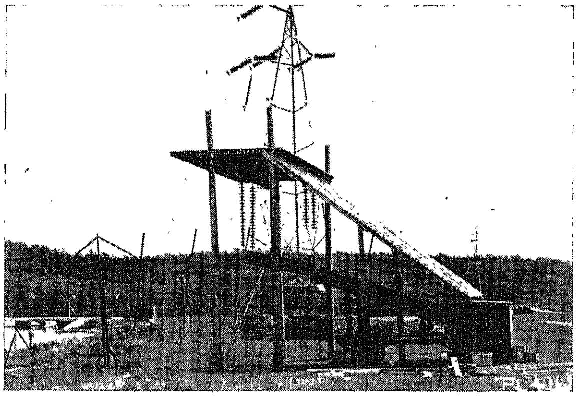

FIG. 1—1,500,000-VOLT IMPULSE GENERATOR CONNECTED TO THE S-19 LINE OF THE CONSUMERS POWER COMPANY

This oscillograph is of the cold cathode type and is provided with many features making for simple and reliable operation in the field. Automatic vacuum control, a fool-proof film changing mechanism, elimination of a large number of joints in the oscillograph, and its small size, are some of the noteworthy improvements incorporated in this equipment. The oscillograph is mounted in a special body Ford truck intended to supply not only transportation for the oscillograph, but also power for its operation. Because of lack of time, the oscillograph went into service before its power supply equipment was complete so that it was necessary to use a portable gasoline driven generator for the oscillograph power supply.

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*Both of the General Electric Co., Pittsfield, Mass.

1, 2. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

Tests were made using the station type Thyrite arrester under a variety of circuit conditions. In all the tests described here, the arresters were located at Tower 14, a distance of 1.46 miles from the lightning generator at Croton Dam. This distance was great enough so that the arrester operation was practically complete before reflections from the impulse generator could return to the arrester.

Negative impulses were used in all of the tests, because the negative wave attenuates less rapidly than the positive, and favorable results required the highest test voltage possible at the arrester.

Thyrite Arrester Performance

When applying arresters, an exact knowledge of the arrester performance for any assumed condition is desirable. Calculations[3,4] have been made for the thyrite arrester which show methods for determining the arrester performance under assumed conditions. The volt-ampere curve for the thyrite arrester is a straight line on log-log paper and is conveniently represented by the equation $E = C I^{(1-a)} = C$ where E and I are the arrester voltage and current respectively, while a and C are constants of the thyrite material. The exponent a is a fixed value of 0.72, while C is the resistance of the arrester with one ampere flowing and obviously varies directly with the number of thyrite units in series.

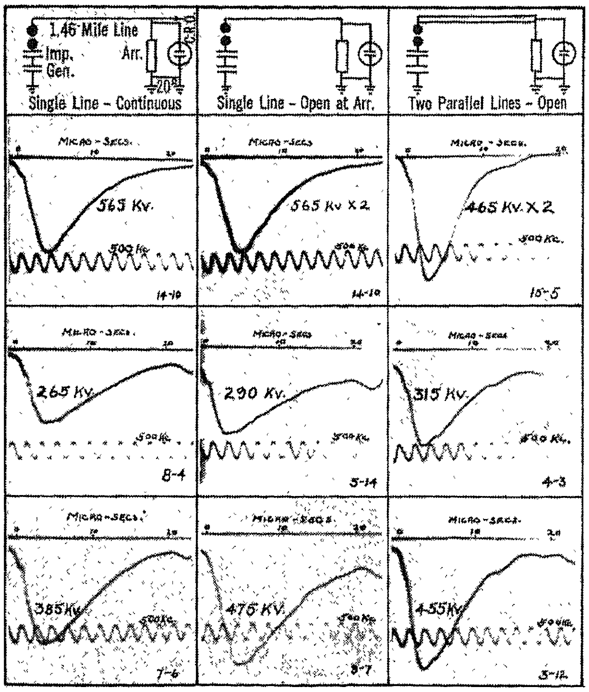

FIG. 2—ARRESTER PROTECTIVE CHARACTERISTICS AS MEASURED AT TOWER 14

Oscillograms—Top row—With no arrester connected
Middle row—69-kv. arrester
Bottom row—115-kv. arrester

To check the calculations which have been published for the arrester, various numbers of arrester units were studied so that all arrester ratings from 15 to 220 kv. have been tested under conditions representing closely those of service. Three conditions were studied for each arrester rating, one corresponding to the arrester at the end of a single circuit line, another with the arrester on the bus at the end of a double circuit line, and the third with the arrester at an intermediate point in a line. These three conditions are illustrated in the circuit diagrams of Fig. 2.

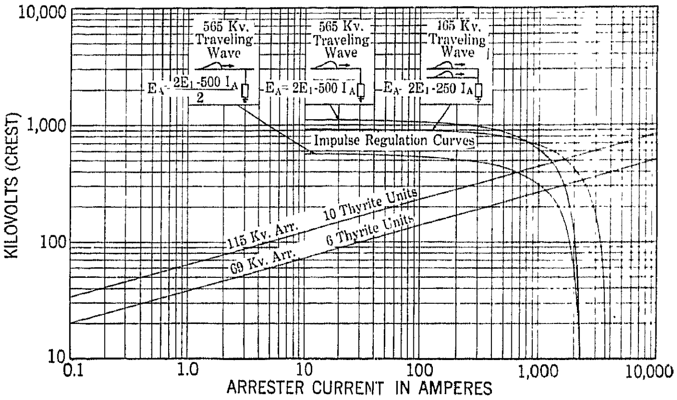

FIG. 3—CALCULATED ARRESTER PERFORMANCE FOR APPLIED IMPULSES. CORRESPONDING CATHODE-RAY OSCILLOGRAMS SHOWN IN FIG. 2

The potential without the arrester at Tower 14 is shown in the upper oscillograms in Fig. 2. The middle oscillograms (8-4, 5-14, 4-3) show the potential measured across the terminals of 6 thyrite arrester units and the lower oscillograms show the potential across 10 units. These units form one leg of a 69-kv. and 115-kv. arrester, respectively, for grounded neutral systems.

The calculated values for the two arresters may be obtained from Fig. 3. The impulse regulation curves are plotted for the three conditions indicated, using the usual surge impedance value of 500 ohms, and the incoming wave voltages corresponding to those used in Fig. 2. The intersections between the arrester characteristics and the impulse regulation curves give the maximum arrester voltages for the various conditions.

The comparison between the test and calculated values are given in the table and the agreement is quite good. If, however, the surge impedance of the line is taken as 530 which is the calculated single line value, the agreement is even more favorable as the effect of increased surge impedance is to reduce the voltage across the arrester. It is also probable that the arresters have a slightly better exponent than 0.72 which also has the effect of giving test results which are more satisfactory than those shown by the calculation.

THYRITE ARRESTER PERFORMANCE
COMPARISON OF CALCULATED AND TEST VALUES USING SURGE IMPEDANCE OF 500 OHMS

Circuit condition	(1) Single line continuous		(2) Single line open		(3) Parallel lines open	
	Calculated	Test	Calculated	Test	Calculated	Test
69-kv. arrester	280	265	305	290	340	315
115-kv. arrester	400	385	475	475	505	455

It is to be noted that the arrester gap breakdown is not readily apparent, although a slight oscillation appearing on the original negatives indicates that the breakdown occurred at from 30 to 35 kv. crest per 11.5 kv. arrester unit. Before these oscillograms were taken in the field it was expected that when gap breakdown occurred a sudden change in potential would take place which on account of its steepness might occasion turn-to-turn stresses in nearby inductive apparatus. The oscillograms from the lowest to the highest rating show the arrester volt-time characteristic to be free from such a sudden change of potential. Laboratory tests do show such a sudden change in potential at gap breakdown, but this effect may be due to the fact that true traveling wave conditions are not reproduced in the present laboratory equipment. Other tests[5] with two arrester units in series made on the Pittsfield 66-kv. line of the Turners Falls Power Company agree in giving an arrester breakdown of about 35 kv. per unit with a wave reaching arrester breakdown in one microsecond, and in not showing a sudden drop in potential after gap breakdown.

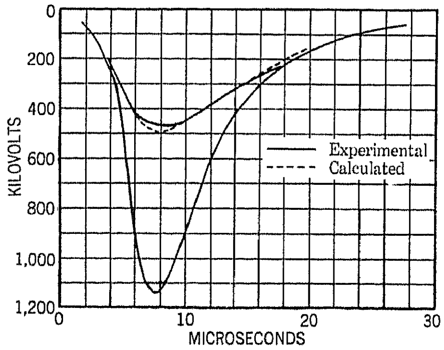

FIG. 4—COMPARISON OF CALCULATED AND MEASURED VOLT-TIME CURVES OF 115-KV. THYRITE ARRESTER

Using methods of calculation previously referred to[4] the wave shape of voltage across the arrester for any of the three given conditions may be calculated and found to agree with the test results. In Fig. 4 the original wave without the arrester is reproduced together with the voltage across the 115-kv. arrester when at the end of the transmission line. These are the top and bottom oscillograms in the middle row of Fig. 2. The calculated volt-time characteristic of the arrester is also shown.

EFFECT OF GROUND RESISTANCE

The oscillogams referred to dealt with the potential across the arrester. In practise it is frequently necessary to know the effect of ground resistance in the arrester circuit.

The tower footing resistance at Tower 14 as measured with a megger was 35 ohms. A 12-unit thyrite arrester was located about 90 ft. away with a driven pipe ground about 15 ft. from the arrester. This ground tested 50 ohms. The tower footing and the arrester ground were connected together with a cable approximately 100 ft. in length. This test set-up is shown in Fig. 5, together with the oscillograms obtained. The S-19 line was open at Tower 14, giving an open circuit potential of 1,130 kv., as shown in Fig. 2.

With the oscillograph connected to a ground remote from the other grounds, the voltage across the arrester plus the drop in the ground resistance was taken and is shown in the first oscillogram at the left of Fig. 5. On the tower at the time this test was made were six-suspension insulators spaced 4¾ in. which did not flash over until the string was later reduced to five insulators. The upper oscillogram on the right (10-2) shows the wave being chopped on the tail by the insulator flashover.

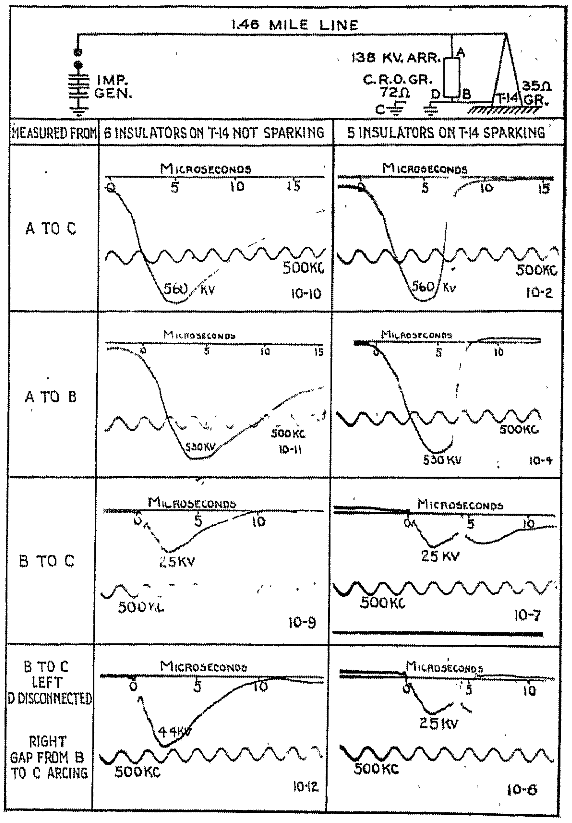

FIG. 5—OSCILLOGRAMS OF ARRESTER PERFORMANCE AS AFFECTED BY ARRESTER GROUNDS

Also showing protection of insulators on Tower 14. Tests made using 138-kv. thyrite arrester

The second group of oscillograms on the left and right (10-11 and 10-4) show the arrester potential with six and five insulators on the tower. The difference of 30 kv. between the crest potential of the first and second set of oscillograms is in the *IR* drop due to the ground resistance. The third set of oscillograms (10-9 and 10-7) taken between the arrester ground and the separate oscillograph ground shows a crest potential of 25 kv. which corresponds well to the difference between the first two oscillograms having crest values of 560 and 530 kv. respectively. Oscillogram 10-9 also determines the point on the arrester voltage wave where the arrester began to take current. This point may be seen more clearly by referring to Fig. 6 showing the plot of the second and third oscillograms of Fig. 5 in which the insulators on the tower did not arc over. The breakdown of the arrester gap does not appear on the oscillogram, the rise being quite smooth except for a slight oscillation during the time of breakdown. This oscillation is so small that its presence is deduced only from the decreased density of the oscillogram on the front of the wave. Since the crest current through the arrester will occur at the time of crest potential across the arrester, the ground voltage and the arrester voltage curves of Fig. 5 are plotted so that the zero of the time scale occurs at the voltage crests. At a time of 2.2 microseconds before the crest of the arrester potential is reached the gap breakdown occurs at a potential of approximately 375 kv.

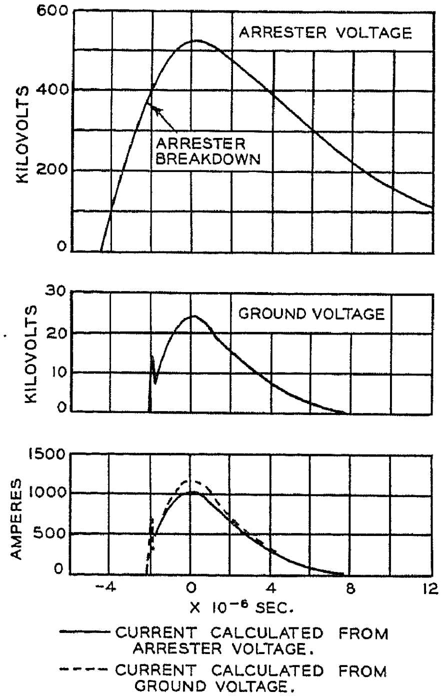

FIG. 6—CURVE SHOWING ARRESTER BREAKDOWN AS DETERMINED FROM OSCILLOGRAM OF GROUND RESISTANCE VOLTAGE

Arrester current and ground current curves

Using the standard volt-ampere curves for the 138-kv. arrester for grounded neutral service a curve of current is plotted, as shown by the full lines in the lower portion of Fig. 6. The dotted lines show the current determined from the ground resistance and the ground voltage curve. The agreement is seen to be very satisfactory. It is well to note in connection with Fig. 6 that the arrester current reaches 1,150 amperes which constitutes quite a satisfactory test on an arrester rated 138 kv.

Referring again to Fig. 5, an oscillogram (10-12) was taken with the arrester ground disconnected, with the result that 44 kv. was measured across the tower footing resistance. Noticeable oscillations appear in the front of the ground voltage waves apparently resulting from reflection through the length of the ground wire. With both the tower ground and the arrester ground connected in parallel and with the conductor insulation reduced to 5 units, oscillogram 10-7 shows two voltage peaks of about equal magnitude. The first occurs when the arrester current flows to ground and the second when the five insulators flash over on the tail of the wave. In oscillogram 10-6 the sparkover of a small gap connected between the oscillograph ground and the tower ground is shown. This oscillogram is interesting since it shows time lag displayed in completing the sparkover of the gap.

The oscillograms of Fig. 5 indicate that when making arrester calculations one is justified in adding the ground resistance to the arrester resistance to determine the total protective characteristic of the arrester. The straight line characteristic of thyrite is very well adapted to this form of calculation. However, if the arrester ground and the apparatus grounds are bused together the potential on the apparatus will be independent of the ground resistance. The connections between the grounds should be as short as possible. The protection of an insulator string on a nearby tower has been demonstrated and flashover shown when the insulation is reduced below a certain limit.

EFFECT OF FLASHOVER OF THE LINE

There has been considerable discussion concerning the possible effect on the arrester protective characteristic of a flashover of an insulator string located at some distance from the apparatus. To simulate this condition the insulation on Tower 11 was reduced to 5 disks and the voltage at Tower 14 measured with the oscillograph.

In order to get a reasonable length of chopped wave, the front of the surge was slowed down by inserting an inductance coil between the surge generator and the line. The wave was chopped at Tower 11 where the tower footing resistance was 35 ohms.

The upper left oscillogram of Fig. 7 (11-14) shows this chopped wave as measured at Tower 14, where the line was open. The reflections back and forth between the point grounded by the spark over and the open end of the line are clearly evident.

The sparkover at Tower 11 evidently occurs after the reflection from Tower 14 arrives back at that point, since the wave at Tower 14 is 8 microseconds long before sparkover occurs, while the distance between Towers 11 and 14 is 1,663 ft. The subsequent reflections apparently have a period of 8.5 microseconds, which, on the assumption that the length of the line equals one-quarter wavelength gives a propagation velocity of approximately 800 ft. per microsecond. Thus the beginning of the original impulse which had a crest value of about 480 kv. and wave front of 10 microseconds reached the open end of the line at Tower 14 and began to reflect, the voltage at the open end always being doubled. The reflected voltage wave traveled back toward the impulse generator adding its potential to that of the original incoming wave so that the potential at Tower 11 was about 550 kv. when the insulator flashed over. For two microseconds after the flashover occurred on Tower 11 the voltage at Tower 14 continued to rise on reflection, because of the time taken for the wave of reduction, due to the flashover on Tower 11, to travel back to Tower 14. Calculation of the reflection at Tower 14 indicates a potential of about 800 kv., although but 720 kv. was measured. The difference is probably due to corona loss. Thus a certain portion of the energy of the original wave was trapped between the flashover at Tower 11 and the open circuit at Tower 14. Since, however, the arc on Tower 11 has resistance which is to be added to the tower footing resistance, the voltage at Tower 11 was not zero, so that a certain small portion of the energy trapped will be absorbed in this resistance and some of it transmitted to the rest of the line. Some may also be received from the impulse generator end of the line because other oscillations are to be found trapped between Tower 11 and the impulse generator. Because of these and other losses of energy the oscillation decreases rapidly in amplitude.

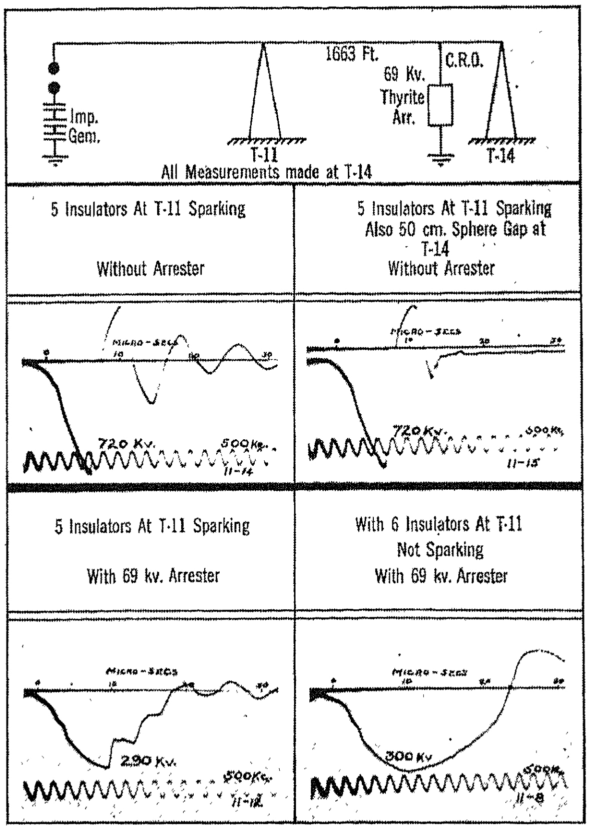

Fig. 7—Oscillograms showing the Effect of Arrester on the Tail of a Wave Chopped by the Flashover of an Insulator 1,663 Ft. from Tower 14

When a 6-unit 69-kv. arrester was connected between line and ground the oscillations as measured at Tower 14 were quite different. The lower left oscillogram (11-12) shows a maximum potential of 290 kv. while the original traveling wave had a crest of about 480 kv. This means that instead of a positive* reflection taking place at Tower 14, a negative reflection occurred such that the crest of the reflected wave was equal to the difference between 290 and 480 kv. or 190 kv. Up to the time when the arrester began to take current from the surge, the reflection was positive, but as the arrester resistance decreased with greater current flowing through it, a point was reached where the arrester resistance was equal to the line surge impedance and no reflection occurred. A little later at higher voltage the arrester resistance became less than the surge impedance and the reflection changed from a positive reflection to a negative reflection, passing through zero at the instant when the arrester resistance equaled the surge impedance of the line.

As a result of the decrease in potential due to the arrester reflection, the flashover of the insulators on Tower 11 was delayed about 2½ microseconds longer than when the arrester was not used. The insulator flashed over on the tail of the resultant potential which appeared at Tower 11.

This test indicates that in cases where a flashover takes place on the line the arrester resistance will be such that most of the dangerous oscillations due to double reflections will not take place.

The lower right oscillogram (11-8) of Fig. 7 shows the potential allowed by the arrester when the insulators on Tower 11 did not flash over. This oscillogram shows the negative reflection returning from the impulse generator after about 22 microseconds. The upper oscillogram (11-15) is interesting in that a 50-cm. sphere gap arced on the third reflection, while the insulator on Tower 11 sparked over on the first reflection. From standard curves the voltage at the time of sparkover is less than the 60-cycle spark potential of the gap. The early stages of arcover occurred at higher voltage although the arcover was not completed until the third half cycle.

Effect of Twin Arrester Stacks

It has been customary to divide the total length of high-voltage arresters into two, or sometimes three parallel stacks, electrically connected in series. This practise has been criticized occasionally because of the number of turns or bends in the arrester circuit. It has also been maintained that the total length of arrester circuit was the criterion rather than the number of bends. In Fig. 8, two oscillograms are shown which were taken with the idea of determining, if possible, the effect of subdivision of an arrester into two stacks. For this purpose a 12-unit 138-kv. thyrite arrester was used, as shown in Fig. 9. This figure also shows the location at Tower 14 where all of the arrester tests were made.

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*A positive or a negative reflection at a point means a reflection with the same or opposite polarity respectively, as that of the incoming wave which is being reflected.

The oscillograms show that the arrester potential is not measurably increased by dividing the arrester into two stacks. The additional length of circuit involved in such subdivision is too small to give a measurable indication on the oscillogram. This additional length was about 12 ft. being equal to the height of an arrester stack plus the connection between the two stacks. In the case of a three-stack arrester there would be less increase in length of circuit through the arrester from the line-to-ground terminal, compared to a single-stack arrester having the same number of units.

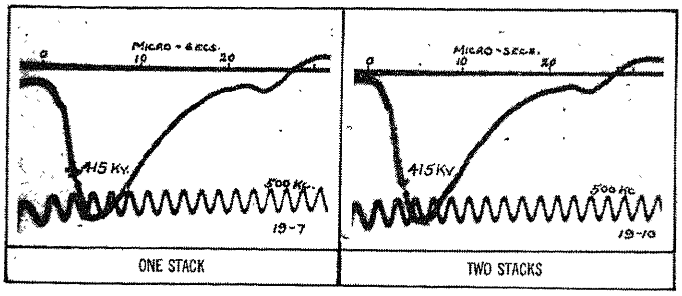

FIG. 8—OSCILLOGRAMS SHOWING THE EFFECT OF A TWO-STACK ARRESTER COMPARED WITH A SINGLE-STACK ARRESTER

Although this increase in length did not make an appreciable increase in potential, with a 138-kv. arrester, yet with a low-voltage arrester such an increase in circuit length gives rise to an appreciable increase because of the lower potential allowed by the arrester. Thus a 10-kv. increase could not be detected on a 220-kv. arrester characteristic, but would be of importance in the case of a 3-kv. arrester.

PROTECTION AFFORDED TRANSMISSION LINE BY STATION ARRESTER

When attempting to coordinate apparatus insulation in stations with adjacent transmission insulation the use of a standard level of insulation extending out from the station for one-half mile has been suggested. From the standpoint of the lightning arrester protection the use of a single point where reference level insulation is maintained is superior. This level can well be established by the use of a horn or other gap close to the arrester and to the transformer to be protected. The gap may be set according to the values suggested by Montsinger and Dann,[6] which have a high enough spark potential so that the arrester will prevent the gap from sparking for anything but a severe direct stroke at the arrester. The use of such a gap gives the designer of both the arrester and transformer a definite condition to follow which may be stated in terms of volts and time.

If, however, the line insulation is kept at a coordinated level for a distance of one-half mile from the station, beyond which the insulation is increased materially, an arrester at the station will be unable to offer protection to insulators one-half mile away. As the arrester is approached the protection becomes better, and calculations are readily made for any given situation which will show the degree of protection offered at any location.

In order that the calculations might be checked tests were made on the S-19 line with the arrester at Tower 14, the impulse generator being connected to the line to give a steep voltage rise. Two arresters were investigated; one a 6-unit 69-kv. arrester and the other a 12-unit 138-kv. arrester. Tests were made using each arrester with the transmission line open and closed at Tower 14. Oscillograms were taken at Towers 14, 13, and 10, the distance of Towers 13 and 10 from Tower 14 being 500 and 2,180 ft. respectively. On each of the three towers when oscillograms were taken the insulation was reduced until the insulators flashed over. An oscillogram was taken with and without flashover.

In Fig. 10 are given the oscillograms taken with the line open at Tower 14. The oscillograms with the line closed are similar, except the potentials are decreased as indicated in Fig. 2. In general with the line continuous the insulation on Towers 14, 13, and 10 could be reduced by one insulator before flashover occurred.

At Tower 14 where the arrester is located, oscillogram 10-11 shows that 6 disks were protected by a 138-kv. thyrite arrester, the crest potential being 530 kv. The next oscillogram 10-4 shows that 5 disks arced over on the tail of the wave. Moving on to Tower 13, oscillograms 16-11 and 16-7 show that the arrester at Tower 14 protects 8 disks but 7 disks arcover on the tail of the wave having a crest potential of 660. At Tower 10 the conditions are the same as at Tower 13. These results show that for this condition the arrester protection does not extend back from the arrester for a distance of more than 500 ft. If the incoming potential had been higher the effect of the arrester would have been felt at a

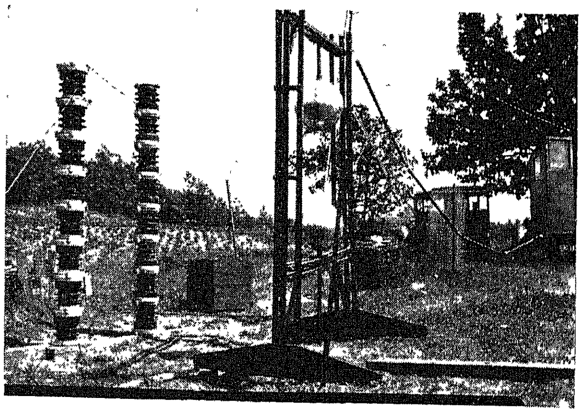

FIG. 9—THYRITE ARRESTER SET UP IN TWO STACKS

Tower 14 in the background. Oscillograph truck at extreme right greater distance from the arrester. When comparing insulator disks flashed over, it should be remembered that the disks used were spaced 4¾ in. so that seven of these units are approximately equal to six units spaced 5¾ in.

The tests made with the 69-kv. arrester show that the zone of protection extends somewhat further than was the case with the arrester discussed in the last paragraph. At Tower 14 the arrester protected four disks but three disks arced over on the tail as seen in oscillograms 10-15 and 11-2. The crest potential measured at the tower was 315 kv. which was 25 kv. higher than the potential measured across the arrester terminals as shown in Fig. 2. At Tower 13, oscillograms 16-3 and 16-6 show a crest potential of 480 and 450 kv. with five disks protected and four disks arcing on the front. At Tower 10 reference to oscillograms 18-3 and 18-2 will show that the effect of the arrester is negligible although on the first oscillogram seven disks did not arc over, while on the second seven disks did arc over. It is clear, comparing the 138-kv. arrester results with those for the 69-kv. arrester measured at Tower 10, that the arrester is having no effect on the insulator flashover. Of course the results would be somewhat different with waves having longer tails and with waves having steeper fronts.

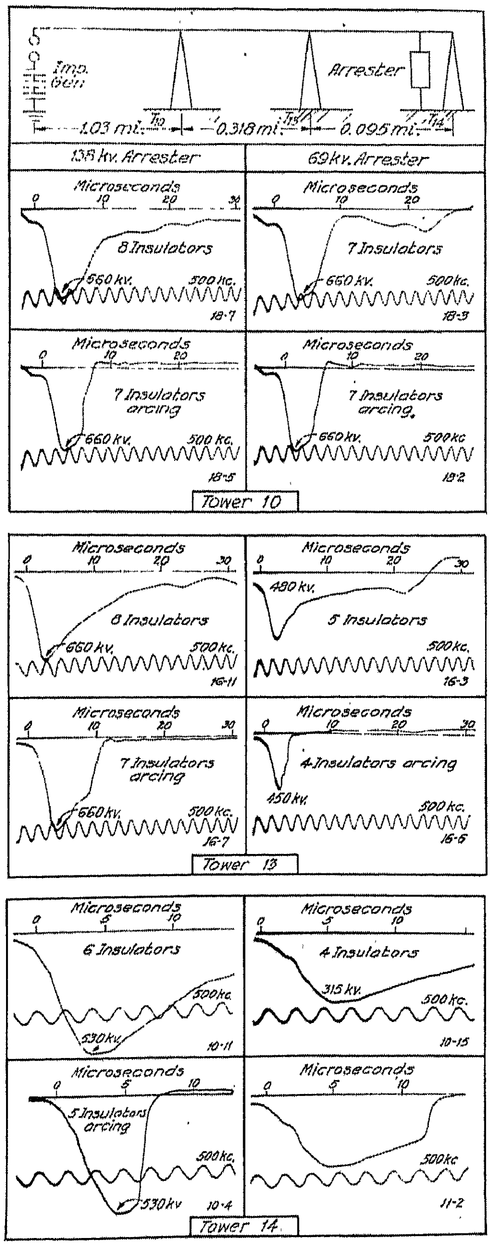

FIG. 10—OSCILLOGRAMS SHOWING THE DEGREE OF PROTECTION OFFERED TO INSULATORS ON INCOMING LINE WITH ARRESTER IN THE STATION

Tests with 138-kv. and 69-kv. arresters

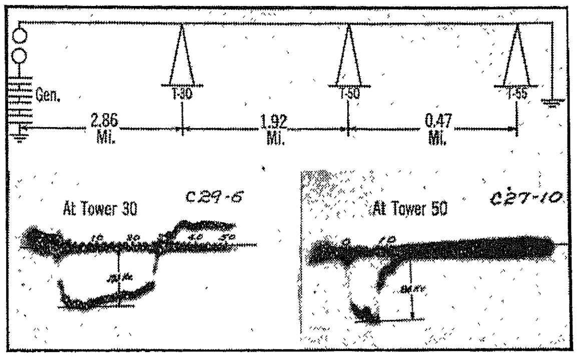

FIG. 11—OSCILLOGRAMS TAKEN AT TWO DISTANCES FROM A GROUNDED POINT SHOWING THE PROGRESS OF THE NEGATIVE REFLECTION BACK ALONG THE LINE

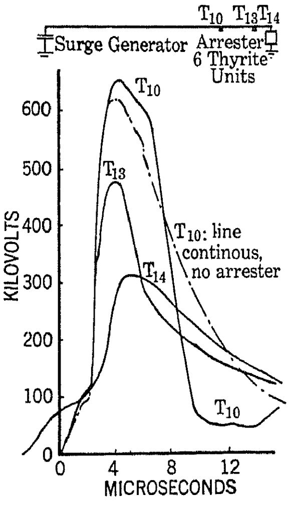

FIG. 12—PLOT OF THE OSCILLOGRAMS OF FIG. 10 SHOWING THE PROGRESSIVE CHANGE IN THE FORM OF THE WAVE AS IT TRAVELS ALONG THE LINE

If the arrester had reduced the potential at Tower 14 to zero, the zone of protection would not have been materially extended. The oscillograms in Fig. 11 taken in connection with another investigation show this condition plainly. With a short circuit to ground at Tower 55 the potential at Tower 50 was 86 kv. and 103 kv. at Tower 30. The oscillogram at Tower 50 shows the complete front of the wave, the tail being cut by the negative reflection at Tower 55. An insulator flashover at Tower 50 on the tail might have been prevented by the ground at Tower 55, but a flashover on the front would not have been so prevented.

In Fig. 12 the oscillograms for the 69-kv. arrester shown in Fig. 10 have all been plotted to the same scale. The broken line shows the voltage at Tower 14 with the line continuous. With the line open at Tower 14 all of the voltage ordinates should be doubled to indicate the open circuit potential at Tower 14. It will be noticed that the crest potential at Tower 10 is somewhat higher than the potential with line continuous measured at Tower 14.

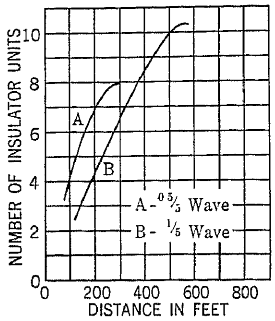

Fig. 13—Protection Offered by an Arrester at Points Back Along the Incoming Line

Traveling waves assumed to be 0.5/5 microsecond and 1/5 microsecond with a crest of 1,500 kv. Number of insulators used is based on front of the wave flashover. (See references 7 and 8 for impulse ratio curves). Calculations based on insulators spaced $5\frac{3}{4}$ in.

The difference in crest potential at Tower 10 is due to the positive reflection which occurs from the end of the line before the arrester goes into operation. This difference would be still greater if it were not for the effect of attenuation. The effect of arrester may be plainly seen in that the tail of the wave is reduced. At Tower 13 both the crest and tail are reduced, while at Tower 14 the curve becomes practically the arrester characteristic itself.

These oscillograms indicate clearly that the arrester cannot be expected to offer much protection more than 500 ft. away from the arrester and if the wave front had been steeper still less protection would have been given by the arrester to an insulator string 500 ft. back from the arrester. Front of the wave flashover occurs frequently in practise while most of the flashovers shown in Fig. 10 were on the tail of the wave. This means that still steeper waves at higher crest potentials would have flashed over in less time and the arrester would need to be closer to give protection. As an illustration of the effect of wave front, two curves have been plotted in Fig. 13 showing the distance from an arrester that protection is offered if the insulators are considered as arcing on the front of the wave. The arrester is assumed to hold the potential constant at 350 kv. while the incoming traveling waves are assumed to have straight line fronts reaching a crest potential of 1,500 kv. in 0.5 and one microseconds. The tail is assumed to decrease to 750 kv. in five microseconds. The curves show that for the 0.5 microsecond front no protection is offered at a distance of more than 250 ft. against front of the wave flashover. With the one microsecond front this distance is increased being equal to approximately one-half the length of the wave front expressed in feet.

Flashover on the tail of the wave requires a lower potential per insulator, so that a wave having the same crest potential will flash over more insulators on tail of the wave flashover than on front of the wave flashover. A curve showing the protection offered by an arrester on tail of the wave flashover is given by Fortescue.[9]

In practise flashovers may occur on either the front or the tail, but if the over voltage is high as in a direct stroke, the flashover at the point of inception will be on the front. Waves which travel in from a distance will be more likely to flash over on the tail at the point where decreased insulation is met, unless the difference in insulation levels is great. When applying transmission line arresters to prevent line flashovers the arresters should be at the point of inception and to prevent front of the wave flashovers, need to be close together, preferably on each tower.

Potential Beyond the Arrester

With the line continuous to Grand Rapids a set of oscillograms given in Fig. 14 similar to those shown in Fig. 10 were taken. One oscillogram (20-16) was taken at Tower 29 and shows the appearance of the wave after traveling beyond the 69-kv. arrester for a distance of 1.38 miles. The oscillogram (8-4) on the left is the original oscillogram taken at Tower 14. The crest potential decreased from 265 kv. to 240 kv. the wave shape being practically the same as at the arrester. The decrease in potential is, of course, caused by attenuation.

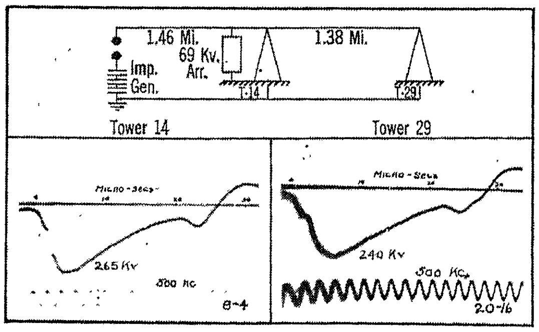

Fig. 14—Oscillograms showing Effect of Distance on Transmitted Wave

These oscillograms demonstrate clearly one of the fundamentals of wave reflection and transmission. The transmitted wave beyond the arrester is the wave measured across the arrester itself. The reflected wave is the difference between the original applied wave and the voltage across the arrester. The reflected wave returns toward the point from which it came, while the transmitted wave passes on never to return, unless a reflection point is reached before the losses have reduced the wave to zero as it travels along the line.

ARRESTER LOCATION AND CONNECTIONS

When discussing the application of lightning arresters it has long been pointed out that the connection to the arrester and to ground should be kept as short as possible and that the arrester should be located close to the transformer. Oscillograms given in Fig. 15 were taken to show the effect of arrester connections.

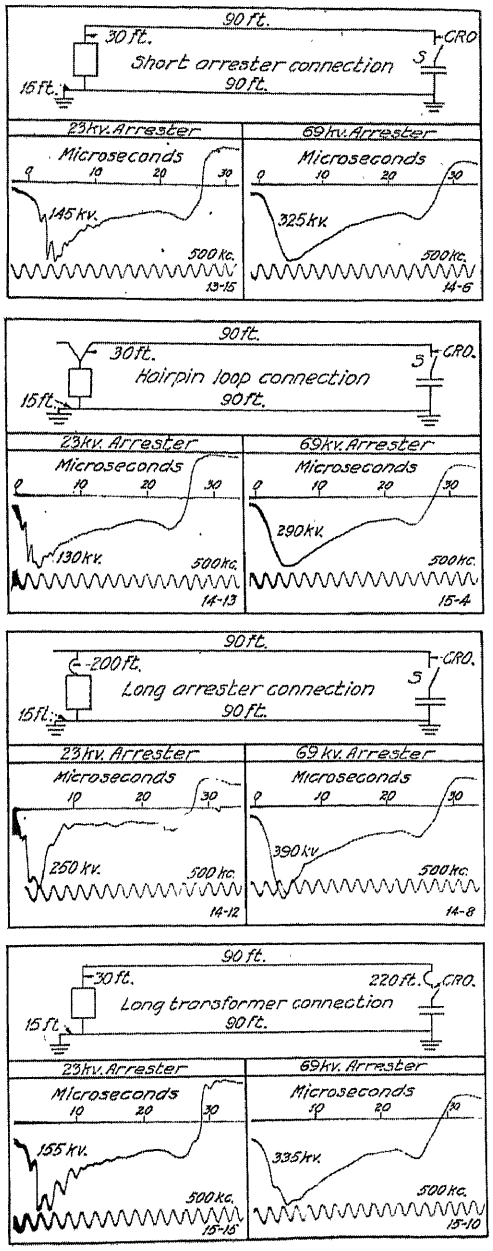

FIG. 15—OSCILLOGRAMS SHOWING THE EFFECT OF LENGTH OF ARRESTER CONNECTIONS AND LOCATION WITH RESPECT TO THE TRANSFORMER

The test was made with a two-unit thyrite arrester rated 23 kv. and with a 6-unit arrester rated 69 kv. The arrester was located at Tower 14 as in the other tests. The distances involved are shown in the figure. The distance from the arrester to the transmission line was 30 ft. while the distance from the arrester to the arrester ground was 15 ft. The measured arrester ground resistance was 40 ohms while the tower footing ground resistance was 35 ohms. The two grounds were connected together by a conductor 90 ft. in length. The line was open at Tower 14 and a conductor run down to a capacitor which represented the electrostatic capacity of the transformer to be protected. Tests were made with the oscillograph connected between the top of the tower and the ground bus, so that the oscillograms do not include the effect of the tower footing resistance, except as it changes the total current flow through the arrester which will, of course, change the arrester potential slightly.

Since the oscillograms taken with the switch *S* (Fig. 15) closed are all of the same relative characteristics as with the switch open, only those with the switch open will be reproduced. There is a tendency for the crest voltage to be slightly higher with the capacitor connected, this being due to the manner in which the reflections between the arrester and the capacitance add. With different values of capacitance and different distances between the arrester and the capacitor the voltage might be more or less than that without the capacitor.

The first pair of oscillograms, 13-15 and 14-6 were taken with the connections shown and give a crest potential of 145 kv. at the oscillograph with the 23-kv. arrester, and 325 kv. with the 69-kv. arrester. Using the hairpin loop the voltages are reduced to 130 kv. and 290 kv. respectively, as seen in oscillograms 14-13 and 15-4 in the second group. The reason, of course, for this reduction in potential is that the line is brought down to the arrester so that the time involved in getting from the transmission line through the arrester and to the ground is made as short as possible. In this connection the arrester is acting as a resistance between line and ground, which in itself does not delay the impulse in reaching the ground. Only distance delays the impulse unless time is involved in changing the arrester from an open circuit to a resistance which is not the case for the arrester used in this test. In order that the impulse on the line be reduced it is necessary that a negative reflection return from the ground through the arrester so that the time delay involved is the time taken for the wave to travel from the line through the arrester to ground and back up to the line. In the meantime the surge on the line has not been waiting, but has traveled on toward the transformer. If the length of the arrester connection line to ground was made equal to half the length of the wave front of an incoming wave, the potential at the transformer would become double the crest value of the incoming wave by reflection and the arrester wave of reduction would reach the transformer only in time to cut the tail of the wave.

To illustrate the effect of long arrester circuit to ground the third group of oscillograms was taken in which 200 ft. of conductor was inserted between the arrester and the transmission line. Its effect is clearly seen on oscillograms 14-12 and 14-8. From the best case with the hairpin loop and 23-kv. arrester, the transformer voltage has increased from 130 to 250 kv. and with the 69-kv. arrester from 290 to 390 kv. With steeper waves or with longer arrester circuit these differences would be increased up to the time that the original wave crest was reached.

The condition of a long arrester circuit is similar to those existing in connection with Fig. 10, in which it was shown that the arrester could not give any protection at Tower 10 when the arrester was at Tower 14, 2,180 ft. away.

The fourth group of oscillograms shows the effect of adding 220 ft. to the distance between the arrester and the transformer. These oscillograms may be compared with the corresponding ones of the first group. Since the arrester circuit is the same, it will be seen that only 10 kv. was added to the transformer voltages by such an increase in circuit length. With either a steeper wave or with more distance a greater increase in potential would have been noted. It is of interest to note that the potential previously measured directly at the arrester terminals was always 122 kv. with the 23-kv. arrester and 290 kv. with the 69-kv. arrester. Arrester volt-ampere curves show that since the open circuit potential without the arresters was 1,130 kv. the current through the 23-kv. arrester was over 2,000 amperes, while that for the 69-kv. arrester was about 1,500 amperes.

The oscillograms indicate that the length of the arrester connection is more important than the length of the connection between the arrester and the transformer. This is true because with long arrester connections the transformer reflects the incoming wave, while with short arrester connections and long transformer connections the transformer reflects only the arrester potential. If it is necessary to choose one or the other, naturally the second condition is to be preferred. To be assured of the best results from the arrester it should be connected close to the transformer, the arrester circuit being as short as possible between line and ground.

The effect of the length of circuit is not dependent on arrester rating. It depends on the steepness of surge. However, the question as to whether or not a certain length of lead is seriously objectionable does depend on arrester rating since an increase in potential which is negligible on a high-voltage arrester may well mean insufficient protection with a low-voltage arrester.

When considering the length of the arrester circuit the entire length from the line to ground must be considered. If the arrester is connected to an extensive steel structure which is thoroughly grounded, the arrester circuit distance can be taken from the line to the steel structure. In cases where both the arrester and the transformer are elevated but are bonded together, as may be the case in a small industrial substation, the voltage in the transformer insulation will be the arrester voltage if the leads are short. The potential on the line will be higher than this if the ground connection is long but the transformer will be protected.

CONCLUSION

Tests have been made under conditions where true traveling waves from a 1,500,000-volt impulse generator were impressed on lightning arresters of various ratings. It was demonstrated that the calculated results agree closely with the test results for a variety of circuit conditions, so that one may predict with assurance the performance of an arrester of the thyrite type.

As this is believed to be the first time such a demonstration of arrester performance has been made in the field, it is hoped that these results will prove of considerable benefit to those responsible for the application and operation of lightning arresters.

These arrester tests were made as a part of a large program of investigation of transients on transmission lines, conducted jointly by the Consumers Power Company and the General Electric Company.

In the field the investigation was in charge of E. J. Wade for the General Electric Company and J. R. Eaton for the Consumers Power Company. O. Brune was responsible for the theoretical side of the work, while Messrs. E. J. Shimek, C. C. Tanner, D. Blackwell and D. Geiger comprised the impulse generator, cathode-ray oscillograph, and transmission line groups. The photographic work, compiling reports, and similar work was performed by W. J. Rudge. All of these men are to be complimented on both the character and amount of work performed.

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Discussion

R. H. Park: Messrs. McEachron, Wade, and others have shown that it is now possible to calculate the performance of lightning arresters as far as concerns the effect of voltages due to lightning. In determining the effect of normal frequency voltages, it is usually only necessary to know the maximum voltage that an arrester can safely withstand continuously. I understand that this figure is given by the ungrounded arrester rating. For grounded circuits, these data would appear to be practically all that is required to permit intelligent application of arresters.

On the other hand, it would appear, on the basis of our present information, that in the case of ungrounded circuits the arrester is practically certain to be subjected to overvoltages resulting from arcing grounds. I believe that, according to present understanding, the voltage during an arcing ground periodically builds up for a few cycles and then discharges through the arrester, the discharge energy being roughly proportional to the length of line affected. If this is the case, it would follow that any arrester should be capable of discharging the voltages occurring during arcing grounds provided that the length of line involved is short enough, while the maximum safe length of line would depend upon the type of arrester used. These considerations suggest the desirability of obtaining quantitative information in regard to the performance of arresters when subjected to voltages due to arcing grounds.

Series Resistance Method of Increasing Transient Stability Limit

Series Resistance Inserted into Generator Circuit Loads Machines During Fault and Retards Subsequent Overswing

BY R. C. BERGVALL[1]

Associate, A. I. E. E.

Synopsis.—*This paper considers the effect of resistance inserted in series with generators during and after a short circuit to increase the transient stability limit. The series resistance loads the machines during a fault and greatly reduces the normal tendency to overspeed. After the fault has been cleared the series resistance exerts a powerful influence in retarding excessive over-swing by taking advantage of the low generator stored energy relative to that of the load center. It is shown that by using series resistance a typical hydro system can maintain synchronism through a three-phase fault with speeds of circuit breaker and relay operation now available. A three-phase resistor normally short-circuited by a circuit breaker and connected into the neutral of each generator provides the most desirable arrangement. The cost will generally be much lower than the combination of other measures commonly taken such as high inertia generators, low reactance of generators and transformers, damper windings, transformer neutral impedance, special simultaneous relay schemes or other remedial measures such as lightning proof lines, and the expenditure can be deferred until it is justified by operating experience.*

* * * * *

THE demand for service continuity from large power stations that transmit their output over long transmission lines has resulted in the development of high-speed circuit breakers and relays. For a short circuit within 10 per cent of the end of a transmission line the circuit breaker nearest the fault can be opened in approximately 8 cycles and the circuit breaker at the other end of the line in 16 cycles, the delay being necessary to obtain proper relay selectivity with the other parallel line. Faults within the 80 per cent zone at the middle of the line can be cleared simultaneously from both ends in 8 cycles. Various methods of obtaining simultaneous circuit breaker tripping for the severe type of faults within the 10 per cent end zones have been considered, but have not been extensively adopted on account of cost and the possibility of dropping the entire station due to faulty operation.

Hydroelectric units have a relatively low stored energy per kilowatt and in general will not stay in synchronism for three-phase faults within the 8-cycle simultaneous clearing zone, or for double line to ground faults within the 8-16-cycle sequential clearing zone when transmitting rated kilowatt output. The electrical output is greatly decreased during short circuits on account of the low power factor of the fault current circuit while the prime mover input remains substantially constant for the time intervals under consideration. The difference between the mechanical input and the electrical output must be expended in accelerating the generator rotor, and depending upon the stored energy, the severity of the fault, and the time of circuit breaker operation, a velocity away from synchronism is imparted to the rotor that must be removed by the synchronizing forces present after the fault is cleared, without permitting sufficient over-swing to result in loss of synchronism.

This paper considers the effect of automatically inserting a relatively high value of series resistance into the generator circuit immediately upon the occurrence of a short circuit, and of removing the resistance a predetermined time after the fault has been cleared. The series resistance can be inserted into the fault current path in about six cycles and will increase the generator output to around full load for the remaining two to ten cycles of fault duration, thereby preventing the rotor from attaining an excessive velocity away from synchronism during the fault. In addition the series resistance is beneficial after a fault has been cleared because it heavily loads the hydro-generators at the expense of delivering less power to the load center. The hydro-generators, which have a relatively low stored energy compared to the load center, are retarded at a much greater rate than the load center so that the series resistance tends to draw them together. The added losses will cause a slight decrease in system frequency, but this is of no consequence as the governors can restore the frequency as long as synchronism is maintained.

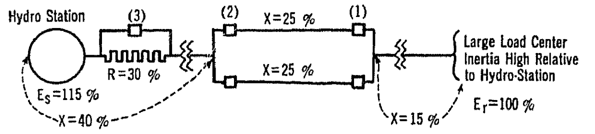

FIG. 1—HYDROELECTRIC TRANSMISSION SYSTEM USED

Example: 60 cycles. Per cent impedances based on generator-kw. rating, represent typical proportions. The 30 per cent series resistor is normally short-circuited

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The action of the series resistance during and after a fault will be considered in detail with reference to the hydroelectric transmission system shown in Fig. 1. The impedance proportions are typical and the stored energy of the hydro station is low relative to the load center. A 30 per cent resistance short-circuited by circuit breaker (3) is shown in series with the generators. Actually the preferred location is in the neutral of each generator in accordance with Fig. 2. The per cent resistance on the generator rating will then be the same regardless of the number of machines in service.

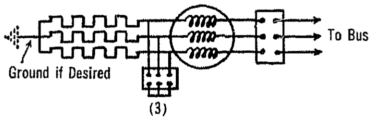


Fig. 2—Preferred Location of Series Resistor in the Neutral-Phase Leads of Each Generator

The conditions produced on the system by a three-phase fault near the load end, with and without the series resistance are compared in Figs. 3, 4, and 5 for 8-16-cycle sequential circuit breaker clearing. The initial angular displacement for 100 per cent load and both lines in service is 31 deg. When the short circuit occurs the generator electrical output is decreased from 100 per cent to 12 per cent on account of the low power factor in the fault current circuit. The prime mover continues to apply 100 per cent torque and the difference, or 88 per cent torque, accelerates the rotor.

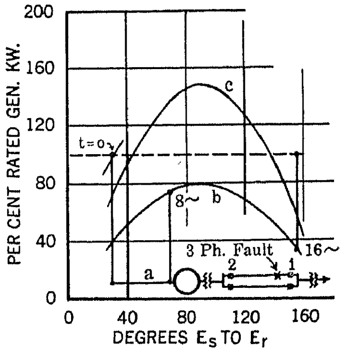


Fig. 3

Fig. 3—Generator Power-Angle Diagram. No Series Resistance

Curve	Time	Fault	Bkr. 1	Bkr. 2
a.	0-8 ~.	On. . . .	Closed. . .	Closed
b.	8-16 ~. . . .	On. . . .	Open. . . .	Closed
c.	16-∞, ...	Off. . . .	Open.	Open

After fault is cleared there is no restoring power as curve *c* is below 100 per cent line

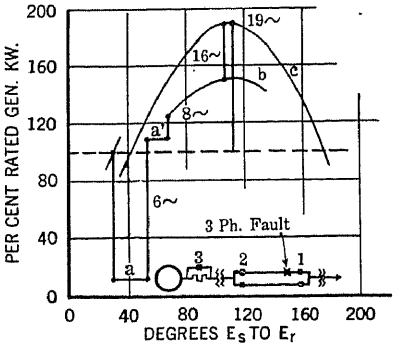


Fig. 4

Fig. 4—Generator Power-Angle Diagram. 30 Per Cent Series Resistance Inserted in 6 Cycles

Curve	Time	Fault	Bkr. 1	Bkr. 2	Bkr. 3
a. . . .	0-6 ~. . .	On. . .	Closed.	Closed.	Closed
a. . . .	6-8 ~. . .	On. . .	Closed.	Closed.	Open
b. . . .	8-16 ~. .	On. . .	Open. . .	Closed.	Open
c. . . .	16-* ~. .	Off. . .	Open. . .	Open. .	Open

*Breaker 3 will be reclosed in from 30 to 200 cycles depending on the system. Operation then will be on curve *c* in Fig. 3.

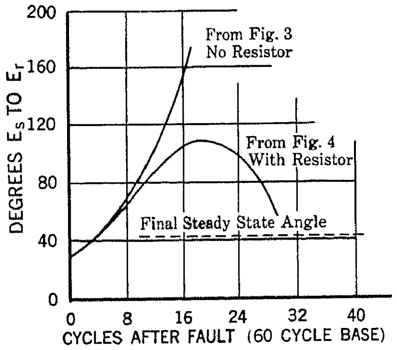


Fig. 5

Fig. 5—Angular Displacement vs. Time After Occurrence of Fault

Without series resistance the hydro generators are 180 deg. out of phase in 18 cycles. With series resistance synchronism will be maintained by a wide margin

Figs. 3-4-5—Three-Phase Fault Near Receiving End of Hydroelectric System Shown in Fig. 1 With and Without Series Resistance. Circuit Breaker 1 Opened in 8 Cycles, Circuit Breaker 2 Opened in 16 Cycles

Fig. 3 shows conditions without the use of series resistance. The angular displacement at eight cycles has increased to 69 degrees and the opening of circuit breaker (1) transfers operation to curve *b*. The system voltage is now partially restored and some power is delivered to the load center. This accounts for the maximum power of curve *b* being 80 per cent even though the fault is not entirely removed. The difference between the ordinates of curve *b* and the 100 per cent line represents the percentage of full-load torque accelerating the rotor and at 16 cycles the angular displacement is 176 degrees and synchronism has been lost.

Fig. 4 shows the same fault and the same line circuit breaker sequence with 30 per cent resistance inserted in series with the generators six cycles after the fault occurred by the opening of circuit breaker (3). In the six- to eight-cycle period the fault current produces losses in the resistor that increase the generator output from 12 per cent to 110 per cent even though no power can be delivered to the load center through the three-phase fault. The opening of circuit breaker (1) at eight cycles transfers operation to curve *b*. The losses in the series resistance plus the power delivered to the load center increases the maximum power to 152 per cent as compared to 80 per cent without the resistor. At 16 cycles circuit breaker 2 opens and the generator output will be in accordance with curve *c*, Fig. 4, which has a crest of 190 per cent as compared to 150 per cent for the corresponding curve *c*, Fig. 3, without series resistance. The shaded area below the 100 per cent line representing prime mover input is a measure of the energy expended in accelerating the rotor, and the equal shaded area above the line indicates that this energy has been absorbed and the crest of the angular swing away from synchronism has been reached. It is evident that there is ample area available under curve *b* and synchronism would have been maintained even though circuit breaker 2 had not opened for a considerably longer period of time. The proper time for short-circuiting the series resistance will be discussed later. When this is done operation will be transferred to curve *c*, Fig. 3, which represents normal conditions with one line out of service.

The angular displacement *vs.* time with and without the use of series resistance is shown by Fig. 5. These curves were calculated by point-by-point methods using the power angle diagrams of Figs. 3 and 4 and typical hydro generator stored energy. It is evident that synchronism will be lost without the use of series resistance and will be maintained by a wide margin if it is used.

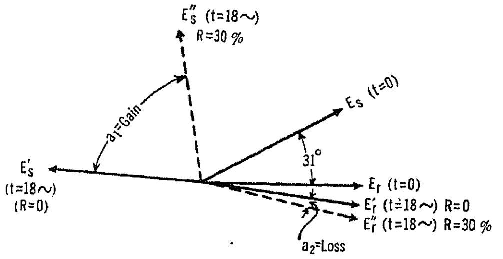


FIG. 6—VECTOR POSITIONS OF E_s AND E_r 16 CYCLES AFTER FAULT

Angle between E_s and E_r decreased by series resistance. Low inertia generators retarded at greater rate than high inertia load center slows down

The foregoing indicates the beneficial effect of loading the generators by inserting series resistance in the circuit during the fault period. After the fault has been cleared the added series resistance component, even though it increases the total circuit impedance, exerts a substantial stabilizing influence because the stored energy of the hydro generators is less than that of the load center. This action will be analyzed in detail.

The generator and receiver circle diagrams shown in Fig. 7 apply to the system of Fig. 1 during single-line operation immediately after a fault has been cleared. With no series resistor the maximum power from the generator and receiver circles is practically the same. The addition of 30 per cent series resistance as shown by the dotted curves increases the maximum generator output from 150 to 190 per cent, but decreases the maximum received power from 140 to 92 per cent due to the added transmission loss. The power angle diagrams for these conditions calculated from Fig. 7 are shown by Figs. 8 and 9.

Fig. 8 shows the generator output plotted against angular displacement with and without series resistance. The area above the 100 per cent line represents the energy available for retarding the rotor after fault has been cleared. The 30 per cent series resistance greatly increases this area and also permits a maximum angular swing of 188 deg. instead of 138 deg.

Fig. 9 shows the corresponding receiver power angle diagrams indicating that the received power is considerably reduced by the series resistance loss.

The influence of the series resistance on the angular positions of the generator and receiver voltages with typical stored energy proportions between the hydro generators and the synchronous machines at the load center is illustrated by Fig. 6. The line circuit breakers were assumed to operate in 8 and 16 cycles respectively. The initial angle with both lines in service was 31 deg. Without series resistance the energy represented by the shaded area under the 100 per cent line in Fig. 3, acting on the relatively small hydro generator stored energy moves E_s to E_s' in 16 cycles. Using the series resistance the shaded area above the 100 per cent line in Fig. 4 beings to retard the hydro generators after six cycles so that E_s only moves to E_s'' in 16 cycles. The series resistance, therefore, decreases the angular displacement of the generators by the large angle a_1.

Considering the load center the series resistance reduces the delivered power as may be ascertained from Fig. 8. Due to the relatively high load center inertia this reduction in delivered power only moves E_R apart from E_s by the small additional angle a_2. At the end of 16 cycles the total angular displacement has been decreased $(a_1 - a_2)$ deg. due to the insertion of 30 per cent series resistance, and the velocity away from synchronism has also been practically removed as may be ascertained from Figs. 4 and 5.

FIG 7.

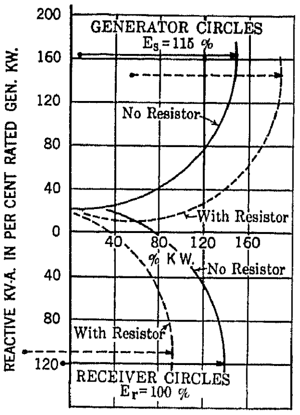
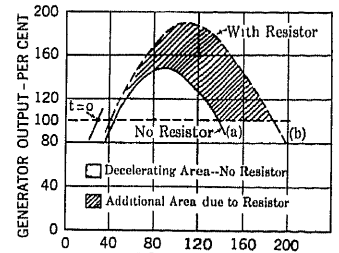


FIG 8.



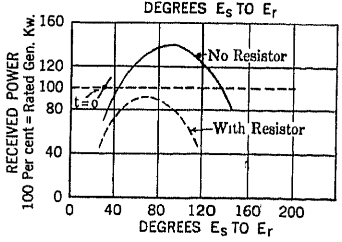


FIG. 9

FIG. 7—CIRCLE DIAGRAMS WITH ONE TRANSMISSION LINE SERVICE

FIG. 8—GENERATOR POWER-ANGLE DIAGRAM

FIG. 9—RECEIVER POWER-ANGLE DIAGRAM

EFFECT OF 30 PER CENT SERIES RESISTANCE ON CHARACTERISTICS OF HYDROELECTRIC TRANSMISSION SYSTEM IN FIG. 1 DURING SINGLE-LINE OPERATION AFTER FAULT HAS BEEN CLEARED

A test set-up was made in the shop using proportions similar to the hydroelectric transmission system shown in Fig. 1 for the purpose of checking our methods of calculation. Fig. 10 is an illustration of the shop system. The hydro station was represented by a 100-kva. synchronous motor-generator set with the d-c. end acting as a prime mover. Sufficient external resistance was used in series with the motor armature to give it approximately constant torque characteristics over a sufficient range of speed. The detailed test data cannot be presented in this paper on account of space limitations. The tests checked calculations closely, and showed that for the same fault and time of circuit breaker operation, up to greater than 50 per cent more power could be delivered and still maintain synchronism by the use of series resistance. The tests showed that quick response excitation functions very effectively in conjunction with the series resistance. The series resistance would hold the system in synchronism until the internal machine voltages could be actually built up to produce an exceedingly rigid system by the time the series resistance was removed.

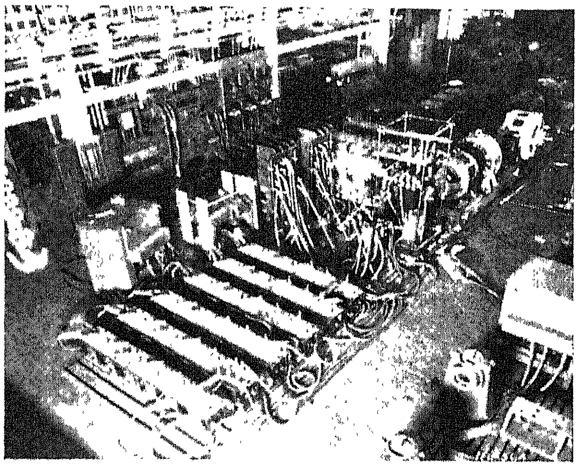


FIG. 10 SHOP TEST USING 100-KVA. GENERATOR

For illustrative purposes the preceding discussion dealt with a simple system having infinite capacity at the load center, except for the discussion regarding Fig. 6. The transients produced by a two-phase-to-ground short circuit on a system where the synchronous equipment at the load end is smaller in proportion to the hydro station than is the case on the usual system are shown in Fig. 11. The angular oscillations were calculated by point-by-point methods in conjunction with the a-c. calculating board. The dotted line shows the rapidity with which the hydro generators go out of synchronism with reference to the synchronous machinery at the load end if no series resistor is inserted. The full lines show the angular oscillation with 30 per cent series resistance on the generator rating inserted in the circuit during the 6-25 cycle period following the fault. It is evident that synchronism will be maintained, as the angular deviation has been restored to about normal in 60 cycles and will stabilize itself after a few oscillations. The downward slope of the curves indicates that the system frequency has decreased slightly due to the losses while the series resistance was in circuit, but the slight decrease in frequency is of no consequence as long as synchronism is maintained.

Three-phase faults, which are the most severe type, were used for the analysis of faults on the system of Fig. 1. Single-phase and double line-to-ground faults do not lower the generator output to such an extent as three-phase faults and these faults in the 8-16-cycle tripping zones could be handled even if the series resistance were not inserted until the fault was closed. The increased area under the generator power-angle curve as shown by Fig. 8 would be sufficient to prevent over-swing, and inserting the resistance during the fault period, while not as effective as for three-phase faults, has beneficial effect and serves to increase the margin of stability. It is evident that if a system can be designed to withstand the severe three-phase faults, it will as a consequence withstand other faults by a wide margin.

The magnitude of the series resistance can be varied over a wide range with practically the same results. The resistance should be inserted as soon as possible after a fault occurs. The length of time that it is left in the circuit will depend on the individual system under consideration.

A number of methods for determining the time of removing the resistor will be considered:

1. Remove the resistor at the crest of the first swing, or slightly afterwards, but before the rotor has gathered enough return velocity due to large area under curve *c*, Fig. 4 to go out on the next swing along the normal operation curve in Fig. 3.

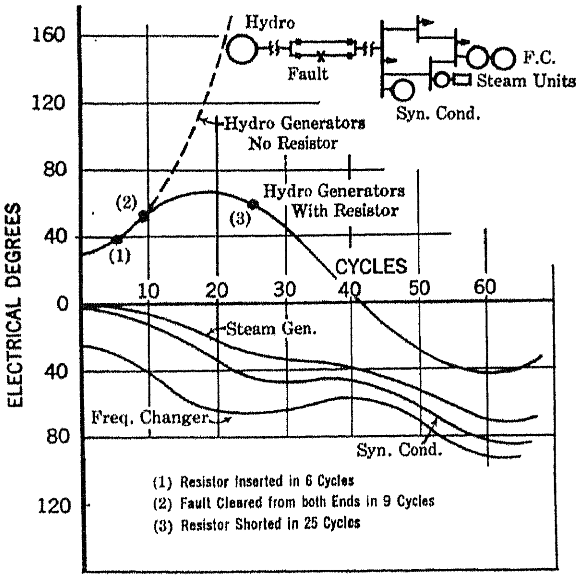


FIG. 11—TRANSIENTS ON TYPICAL SYSTEM SHOWING EFFECT OF INSERTING SERIES RESISTANCE

From point-by-point calculations made in conjunction with a-c. calculating board

2. Leave the resistor in the circuit until the severe oscillations are damped out, which occurs in two or three seconds. The resistor can then be short-circuited without producing any appreciable disturbance as there is only four degrees angular difference with and without the series resistance at 100 per cent load as shown by Fig. 8. Quick response excitation will in this time have

increased the internal voltages so that the crest of the the curve *a*, Fig. 8, will approach curve *b* forming a rigid system for a short time after the resistance is short-circuited.

3. Short-circuit out the series resistance in steps and gradually "notch" from curve *b*, of Fig. 8 curve *a*.
4. There is usually a number of machines in a station and the neutral circuit breakers short-circuiting the resistors can be closed successively, producing the same effect as cutting the resistance out in steps.

The series resistor used in the neutral of each generator as shown by Fig. 2 will have approximately the same physical size as an ordinary grounding resistor. The circuit breaker short-circuiting the resistor need not have a high rupturing capacity and the duty is light since it is shunted by resistance.

CONCLUSION

The foregoing indicates that synchronism can readily be maintained on a hydro system of normal proportions for any type of a fault with the speeds of circuit breaker and relay operation now available, by inserting series resistance in the circuit for a short time. The cost of the series resistor and its circuit breaker will generally be less than the combination of other measures such as high rotor inertia, low generator and transformer reactance, damper windings, impedance in the transformer neutrals and special simultaneous operation relay schemes that have been considered in the past.

After confidence has been established in the series resistance method of maintaining stability, consideration may be given to building lower cost transmission lines that will be expected to flash over, instead of trying to solve the stability problem by building lightning proof lines.

Unlike most of the other measures for improving stability the resistor installation can be deferred until operating experience indicates that the added expenditure to maintain service is justified.

Discussion

S. B. Crary: The use of series braking resistors increases the generator output at a sacrifice of the receiver or motor input and of the simplicity of the cycle of operation while the fault is on and after it is cleared. This method then does not belong in the category of those methods which make the system inherently more reliable, such as high-speed switching, fast relaying and lightning proof lines; but in a class of methods which attempt to increase the reliability at a sacrifice of other characteristics of the system.

The effect of series braking resistors has been analyzed over a whole range of switching times rather than at a particular set of switching times as was done by Mr. Bergvall. By doing this a more complete picture of the problem is available.

Fig. 1 illustrates graphically the results of calculations made to determine the relative gain that is obtainable by the use of a series braking resistor. Calculations were based on a typical 60-cycle system using the reactances shown, which were the same as used by Mr. Bergvall. A 30 per cent series resistor, which is about the optimum value, was used and the same as used by Mr. Bergvall for this particular system. Conditions as assumed are ideal for the series resistor, in that an infinite bus receiver was assumed, which is a system so large compared with the generator that its frequency or voltage does not change during any fault conditions, and is not affected by the decreased amount of power that it receives from the generator while the series resistance is inserted in the circuit. The theoretical gain that can be realized for a sequence of switching of 6, 8, 16 cycles (6 cycles being the time required for breaker 3 to open, 8 cycles for breaker 1 to open, and 16 cycles for breaker 2 to open) is in the neighborhood of 50 per cent. This total gain, however, is not all necessary as the rated output of this typical system is unity.

The practical gain to be realized from the use of the series braking resistor decreases with faster switching time. It has already become practical to consider actual switching times under favorable conditions of the order of 6 cycles or less as is shown in the paper by Messrs. Spurck and Strang. At the point of rated output for the particular system being considered, 6 cycles switching is sufficiently fast to allow the system to ride through a three-phase fault at full load. The curves given on this figure are for a fault at the receiving end of the transmission line, which is just as severe as a fault at the generating end of the line as breaker 2 is not opened until the end of 12 to 16 cycles for the case of a fault at the receiving end as against opening in 6 to 8 cycles for a fault at the generating end.

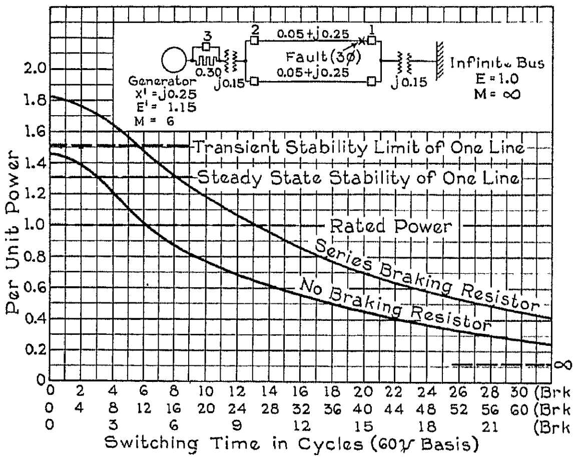


FIG. 1—POWER THAT CAN BE CARRIED THROUGH THE FIRST SWING WITH A THREE-PHASE FAULT WITH AND WITHOUT SERIES BRAKING RESISTORS

Calculations show that these faults are of equal severity. Faults outside of the 10 per cent zone at either end will be cleared at both ends in 6 to 8 cycles so that the curves given above are for the most severe faults that could be obtained on the transmission lines. For three-phase faults in the center section of either line the theoretical gain to be realized from the series resistance over the case when no braking resistors are used is reduced to 34 per cent for 8-cycle switching of the line breakers and 6-cycle switching of the resistor breaker. This reduction in the theoretical gain for a fault in the mid-section is accompanied by a reduction in the practical gain also, for 7 to 7½ cycle switching of the line breakers at either end is sufficiently fast to allow the system to carry full rated output through the first swing without the use of braking resistors for this particular location of the fault.

With the recent improvement in switching times the possibility of three-phase faults occurring on a system has been reduced. Even in the past three-phase faults have been fairly infrequent. For faults other than three-phase, the power limit is greater. As Mr. Bergvall has stated "if a system can be designed to withstand the severe three-phase faults it will as a consequence withstand other faults by a wide margin." High-speed switching then for this particular type of system will normally give a margin of power limit above the rated output and, therefore, eliminates the need of braking resistors and the accessory apparatus that must be used with them.

If the system under consideration has slower speeds of switching, it is problematical whether it would be good engineering to equip the generating station with series braking resistors, the necessary additional circuit breakers, and the required control equipment to protect against the occurrence of three-phase faults, when the manufacturers are already in a position to supply breakers and relays with an over-all operating time of approximately 8 cycles or better.

All of the foregoing has been based on a system having a comparatively large receiving end inertia as compared with that of the sending end.

Consideration, however, should also be given to a system which depends considerably on the hydro-station for its power when the hydro-station is operating at or near full load. Such a system would suffer from the loss of the hydro-station sufficiently so that a large portion of the load would have to be dropped, while with the system of the previous case the loss of the generating station would not necessarily be accompanied by a loss of load.

With a system having a comparatively large hydro-station as compared with the load center, conditions are different from that of the case when the load center has a relatively large inertia. A system of this type may have a receiving end inertia which is only twice as large as the generating station. Upon the occurrence of a fault on the lines connecting the hydro-station to such a load, the motor may swing to a considerable angle away from the generator before the fault is cleared, so that it may not be possible for synchronism to be maintained when the restoring forces become effective, *i. e.*, upon the clearing of the fault. The stability problem of such a system is much more acute as may be seen by referring to Fig. 2. This is the type of system which needs a means of improving its stability limit. The stability characteristics of such a system have been studied.

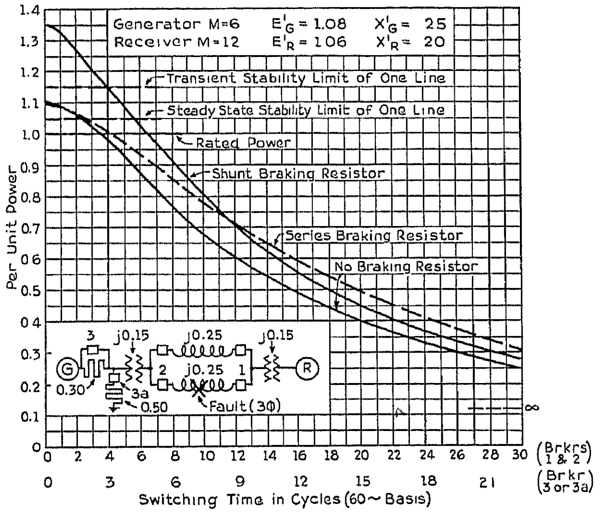


FIG. 2—POWER THAT CAN BE CARRIED THROUGH THE FIRST SWING WITH A THREE-PHASE FAULT WITH AND WITHOUT BRAKING RESISTORS

Mr. Bergvall in his paper selected as his second case a system having a finite receiving end inertia. As the constants of the system were not given, a system was set up having the constants as shown on Fig. 2. This figure gives the switching time curves for a system having an equivalent receiving end inertia of twice the sending end inertia. The load is taken as 160,000 kw. The hydro capacity is taken as 100,000 kw. and the steam capacity as 60,000 kw.

Mr. Bergvall in his calculations used a two-phase to ground fault, at the center of the line, however, since a three-phase fault was considered in the previous case it seems logical to examine the case of three-phase faults on this system as well, so the results will be more closely comparative. Fig. 2 is a switching time curve for this system with a three-phase fault in the center of the line. On such a system the series braking resistor gives a much smaller gain (10 per cent at 8 cycles) in the stability limit for the switching sequence assumed (the braking resistor breaker opens in ¾ of the time it takes the line breakers to open) the gain in power limit is reduced due to the smaller relative inertia of the receiver. This lower receiving end inertia is affected more by a decreased power input, which is brought about by the insertion of the series resistance, than it would be if it were very large. A decrease in power input causes the motor to slow down and therefore the generator must slow down at a faster rate in order that the angle between them will be decreased and synchronism maintained. This results in a greater dip in frequency and receiving end voltage than would normally be obtained. The standards of system requirements are already fast traveling beyond the point at which it is only necessary to consider means of holding the system together, but it also becomes necessary to keep the voltage and frequency at as near a constant value as possible.

There are other methods by which the stability limit may be increased for this particular type of system that parallel the series resistance method somewhat in principle. Fig. 2 has included on it a curve for a shunt braking resistor, this braking resistor to be thrown across all three phases to ground after the occurrence of a fault. The particular value of resistor used in calculating the above curve is 50 per cent. That is, with 50 per cent of normal voltage across the resistor, the loss in the resistor will be normal or 100 per cent kw. The speed at which this resistor was connected to the circuit was taken as 6 cycles for 8-cycle switching of the line breaker, the same time as Mr. Bergvall selected for inserting the series resistance after the occurrence of the fault. A breaker can be built for this duty such that the necessary relay time and the speed of closing is 6 cycles, or less. The shunt braking resistor curve shows a greater power limit than the series for switching times less than 11 cycles when used in this particular system.

The use of either series or shunt braking resistors, if used at all, should be strictly limited to the period during which the station is operating at or near full load. The value of resistance selected to result in any appreciable gain in power limit at full load must, of necessity, be of such a value as to load the generators down considerably. The loss in the resistor during a large portion of the period while it is connected in the circuit is the rated kw. output of the station. If the generator or the generators of the station are lightly loaded, switching on the braking resistors with the occurrence of a fault would increase the magnitude of the oscillation rather than decrease it. This objection, however, could be solved by the use of a wattmeter control scheme.

In general, therefore, the following conclusions may be made as to the benefit to be derived from the use of the shunt and series braking resistors.

Approximately 75 per cent of the generating capacity of the country is steam whose stability problems can in most cases be solved by other means. Out of the remaining 25 per cent of hydro capacity those stations having stability problems may be grouped into two general classes; systems with relatively large receiving end inertias, and systems with relatively small receiving end inertias.

a. For systems having a relatively large receiving end inertia, high-speed switching will, in general, be sufficient to carry the system through the most severe faults.

b. For systems having a relatively small receiving end inertia, series resistance method shows very little gain in power limit with the shunt resistor method showing a slightly greater advantage for the switching times now available.

J. W. Anderson: As pointed out by Mr. Bergvall, hydroelectric generators because of their relatively low inertia have a very marked tendency to overspeed upon loss of load immediately after a short circuit occurs. Some actual figures of acceleration will serve to emphasize this effect. Tests made some time ago at Conowingo to determine governor action upon sudden loss of load showed that generator acceleration during the first half-second after dropping full load averaged over 900 elec. deg. per sec., or, in other words, the generator advanced more than a full cycle in the first half-second after dropping full load. During this period the governor had closed the gates only a negligible amount. A similar test dropping half load indicated an average acceleration of about 500 deg. per sec. for the first half-second after loss of load. However, for these tests the load was dropped by opening an oil circuit breaker which entirely disconnected the generator from the system. Actually under the usual short-circuit condition of one or two phases faulted to ground, some synchronizing torque will exist between the hydroelectric generator and the steam system which will tend to retard acceleration of the former. An oscillogram of an actual two-phase-to-ground fault on the Conowingo system which occurred at a time when four generators were on the line carrying approximately full load indicated that the waterwheel generators during the 30 cycles on the Philadelphia system immediately following the fault accelerated at an average rate of about 580 deg. per sec., or in other words, Conowingo was 180 deg. out of phase with Philadelphia in a little less than 20 cycles. These figures, of course, reflect the slight tendency for the steam system to slow down referred to by Mr. Bergvall.

The use of resistance to load hydroelectric generators on the occurrence of a short circuit is apparently an effective method of preventing overspeeding of waterwheels to the point where instability results. The idea of the installation of resistors to prevent excessive overspeeding has, of course, been under consideration for some years and at least one installation of a resistor in the neutral of high-voltage transformers has been made. It was formerly thought by some engineers that the resistance of an arc might be high enough to exert an appreciable effect in this direction and some stability calculations have been made using various assumed values of arc resistance. However, both accidental faults and staged short-circuit tests have indicated that arc and ground return resistance are not sufficiently high to produce the braking effect required to preserve stability.

The installation of resistors in the individual generator circuits offers some advantages over the installation of resistors in the neutral of high-voltage transformers, since with the individual resistors the amount of resistance in use varies automatically with the number of generators in service, and maximum advantage can be taken of graded transformer insulation since overvoltages accompanying the installation of transformer neutral resistors are eliminated, as are the effects of these voltages on lightning arresters and transmission lines. Another advantage of individual resistors in generator neutrals is their effect in reducing the terminal voltage of the generators, thus forcing the voltage regulating system to produce a higher internal machine voltage.

Resistor installations of the type proposed by Mr. Bergvall may constitute the only means of preserving stability or they may supplement or be supplemented by other methods such as the use of high-speed oil circuit breakers, high-speed excitation systems, damper windings, and apparatus of special design. In installations where the resistors would permit the use of generators of normal design, transformers of normal reactance, and normal speed oil circuit breakers and relays, it is probable that the resistors will be justified economically by a wide margin.

The choice of a suitable resistor for a given installation may require careful consideration. Although the amount of resistance is automatically varied with the number of generators in service, the total short-circuit current supplied to any fault will not vary in direct proportion to the number of generators in operation and the amount of short-circuit current supplied by any generator will vary with the number of generators in service for a given fault location. Further, to secure best results, the value of resistance should change for each type and location of fault. Under certain system conditions it may be desirable to operate hydroelectric units at reduced loads with the result that generators in service will operate at partial gate openings and a resistor designed for sufficient retardation of full gate opening may cause too great a retardation at partial gate openings. It is probable, however, that some average value of resistance can be found which will satisfactorily meet all conditions. Certainly in the interests of simplicity and good operation it would be undesirable to alter the resistor set-up for different operating conditions.

As Mr. Bergvall points out, the use of resistors may reduce the necessity for solving the problem of lightning-proof lines. We do not, however, subscribe to his suggestion that consideration be given to building lower cost transmission lines that will be expected to flash over. Although it is true that a saving of a few per cent such as might be obtained by building a line of reduced insulation might be greater in a long, high-voltage line than the cost of the resistor installation, it would appear that in any high-voltage, high-power installation where stability is a problem that required high grade of service would not permit deliberately inviting flashover even though there was no actual interruption to service.

G. D. Floyd: The idea of inserting resistance into a system at time of fault is not new, but the advantages of placing this resistance in the neutral of each generator in a station have not been investigated. The apparent simplicity of the device, which is nothing but a dynamic brake can be easily appreciated, and for this reason alone the arrangement is likely to find favor more quickly than other changes in generator or system layout, most of which added in greater or less degree to the capital cost of an undertaking, and had to be defended before they could be specified. Anyone who has had to do with the design of a power plant or layout of a system will welcome an improvement on which capital outlay can be deferred, but where the contingency of instability has been provided for. Objections are seldom raised where provision is made in a design for something which does not involve immediate capital outlay.

The very simplicity of the arrangement suggested by the author may, however, be deceiving, and each individual case where the use of the series resistor is contemplated should be examined carefully, before assuming that its use will prevent instability. A case in point is a system having hydraulic generators, a transmission line, and a large proportion of synchronous load, such as pulp grinders. It would appear from Mr. Bergvall's analysis that conditions at such a load may be made worse actually, by use of series resistors at the generating end.

In the calculations and tests covered by the paper, zero resistance at point of fault may be inferred. This is justified for three-phase faults, but in the case of single and double line-to-ground faults where there may be high resistance at fault location, the added resistance in the generator leads may cause a drop in speed of the unit sufficient to cause loss of synchronism. Cases are known where there was sufficient resistance at point of fault to cause drop in speed of the generators, where no other resistance was present except that of the line itself. The same argument would apply for those systems where the high-voltage neutral is grounded through impedance made up of appreciable resistance, unless special provision were made to allow the resistor in the generator neutral to be introduced only in the case of three-phase faults.

H. S. Phelps: In concluding his paper Mr. Bergvall states that following the establishment of confidence in the series resistance method of maintaining stability, consideration may be given to building lower cost transmission lines that will be expected to flash over instead of trying to solve the stability problem by building lightning proof lines.

That conclusion must, of course, be studied in conjunction with such papers as those presented by Harrison and Silver and Conwell and Warren presented as a part of the Symposium on Coordination of Power and Telephone Plant, see page—.

In the former paper the authors indicate various ways by which system reliability has been improved and likelihood of line flashover has been decreased.

In the latter paper the authors consider in some detail the influence on the coordination problem of such a proposal as that offered by Mr. Bergvall. It is discussed in their paper in the section entitled "Factors Controlling the Frequency of Occurrence of Induced Voltages."

It is, therefore, evident that before reaching a decision which would favor increasing the probable number of line flashovers there are several related problems that must be considered in order to assure that a satisfactory solution has been reached in a particular situation.

R. H. Park: Mr. Bergvall has presented a valuable analysis of the benefits to the stability of a hydroelectric transmission system which result when a resistor is momentarily inserted in series with the generator leads on the occurrence of a short circuit on the high-voltage lines. An alternative procedure would be to momentarily connect a shunt resistor across the generator terminals on the occurrence of a line fault. I should like to ask Mr. Bergvall what he considers to be the relative advantages and disadvantages of these two arrangements.

R. C. Bergvall: Mr. Crary's discussion is based on two figures showing the transient power limit plotted against switching time. Unfortunately, he varied the switching time of the series resistor whereas in any practical installation this time would remain constant at 6 cycles or less. This assumption places the series resistance method in an unfavorable light and, therefore, general conclusions cannot be drawn based on these figures.

I recognize that there are limitations to the use of series resistance in maintaining stability and each individual system must be considered by itself in applying the series resistance method. Fig. 11 in the paper represents such an analysis made in conjunction with the a-c. calculating board and shows the effectiveness of the series resistance in maintaining stability with a 2-phase-to-ground fault.

As indicated by Mr. Anderson and Mr. Floyd, the principal advantage of the series braking resistance method is that the expenditure can be deferred until operating experience indicates that service improvement will justify it. The use of damper windings, special reactance in generators and transformers, etc., is an expenditure that must be made at the time of original construction when the necessity for such expenditure may not be evident.

In analyzing an individual system it would be well to give consideration to the shunt resistance proposed by Messrs. Park and Crary as there may be a field for both the shunt and series resistance method.

Oil Circuit Breaker Tests—Philo 1930

Aims, Setup and Results from a System and Operating Viewpoint

BY PHILIP SPORN* and HARRY P. ST. CLAIR*
Fellow, A. I. E. E. Member, A. I. E. E.

AIMS AND OBJECTS OF TESTS

BEFORE undertaking the Philo 1930 132-kv. oil circuit breaker tests, which involved the placing of short circuits directly on the 132-kv. bus of the largest generating plant on the interconnected system of the American Gas and Electric Company group, careful consideration was given to the benefits to be obtained. Among these considerations were the following:

First, the proposed tests offered a means of determining positively whether a high capacity 132-kv. breaker would be able to withstand its full rated duty. This in itself was considered of great value. In addition, past experience justifies the conclusion that the carrying out of such tests almost invariably results in the discovery of one or more weak points in the breaker assembly; the remedies which are devised to overcome such weaknesses often constitute important advances in the art of circuit breaker design. An instance of this kind was provided by the tests on the 132-kv. breakers made at Canton, Ohio,[1] in 1925 and 1926, in the course of which tests weakness showed up in the explosion chamber assembly. The result was not only an immediate redesign of the explosion chamber assembly, but the beginning of the development which later, as the result of the Canton tests as well as subsequent tests on other systems, produced the non-metallic explosion chamber.

Second, the tests promised to materially contribute to the development of high speed in the explosion chamber type of breaker, a considerable number of which is in service on the Ohio Power Company's System and on the systems of other subsidiaries of the American Gas and Electric Company. At the time these breakers were installed speed as an item in the performance of a breaker was, even from an academic standpoint, of only minor interest. The closing speed was, of course, considered, but only from the standpoint of the breaker's ability to successfully synchronize large generators. At the present time, however, the high-speed breaker is recognized as a very important development which promises considerable relief from the damaging effects of system disturbances, such as flashovers, insulator failures, etc., and by means of it system stability in many cases may be greatly improved. In addition, the high-speed breaker is expected to prove of great value from the standpoint of service to customers by shortening the time of voltage surges and dips. This is particularly important to many customers using motors which either inherently or by arrangement of control apparatus have a tendency to drop off the line on voltage dips.

Again, the tests offered an opportunity to further demonstrate the feasibility of the multi-duty cycle to replace the present standard duty cycle for circuit breaker performance. As power systems grow more extensive and standards of service are raised, it is becoming more and more evident that the present standard duty cycle of two breaker openings separated by a two minute interval is hopelessly inadequate. It is well known that during storms, breakers are frequently called upon to operate many times without any intervening inspection, and that the interval of two minutes is entirely too long, and cannot be tolerated in most well-operated systems. In the Canton tests previously referred to, several groups of as many as six to eight shots with considerably less than two minute intervals were made on different breakers, the successful outcome of which indicated that a more severe duty cycle could be adopted. One of the aims of the Philo tests was, therefore, to carry out further tests of a similar nature to provide further evidence of the feasibility of changing the present duty cycle.

It was felt, too, that the carrying out of heavy duty short-circuit tests directly on the high-tension bus of a large generating plant would give valuable information not only on the performance of a breaker located at such a point, but also on the performance and operation of the plant itself under short-circuit conditions. Carrying out the tests at this location also made it possible to obtain experimental confirmation of a recently enunciated theory regarding recovery voltage phenomena in oil circuit breaker operation.[2] In order to obtain this experimental confirmation it was necessary to carry out short-circuit tests with various numbers and lengths of 132-kv. lines connected to the test bus, a condition which was met by the Philo setup.

SYSTEM SETUP AND ARRANGEMENTS FOR PHILO OIL CIRCUIT BREAKER TESTS

The system setup for the Philo short-circuit tests is shown in Fig. 1. It will be observed that practically the entire system of the American Gas and Electric Company group, together with some of the important interconnections was tied together in a practically normal setup.[3] No special provisions were made to take care of any portions of the system load from sources not connected with the system supplying the tests; normal interconnections with West Penn. Power Company

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*Both of American Gas and Elec. Co., 30 Church Street, New York City.

[1] For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

on the east, with Chicago on the west, and with North Carolina on the south, were maintained. With this setup a short circuit of over 1,500,000 kva. was obtained at the Philo 132-kv. bus. The major portion of this total, or approximately 1,000,000 kva., was supplied by the Philo plant itself, while the remainder was supplied by the various generating plants distributed at more or less distant points throughout the system. These remote generating plants in some cases maintained only their usual light load (Sunday) running capacity, but in other cases, such as at Windsor, Cabin Creek, Kenova and Glen Lyn, one or more additional generators were put on the line to increase the available capacity for the tests. The tests made at Canton[1] in 1925 and 1926 involved only about one-half of the short-circuit kv-a. obtained in the Philo tests and this was supplied from several generating plants all located at some distance and connected by transmission lines. Therefore, in severity of duty on any one generating plant and on the breaker, the Philo tests represented a step of a considerable proportion.

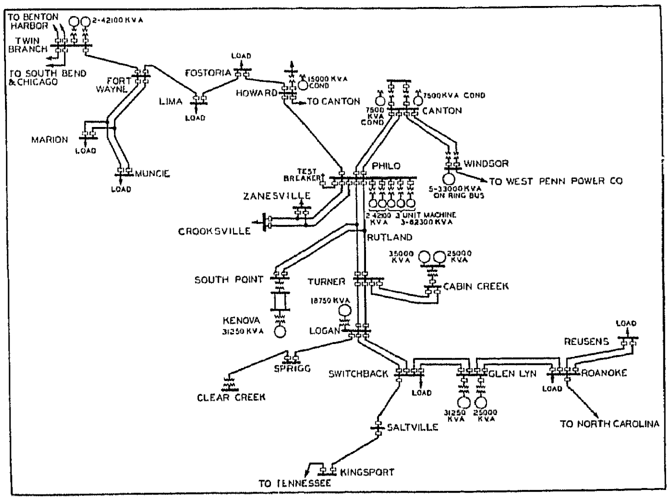


Fig. 1—Diagram of the American Gas and Electric Company 132-Kv. Interconnected System as Set Up for the Philo Oil Circuit Breaker Tests

Figs. 2 and 3 show the location of the test breaker with respect to the plant and switch yard respectively. The test breaker was mounted on a concrete mat foundation located on a cinder fill and was connected by means of short overhead 250,000 cm. copper conductors to the end of the 132-kv. reserve bus. By a special arrangement of the main, reserve and transfer buses, and by using both the transfer breaker and one line breaker, it was possible to place two breakers in series to serve as backup breakers during closing-opening tests and as closing and backup breakers respectively during opening tests.

In Fig. 3 immediately to the right of the test breaker, in the background, may be seen the temporary building which was erected to house the magnetic and cathode-ray oscillographs, together with dark room and other necessary equipment.

Results of Tests

A companion paper[4] gives detailed quantitative data on the results of the Philo tests with particular emphasis on an analysis of the effects of varying rates of rise of recovery voltage on the breaker performance. Briefly, however, the tests comprised a total of 81 shots divided into three separate groups or programs carried out on three separate days. In order that the tests might be carried out at full rating, a 60-in. tank breaker having a rating of 1,500,000 kva. was used.

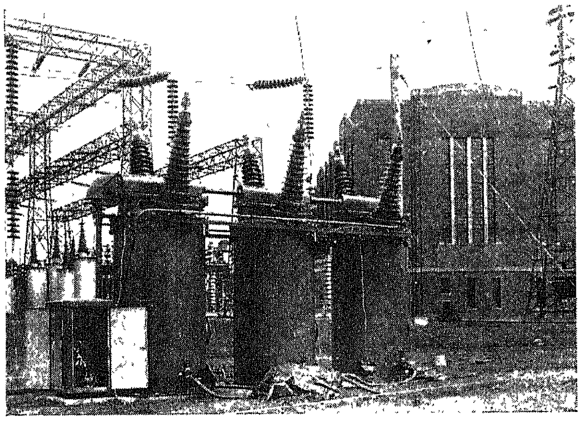


Fig. 2—View of Test Oil Circuit Breaker and Philo Plant

The first group, consisting of 28 shots on the breaker equipped with standard butt type explosion-chamber contacts, included 7 shots with Philo generators alone supplying the short circuit and with no transmission lines connected to the short-circuited bus. These shots, of which 4 were made with full capacity of Philo plant, undoubtedly imposed a severe duty on the breaker as evidenced by length and duration of arc. The greatest distress was shown by the breaker on two of the shots in which approximately two-thirds of the Philo capacity

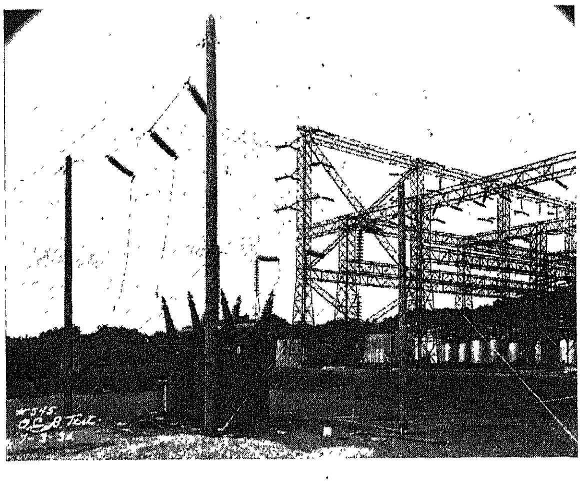
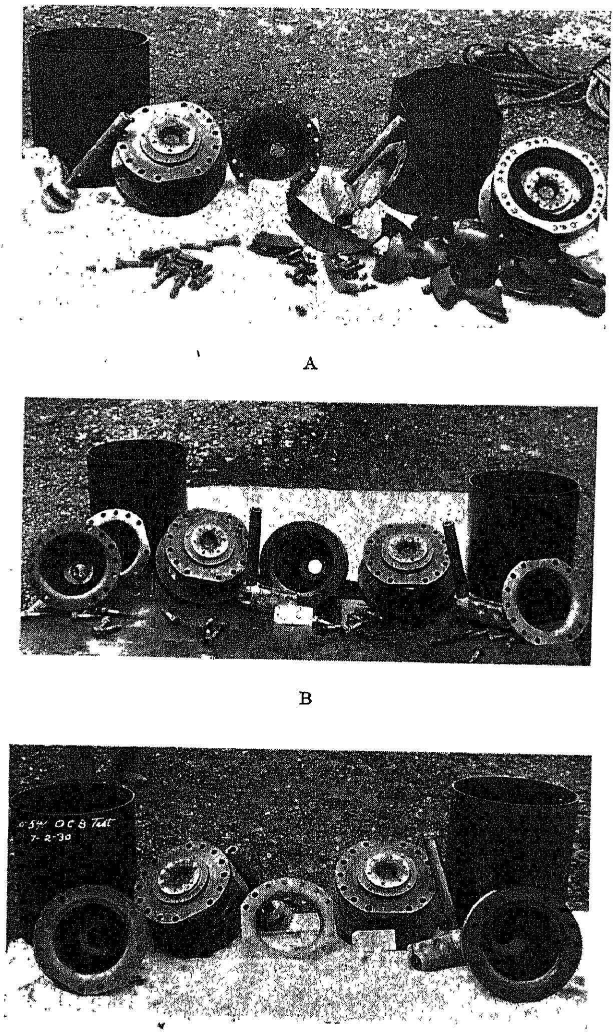


Fig. 3—View of Test Oil Circuit Breaker and Philo 132-Kv. Switchyard

was used. While the breaker succeeded in rupturing the arc on both of these shots before the opening of the backup breaker, there was evident an unusual amount of smoke and some oil discharge from breaker vents, and the oscillographic records showed arcing continued to the end of the stroke. Subsequent tests under the same recovery voltage conditions but with maximum Philo plant capacity showed no repetition of this distress, the arc being cleared considerably before the end of the stroke. In addition to a number of shots taken with part or all of Philo capacity and with from one to three open transmission lines connected to the short-circuited bus, five shots were taken with the maximum available system capacity, resulting in a duty on the breaker either at or above its full rating.



A

B

C

Fig. 4—View of Contacts of Standard Butt Type Explosion-Chamber Oil Circuit Breaker After Test of June 29

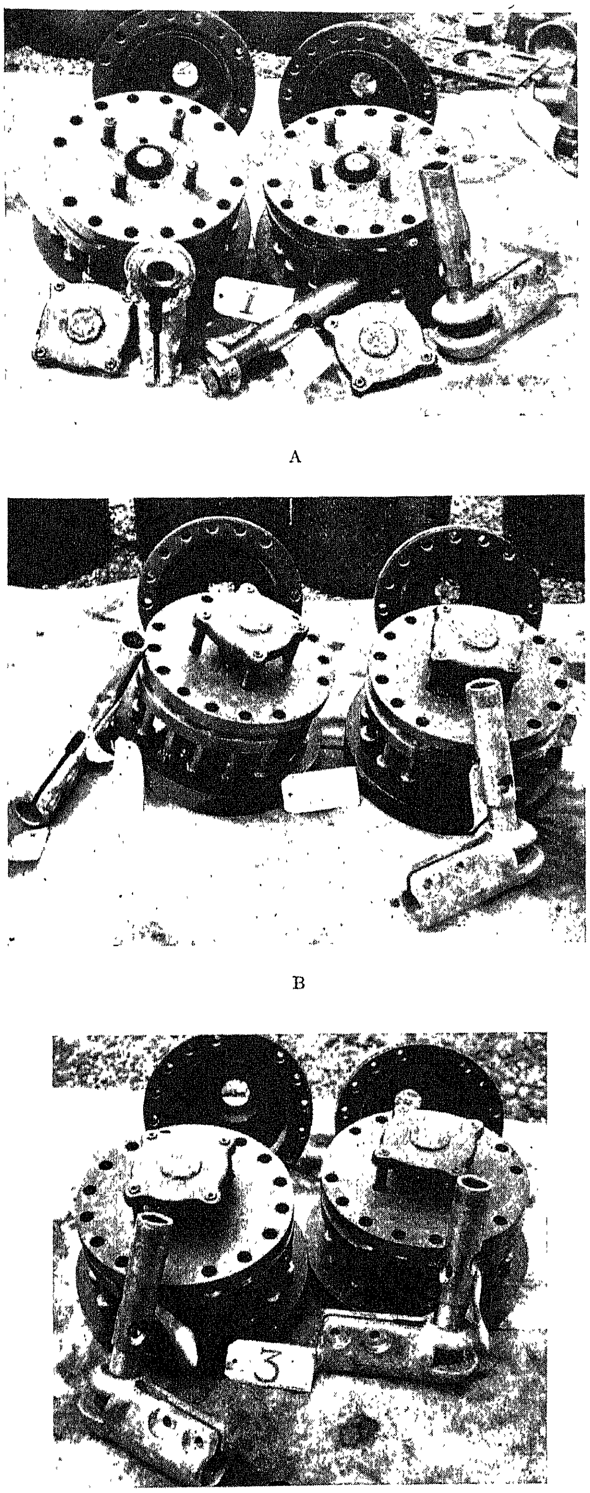
Inspection at the conclusion of the 28 shots revealed one of the explosion chambers completely shattered, with evidence from the manner of burning on the contact rod that the breaker had been operated a number of times in this condition. While it was thought that the breaking of the explosion chamber might have occurred on the 12th or 13th shot, on which the maximum distress was apparent, no other evidence as to the particular shot on which the breakage occurred was available.

Fig. 4, *A*, *B*, and *C*, shows all of the contacts of this breaker as they appeared at the conclusion of the tests. It will be noted that except for the broken explosion chamber on phase 1, all of the contacts were in excellent condition considering the number of shots that had been taken on the breaker.

The second group, consisting of 40 shots on the same breaker equipped with the new "oil-blast explosion chamber"[5] included a total of 16 shots with the maximum rate of recovery voltage rise obtained as above with no lines connected to the bus. Seven of these shots were made with the full capacity of the Philo plant behind the short circuit giving a duty on the breaker varying from a little below 1,000,000 kva. to 1,285,000 kva. interrupted. Six shots were made with the maximum available system capacity, all of them at a duty of more that the full breaker rating of 1,500,000 kva. with a maximum of 1,735,000 kva. on the last shot.

The performance of the breaker was satisfactory throughout and all of scheduled shots were taken notwithstanding the coincidence that on the last test a contact rod was lost through breakage of a crosshead. This breakage was due to abnormal stress resulting from the use of a rod which was too short to be properly threaded into the crosshead. Oscillograms showed that the breaker carried its full three-phase short-circuit current on this last test, although, of course, one phase would not have closed had any further tests been made. This and other breakage occurring on the contacts of the same phase on this last shot are described in another paper[6] and are illustrated in Fig. 5, shown herewith. It will be noted that the contacts on the other two phases appeared in good condition following the total of 40 shots.

As a final group of tests it had been planned to take around 20 shots on the same breaker equipped with the older type segmental contacts and explosion chambers. Due, however, to the occurrence of distress resulting in a broken bushing on the breaker (Crooksville No. 1) used as a closing breaker for opening shots on the test breaker, the program had to be cut short with only 13 shots. After taking 6 shots at intervals of from ¾ minute to 1½ minutes with light duty both in generating capacity and rates of recovery voltage rise, a total of 6 shots was made at intervals of from 1 to 3 minutes with the maximum available system capacity. On all of these 6 full system shots, which were handled by the breaker without apparent distress, the duty was approximately 1,400,000 kva., or slightly under the rated capacity of 1,500,000 kva. As stated above, the tests were cut short after the completion of the 13th shot. This shot, intended to be the first of a series, was taken with the full capacity of the Philo plant and with medium rate of rise of recovery voltage as obtained with one short line connected to the short-circuited bus.



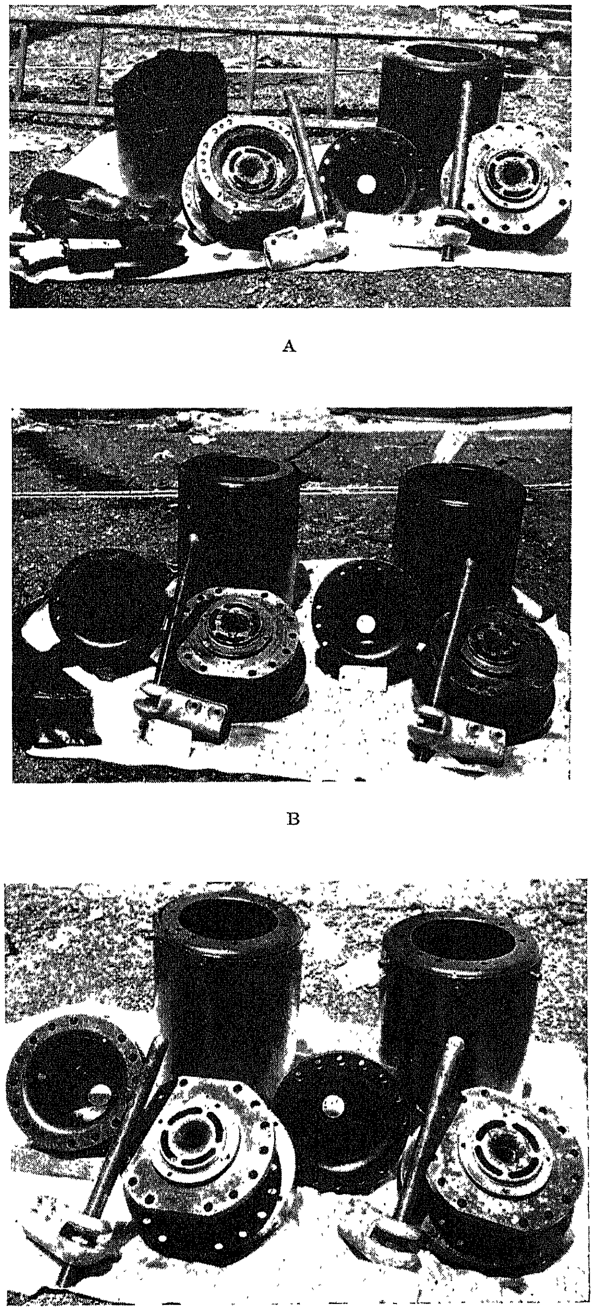
A

B

C

Fig. 5—View of Contacts of Oil-Blast Explosion-Chamber Breaker After Tests of July 4

As in the tests on the standard butt type contacts, inspection after the tests disclosed one of the explosion chambers shattered. Some idea of the force with which this explosion chamber blew up was evidenced by holes in both thicknesses of the tank linings made by pieces of the explosion chamber. Fig. 6A shows the shattered explosion chamber on phase 1, together with the remaining contacts on all three phases of this breaker following the tests. It will be seen that the remaining contacts came through in fairly good condition.



A

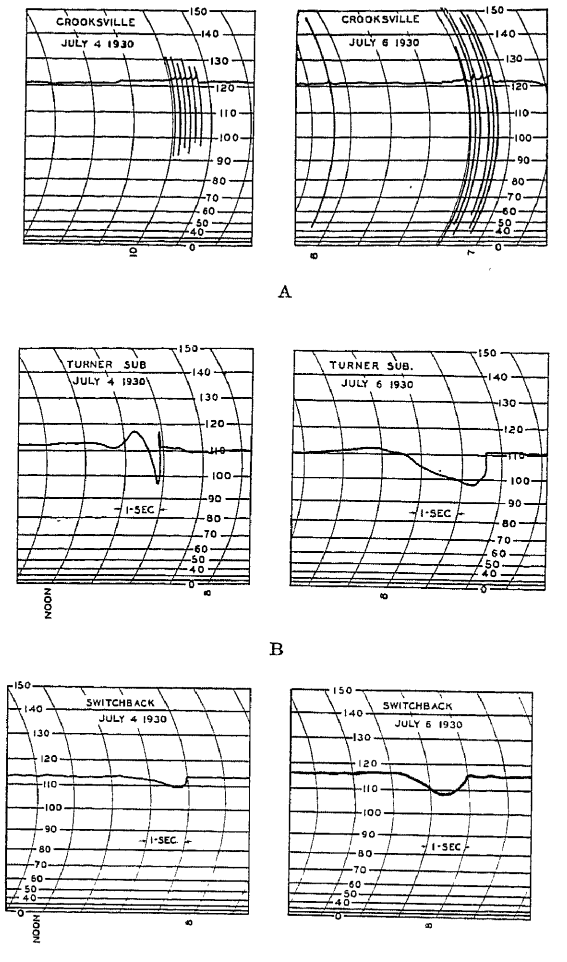
B

C

Fig. 6—View of Contacts of Segmental-Contact Explosion-Chamber Breaker After Tests of July 6

The two explosion-chamber failures encountered on these tests were apparently the result of a general weakness on the part of the chamber structure and were not due to other faults of design. It is believed that subsequent changes considerably increasing the strength of the chambers, will be sufficient to prevent the recurrence of such failures.

In addition to the three groups of interrupting tests just described, several charging current interrupting tests were made with the circuit breaker equipped with the oil-blast explosion chambers. These tests were made immediately following the group of 40 shots and consisted of the interruption of charging current on individual 132-kv. lines, varying in length from the short line from Philo to Crooksville to a line of 128 miles in length from Philo to Windsor. These tests were carried out without incident, the charging current involved being interrupted with comparatively short duration and length of arcs.



A

B

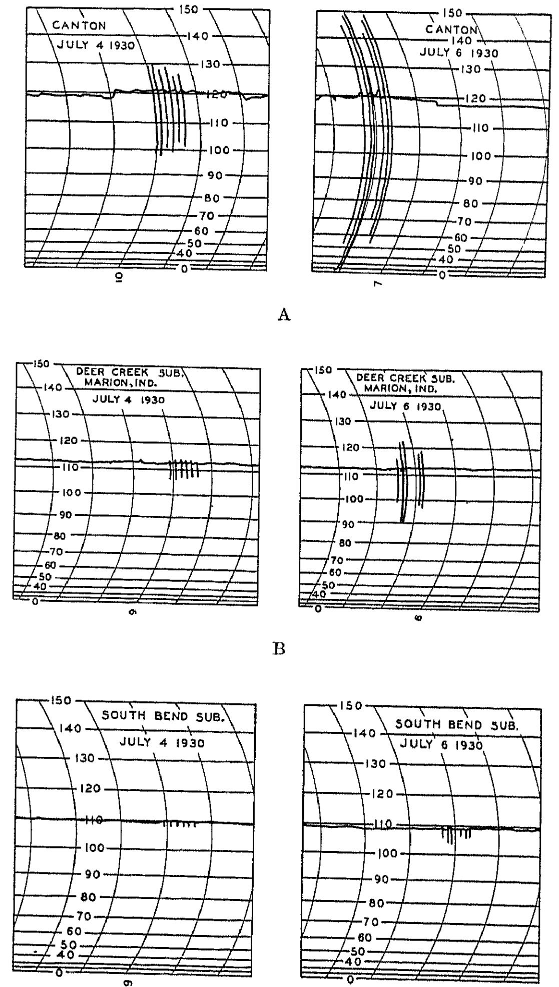
C

Fig. 7—Voltage Charts Taken at Crooksville, Turner, and Switchback Substations, Showing Comparative Voltage Dips Caused by High-Speed and Ordinary Circuit Breakers Tested at Philo, July 4 and July 6 Respectively

Effect on System Operation During Tests

On the whole the test programs were carried out smoothly and with very little delay or deviation from the schedules laid out in advance. Also disturbances on the system due to the tests were not very severe and caused very little trouble or complaint except at nearby points such as the city of Zanesville. At this place voltage dips were naturally fairly heavy and as a result of some of the heavier or more prolonged short circuits, a number of complaints of voltage surges were received.

During the tests, recording voltmeters, some of them equipped with a high-speed tripping device, were in service at all of the important substations on the 132-kv. system. Typical charts from these voltmeters are shown herewith in Figs. 7 and 8. Fig. 7A shows comparative records obtained on a quick-trip recording voltmeter at Turner Substation on July 4 with the high-speed breaker, and on July 6 with the segmental contact breaker. While inertia and over travel of the voltmeter pen on this type of instrument are responsible for some of the variations in these charts, nevertheless a very clear comparison is obtained between the severity of disturbances caused by the high-speed and ordinary circuit breaker. A similar comparison, both as to



A

B

C

Fig. 8—Voltage Charts Taken at Canton, Marion, and South Bend Substations, Showing Comparative Voltage Dips Caused by High-Speed and Ordinary Circuit Breakers Tested at Philo, July 4 and July 6 Respectively

severity and duration of the voltage dip is evident in the charts obtained at Switchback reproduced in Fig. 7B.

A very striking comparison of the severity of voltage dips caused by the tests at a point very close to the Philo plant is afforded by the chart shown in Fig. 7C taken at Crooksville. While it is obvious that the exact extent of the voltage dip is not obtainable from the moving pen instrument since inertia plays a large part in the record obtained, at the same time the comparative length of the lines drawn on the charts during the tests on the high-speed and low-speed breaker respectively, may be taken as a rough indication of the severity of such disturbances on rotating equipment connected to the system, since this equipment is also subject to inertia effects. In other words, a voltage dip, whose duration is too short to allow the recording voltmeter pen to follow it all the way down, may also be too short to cause a synchronous machine to fall out of step.

Figs. 8A, B, and C show voltage charts taken at Canton, Marion, and South Bend, respectively. The Canton charts show almost the same degree of disturbance as Crooksville, whereas the voltage surges at Marion and South Bend are successively less severe. In each case, however, the comparison between the effects of the high-speed and the low-speed breakers is very clearly indicated.

Benefits Obtained

In general the objects for which the tests were undertaken were quite fully realized; the tests have been and will be of real benefit from the standpoint of the power system in many ways. Some of these are the following:

For one thing, the tests have without question increased confidence in the higher ratings of modern breakers. A few years ago 132-kv. breakers of 1,500,000 kva. rupturing capacity were considered adequate for any location on the system, and, as a matter of fact, higher ratings were not available. Unforeseen developments, however, such as the installation at the Philo plant of three-unit turbines of much higher capacity than the single-unit machines originally planned have made it necessary, to employ breakers of considerably more than 1,500,000 kva. rating. At the same time later developments in the art of designing and building breakers have given the manufacturers sufficient confidence to offer breakers of 2,500,000 kva. rating. Not only has a number of breakers of this rating with 72-in. tanks been purchased, but it has been possible, with certain changes to re-rate existing 72-in. tank breakers at 2,500,000 kva. While these ratings for the 72-in. tank breakers had been accepted, it was felt that such ratings involved rather liberal extrapolation of existing knowledge into unexplored regions, and quite naturally the acceptance was accompanied by a certain amount of skepticism. In view of the recent Philo tests, however, in which a duty of 1,735,000 kva. was successfully handled by the 60-in. tank breaker, there is little question but that the 2,500,000 kva. rating can be obtained on the 72-in. tank breaker. This positive assurance of obtaining the desired circuit breaker ratings is, of course, very valuable in that it is now possible to engineer the system along the most desirable lines without being hampered by fears of circuit breaker limitations.

Again, the definite realization of the 8-cycle breaker, as demonstrated by the tests, marks a forward step of real practical value to the power system. In the past the types of relay systems used have employed such a large element of time in most cases that the saving in breaker time was of no great importance. At the present time, however, more and more instances are being encountered where it is desirable and necessary in meeting continually improving standards of service to devise means of reducing relay time to an absolute minimum. In such cases the time consumed in breaker operation becomes a very large factor.

On the system of the American Gas and Electric Company, at least two instances of the type just mentioned have occurred and are under consideration at the present time. One of these, a fairly typical situation, occurs in the Appalachian Electric Power Company territory on the portion of the 132-kv. system east of Glen Lyn, where the important loads to the east and south, including Roanoke and Lynchburg, are supplied from the long 132-kv. double-circuit lines from the Glen Lyn station. A very long single-circuit 132-kv. line, passing by and supplying Fieldale and Martinsville, extends all the way to the North Carolina state line where it ties in with the Carolina Power and Light Company. Faults on this single-circuit line or on the double-circuit line from Roanoke to Glen Lyn result in severe voltage dips at Roanoke and Lynchburg, due to the absence of any appreciable amount of generating capacity at either of these points. These voltage dips have been the cause of serious disturbances to service on a number of power users employing synchronous apparatus which either drops out inherently or is designed to be dropped off the line in case of loss of voltage. While some progress has been made in the way of remedying this difficulty by suitably revamping the users' control apparatus, at the same time many of the cases cannot be taken care of in this way and the necessity of doing everything possible to reduce the duration of voltage dips remains very urgent.

Progress has also been made in the application of recently developed relays for shortening the time required for clearing sections of single-circuit lines, but there has been little incentive to adopt new and faster relays for the protection of balanced double-circuit lines because the operating time of existing relays for this service is already small compared with the breaker operating time, so that comparatively little reduction in over-all time could be obtained. By employing the high-speed circuit breaker, however, and making use of faster relays which will thereby be justified, it will be possible to cut down by at least half and probably more, the time required to clear one circuit of balanced double-circuit lines.

The high-speed circuit breaker offers great possibilities in reducing the percentage of two-line outages on double-circuit balanced lines. As is well known, the percentage of outages on double-circuit lines in which both circuits are involved is small compared with those in which a single circuit only is involved. However, recent experience seems to indicate that the percentage of two-circuit outages is increasing. This may be due to power system growth without accompanying improvements in the way of reducing relay and breaker operating time, with the result that the greater concentration of power in the arc following the flashover increases the tendency for the flashover to be communicated to the second circuit. It has been suggested, therefore, that if one circuit on a balanced double-circuit line could be cleared very quickly by means of fast relays and a high-speed breaker, the number of outages involving both circuits could be substantially reduced.

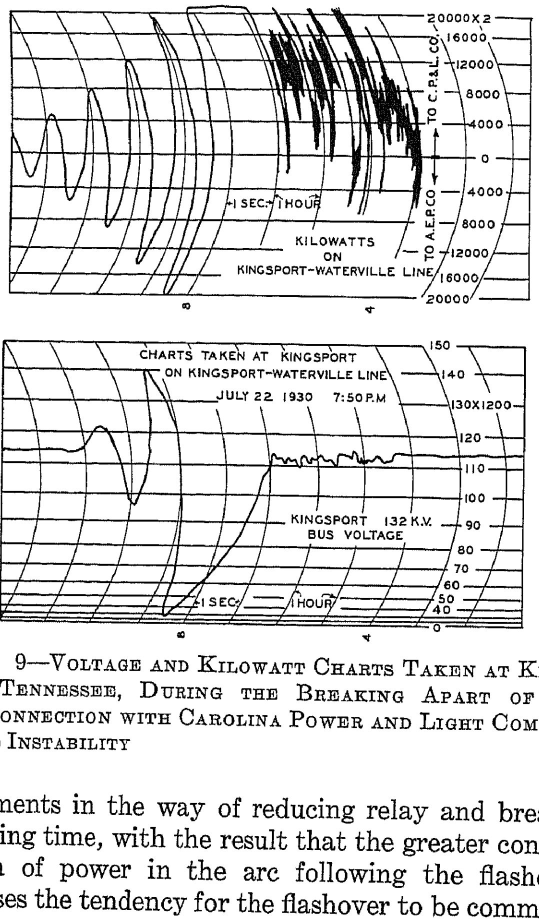


Fig. 9—Voltage and Kilowatt Charts Taken at Kingsport, Tennessee, During the Breaking Apart of the Interconnection with Carolina Power and Light Company Due to Instability

While the principles governing the phenomena of static stability limits in transmission lines have been well known for some time, comparatively few actual cases of static instability have been reported. It may be of interest, therefore, to describe a case which has occurred on the Appalachian Electric Power Company's System at Kingsport, Tenn., where an interconnection is made with Carolina Power and Light Company. This interconnection may be characterized as a comparatively weak tie between two large systems. While it would be highly desirable to maintain the interconnection during emergencies on either system, at the same time certain disturbances may occur on one of the systems resulting in a burden on the interconnection in excess of the static stability limit of the lines involved. Such a disturbance occurred on the Carolina system, with the result that the two systems pulled entirely apart and voltage at Kingsport dropped to practically zero. A graphic record of this stability breakdown is reproduced herewith in Fig. 9 in the form of power and voltage charts taken at Kingsport by means of quick-trip high-speed recording instruments on the interconnecting line to Waterville. It will be noted that the power goes considerably beyond the maximum scale reading of 40,000 kw. and that the voltage drops to zero in approximately two seconds.

In order to more definitely establish the value of kilowatts at which these two systems pulled apart and to determine the effect of using instantaneous relays to trip the line instead of the induction type relays previously used, the conditions on the systems in Tennessee causing the excessive loading of these interconnections were deliberately reproduced and a record was obtained as shown in Fig. 10. It will be noted that the total time from the beginning of the voltage dip to its recovery after separation of the two systems was reduced to less than one second as compared with two seconds in the previous case. Also the kilowatt charts, this time with a maximum scale of 80,000 instead of 40,000 kw., showed a power swing to approximately full scale or 80,000 kw. which checks fairly well with approximate calculations of the stability limit for these interconnections.

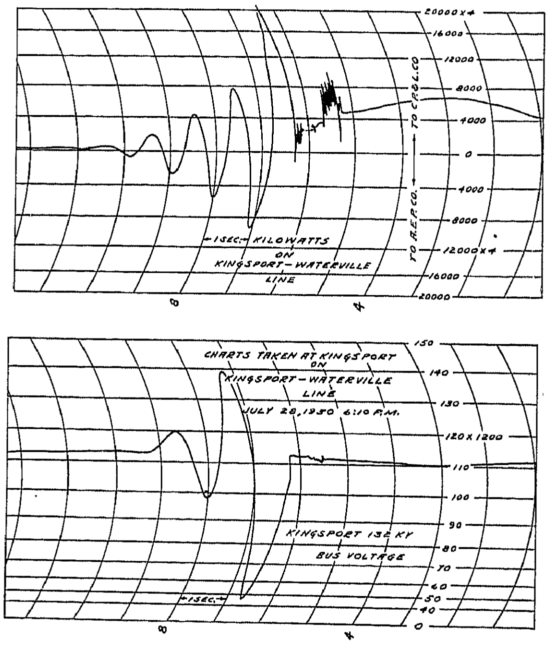


Fig. 10—Voltage and Kilowatt Charts Taken at Kingsport During the Breaking Apart of the Carolina Power and Light Company Interconnection After the Installation of Instantaneous Relays

Since such a case of trouble necessitates the separation of the two systems in any event, it would obviously be of distinct advantage to bring about the separation before the complete loss of voltage occurs due to instability. In order to accomplish this some relay scheme based preferably upon power flow, or perhaps on current only, could be employed to open up the line when power or current reaches values at which pulling apart from instability will be likely to result. To be effective, however, the scheme will have to include a high-speed circuit breaker in order to quickly open the line once the load reaches the predetermined critical value.

The carrying out of the Philo tests has undoubtedly resulted in a distinct benefit in the training and experience obtained by the operating personnel of the Philo plant in handling the equipment under short-circuit conditions, particularly the new 165,000-kw. three-unit turbine generator. This experience will be useful in giving a greater degree of confidence in the ability of the plant and equipment to withstand heavy short circuits, particularly since subsequent inspection of all the Philo generators showed no damage to any machine.

The Philo tests, particularly when viewed in the light of many other groups of tests, such as those carried out at Canton and on other systems in the past few years, will it is hoped result in a concerted effort to bring about the adoption of a new duty cycle for oil circuit breaker performance. Among operating people it is hardly questioned that the multi-duty cycle is necessary since breakers are frequently required to stand repeated shots during lightning storms. The carrying out of the Philo tests in which the breaker operated through a long series of shots without intermediate inspection or change of oil indicates that a multi-duty cycle is practicable.

The authors wish to acknowledge the cooperation of The Ohio Power Company and the interconnected system without which the full value of the tests could not have been realized, and the valuable assistance and cooperation of the late Mr. Joseph A. Bergin, Superintendent of the Philo Plant.

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Discussion

For discussion of this paper see page 521.

The Oil-Blast Circuit Breaker

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Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis.*—The principle of operation of oil circuit breakers has been conceived to the present time as deionization of the arc stream during the time required for the recovery voltage to appear across the circuit breaker terminals. An alternative principle is here developed in which the arc products are swept away by the scavenging action of an oil blast. As a logical development from this conception a new type of oil circuit breaker is described which owes its efficiency to careful provision for a scavenging blast of oil which is driven by the gas pressure from one break and which displaces the arc in the space between the electrodes of another break.*

THE oil circuit breaker has been used for more than thirty-five years as a circuit interrupter for alternating current circuits, and still is the most practical, economical, and effective device for that purpose so far developed. In recent years much study has been given to the determination of the fundamental characteristics of the arc in oil as encountered in oil circuit breakers. One of the first important facts brought to light by these studies was that an unconfined arc in oil is extremely erratic in length, duration, and amount of gas formed. To illustrate this point, the arc lengths and arc duration of a typical plain-break breaker for a number of tests are shown in Fig. 1. It is apparent, from a study of this figure, that the arc length and arc duration are subject to capricious variation so that any prediction of their behavior must be a matter of probability and any effective design must be based on maximum values. Even the maximum arc lengths are different in different sizes of breakers at the same currents and voltages, and in the same breaker are different with various opening speeds. These maximum values for various conditions and voltages have been determined from actual tests and are used as the design basis in plain-break breakers.

It is not difficult to see why this variation exists. Photographs of the operation of plain-break oil circuit breakers have been made which show the formation of gas bubbles surrounding the arc, (see Fig. 2). These gas bubbles are rather poor insulators and only as portions of the arc stream are accidentally interrupted by oil spray is their dielectric strength increased to the point where the arc will go out. Even so, the plain-break oil circuit breaker has given an excellent account of itself for many years and only where increased rupturing capacity or higher speed is required has it been displaced by more complicated structures.

As early as 1906 explosion-chamber breakers were developed, as in Fig. 3, in which the arc stream was confined and drawn through a hole into which fresh oil and gas from the explosion chamber discharged. This greatly increased the rate at which the arc gases were replaced by dielectric and raised the dielectric strength of the arc path, multiplying the rupturing capacity of a given circuit breaker structure by several fold.

Recently an excellent analysis of oil circuit breaker phenomena has been presented[1,2] in which the qualitative factors governing deionization rates in arcs have been explained. The principle of confining the arc and maintaining a supply of oil in its immediate vicinity has been confirmed, although the means suggested differ considerably in operation from the method employed in the explosion chamber. The importance of low dissipation of energy in the arc in keeping down the mechanical stresses accompanying an interruption has been set forth, and a new structure has been developed for the purpose of accelerating the deionization of a mixed gas-and-oil arc path, so as to raise its dielectric strength during the short interval while the restoration of voltage across the breaker contacts is taking place. The deionization processes suggested, however, operate vigorously throughout the current cycle, requiring, for an arc of given length, high voltage and energy dissipation and causing, therefore, a rapid evolution of gas. This is unavoidable where effective deionization at the current zero must be secured in a fairly homogeneous arc path. But the arc path at the time of rupture need not be homogeneous and it is the purpose of this paper to describe a process operating along entirely different lines.

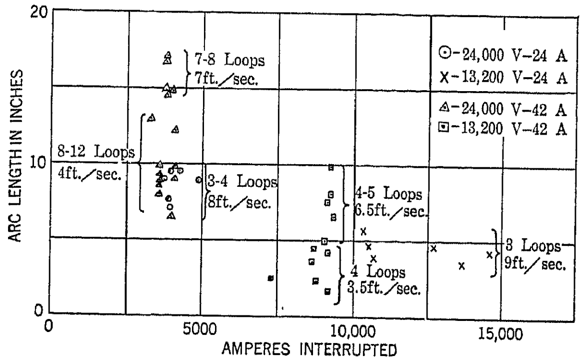

FIG. 1—ARC LENGTH VS. CURRENT FOR PLAIN-BREAK OIL CIRCUIT BREAKERS OF DIFFERENT PROPORTIONS

The arc path at the time of rupture for existing circuit breakers has an effective dielectric strength of from about 2,500 volts per in. in plain-break circuit breakers up to 10,000 volts for the deion grid breaker and as high as 15,000 volts for standard explosion-chamber circuit breakers. Clean oil has a dielectric strength of over 200,000 volts per in., with a strength under impulse over 600,000 volts per in. The introduction of any considerable amount of solid oil into the gap at the current zero, therefore, holds possibilities of great improvement over other methods of interruption.

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[1] *Extinction of a Long A-C. Arc*, by J. Slepian, A. I. E. E. TRANS., Vol. 49, 1930, p. 421.

[2] *The Use of Oil in Arc Rupture*, by B. P. Baker and H. M. Wilcox, A. I. E. E. TRANS., Vol. 49, 1930, p. 431.

In order that we may know accurately the time available for introducing dielectric following the current zero, an analysis has been made of the rate at which voltage may be expected to appear across breaker terminals. These results have been presented in another paper.[3] From this analysis it appears that high rates of applied voltage need be looked for only across breakers connected directly to generator or transformer terminals or to current limiting reactors. Under the worst conditions line voltage may be impressed across a 15-kv. breaker in 10 microseconds. The rate of voltage rise is then

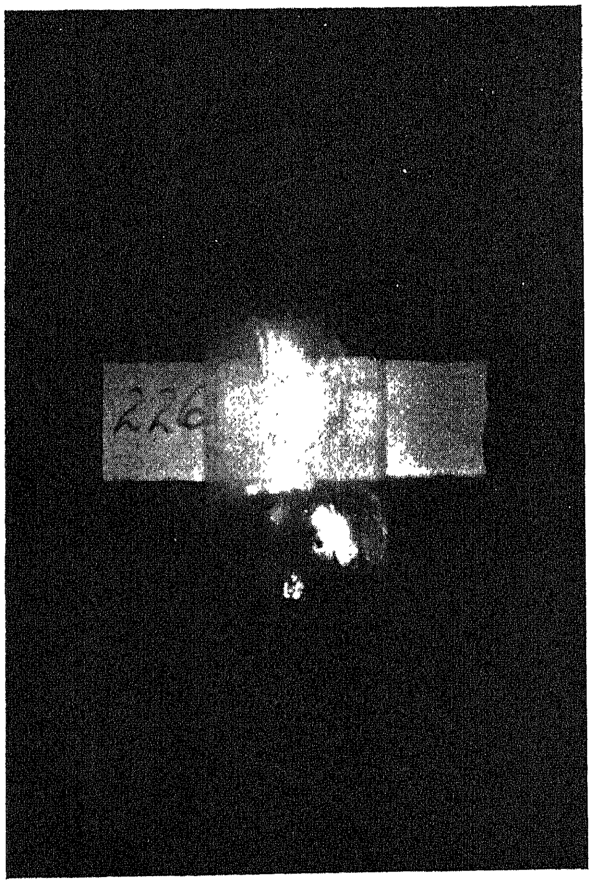

FIG. 2—TYPICAL SHAPE OF GAS BUBBLE IN PLAIN-BREAK OIL CIRCUIT BREAKERS

$$\frac{\sqrt{2} \times 15{,}000}{10 \times 10^{-6}} = \sqrt{2} \times 1.5 \times 10^{9} \text{ volts per sec.}$$

For the extremely short time involved, the oil in an oil circuit breaker may be said to have the dielectric strength it presents to impulse voltages, or about $\sqrt{2} \times 6 \times 10^5$ volts per in. Therefore, if fresh dielectric is to be introduced at a rate equal to the rate of voltage rise, it must be introduced at the rate

$$\frac{\sqrt{2} \times 1.5 \times 10^{9}}{\sqrt{2} \times 6 \times 10^{5}} = 2.5 \times 10^{3} \text{ in. per sec. or 200 ft.}$$

per sec. This velocity corresponds to a pressure of 255 lb. per sq. in., which is not an unusual explosion-chamber pressure.

A high-voltage transformer will have a voltage recovery rate corresponding to $\sqrt{2} \times 5 \times 10^9$ volts per sec. while the rate for a long distance 220-kv. transmission line will be about $\sqrt{2} \times 0.8 \times 10^9$ volts per sec. even though the full voltage corresponds to 220 kv. line to line. It is quite possible to design a circuit breaker so that oil will sweep away the arc at the rate required by the foregoing calculations.

The effectiveness of the explosion-chamber breaker may be explained in this way. Fig. 4 shows a standard high-speed explosion-chamber circuit breaker in four positions. In the first diagram the breaker is shown in the closed position. In the second, the contacts have separated and a small arc has been drawn which has decomposed enough oil to place the remaining oil in the explosion chamber under considerable pressure. In the third diagram, the moving contacts have reached the explosion-chamber throat and oil is being crowded in upon the arc in the throat. In the fourth diagram the oil has divided the gas bubble into two parts, interposing a solid wall between them, and thus effectively opened the circuit.

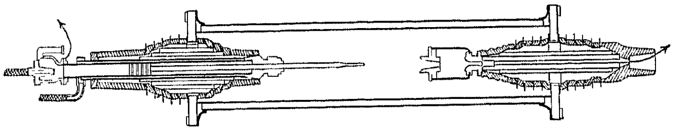

FIG. 3—SECTION OF EARLY OIL CIRCUIT BREAKER SHOWING EXPLOSION CHAMBER

As the gas bubble expands, driving the oil through the throat of the explosion chamber, the pressure varies from hundreds of pounds in the large part of the volume of the explosion chamber to a very low value actually in the throat. That is, the potential energy of pressure has been converted into kinetic energy of velocity. At the same time, since the pressure is low in the throat, the oil can very easily rush into this space so as to cut the arc in two.

That this is actually what happens is proved very well by Fig. 5, in which, for a large number of tests at widely different voltages, the arc is uniformly extinguished while the contact tip is passing through the explosion-chamber throat.

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[3] *Circuit Breaker Recovery Voltages: Magnitudes and Rates of Rise*, by R. H. Park and W. F. Skeats, A. I. E. E. TRANS., March 1931, p. 204.

In order to make this action even more sure and positive, the form of explosion chamber shown in Fig. 6 has been developed. It contains an upper butt contact *A*, a floating contact *B*, and a hollow contact *C*. In the closed position current is carried by these three members firmly pressed together. When the breaker is tripped, the floating and hollow contacts, *B* and *C*, start down, driven by the opening springs, thus drawing a short arc between *A* and *B*. This arc forms gas and places the oil in the explosion chamber under considerable pressure for which there is very little relief until the floating contact reaches its stop. The secondary break then occurs between contacts *B* and *C* and a second arc is drawn between these contacts. But as the oil glides through the passageway in the contact rod, this arc is carried before it and exists alongside it without mixing, and hence with a minimum of loss throughout the current cycle. Then, as soon as the current zero is reached, the arc products are displaced by a smooth flowing wall of solid oil, the dielectric strength of which has not been impaired by carbonization or entrained ionized gas.

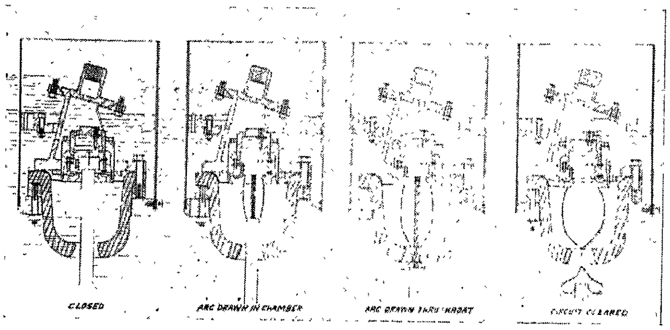
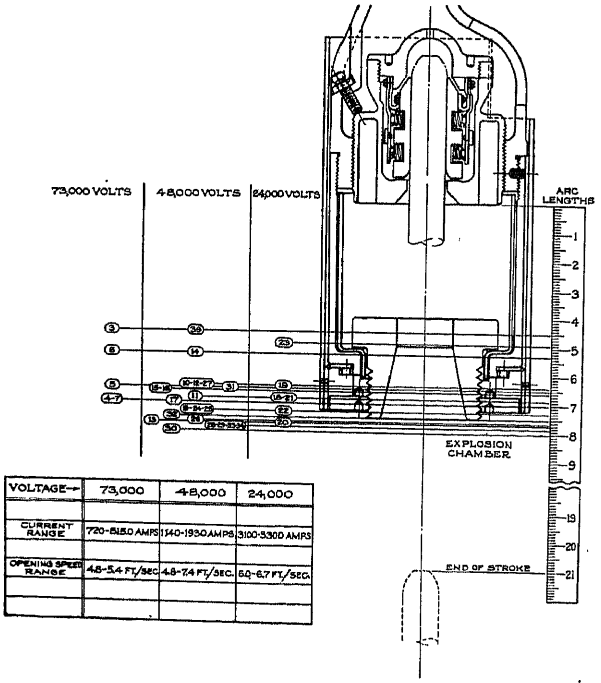


FIG. 4—DIAGRAM SHOWING OPERATION OF EXPLOSION CHAMBERS



VOLTAGE→	73,000	48,000	24,000
CURRENT RANGE	720-815.0 AMPS	1140-1930 AMPS	3100-3300 AMPS
OPENING SPEED RANGE	4.8-5.4 FT./SEC.	4.8-7.4 FT./SEC.	6.0-6.7 FT./SEC.

FIG. 5—SKETCH SHOWING METALLIC EXPLOSION CHAMBER AND VARIOUS ARC LENGTHS FOR DIFFERENT VOLTAGES. CIRCLED NUMBERS ARE SERIAL TEST NUMBERS

Several beneficial effects are obtained by the use of this modification. In the first place, the necessary arc length at generator voltages is reduced from something slightly in excess of the inside length of the explosion chamber to a little over an inch per break. This shorter arc length can be more rapidly established with a given contact speed, which is a decided advantage where it is particularly important to cut down the duration of short circuit. Furthermore, both on account of the shortness of the arc and its brief duration, the energy which it dissipates is much reduced. The importance of reducing the energy from the point of view of (1) keeping down the mechanical stresses and oil deterioration resulting from a given interruption, (2) increasing the interrupting capacity of a structure of given size, and (3) minimizing the demonstration accompanying an interruption, has been brought out by other investigators as well as the authors. In addition, the shorter arc duration results

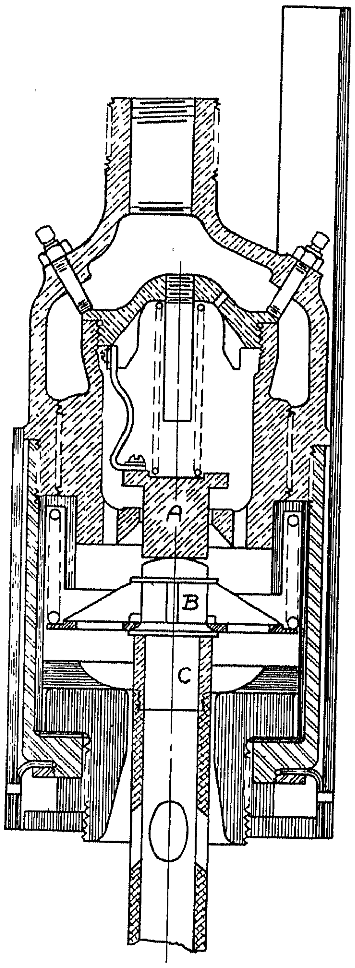


FIG. 6—SPECIAL EXPLOSION CHAMBER

in less burning and longer life of the contact parts and of the oil. Also, both arcs are quite effectively kept away from insulating material so that burning of insulating parts is reduced. These two latter effects operate to cut down inspection and replacement requirements. Finally, the oil is more effectively directed into the second gap, so that the current range over which the chamber operates effectively is increased.

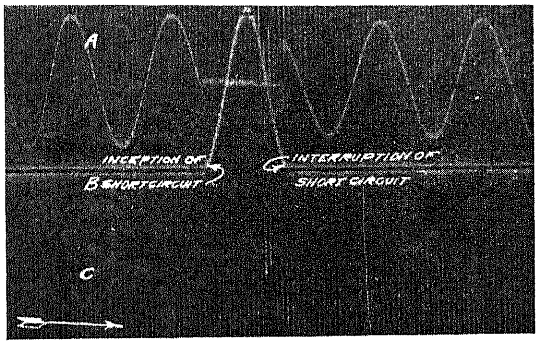


Fig. 7—Oscillogram Showing Interruption of 24,800 Amperes at 13,200 Volts, 24 Cycles, by Circuit Breaker with Oil-Blast Explosion Chambers

Curve *A* Voltage across contacts of breaker
Curve *B* Line current
Curve *C* Pressure in tank

Over 250 tests have been made at currents from 900 to 25,600 amperes at 13,200 volts and up to 56,400 amperes at lower voltages. Above 5,000 amperes the arc is uniformly extinguished at the first current zero occurring after the secondary contacts have been separated by as much as an inch, and in many cases the arc has been extinguished with a contact separation of but one-quarter of an inch. Fig. 7 is a typical oscillogram showing the operation of this circuit breaker.

The need for high speed of interruption is fully as urgent at transmission voltages as at generator voltages. Accordingly, a somewhat larger explosion chamber was modified to correspond to Fig. 6. This arrangement was found to be effective up to 44,000 volts, as is demonstrated by the data in Table I. By comparison of the data shown in this table with those given by Fig. 5, the improvement is apparent. Whereas with the standard explosion chamber, arc lengths varied from four to eight

TABLE I

TESTS OF OIL-BLAST CIRCUIT BREAKER FHKO-139-54-B-110 KV., 600 AMPERES

Feb. 6, 7 & 18, 1930

Volts	Amperes	Arc duration Cycles	In. per break	Volts per in.
22 kv.	1570	0.9	0.9	13.2 kv.
22 "	1600	0.9	0.8	13.75 "
22 "	1740	2.2	2.6	4.23 "
22 "	1880	0.8	0.9	12.2 "
22 "	1820	1.5	*	
22 "	1730	1.2	1.3	8.46 "
22 "	1530	1.2	1.3	8.46 "
44 "	3870	0.5	0.75	29.7 "
44 "	3930	0.5	0.5	44.0 "
44 "	3240	1.5	1.7	13.0 "
44 "	5000	0.5	0.5	44. "
44 "	3620	0.5	0.5	44. "

*No record obtained.

TABLE II

TESTS OF OIL-BLAST CIRCUIT BREAKER FHKO-139-54-B-110 KV., 600 AMPERES

March 18 & 19, 1930

Volts	Amperes	Arc duration Cycles	In. per break	Arc energy Kw-sec.	Kva. x 1000	Watt-sec. per Kva.	Volts per In. in arc
66 kv.	2040	2.25	5.75	23	194	0.118	5.75 kv.
66 "	2590	1.85	7.50	32	171	0.187	4.55 "
66 "	2920	2.25	5.25	28	193	0.145	6.3 "
66 "	2830	3	7.50	35	187	0.187	4.4 "
66 "	2020	1.75	6.00	25	193	0.130	5.5 "
88 "	1890	3.75	10.5	58	166	0.350	4.2 "
88 "	2020	2.75	6.25	15.2	177	0.086	7.0 "
88 "	1890	2.62	5.25	14	166	0.084	8.4 "
88 "	2210	2.25	7.50	36	194	0.185	5.9 "
88 "	1950	3.62	11.00	50	171	0.292	4.0 "
110 "	1740	3.25	8.75	29	191	0.152	6.3 "
110 "	1560	2.75	6.00	19	172	0.110	9.2 "
110 "	1650	3.75	9.00	36	181	0.199	6.1 "
110 "	1540	3.25	8.00	21	169	0.124	6.9 "
110 "	1690	2.25	5.25	11	186	0.059	10.5 "
132 "	1350	2.25	4.50	19	178	0.107	14.7 "
132 "	1570	3	6.50	18	207	0.087	10.2 "
132 "	1480	4.25	10.75	41	195	0.210	6.1 "
132 "	1250	3.5	7.25	18	165	0.109	9.1 "
132 "	1340	5	12.75	56	177	0.316	5.2 "
132 "	1220	4	17.80	22	161	0.137	3.7 "
132 "	790	4.5	18.70	25	104	0.240	3.5 "
132 "	1075	6	21.90	65	142	0.458	3.0 "

Contact burning negligible. No load speed 7.8 ft./sec.
Oil deterioration negligible.
No measurable pressure above oil.

inches, those in Table I vary from one-half to two and one-half inches.

At 110 kv., however, it was found more desirable to substitute molded explosion chambers for the steel ones used at the lower voltages. The detailed arrangement of these chambers is shown in Fig. 9. Fig. 10 shows an oscillogram typical of the behavior of this combination. The arc duration is about three cycles, and both arc drop and arc energy are very low. The results of a considerable number of tests are shown in Table II.

In comparing these data with those shown in Fig. 5 for tests on a standard explosion chamber at 73 kv., it should be noted that all tests recorded in Table II were made with one terminal of the breaker grounded, whereas the tests of Fig. 5 were made with the transformer neutral grounded. In addition, the recovery rate of the plant used for the tests of Fig. 5 was somewhat lower than the rate of the plant used in the more recent tests.

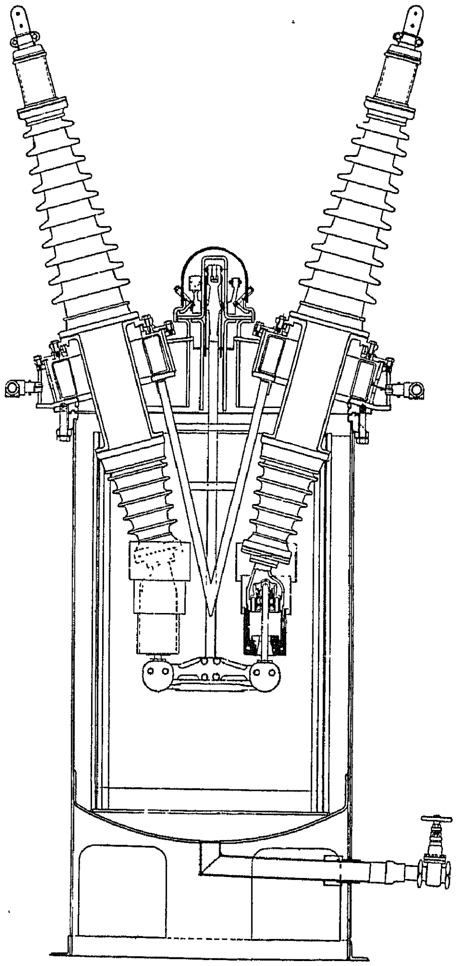


FIG. 8—TYPE OF 110-KV. OIL CIRCUIT BREAKER USED IN TESTS WITH OIL-BLAST EXPLOSION CHAMBERS

On July 4 the explosion-chamber arrangement just described was installed in a three-phase oil circuit breaker at the Philo plant of the American Gas and Electric Company. The breaker rating was as follows: FHKO-139-60B, 132,000 volts, 600 amperes. This oil circuit breaker interrupted a large number of short circuits. These tests are being reported in another paper.[4] The results of these tests have been calculated and plotted in terms of calculated recovery rates for the systems attached, as shown in Figs. 11 and 12. In Fig. 11 is shown a comparison of the arc lengths for the butt-contact explosion chamber and the oil-blast explosion chamber. The arc length with the oil-blast is much less than with the butt contacts. Fig. 12 shows the actual over-all time for these two circuit breakers figured from the application of current to the trip coil.

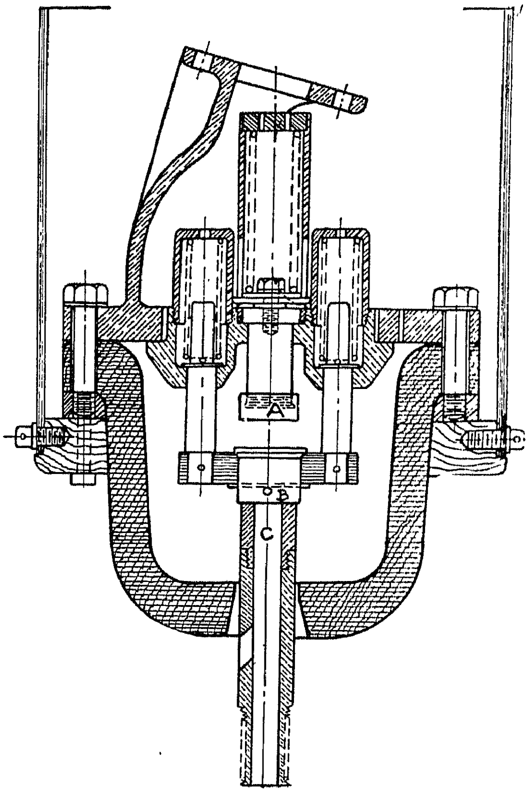


FIG. 9—OIL-BLAST EXPLOSION CHAMBER FOR 110-KV. OIL CIRCUIT BREAKER

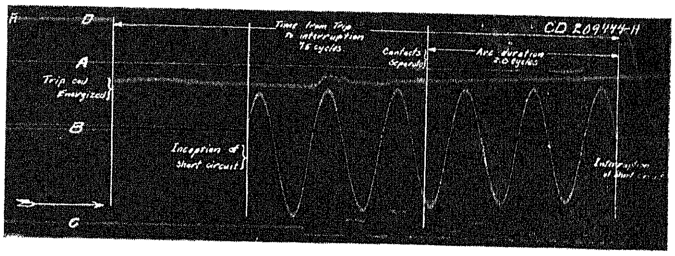


FIG. 10—OSCILLOGRAM SHOWING INTERRUPTION OF 1,560 AMPERES AT 110,000 VOLTS, 60 CYCLES, BY BREAKER WITH OIL-BLAST EXPLOSION CHAMBERS

Curve *A* Voltage across contacts of breaker
Curve *B* Line current
Curve *C* Breaker travel between impulses equals ½ in.
Curve *D* Trip coil current

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[4] *Circuit Breaker Field Tests on Standard and Oil-Blast Explosion-Chamber Oil Circuit Breakers*, by R. M. Spurck and H. E. Strang, p. 513.

The gain in the case of the oil-blast contacts is more impressive when it is remembered that the mechanical time required to separate the contacts is the same in both cases. Of the single-phase laboratory tests, those at 88 kv. are probably the most comparable with those at Philo, which were made with the same explosion chambers, (Fig. 11). The maximum arc lengths were about the same, although the recovery time of the test transformers is only 60 per cent of the shortest time in the Philo tests. An especially reassuring feature of these tests is that the factory testing plant appears to stress the circuit breaker as severely as a large central station connection providing two and one-half times the short-circuit energy.

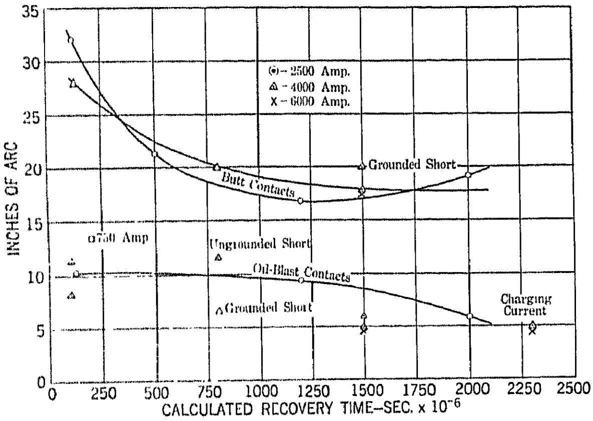


Fig. 11—Tests at Philo, Ohio, on the FHKO-139-60B, 132,000-Volt, 600-Ampere, Oil Circuit Breaker, Showing the Maximum Arc Length vs. Recovery Time

During the tests it has been found that the circuit has, in many cases been successfully opened with chamber pressures less than those calculated above as theoretically necessary. This is explained by a more complete analysis of the oil flow during the arcing period.

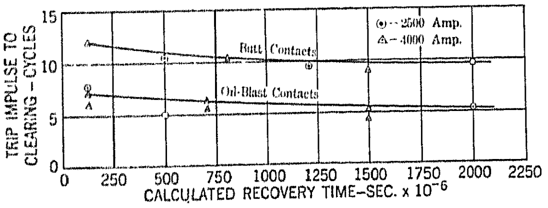
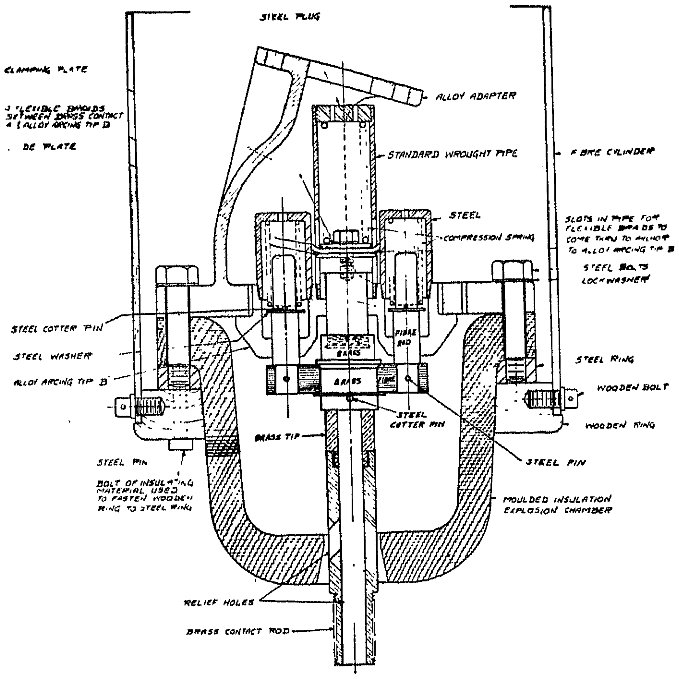


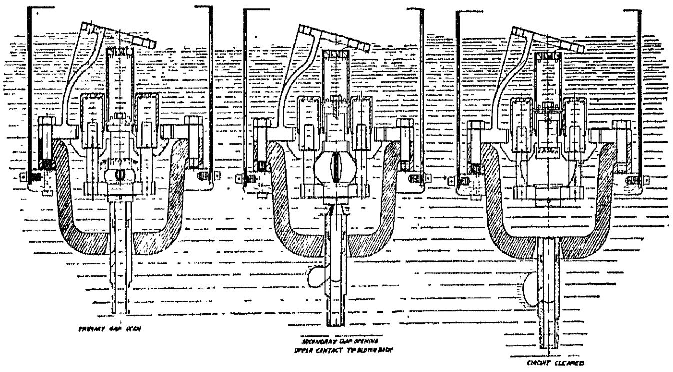
Fig. 12—Tests at Philo, Ohio, on the FHKO-139-60B, 132,000-Volt, 600-Ampere, Oil Circuit Breaker Showing the Duration of Short Circuit vs. Recovery Time

Fig. 13 shows a series of drawings representing the locations of arcs, gas, and contacts during an operation of the breaker. Fig. 13A shows the contacts in the closed position. In Fig. 13B the upper contacts have parted, establishing the pressure-generating arc. The lower contacts have parted in Fig. 13C and the current is still considerable, giving a fat arc, which is, however, beginning to be carried down the contact tube, which has been freed of oil, and gas is commencing to escape from the vent holes. As the arc dwindles with the cyclic decay in current, the oil moves radially inward along an area many times larger than the bore of the contact tube. At the current zero, Fig. 13D, the velocity of gas moving downward into the tube becomes many times the velocity of the inward-flowing oil. In effect the rate at which solid oil dielectric replaces ionized gas is the velocity with which the gas can go through the bore of the contact rod. The gas may pass through the contact tube at the velocity of sound, 2,300 ft. per sec. in oil circuit breaker gas at ordinary temperatures, while at arc temperatures it probably exceeds 10,000 ft. per sec., which accounts for the extraordinary rate at which the space once occupied by the arc recovers its dielectric strength.



CLOSED POSITION

A



B C D

Fig. 13—Diagram Showing Locations of Arcs and Gas in Oil-Blast Explosion Chamber

In old types of plain-break oil circuit breakers, the fact that the voltage divided by arc length gave values of approximately 2,500 volts per in., or approximately one per cent of the strength of the oil, has frequently caused comment. With a breaker of the type described above, the voltage across the working gap reaches, in actual test, values of 44,000 volts per in., which is quite comparable with the ultimate strength of the oil and considerably higher than should be allowed across the internal dielectric in the open position of the breaker.

Since there has been no need for mixing oil and arc in the main gap, the parts are so made as to permit a smooth flow. The arc then burns in quiet gas with a minimum of arc drop. It decomposes a minimum amount of oil and serves as an excellent medium for carrying the current with a minimum loss until the arrival of a current zero.

The arc energies in the tests have been too low to measure satisfactorily from oscillograms. Data are being collected by means of a ballistic galvanometer by which it is hoped that more refined comparisons of circuit-breaker behavior can be made in the future.

The principle of employing solid oil instead of mixed oil and gas as the interrupting medium in oil circuit breakers is not limited to the larger types. Smaller types employing this principle are under development and tests already made indicate that large decreases in arc length and increases in interrupting capacity will be obtained in all cases.

In developing the oil-blast principle it has not been necessary either to prove or disprove any of the numerous theories which have been advanced to explain the operation of plain-break oil circuit breakers. The plain break is simply set aside with all its vagaries and mysteries. The arc with its varying gas composition, varying pressure, uncertain temperature, varying conductivity, unknown state of ionization, etc., is merely swept away and its place taken by solid oil of known and high dielectric strength, something that is understood and can be tested with certainty by any laboratory.

As relative newcomers in the oil circuit breaker field, the authors would have been utterly unable to obtain the results which have been yielded by this investigation without the splendid cooperation of the entire switchgear staff of the General Electric Company. In particular, Messrs. W. K. Rankin and W. E. Paul have contributed many of the ideas on which this paper is based, while the active cooperation of Mr. G. F. Davis and his testing staff has made possible many checks upon the theory as well as the determination of the performance characteristics of the final designs.

Discussion

For discussion of this paper see page 521.

Circuit Breaker Field Tests

On Standard and Oil-Blast Explosion-Chamber Oil Circuit Breakers

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Associate, A. I. E. E.

and

H. E. STRANG*
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INTRODUCTION

THE principle of interrupting short circuits inside an oil-filled explosion chamber immersed in the oil in a circuit breaker tank has been in use many years and its efficacy recognized. Nevertheless, many tests made in the High Capacity Laboratory and on power systems indicated that there was in breaker performance, some factor not thoroughly understood. It has been known that the apparent severity of duty on an oil circuit breaker during interruption is not entirely a function of line voltage and short-circuit current. It has also been recognized that at the instant of interruption, the recovery voltage reaches a value frequently in excess of normal line voltage, and it was considered probable that the recovery voltage characteristic had something to do with variations in breaker performance. An analysis of data collected from field and factory tests made during the last ten years, showed that variations in breaker performance were not caused solely by differences in the magnitude of recovery voltage. Calculations were therefore made on the probable *rate* of such voltage rise under the various prevailing test conditions and it was found that this rate, together with the magnitude, furnished a satisfactory explanation for much of what had previously been considered erratic behavior of test breakers. A very complete discussion[1] of the effect of circuit arrangements on the magnitude and rate of rise of recovery voltage has been presented to the Institute, with examples of the effect on oil circuit breaker performance which results from such variations.

A companion paper[2] describes the development of a modification of the conventional explosion-chamber assembly on which the tests in the factory indicated a reduction in arc duration for all rates of rise of recovery voltage and a definite improvement in performance.

It is the object of this paper to describe field circuit interrupting tests on an oil circuit breaker planned for the purpose of determining:

1. The actual rate of rise of recovery voltage across the terminals during the interrupting period under varied system conditions.

2. The performance of a new oil-blast explosion-chamber type of oil circuit breaker during interrupting tests of various current values and varied rates of rise of recovery voltage.

3. The performance of a standard butt contact type of explosion-chamber oil circuit breaker during interrupting tests of various current values and varied rates of rise of recovery voltage.

4. The performance of a standard segmental contact type of explosion-chamber oil circuit breaker during interrupting tests of various current values and varied rates of rise of recovery voltage.

5. The contact burning, deterioration, and other changes in the condition of the breaker after repeated interrupting tests in order to form a basis for recommended operating duty.

SYSTEM

The tests were made in the summer of 1930 on the 132,000-volt system of the Ohio Power Company at their Philo Station, Zanesville, Ohio. The extensive facilities of that system, involving an investment of many millions of dollars, were offered as a proving ground by the courtesy of the American Gas and Electric Company. The concentration of power possible at this point was the greatest ever made available for short-circuit tests at any voltage, amounting to nearly 1,750,000 kva. Thus it was possible to test for the first time a triple pole 132,000-volt oil circuit breaker at short-circuit currents not only approximating its rating but far in excess of the rating. Power concentration sufficient to test breakers at the duty possible at Philo is not to be found in any manufacturer's testing plant and probably also not available in the higher voltages on any other system.

Inasmuch as another paper[3] describes the system connections and test set-up, this paper will discuss such features only generally.

In order to supplement factory investigations of the effect of recovery voltage characteristics, arrangements were made to control the rate of voltage rise during these tests. A convenient means of manipulating this characteristic was presented by transmission lines which could be connected to or disconnected from the high-voltage bus to which the test breaker was connected. Under this condition, the recovery voltage characteristic is the sum of several outgoing and reflected voltage waves. The initial rate of rise is determined by the

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*Both of the General Electric Co., Philadelphia, Pa.

[1] *Circuit Breaker Recovery Voltages—Magnitudes and Rates of Rise*, by R. H. Park and W. F. Skeats, A. I. E. E. JOURNAL, December, 1930, p. 1017.

[2] D. C. Prince and W. F. Skeats, p. 506.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

[3] P. Sporn and H. P. St. Clair, p. 498.

short-circuit current and the number of lines connected to the bus, and the time required to reach a maximum value also involves the length of the lines. The severity of the recovery voltage characteristic is thus reduced as the number and length of lines connected to the bus are increased.

In order to best determine the effect of this difference in rate of rise, the two possible extreme conditions and one intermediate condition were chosen, as follows:

1. Calculated rate of rise of recovery voltage 270 volts per microsecond.

Three lines connected as follows:

One Rutland line 64.7 mi. long.
One Canton line 73.0 mi. long.
One Howard line 80.6 mi. long.

These lines were left open at the far end except on full system capacity tests during which they delivered power to the short circuit.

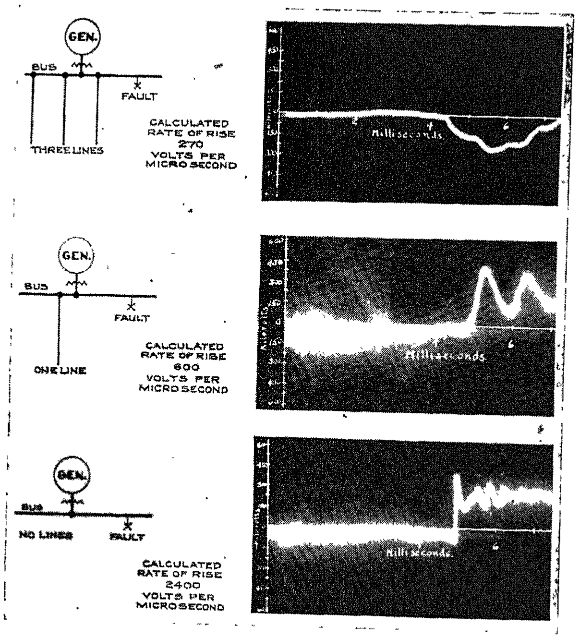

Fig. 1—Rate of Recovery Voltage Rise Under Varying System Connections

2. Calculated rate of rise of recovery voltage 600 volts per microsecond.

One Crooksville line 21.8 miles long connected.

3. Calculated rate of rise of recovery voltage 2,400 volts per microsecond.

No lines connected.

Representative cathode-ray oscillograms taken under these three conditions are shown in Fig. 1.

BREAKER TESTED

The breaker chosen for these tests was a triple-pole, 132,000-volt type FHKO-139-60B, having an interrupting rating of 1,500,000 kva. and operated by a standard SD-19 trip-free solenoid.

In one series of tests, this breaker was equipped with the oil-blast explosion chambers described in Mr. D. C. Prince's paper referred to above. Such a chamber in the form used for the tests is shown schematically in Fig. 2 and as designed for regular service in the photograph Fig. 3. In these chambers, the upper contact is similar to the conventional butt type with compression springs to provide proper contact pressure.

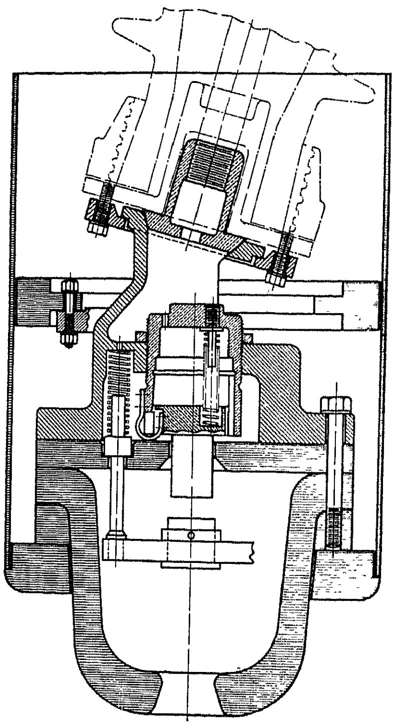

Fig. 2—Oil-Blast Contacts for First Series of Tests

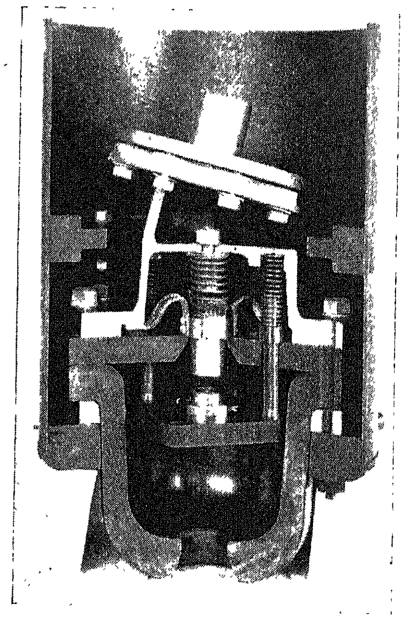

Fig. 3—Typical Oil-Blast Contact Explosion-Chamber Assembly

Between this upper contact and the moving contact rod, is supported a movable horizontal insulating baffle, having mounted in its center a conducting cylindrical section arranged in such a way that when the breaker is in the closed position, this section provides the connection between the moving contact rod and the upper contact. This intermediate baffle, with its conducting center cylinder, is supported and its movement restricted by four studs attached to the cover of the explosion chamber. Compression springs between the movable baffle and the upper explosion chamber support tend to force the baffle downward. The main movable contact is a hollow tube, 1¾ in. outside diameter and 1¼ in. inside diameter, fitted with removable arcing tips open on the ends. The action during opening is such that as the crosshead moves downward, the contact between the upper fixed member and the upper face of the conducting cylinder separates, and an arc is formed between them. This arc decomposes the oil surrounding it and forms gas which creates a pressure in the explosion chamber. By this time, the main contact has moved downward such a distance that the intermediate baffle has reached the limit of its permissible stroke, and the moving contact separates from the lower face of the conducting cylinder in the baffle. This action uncovers the opening in the end of the hollow moving contact and permits the pressure already generated by the opening of the first gap, to blast a continuous stream of clean oil through this hollow contact. This continuous stream of high dielectric oil, obviously, must pass across the arc formed between the contact rod and the lower face of the conducting cylinder of the moving baffle and in so doing, introduces insulation into that arc stream at an extremely high rate.

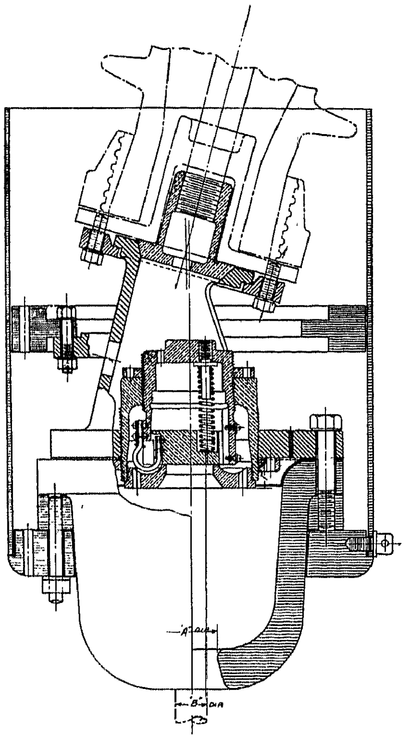

Fig. 4—Butt Type Contacts for Second Series of Tests

The same chamber of molded compound which has been standard for the last two years for high-voltage breakers is used. In common with previous designs, there are no pockets which can trap carbon or foreign material, and such parts as are subjected to burning are easily renewable; also, a vent is provided in the upper part of the contact to allow the escape of gas which collects in the chamber during interruption and to permit it to refill through the throat opening with fresh oil.

STANDARD EXPLOSION-CHAMBER BUTT CONTACTS

The standard explosion-chamber butt contacts used in the second series of tests were similar to the contacts in the oil-blast explosion chamber, except that the moving baffle was omitted and contact in the closed position made directly between the hollow moving member and an upper fixed contact. This chamber is shown schematically in Fig. 4. These contacts were mounted in the same breaker used on the first series of tests on the oil-blast explosion chamber.

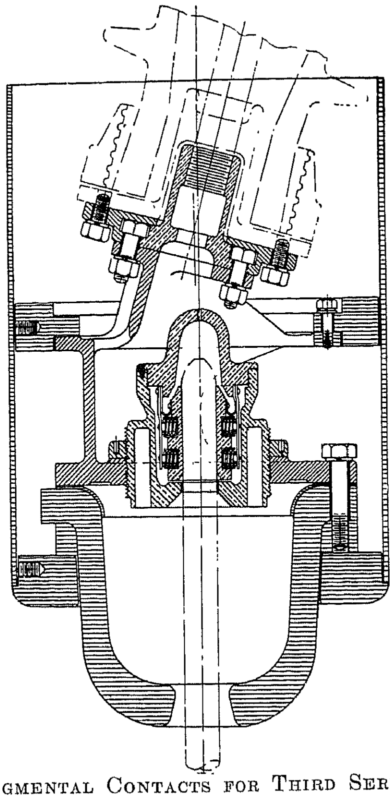

Fig. 5—Segmental Contacts for Third Series of Tests

STANDARD EXPLOSION-CHAMBER SEGMENTAL CONTACTS

In the third series of tests, the breaker used for the previous series of tests was equipped with standard explosion chambers with segmental contacts. Such a chamber is shown in Fig. 5.

TEST PROCEDURE

A very definite program was laid out for each series of tests covering a wide range of short-circuit current on both CO and OCO duty cycles.

During all the tests, the breaker was tripped by means of bushing current transformers which operated instantaneous solenoid relays, the contacts of which completed the d-c. trip coil circuit. No attempt was made to trip the test breaker by means of an auxiliary switch on a closing or back-up breaker; so these results show actual breaker operating time. Closing-opening tests, of course, required no other breaker for switching under short-circuit conditions. On opening tests only, however, a breaker in regular service at the Philo Station was used to throw on the short circuit.

TESTS ON OIL-BLAST EXPLOSION CHAMBER

A series of 40 tests was made on the breaker equipped with the oil-blast explosion chambers shown in Fig. 2. The results of this series of tests, arranged in groups according to the rate of recovery voltage rise, are summarized in Table I. This entire series of tests was run off in about six hours, including the necessary time required for switching to change the system conditions. The maximum value of short-circuit current which was recorded was 1,735,000 kva. at 140,000 volts and many of the tests were at values of short-circuit current in excess of the breaker's published rating. The last six tests listed, which were at interrupting values in excess of the rating of the breaker, were made as rapidly as the oscillograph film holders could be loaded, or approximately two minutes between tests. This completed the originally contemplated program. No inspection of contacts was made during the entire series of tests.

Oil samples taken and tested by the regular maintenance crew of the Ohio Power Company averaged 30 kv. before the tests. The average of four samples from different parts of each tank following the tests was 28 kv. and a comparison showed almost no discoloration of the oil. The average tank pressure was 8 lb. per sq. in. with a minimum of zero and a maximum of 11 lb. There was not a drop of oil thrown and only a slight amount of gas during this series.

After the tests were completed, an inspection of the breaker disclosed that one pole was failing to make contact because of breakage in the end of the crosshead which allowed the contact rod to drop to the bottom of the tank. This breakage was near the tapped hole in the end of the crosshead into which the contact rod is screwed and was caused by the use of too short a contact rod, which did not permit fastening properly into the threaded socket provided in the crosshead. This caused an unusual strain to be put on the crosshead socket and broke it off. A subsequent check of the oscillographic

TABLE I
OIL-BLAST CONTACTS

Rate of recovery voltage rise	Duty	Line volts kv.	R m. s. amperes initial in arc	Kva. interrupted	Short-circuit duration cycles	Breaker operating time cycles	Arc duration cycles	Arc length in.
270 Volts per microsecond	CO	140	2,550	617,000	5.5	5.2	3.0	4.0
	CO	140	3,450	835,000	5.5	5.2	3.0	4.2
	OCO	140	No current records					
	OCO	140	3,000	725,000	6.0	5.7	3.0	4.5
	CO	140	3,550	860,000	6.0	5.7	2.8	6.0
	CO	140	4,900	1,188,000	6.0	5.2	3.2	5.0
	CO	140	4,450	1,080,000	6.0	5.5	2.5	5.1
	CO	140	4,300	1,042,000	5.0	4.5	1.6	2.7
	OCO	140	4,250	1,030,000	5.0	4.5	2.0	3.0
	CO	140	6,950	1,690,000	6.0	5.2	2.4	5.3
	CO	140	6,350	1,540,000	5.0	4.2	2.0	2.5
	OCO	140	6,800	1,650,000	6.0	5.5	3.0	5.0
	OCO	140	6,550	1,590,000	5.5	4.5	2.4	3.5
	OCO	140	6,550	1,590,000	6.0	5.7	2.9	5.2
	OCO	140	7,150	1,735,000	6.0	5.5	3.0	6.4
600 Volts per microsecond	CO	140	3,250	787,000	7.0	5.7	4.5	8.0
	CO	140	2,700	650,000	7.5	7.0	4.7	9.1
	OCO	140	3,250	788,000	7.0	6.7	3.9	8.0
	OCO	140	3,000	726,000	7.0	6.5	4.0	8.0
	CO	140	3,650	885,000	7.0	6.2	3.7	6.5
	CO	140	3,450	835,000	7.0	6.2	3.5	8.5
	CO	140	4,700	1,140,000	6.0	5.5	2.7	5.3
	CO	140	4,050	983,000	6.0	5.7	2.7	5.5
	OCO	140	3,900	946,000	6.5	5.5	3.6	7.1
2,400 Volts per microsecond	CO	140	2,800	680,000	7.0	6.2	4.2	8.5
	CO	140	3,300	800,000	6.5	6.2	4.1	8.2
	OCO	140	3,550	860,000	6.5	6.2	3.5	6.0
	OCO	140	3,100	750,000	7.0	6.5	4.0	8.1
	CO	140	2,100	510,000	8.0	7.2	4.7	10.5
	CO	140	2,700	655,000	8.5	7.7	5.5	11.5
	CO	140	3,100	752,000	7.0	6.2	4.2	8.5
	CO	140	730	177,000	9.0	8.0	5.7	9.3
	OCO	140	800	194,000	8.5	7.2	5.0	8.0
	CO	140	5,300	1,285,000	6.5	6.2	4.0	8.1
	CO	140	4,100	995,000	6.5	5.7	2.7	6.0
	CO	140	4,100	995,000	8.0	7.0	4.6	14.5
	CO	140	3,800	922,000	6.0	5.5	3.2	8.0
	OCO	140	3,650	885,000	6.5	6.0	3.5	6.7
	CO	140	4,500	1,090,000	8.0	7.7	4.4	13.3
	CO	140	4,400	1,068,000	6.5	6.2	3.2	7.0

records indicated that this breakage occurred during the last test and that the breaker was, therefore, in this damaged condition only during the four charging current tests.

On the same pole, all the fibre posts supporting the movable baffle in both explosion chambers were found broken. It is probable that this occurred during the last scheduled test, at which time the contact rod support was broken. Materials of increased strength and section have been substituted in the final design to overcome this difficulty.

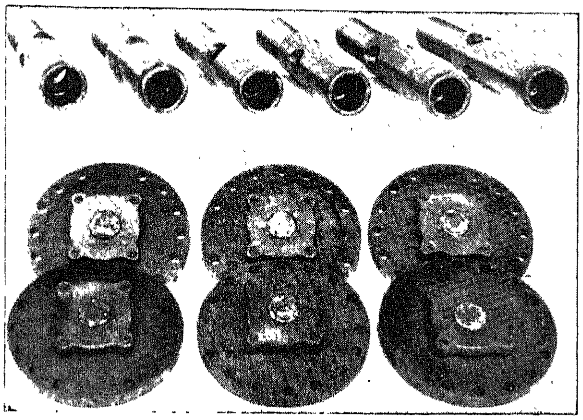

FIG. 6—OIL-BLAST CONTACTS FOLLOWING 40 TESTS

Fig. 6 shows the contacts after all the tests were completed. Actual measurement indicates that a total of less than ¼ in. of contact metal had been burned from the arcing surfaces. In order to determine their suitability for further operation, they were returned to the factory and subjected to a heat run of 1,200 amperes at 60 cycles showing less than a 30 deg. cent. rise.

Fig. 7 shows a pair of typical oscillograms obtained during one of the tests on the modified contacts. These oscillograms record the current in three phases, pressure in three tanks, one line-to-line voltage, two arc voltages, travel record, trip coil current, and cathode-ray oscillograph trip record.

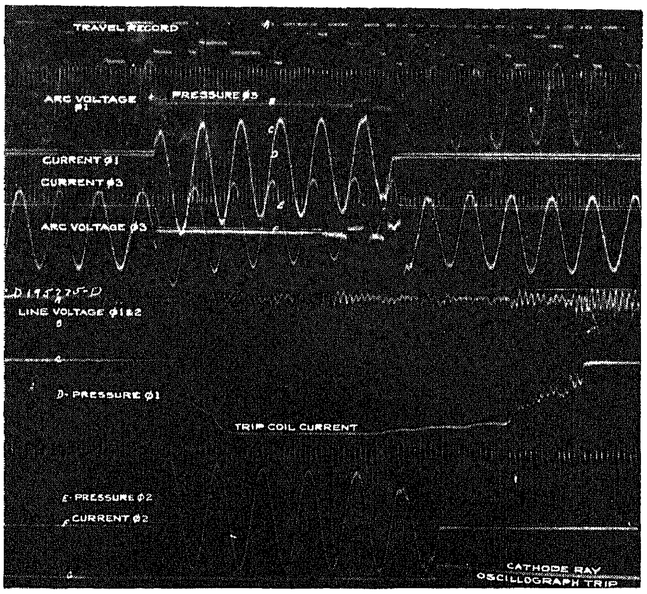

FIG. 7—TYPICAL OSCILLOGRAMS FROM FIRST SERIES OF TESTS

TESTS ON BUTT TYPE CONTACT

A series of 28 tests was made on the standard or butt

TABLE II
BUTT CONTACTS

Rate of recovery voltage rise	Duty	Line volts kv.	R. m. s. amperes initial in arc	Kva. interrupted	Short-circuit duration cycles	Breaker operating time cycles	Arc duration cycles	Arc length in.
270 Volts per microsecond	CO	136	3,250	765,000	9.5	9.2	7.5	18.5
	CO	136	3,200	755,000	9.0	8.5	7.0	17.2
	CO	136	2,600	612,000	9.0	8.5	6.5	16.2
	OCO	136	2,700	635,000	9.5	9.2	5.7	16.2
	OCO	136	2,700	635,000	10.0	9.5	6.0	14.7
	CO	136	3,250	765,000	9.5	9.2	7.2	18.2
	CO	136	2,700	635,000	9.5	9.0	7.2	16.7
	OCO	136	2,550	600,000	10.0	9.5	7.4	14.4
	CO	140	4,500	1,090,000	8.5	8.0	5.5	16.7
	OCO	140	4,400	1,065,000	10.0	9.7	7.5	17.2
	CO	140	4,700	1,138,000	9.0	8.7	5.8	16.7
	OCO	140	4,550	1,100,000	10.0	8.2	5.7	16.6
	CO	140	6,550	1,585,000	8.0	7.5	5.0	16.3
	CO	140	6,000	1,450,000	8.0	7.2	5.0	15.5
	CO	140	6,750	1,640,000	7.5	7.0	4.5	11.5
	OCO	140	6,200	1,500,000	9.0	8.5	7.1	14.7
	OCO	140	6,650	1,610,000	9.0	8.2	5.5	12.0
600 Volts per microsecond	CO	136	2,900	683,000	9.5	9.2	6.0	15.0
	OCO	136	2,750	648,000	11.0	10.5	7.5	17.2
	CO	140	4,000	268,000	9.0	8.5	5.7	17.2
	OCO	140	4,350	1,052,000	11.0	10.2	7.2	18.7
2,400 Volts per microsecond	CO	136	2,500	590,000	12.0	11.2	8.2	24 0
	CO	136	2,500	590,000	39.0	38.0	36.0	32.0
	OCO	136	2,600	613,000	34.0	33.0	31.0	32.0
	CO	140	4,300	1,040,000	12.5	11.0	9.5	28.0
	CO	140	3,900	943,000	10.5	9.7	7.2	20.4
	OCO	140	4,000	967,000	11.5	9.5	7.7	18.2
	OCO	140	3,750	907,000	11.0	10.2	7.5	18.7

type of contact as shown in Fig. 4. A summary of the results arranged in groups according to the rate of recovery voltage rise is given in Table II. All of the tests in this series were made in about eight hours, including the time used in the switching required to modify the system conditions. Many of the tests were made in groups having intervals between tests of approximately two minutes, which was the approximate time required to reload the oscillographs. No inspection of the contacts was made during the entire series of tests. The contacts which were removed from the breaker following these tests are shown in Fig. 8, and although somewhat burned, were in such shape that they could have been kept in operation for many more tests. The average pressure developed within the breaker tanks was 10 lb. per sq. in. with a minimum of zero and a maximum of 46 lb. per sq. in. Prior to the tests, the oil measured 30 kv. average and after the tests, the average value was 15 kv.

On Nos. 12 and 13 of this series of tests, at a voltage of 140,000 volts and at an interrupting duty of approximately 600,000 kva. with the system connections such as to give the maximum rate of recovery voltage rise of 2,400 volts per microsecond, the breaker arced to the end of the stroke. However, it cleared the circuit without causing an operation of the back-up breaker. Even under such severe conditions only a slight amount of oil was thrown out of the vent pipes on one tank during one test and from two tanks in the other test. In performance as to oil throwing, this compares with no oil throw on any of the other tests in this series up to interrupting duties of 1,640,000 kva.

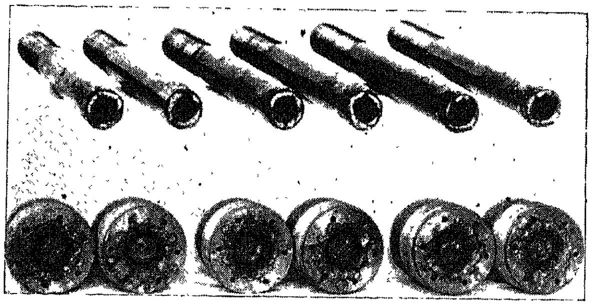

FIG. 8—BUTT TYPE CONTACTS FOLLOWING 26 TESTS

On test 22, after the breaker had interrupted 19 short circuits ranging in value from 750,000 to 1,000,000 kva. and three short circuits equal to or in excess of its interrupting rating, one of the bushings developed a crack in the cement joining the lower clamping ring to the bottom porcelain and permitted oil to leak out. All other bushings withstood the entire series of tests on both types of contacts. This breakage did not prevent completing the series of 28 tests as originally planned.

The inspection of the breaker after the series of tests was completed showed that one of the explosion chambers had broken but on just which test this occurred is not known. It is of interest to note that improved manufacturing methods have resulted in materially increasing the strength of chambers of this type so that even though the operation of the breaker was not seriously impaired, chamber breakages even under severe conditions of interrupting duty are a remote possibility.

TESTS ON STANDARD EXPLOSION-CHAMBER SEGMENTAL CONTACTS

The breaker used on the previously described tests was then equipped with standard molded explosion chambers and segmental contacts. Such a chamber is shown on Fig. 5. As such, it represented the then standard 132,000-volt oil circuit breaker rated at 1,500,000 kva. interrupting capacity on the standard two OCO two-minute interval cycle. A total of 12 tests was made at 140,000 volts. On the first group of these tests, approximately 140,000 kva. was interrupted. The time required for making those five tests was about sixteen minutes. After the group of tests, the system connections were changed and another group of six tests was made and about 1,400,000 kva. interrupted. These tests were made in about ten minutes.

On the last test of this group, which was an OCO test at 1,360,000 kva., the back-up breaker cleared the circuit. Shortly after that, however, and without inspection, the test breaker was again given a CO test. On this test, current appeared only in one phase because of difficulty with the back-up breaker. The current interrupted on the one pole was equivalent to 940,000 kva. An examination of the breaker after the test showed one explosion chamber broken. This had probably occurred on the next to the last test. On none of the tests except the next to the last one was any distress shown or any oil thrown, but on that one, however, some oil was thrown from one pole. Other than that, the only evidence that the breaker had opened was a slight puff of smoke from the gas vents. The oil before the tests were started had a breakdown of 30 kv. and after the tests averaged 23 kv., measurements in both cases made in a standard test box. The contacts were only moderately burned. A summary of these tests is given in Table III.

BREAKING LINE CHARGING CURRENT

The ability of any type of breaker or contact to quickly rupture line charging currents has been a question of considerable interest, although not one of prime importance; therefore, following the series of 40 tests on the oil-blast explosion chamber and before any inspection had been made, four shots were taken interrupting the charging current of two different lines from the Philo station.

The average arcing time during these tests was 2.7 cycles with a maximum of four cycles and an average arc length of 3.2 in.

DISCUSSION OF RESULTS

Table IV gives a summary of the two principal series of tests and indicates clearly the effect of the difference

TABLE III
SEGMENTAL CONTACTS

Rate of recovery voltage rise	Duty	Line volts kv.	R. m. s. amperes initial in arc	Kva. interrupted	Short-circuit duration cycles	Breaker operating time cycles	Arc duration cycles	Arc length in.
270 Volts per microsecond	CO	140	5,800	1,400,000	12.5	14.2	5.7	14.8
	CO	140	5,800	1,400,000	12.5	12	5.0	14.3
	CO	140	5,800	1,400,000	11.5	11	4.5	13.2
	OCO	140	5,650	1,370,000	17.5	15	5.8	14.2
	OCO	140	5,600	1,360,000	17.5	15	5.5	13
	OCO	140	5,600	1,360,000	Cleared by backup breaker			
	CO	140	3,900	940,500	15	13.7	7.7	13.2
2,400 Volts per microsecond	CO	140	No current record					
	CO	140	580	140,000	14	12.5	6.2	11.5
	CO	140	600	145,000	14	12.5	7.2	13
	OCO	140	600	145,000	16.5	15	7.0	12.2
	OCO	140	580	140,000	18	15.7	7.2	12.8
	OCO	140	580	140,000	17	14	6.7	11.3

in rate of rise of recovery voltage on arcing time and arc length. An analysis of these data shows that for comparable values of rate of recovery voltage rise, the average values of arc duration in the oil-blast explosion-chamber breaker are about half the duration obtained with the butt type contact; also, that the average arc length with the modified contact is approximately 40 per cent of the arc lengths with the butt contacts under comparable rates of rise of recovery voltage.

The average speed of the contacts during the opening stroke under load on the oil-blast explosion-chamber breaker was 9.5 ft. per sec., whereas that of the butt type contact chamber breaker was 13 ft. per sec. The conclusion drawn is that although high speed of cross-head travel has never been essential to the proper functioning of explosion-chamber breakers, it is even less important in breakers provided with the oil-blast explosion chambers.

TABLE IV
SUMMARY OF TEST DATA

Butt Contacts

Lines connected to bus	Calculated volts per microsecond	No. of tests	Arc duration cycles av.	Arc length in. av.
3	270	17	6.2	15.5
1	600	4	7.1	17.0
0	2400	7	15.0	24.5
Total 28 tests		Average	8.6	18.0

Oil-Blast Contacts

Lines connected to bus	Calculated volts per microsecond	No. of tests	Arc duration cycles av.	Arc length in. av.
3	270	15	2.6	4.5
1	600	9	3.7	7.1
0	2,400	16	4.1	8.8
Total 40 tests		Average	3.5	6.8

Fig. 9 shows a composite diagram of the butt type and oil-blast explosion-chamber breakers with the values of the range of arc lengths as measured during all the interrupting tests at all rates of rise of recovery voltages compared with the breaker stroke.

Fig. 10 shows graphically the relation which existed between arc duration and current interrupted, for both types of contacts during all interrupting tests and for all rates of rise of recovery voltage. Although the difference in rate of voltage rise had an effect on arc duration as previously shown yet the performance of the oil-blast explosion-chamber contacts is remarkably consistent over a wide range of current and recovery voltage conditions.

General Comments

Owing to previous inability to obtain short circuits of this order of magnitude for test purposes, there has been on the part of purchasers of oil circuit breakers, a certain amount of skepticism regarding the higher ratings. It is therefore interesting that these tests have demonstrated the ability of the standard breaker, without modification of contacts, to meet its rating.

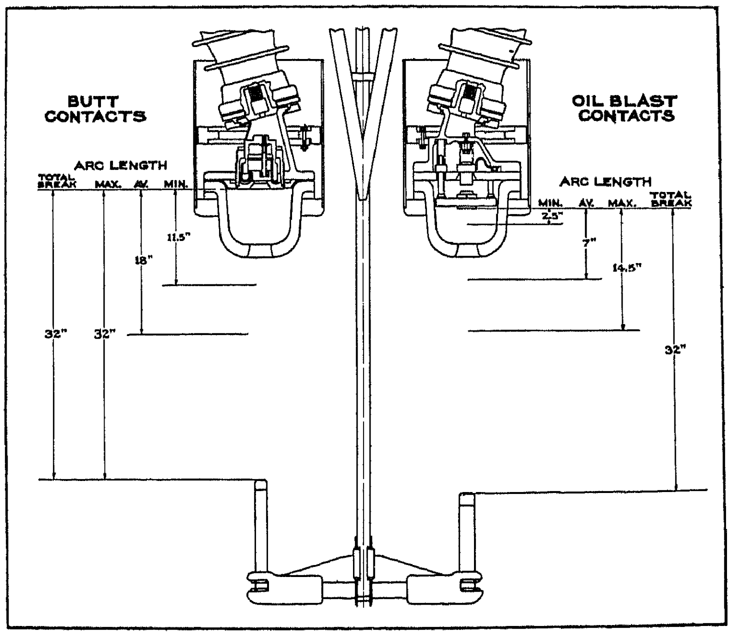

Fig. 9—Comparative Arc Length Diagram

The higher rate of rise of recovery voltage on these tests (2,400 volts per microsecond) is approximately the same as obtained with the high current laboratory test equipment at Schenectady, which is in the order of 3,000 volts per microsecond. Inasmuch as this seems to be about as high a rate as is met in practise, it would appear that breakers which successfully meet their rating in the laboratory are adequate to meet the most severe field conditions and have a very wide margin for normal conditions, where the average rate of rise is probably in the order of 300 to 600 volts per microsecond.

Although the circuit breaker designers' problem has always been to consider the worst possible operating conditions, a better knowledge of what these conditions are is now made available through the newly developed theory of recovery voltage characteristics, and is substantiated by these tests, typical oscillograms of which are shown. Thus, a much more accurate idea may be obtained of the rate at which insulation must be inserted between the electrodes of an interrupting device, and designs are now available in which the required rate of insulation insertion is realized, so that the effects of such wide differences in the rate of recovery voltage rise are minimized.

Fig. 9 indicates the increased factor of safety obtained by applying the oil-blast explosion chamber to breakers of present standard design even under the worst operating conditions. Considering the results so far realized, it may not be too optimistic to expect that this development will lead to more important and desirable improvements in design.

The effect of reduced arc duration on the mechanical construction of high-speed oil circuit breakers is obvious, as it is not necessary to design moving parts for tremendous speeds, hence not necessary to provide extra heavy opening springs to obtain such speeds. Conventional types of operating mechanisms may be used, and when provided with high-speed trips to reduce as far as possible the time lost between energizing the trip coil and the separation of contacts, the full benefit of this very material reduction in arcing time is obtained.

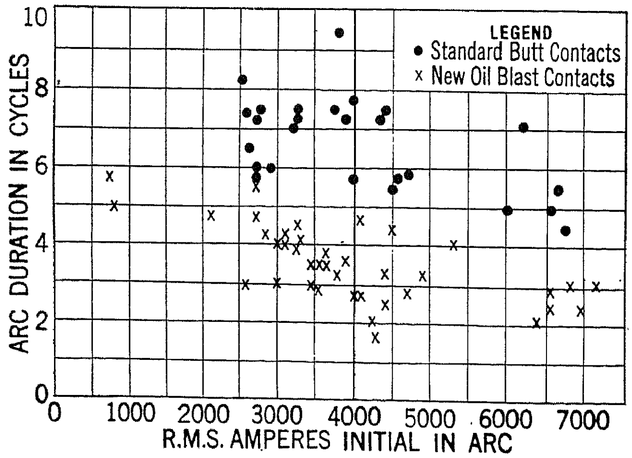

Fig. 10—Comparative Arc Duration

The ability of the oil-blast explosion-chamber contacts to carry currents up to 1,200 amperes without danger of overheating, even after performing severe interrupting duty, has been demonstrated.

The oil-blast explosion chamber has the same over-all dimensions as the older style and can be applied to existing breakers by substituting an assembly similar to that shown in Fig. 9 for the old style explosion chamber but with the proper type of crosshead having larger contact rods. The oil-blast explosion chambers may also be installed on many older style breakers now in operation with the plain break or finger contacts, without involving more extensive changes than are often economically justified by the increased interrupting capacity and shorter operating time.

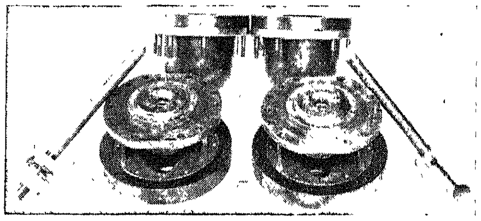

Fig. 11—Segmental Contacts Following Tests

Even in cases where system stability is not a pressing problem, there is an increasing demand for faster operating breakers in order to reduce the duration of shock to connected apparatus. This need has been emphasized particularly by users applying electricity to processes where voltage dips of appreciable duration may affect their finished product. Such shorter duration of faults, and particularly the shorter arcing time, materially reduce the required maintenance on the breakers themselves, and for that reason this development, in addition to being most valuable as a solution to high-speed switching problems, should find a very general application.

Conclusions

A high-voltage high capacity breaker has now been demonstrated to have the ability to interrupt 40 short circuits up to 115 per cent of its rating in a short period of time without inspection or attention and still be in condition to operate. This would indicate that the general impression of an oil circuit breaker as an unreliable device, and one requiring undue maintenance, may in some respects be subject to revision. Such a development appears to meet the requirements set up only a few years ago by some operating engineers for a breaker that could operate 50 times under short circuits without requiring attention.

A standard breaker was given a number of interrupting tests at short-circuit current approximating its rating and gave a very satisfactory performance. It seems pertinent to emphasize again that this is the first demonstration ever made of the interrupting performance of a high-voltage breaker at short-circuit values approximating 1,500,000 kva.

Also, much data were obtained on burning of contacts and oil deterioration under repeated tests at high interrupting duties. These data will be very useful as a basis for possible changes from the standard operating duty cycle.

There is an increasing appreciation of the beneficial effect on operation of short duration of fault, and from that standpoint the minimum possible breaker operating time is to be desired. Nevertheless, the majority of the requirements for fast switching demanded by present day standards of service are fairly well met by a breaker with an operating time of not over eight cycles.

Furthermore, it is not improbable that additional refinements in mechanical construction in order to cut down elapsed time until contact separation, and possibly even more efficient means of arc extinction, will in the future permit the design of breakers having still shorter operating time.

The authors desire to express their appreciation to the organization of the American Gas & Electric Company and the Ohio Power Company for their courage, planning, and cooperation, without which these tests could not have been so successfully carried out. Mr. Skeats' assistance, particularly in comparing test results with his previous work on the theory of recovery voltage, is gratefully acknowledged.

Discussion

OIL CIRCUIT BREAKER TESTS—PHILO 1930

(Sporn and St. Clair)

THE OIL-BLAST CIRCUIT BREAKER

(Prince and Skeats)

CIRCUIT BREAKER FIELD TESTS

(Spurck and Strang)

R. H. Park: Recent developments in the design of oil circuit breakers have made possible very much more rapid switching than could be obtained formerly. With such quick switching the ratio of the direct current to the symmetrical component of short-circuit current is apt to be much greater than formerly, and as a consequence the calculated circuit breaker duty may be affected to a larger extent by the presence of d-c. component.

In the past it has been customary to assume that, as far as involves interrupting capacity, the effect of a displaced current was the same as the effect of a symmetrical current of the same mean square value. Since the d-c. component was generally a small fraction of the total short-circuit current, it actually did not make much difference what assumption was made in regard to this effect. However, with increased switching speeds, it appears to be desirable to review the situation in order to determine whether or not the original assumption was a reasonable one.

Undoubtedly, the only final test of any rule for determining the importance of the d-c. component is one which is deduced directly from actual tests in which the magnitude of the d-c. component is varied. Granting this, it should still be of interest to determine what conclusions may be arrived at on the basis of our present conception of the factors involved. As I understand it, the two principal factors are recovery voltage and arc energy. From the standpoint of recovery voltage, it can be shown that the presence of a d-c. component tends to lower circuit breaker duty. Suppose, however, that we disregard this tendency. It would then follow that the equivalence of a symmetrical and displaced wave is determined by the equality of their arc energies. But the volt-ampere characteristic of an arc at any given separation of contacts may be roughly approximated by the equation $e = k\,i^n$, where n is negative instead of $+1$ as would be the case with a resistor. The power loss in the arc is then $e\,i = k\,i^{n+1}$ and the average power loss per cycle is measured by the root mean $n + 1$ power of the current instead of by the root name square as in the special case then $n = +1$. For instance if $n = -\frac{1}{2}$, the average energy loss per cycle is measured by the root mean square root of the current.

The following table gives the root mean $n + 1$ power of a displaced wave as a function of arch exponent n and amount of displacement.

Per cent displacement	$n = +1$	$n = 0$	$n = -\frac{1}{2}$
0	100	100	100
20	104	103	102
40	115	108	105
60	131	118	112
80	151	134	120
100	173	157	141

These data suggest that the established practise of evaluating breaker duty in terms of the r. m. s. value of the current in the arc may result in an overemphasis of the effect of d-c. component. I should like to ask the authors of the three papers on circuit breakers whether they have any data bearing on this conclusion.

R. Treat: For a long period, the oil circuit breaker was regarded in many quarters as about the crudest and most unsatisfactory piece of equipment with which the electrical industry had to contend.

It is refreshing, therefore, to find from the papers presented and from those which have been presented to the Institute in the last year or two that the present trend in design of circuit breakers is to replace brute strength with some degree of subtlety, based on better knowledge of the phenomena involved.

Frequently in pursuing the paths of progress we find that an advance for the purpose of achieving a definite objective brings with it some corollary advantages which were more or less, if not entirely unexpected and in some cases are more valuable than the original objective. This condition seems to exist, in part, at least with respect to the development of high-speed breakers. While it has been generally appreciated that short duration of short circuits was desirable, it was the system stability problem which brought the real urge for fast breakers and fast relays. Many calculations on a large number of systems indicated very definitely the considerable increase in power limits of transmission systems during short circuits which could usually be attained by reducing materially the duration of the short circuit. The demand became so insistent as to necessitate the developments which have been described in these papers.

A few months ago I had the privilege of witnessing some tests on the New England Power Company's Fifteen Mile Falls system on which are installed high-speed breakers and relays. In some of the tests the total duration of short circuit was on the order of 6 cycles and on many it did not exceed 8 or 9 cycles. It was a revelation to observe the small effect of even severe short circuits when the duration was kept low. From the standpoint of shock to the synchronous apparatus, of disturbance to the rest of the system, of the effect on paralleling communication circuits and of damage at the point of the arc itself, the improvement occasioned by the short duration of the fault was astonishing. It was my own impression and I think that impression was shared by the engineers of the Power Company that high-speed breakers and relays were entirely justified from these standpoints, regardless of any effect they might have on system stability.

The other advantage of short duration of fault lies in the breaker itself. A careful examination of the facts brought out in these papers indicates that the achievement of speed has been accompanied by reduction in severity of short circuit on the breaker structure. It is not too much to hope that some time in the future advantage of this reduction may be had in giving us more breaker rating for the money. A present breaker of given voltage and interrupting capacity might conceivably have its size, that is, height reduced on account of reduced arc length and, therefore, stroke, and at the same time have its interrupting capacity raised possibly 50 or even 100 per cent. Further, the tests reported arouse the hope that there will soon be available breakers in which the period between necessary maintenance inspections can be significantly increased.

I do not by any means have the feeling that the designers of circuit breakers have reached the end and that there is no more progress to be made. Quite the contrary. I do feel, however, that the progress that has been reported to the Institute in the last two or three years is most gratifying and a happy augury of what may be expected in the next few years.

J. Slepian: The great progress in the field of circuit interruption which has come in recent years was due first and foremost, apart from the driving force of necessity—that well known parent of invention—to the clarification and precision of ideas relating to the extinction of a-c. arcs under oil. If further progress is to be made, these ideas must be still further clarified and made still more precise, even to the point of complete expression in mathematical form. The very excellent paper by Messrs. Park and Skeats* presented last fall is an example of the advance toward greater precision, and is to be highly commended. Relapses to the old loose ways of glossing over the happenings in those vital few microseconds after a current zero in which the interruption of the circuit takes place should not be tolerated, however, and I feel compelled, in the interest of progress in this art, to criticize adversely very many statements made in the paper by Messrs. Prince and Skeats. Let me leave the detailed citation of the various incorrect and loose statements to the end of my discussion, however, and proceed first to the discussion of the theory of operation of the explosion pot oil circuit breaker as given by the authors of this paper.

In Fig. 4 of the paper are given diagrams of an explosion pot breaker contact in four positions; the first closed, the next two with different lengths of arc, and the last, with a barrier of oil across which, a little while before, had been the arcing path. Everything has been shown in this figure except what is important from the standpoint of theory, namely, how the circuit is interrupted by the interaction of the arc and the oil. As the authors state earlier in the paper, the space between the contacts which is highly conducting up until a current zero must change over into a good insulator in less than ten microseconds. It is what is happening in these ten microseconds which is important in the circuit interruption, and about which the authors profess to give information. But the first three diagrams of Fig. 4 refer to times thousands of microseconds before the current zero, and the last diagram refers to a time thousands of microseconds after the current zero. There is no doubt that an arc in a continuous gas pocket or bubble, as shown in the second and third diagrams, exists prior to the current zero, and that some time after the circuit interruption, oil will displace gas from the bubble and will form a barrier between the open contacts, somewhat as shown in the fourth diagram, but what is wanted is information as to the circuit interruption period itself. Where are diagrams showing conditions one microsecond after current zero, two microseconds after current zero, and so forth up to ten microseconds after current zero? These are the times in which the space is recovering dielectric strength, and which should be portrayed to explain the authors' theories.

If the authors had attempted to draw such diagrams, the fallacy of the theory which they present would have been evident. The authors account for the growing dielectric strength of the arc space after current zero by the formation of a barrier of oil whose growing thickness keeps pace with the growing voltage impressed upon the electrodes by the external circuit. Using the very large figure of 600,000 volts per in. for the dielectric strength of the oil next to the arc, the authors calculate that this barrier of oil must grow in thickness at the rate of 2,500 in. per sec. to keep pace with the 1.5×10^9 volts per second rate of rise of the voltage impressed upon the electrodes tending to reignite the arc.

The authors believe this speed of formation of an oil barrier is attained through the use of the explosion pot. But consider the first microsecond of the extinction period. An obstructing film of oil must form across the arc space with a thickness equal to $2{,}500 \times 10^6$ or 0.0025 inches, capable of withstanding $1.5 \times 10^9 \times 10^6$ or 1,500 volts. This film must be completely continuous. Any hole or break in it will cause reignition of the arc. In the next microsecond, the film will have grown to 0.005 in. and will be withstanding 3,000 volts, but the film must not wave in its continuity. The slightest hole in it, and the arc will reignite. Even more, the film must not waver in its thickness. If it is anywhere less than the required 0.005 inches, the arc will reignite.

What extraordinary hydromechanics will be required which will project such wonderfully continuous and uniform films of oil across the gas space precisely at a current zero within a microsecond.

The incorrectness of the theory given here is also revealed at once by the experimental fact that the arc length required to interrupt a circuit was found by the authors to vary with circuit voltage. But if the theory given by the authors was correct, the interruption of the circuit, whether of high or low voltage, would be accomplished by the formation of a barrier of oil growing in thickness at the rate of 2,500 in. per sec. and withstanding 600 volts per thousandth inch. For a 15,000-volt circuit, the oil barrier would need to reach a thickness of only 0.025 inches, but it must reach this thickness certainly or the arc will reignite. Increasing the separation of the electrodes will not help (according to the authors' theory) if the required thickness of oil barrier is not obtained, for the authors are not depending upon deionization of the arc space which other writers have stressed. Therefore, in a breaker which is operating according to the authors' theory in a 15,000-volt circuit, the 0.025-in. thick oil barrier must be formed within 10 microseconds. But if this oil barrier is growing in thickness at this rate for the first ten microseconds, it may surely be expected to continue to grow at this rate for some time after, since the pressures which are responsible for the formation of this oil barrier (according to the authors' theory) are continuing to act. It is not unreasonable to expect the barrier to continue to grow for 100 microseconds, and in this time the oil barrier will have attained a thickness of 0.25 in. and will be capable of withstanding 150,000 volts. Hence, since the rate of voltage rise is about the same for high-voltage circuits as for low, any practical breaker which operates according to the authors' theory, and which is capable of interrupting 15,000 volts, should also be capable of interrupting 150,000 volts. It should be 150,000 volts or nothing! But this is in glaring contradiction with the experimental facts.

I cannot leave Fig. 4 of the paper without pointing out the serious misconception of the nature of the arc in the oil circuit breaker which is revealed in diagrams two and three of Fig. 4. The arc is shown in a gas pocket or bubble, but the profound influence of the rapid vaporization and decomposition of the oil-wall boundaries of the bubble are not illustrated. The arc in the gas pocket under oil is in a very different state from an arc of like current in a gas volume of like size and pressure but in a chamber whose walls are of non-gas-forming material such as glass or other refractory. This is revealed by observation of the arc with the camera, as well as by the fact that the alternating-current arc in the gas pocket bounded by oil can interrupt greater volts per inch than the arc in the gas pocket bounded by glass. The arc in the glass vessel has a definitely columnar structure similar to that shown by the authors in diagrams two and three of their Fig. 4. There is an intensely luminous cylindrical core of fairly definite section joining the two electrodes, and surrounded by regions of lesser luminosity. But in the gas pocket in the oil, the camera shows that the columnar structure of the arc nearly disappears. The whole gas bubble is filled with a rather uniform luminosity. The gas projected from the decomposing oil walls breaks up the structure of the arc continually. The highly ionized conducting paths are swirled around in all parts of the gas bubble. This is the action which is responsible for the interrupting capacity of the oil circuit breaker. On this account, in discussing the oil circuit breaker in my paper, *Extinction of a Long A-C. Arc*,* I do not show a columnar structure of the arc in the illustrating diagram.

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*A. I. E. E. Trans., March 1931, p. 204.

What then, is the theory of the explosion pot circuit breaker? What are the gains which it gives over the plain break? In the above mentioned paper, after having pointed out, as I believe, for the first time, the important part played by the rapid evolution of gas from the oil walls of the gas bubble, I showed that the action of the gas could be intensified by keeping the oil walls of the bubble close to the arc at all times. In this way, the rate of evolution of gas from the oil wall is greatly increased, and the distance through which this gas must move to mix with the arc is greatly reduced. The normal growth of the gas bubble must be combatted, as this growth will move the oil walls farther from the bulk of the arc. A method which I showed in my paper for combatting this growth of the bubble was by using oil soaked baffles which stayed close to all parts of the arc even though the liquid oil moved away. A still better method was revealed by Messrs. Wilcox and Baker† in their paper on the deion grid. Still another method which is more obvious, but which was not discussed in my paper, is to use obstructions or baffles which directly prevent the motion of the oil away from the arc, and physically prevent the expansion of the oil bubble. This method has the disadvantage of requiring the development of high pressure, but it is the method which is used in the explosion pot.

In the explosion pot, by the confining of the oil, the gas bubble is prevented from attaining the large size it would have, if the oil were free to move. Hence, within the explosion pot, there is a more intense rate of gas evolution, and greater volts per inch can be interrupted than with the plain break, in those cases where the interruption takes place with the arc entirely within the pot. This action is still more pronounced for that section of the arc which is in the throat of the chamber. Here, the throat walls with the adhering oil press still more closely upon the arc and force the gas bubble section to be small. Furthermore, upon emergence of the electrode, there is at this place an intense blast of gas as the gas stored in the chamber vents itself through the throat as the moving electrode emerges. These are reasons why the likelihood of extinction is greatest at the moment when the electrode emerges from the throat.

There is still a fourth method of keeping the gas bubble around the arc small, which was not mentioned in my paper but which was described in a patent application filed at about the time of its publication, and which has a very direct relation to the oil blast explosion pot described in the present paper. This is to use a hollow electrode, which will permit venting of the gas from the bubble around the arc without requiring the bubble walls to recede from the arc. Thus the bubble around the arc itself is greatly reduced in size.

The question then naturally presents itself as to how much of the advantage of the oil blast explosion pot arises from the venting of the gas bubble around the second arc through the hollow electrode *C*, and how much from the arc between the fixed contact *A* and the intermediary contact piece *B*. May it not be that if the intermediary piece *B* were removed, still better results might have been obtained, due to the earlier venting of the gas within the explosion pot through the hollow electrode, giving a smaller bubble around the arc with a reduced pressure within the pot? It is true that in the tests described by Messrs. Spurck and Strang on the standard explosion pot breaker a hollow moving contact was used, but apparently there were no holes in the sides of these contacts which would permit the gas to escape through the hollow electrode. I would like to ask the authors whether they tried omitting the piece *B* from their present structure with no other essential change, so that venting of gas through the hollow electrode would still be permitted, and what results they obtained. This question is all the more pertinent because in the paper by Messrs. Spurck and Strang, Fig. 9 shows that in many of the operations of the oil-blast explosion pot breaker, the circuit interruption occurred with the moving electrode far outside the pot.

Besides being in contradiction with the explanation given by Messrs. Prince and Skeats in connection with their Fig. 6, it makes it seem extremely improbable that the little short arc between *A* and *B* could have the profound influence upon the long arc between *B* and *C* which the authors claim, particularly when so much of this long arc lies outside of the explosion pot. A diagram explaining the authors' theory in relation to the 14.5-in. arc length shown in Fig. 9 of the Spurck and Strang paper would be very helpful.

Now for the loose and inaccurate statements which I find throughout this paper and which are of the type which I believe are inimical to progress in the art of circuit interruption. On page 506, column one, second paragraph, "These gas bubbles are rather poor insulators, etc." Gases at atmospheric and higher pressures are normally very good insulators. Air at normal pressure, and for ordinary distances, has a dielectric strength of about 50,000 volts per inch. It is the capacity for the rapid recovery of this high dielectric strength which gases enjoy under proper conditions which makes arcs in gases so useful for circuit interruption purposes. It is through the recognition of the great insulation value of gases and the discovery of those conditions which accelerate the recovery of this insulating state that progress is to be obtained.

Continuing with the same sentence, "and only as portions of the arc stream are accidentally interrupted by oil spray is their dielectric strength increased to the point where the arc will go out." How can a spray which consists of discrete drops be able in any way to "interrupt" a gas path? If the drops are expected to coalesce into a continuous barrier, what an extraordinary "accident" that within a few microseconds including a current zero, they should coalesce into a film of the proper continuity and thickness as to interrupt the gas path, and withstand full circuit voltage!

In the next paragraph, "This greatly increased the rate at which the arc gases were replaced by dielectric and raised the dielectric strength of the arc path, etc." Where do the "replaced arc gases" go in the few microseconds in which ability to withstand voltage is obtained? If they themselves do not recover the ability to withstand voltage by rapid deionization, must not their continuity be broken by the "introduced dielectric," and must not the authors show that this "introduced dielectric" really does break the continuity of the gas path?

Beginning at the bottom of page 506, "The arc path at the time of rupture for existing circuit breakers has an effective dielectric strength of from about 2,500 volts per inch in plain-break circuit breakers up to 10,000 volts per inch for the deion grid breaker, etc." This is a very inaccurate statement of the dielectric properties of the arc space near current zero. The dielectric strength is a continually increasing one in time from a very low value to strengths of the order of 50,000 volts per inch. In the deion grid breaker dielectric strength is recovered more rapidly than in the plain break breaker, but it is likely that the ultimate or final dielectric strength is not much greater than in the plain break breaker.

In the same paragraph, "The introduction of any considerable amount of solid oil into the gap at the current zero, therefore, holds possibilities of great improvement over other methods of interruption." It is difficult to see how the introduction of "solid" oil into the gap will increase dielectric strength in itself unless the solid oil actually breaks the continuity of the gas path. This should be mentioned and some reference to the difficulty of causing this introduced solid oil to interrupt the gas path in the short time available, should be mentioned.

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*A. I. E. E. Trans., April 1930, p. 421.

†*Use of Oil in Arc Rupture*, A. I. E. E. Trans., April 1930, p. 431.

Again in the second column of page 507, "Therefore, if fresh dielectric is to be introduced at a rate, etc.," here again the provision that from the very beginning and throughout the period of "introducing the fresh dielectric" the fresh dielectric must always be breaking the continuity of the gas path is not mentioned, even though this requirement seems impossible of practical attainment.

In the next paragraph, "oil will sweep away the arc." Where to, in the few microseconds available?

At the bottom of page 507 it is stated that the experimental fact that the arc in the explosion pot breaker is most often extinguished at the time that the moving contact emerges from the throat of the chamber proves that the authors' theory as to the interrupting action of solid oil is correct. This is not at all the case. All that can be concluded is that probably the arc space of that portion of the arc which is at the throat recovers dielectric strength at current zero much more rapidly than that of other portions of the arc.

At the bottom of the first column, page 508, "But as the oil glides through the passageway in the contact rod, this arc is carried before it and exists alongside it without mixing, etc." Without inquiring into the justification on hydrodynamical grounds of the authors' assurance that the flow of oil at so high speeds as they postulate will be as smooth as they describe, we might ask as to whether this oil which is receiving heat from the arc alongside of it at the rate of thousands of kilowatts per square inch, may not be decomposing into gas, and whether this gas might not be mixing into the arc space and causing some disturbance of the arc.

Continuing, "Then, as soon as the current zero is reached, the arc products are displaced by a smooth flowing wall of solid oil, etc." By what magic is the flowing oil caused to leave the gas space so severely alone up to the current zero, and then immediately after current zero to change its course suddenly and "displace the arc products?" Where are these "arc products displaced" to in the first microsecond, the second, etc.? They must be "displaced" even after the first microsecond, as 1,500 volts are recovered by the circuit even in that short time. A few diagrams would be very helpful.

In Table I the volts interrupted per inch are stated to be the extraordinary value of 44 kv. This, however, is probably obtained by dividing the voltage of the circuit by the length of the second arc which is drawn and wholly neglecting the length of the first arc. But are the authors so sure that the first arc was contributing nothing at all to the recovered dielectric strength of the switch? It would seem that in all fairness they should allow at least 2,500 volts per inch for the "effective dielectric strength" of this first arc, since earlier in the paper they credit the arc in the plain-break breaker with this figure. It might even be expected that the advocates of an explosion pot breaker would allow an even larger figure for "effective dielectric strength" of an arc which is entirely within an explosion pot. In comparing the advantages of the "fairly homogeneous arc path" (p. 506. second column) with the non-homogeneous, will it not be fairer for the authors to make comparison on the basis of the total arc length making up the break?

In the first paragraph of page 512, the authors again state that in actual test 44,000 volts per inch of arc were interrupted. Again may we ask whether the several inches of first arcs drawn have not been omitted in calculating this figure?

In concluding, the authors say in their next to the last paragraph, the arc "is merely swept away, and its place taken by solid oil." May I conclude with whither, and when?

H. M. Wilcox: Messrs. Prince and Skeats are to be congratulated upon their contribution to the theory of arc phenomena in oil, in the form of the oil-blast principle. It is to be regretted, however, that in presenting to the engineering profession a new theory of arc rupture, distinct from existing theories, they have not seen fit to set forth more fully the fundamental principles underlying the theory which they have evolved. A careful study of the data they present leaves some points not sufficiently clear to warrant unconditional acceptance of their theory.

Referring to Fig. 6 of their paper, it is stated that a short arc is drawn between contacts A and B which forms gas and places the oil in the explosion chamber under considerable pressure. This action is evidently the result of volatilization of oil in contact with the arc since there is no other gas-generating agency present. Immediately following this the second arc is drawn between contacts B and C, and it is stated that as the oil glides through the passageway in the hollow contact rod this arc is carried before it and exists alongside it without mixing, remaining at the current zero a smooth flowing wall of solid oil the dielectric strength of which has not been impaired by carbonization or entrained ionized gas. The reason for this marked difference in performance in the contact between oil and arc in the two cases is not apparent from the subject matter of the paper, or in the light of other existing theories of arc rupture in oil.

Again, referring to Fig. 13, the authors leave the impression that the contact tube is discharging gas at high velocity during the current cycle and that this gas is still flowing at the current zero. On the other hand Mr. Spurck, in a companion paper, evidently discussing the same structure, speaks of pressure generated by the opening of the first gap as blasting a continuous stream of clean, high-dielectric oil through the hollow contact, this continuous stream passing across the arc formed between the contact rod and the lower face of the conducting cylinder, thus introducing insulation into the arc stream at an extremely high rate. The authors should clarify these apparently conflicting statements before definite conclusions can be drawn as to the theory they advance.

A comparison of the structure in Fig. 6 with the earlier explosion chamber shown in Fig. 5, leads to an evaluation of the differences between the two structures. Judging from the performance data given for the new chamber, the interruption occurs while the lower terminal of the arc is in the throat of the chamber which is precisely what is claimed for the older chamber. The throat effect has been modified by the substituton of a hollow electrode for the former throat bushing, thus increasing its effective length but tending to decrease the velocity of the oil or gas passing through it, although this may be offset by the increase in pressure due to the reduction in volume of the oil space within the chamber. An intermediate baffle has been added with its attending complications. The oil reservoir space within the chamber has been shortened and, since the arc is extinguished in the throat in both cases, the amount of shortening may be taken as being roughly a criterion of the reduction in arc length on circuit interruption.

In this connection the question arises as to just what is considered as arc length in the test data reported in these papers, particularly in view of the high value of volts per inch of arc interrupted as claimed by the authors. Messrs. Prince and Skeats show an oscillogram (Fig. 10) in which contact separation takes place after five impulses of the breaker travel wave, or a lift rod travel of 2½ inches, the arc duration being considered as starting at this point in the travel. Mr. Spurck in Fig. 9 of his paper, shows clearly arc lengths measured from the point at which the second break takes place and, since the various tabulated data show repeated instances of arc lengths of one inch or less, the conclusion must be drawn that the arc in the first gap has been disregarded in preparing the data given.

There is grave doubt as to whether the authors are justified in ignoring arc lengths of such magnitude in determining the performance of a circuit breaker. From the data given it appears that the first arc drawn is of the order of 2½ inches long per break, giving an arc length of five inches for the two breaks. This arc length represents energy dissipated and is subject in all ways to the ills cited by the authors against arcs drawn under previous existing arc-rupturing methods and should, therefore, be included in determining the volts per inch of effective dielectric strength of the arc path.

Assuming the length of the first arc drawn to be 2½ inches per break or five inches in the two breaks, a comparative study of the two conditions may be made for the 140-kv. tests shown in tabulation I of Mr. Spurck's paper. For the 15 tests made with a rate of recovery voltage rise of 2.70 volts per microsecond, which is a comparatively easy circuit to interrupt, the average effective dielectric strength of the arc path in terms of normal-frequency recovery voltage, as calculated from the tabulated data, was 9,000 volts per inch, assuming that recovery voltage is 100 per cent of initial voltage. Adding five inches of arc length for the first arcs drawn, this value drops to 5,800 volts per inch. For the nine tests made with a 600-volt per microsecond rate of rise of recovery voltage, the average effective dielectric strength of the arc path as calculated from the tabulated data was 5,500 volts per inch, while this value drops to 4,050 volts per inch when the length of the initial arc is included. For the sixteen tests made with a 2,400-volt per microsecond rate of rise, a very "fast" circuit, the average value calculated from the data is 4,500 volts per inch, but this value becomes 3,550 volts per inch when the length of the first arc is included. It will be noted in reviewing these revised values that in changing from the 270 volts to the 600 volts microsecond rate of rise of recovery voltage, the average decrease in performance as measured by volts per inch interrupted, was 1,750 while on changing from the 600 to the 2,400 volts per microsecond rate of rise, the decrease was only 500 volts per inch. It may be assumed then that the interrupting performance at the 600-volt rate was representative of the breaker performance over a relatively wide range of voltage recovery conditions and that the value of about 4,050 volts per inch represents its interrupting ability under average operating conditions when the total length of arc drawn is taken into consideration.

A comparison has been made in these papers between the oil-blast circuit breaker and oil circuit breakers equipped with deion grids. No tests have been made on an operating system at 140 kv. with the latter device but field tests made at 66 kv., at 110 kv. and at 220 kv. indicate that its performance in volts per inch interrupted is substantially constant over this range of voltages so that a comparison may be drawn between the two methods of circuit interruption from these data. Published results of tests with deion grids on the 220-kv. system of the Philadelphia Electric Company at Plymouth Meeting Station, November, 1929 show 12 single-phase, line-to-ground tests, at current values from 1,000 to 3,000 amperes, with an average arcing time of 2.8 cycles, giving an average effective dielectric strength of 7,500 volts per inch of total length of arc in the two breaks.

Published results of tests made with a breaker taken from service and equipped with deion grids, on the 110-kv. system of the Alabama Power Company at Birmingham, Alabama, November, 1929 show 9 three-phase tests and 2 line-to-ground tests, at current values from 2,000 to 4,000 amperes, with an average of two cycles of arcing for all tests, giving an interrupting value of 8,000 volts per inch of total arc length in the two breaks. Two additional single-phase, line-to-ground tests were made at 3,500 amperes, with an average arcing time of 2½ cycles, giving an interrupting value of 12,000 volts per inch. Tests made on the 66-kv. system of the Union Gas & Electric Company at Cincinnati, Ohio in June 1930, hitherto unpublished, show 9 three-phase tests at currents from 1,200 to 6,500 amperes, with an average of 2½ cycles of arcing, giving an interrupting value of 7,000 volts per inch of total arc length.

Magnetic oscillographic equipment only was used on these tests and definite quantitative values for rate of rise of recovery voltages cannot be determined from the records. They were all large systems, however, with many ramifications, and were among the leaders in their respective voltage classes so that the tests may be accepted as representing average conditions to be found in operating service. It will be noted that the average performance of the deion grid circuit breakers on all of these tests ranged from 7,000 to 8,000 volts per inch with a grand average of 7,500 volts per inch of total arc length as compared with a performance of 4,050 volts per inch on the 140-kv. tests with the oil-blast circuit breaker under average circuit conditions, when the full length of arc drawn is taken into account, and a performance of 5,800 volts per inch under the best circuit conditons tested.

Philip Sporn: The paper by Messrs. Prince and Skeats brings out forcefully again, as was brought out some time ago by Dr. Slepian and his associates, how by proper study and research an apparently undesirable characteristic of engineering material can be overcome and turned to advantage. In this case the natural tendency of transil oil to gasify under an arc has been made the means of forcing a supply of fresh oil into the arc path, thus reducing both the time and the energy of the arcing period. The evolution of gas is thus in effect made the means of restoring or regenerating the dielectric strength of the oil in the arc path. In theory it is stated, this is quite different from the deionizing process taking place in the slot of the deion grid. It may, however, be that one general theory could be made to explain both principles. The fact that the arcing periods have been brought down to approximately the same time by the two methods would tend to lend plausibility to this idea.

J. B. MacNeill: Messrs. Prince and Skeats have given an interesting description of the oil-blast circuit breaker. Both the manufacturers and the operators involved are to be congratulated on the tests made on this breaker and the amount of power procured on short circuit. The idea of using tubular electrodes through which hot gases may escape has been given considerable attention in the past, and the following quotation from a patent specification written in 1926 shows the general idea as previously worked on.

> "We are aware that it has been proposed heretofore to employ tubular electrodes through which hot gases emanating from the arc may escape; to enclose these electrodes in a gas-tight chamber; and to initiate compression of gas in such chamber prior to separation of the electrodes to break the circuit of which they form a part."

Some of the ideas advanced seem at variance with well-established facts concerning arc behavior. For instance, pictures given show the arc as forming between the contacts (see Fig. 4 of the Prince and Skeats paper) concentrically all around the periphery of the arcing tip and in such a way that oil or gas passing down the hollow electrode will pass through the arc stream. This does not happen, but on the other hand, the arc will form a crater at some point on the arc tip which will move about but will never permit the arc to envelop the tip. What actually will occur is an arc holding to one spot on the tip momentarily with gas and oil rushing past it partly into the hollow electrode and partly out of the explosion pot throat through the regular contact clearance. The effect is entirely similar to the old-fashioned explosion pot action, except that the relation of arc gas and oil to the crater on the moving contact is more intimate.

The claim for very high voltages ruptured per inch of arc length given in this paper is not borne out by the results of actual tests given in the Spurck and Strang paper. The claims seem to be based on the inadmissible assumption that the arcs between contacts *A* and *B* (Fig. 6 of Prince and Skeats paper) can be disregarded. At any rate, one finds no data to substantiate claims for 44,000 volts per inch of arc and it appears that a reasonable average is more nearly 4,000 volts per inch.

Messrs. Prince and Skeats wave aside the older forms of breakers, "The plain break is simply set aside with all its vagaries and mysteries." But still the new device they propose as shown in Fig. 9 of the Spurck and Strang paper gives arc lengths on the same voltage and system location varying all the way from 2.5 in. to 14.5 in. The device, therefore, does not give the consistent operation under all conditions which is claimed for it in the papers.

Incidentally, the Prince and Skeats paper predicates arc rupture on action within the explosion pot, or at the worst when the arc tip is leaving the pot. Reference to Fig. 6 of the Spurck and Strang paper, however, shows that the average arc length at rupture gives the arc tip well outside the pot and in the worst case of 14.5 in. arc length between contacts *B* and *C* the greater part of the arc is outside the pot. What becomes of the theory of arc interruption given in such cases?

It is pointed out by Messrs. Sporn and St. Clair that an explosion pot was ruptured probably on the 12th or 13th shot, but that the test was continued through a series of 28 shots. The remarkable point is that on the shots succeeding the chamber destruction the breaker continued to give performance on the same rate of recovery voltage rather better than on the earlier shots when the explosion pot was intact. Even on the 600-volt per microsecond circuit on succeeding shots of the same series the operation with the broken pot is comparable to that on the early shots made with the less severe circuit. The longer arc time on the 2,400 volt per microsecond is naturally attributed to the relatively severe circuit.

On page 512, Prince and Skeats say, "the arc energies in the tests have been too low to measure satisfactorily from oscillograms." This is certainly not because of low arc energy. For illustration see Fig. 7 of Mr. Spurck's paper, showing the operation of the *G* oil-blast breaker, in which the arc voltage is easily 8,000 volts on the cycle preceding rupture. Surely arc voltages of this nature indicate large arc energy and are well within the accuracy limits of ordinary instruments.

One is inclined to question the desirability of the extra breaks introduced within the explosion pots and shown as *A*-*B* on page 508 of the Prince-Skeats paper. Two more contact surfaces exceedingly inaccessible for inspection and maintenance are introduced and the number of replacement parts is increased. Furthermore, all these contacts are of the "butt" type with no relative motion between them while in contact and, therefore, subject to welding when closed on heavy currents. We must assume that the authors have found the complication necessary even though the discussion in the papers does not make the necessity of it quite clear.

In conclusion, I wish to congratulate the authors on having made improvements in explosion pot breakers; the fact remains, however, that the results obtained still permit arcing outside the pots and, therefore, in some cases do not meet the claims that have been made for years even on old constructions.

J. N. Mahoney: The several papers on this general subject, indicate that the general trend of development of devices for improving the process of a-c. circuit interruption, follows the methods investigated, tried out, or adopted in connection with high power and moderate voltage d-c. circuit interruption. This is a natural trend as the problem of interrupting a d-c. circuit of equivalent power and voltage is more difficult than alternating current because the d-c. power does not cyclically pass through zero as in the individual legs of any a-c. circuit.

The third sentence first paragraph of the paper by Messrs. Prince and Skeats, states that "One of the first important facts brought to light by these studies was that an unconfined arc in oil is extremely erratic in length, duration and amount of gas formed." It has been the writer's experience that an unconfined or undirected arc, in either air or oil of either direct or alternating current of any considerable power, is likely to be extremely erratic and out of definite control both as to the problems of insulation and interruption.

The whole problem is to efficiently confine and direct the arc while limiting its volume and effects. All of the schemes described in the several papers and discussed herewith have these general objects in view.

The use of alternating current and oil immersion reduced the dimensions of switching apparatus in a ratio of at least ten to one, compared with the use of direct current in air, at the higher transmission voltages.

Many of the schemes suggested or tried out for d-c. circuit interruption in the last 40 years, and used mostly in special or experimental applications because of the relatively moderate voltage of present d-c. circuits, are now being brought forward in various combinations for the higher powered and higher voltage a-c. circuit interruption.

One of the oldest arc directing means is the magnetic blow-out, inherent or applied. This was supplemented by the so-called "arc chute," for confining the arc, and later by "arc horns," for directing the arc, and arc splitting plates, both conducting and non-conducting, for the purpose of reducing the arc gas volume and effects.

One of the design defects of an otherwise very good and much used line of d-c. railway power plant breakers, was in having the blow-out coils in shunt to the main contacts and no provision for clearing an arc starting on the main contacts, because of the blow-out coil circuit impedance.

This caused the industry to adopt the so-called "carbon circuit breaker" for heavy current moderate voltage service largely because of the superior arrangement of the main, auxiliary, and arcing contacts which caused the arc to travel upward away from all contacts by means of the inherent magnetic and expulsion blow-out action.

Modern air type breaker designs combine the structural arrangements of the carbon type with powerful series type magnetic blow-out and liberal arc chutes containing energy reducing and absorbing means to quickly quench the arc. Some of the lower voltage forms include a large number of arcing circuit gaps to utilize what we used to call the "inherent arc counter electromotive force" of the several series gaps as indicated by arc stream exploration.

About 20 years ago in certain 5,000-volt d-c. railway experimental work we tried out the water-break type of breaker with good results. Later in circuit interrupter development for auxiliary circuits on 3,000-volt railway equipment, a multiple metal-screen enclosure for directing and condensing the arc gases, similar in principle to that used in the Davy miners lamp, was found to be effective in confining an arc in air. This device was practically the same as that shown in Fig. 5 of Mr. Torok's paper—*Experimental Lightning Protector for Insulators*.

A successful application of the air-blast principle of circuit interruption has been used on the pressure governor switches of electric railway air compressors for the past twenty years. It was found necessary to have an appreciably persistent blast of air to prevent arc re-establishment on inductive circuits.

An old application of the carbon dioxide blast circuit interrupter utilized a small metal container of the compressed gas as an arcing contact. These containers were automatically replaced for a limited number of circuit interruptions.

With reference to the paper by Messrs. Spurck and Strang, my experience indicates that both the rate and magnitude of the recovery voltage rise as well as the magnitude of the current determine circuit interrupter duty in oil, air, or other atmosphere on either alternating or direct current.

One of the most severe field test setups of my experience was clearing a short circuit made at the load end of several underground cables in parallel with a breaker cut in between the bus and the battery of cables. Ordinarily a breaker is only required to handle a short circuit through one of these cables but in this setup the entire output of the bus was temporarily carried by this test breaker. The current magnitude and rate of voltage rise were both high, the rate approaching that of a simple d-c. circuit containing resistance only.

W. F. Skeats: The first three illustrations are for the conditions of Fig. 1, in Messrs. Spurck and Strang's paper. The first one, Fig. 1, is the recovery voltage characteristic for the last phase to clear of a three-phase grounded short circuit at the full capacity of the station and with three lines, varying from 62 or 63 to about 80 miles in length, connected to the bus. The timing of the phenomena is correct, but the calculated amplitude is a little greater than that shown by the cathode-ray oscillogram, a part of the discrepancy being accounted for by the decrement of the short-circuit current up to the time of interrruption.

Fig. 2 shows a comparison for the same conditions except that only about 65 per cent of the station generating capacity was used, and only one line, twenty-two miles long, was connected to the bus. The calculation was not carried any further, because the line had a tap which made the figuring very complicated, due to reflections. The check, so far as it goes, is good, however.

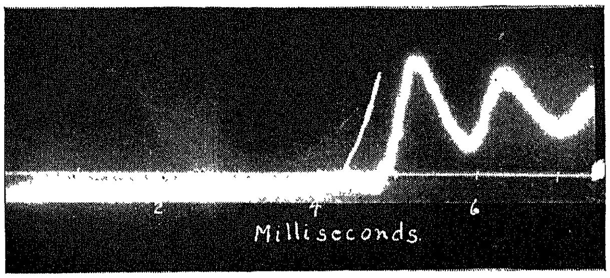

Fig. 2—Recovery Voltage Characteristic for Last Phase to Clear of a Grounded Short Circuit with One Line Connected. 730,000-Kva. Short Circuit

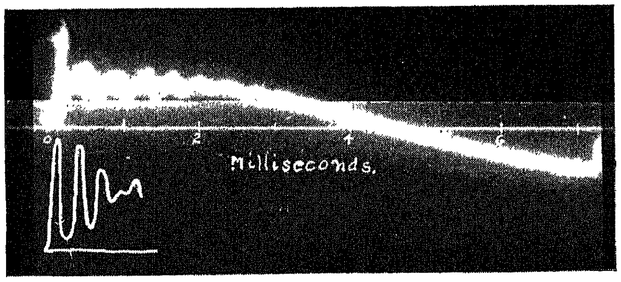

Fig. 4—Recovery Voltage Characteristic for Second Phase to Clear of a Grounded Short Circuit with No Lines Connected. 860,000-Kva. Short Circuit

Fig. 3 is for the same generating capacity as in Fig. 2, but with no transmission line connected to the bus. This curve is a composite of two oscillations at somewhat different frequencies. The calculated frequencies and amplitudes are approximately the same as those of the test curve, although a slight difference in the ratio of the two frequencies and probably a difference in the rate of decay of the two oscillations in the cathode-ray oscillogram causes the curves to have a slightly different appearance.

Fig. 4 is for the case of the second phase to clear, the conditions otherwise being the same as for Fig. 3.

Oscillations appear at two different frequencies because of the mutual capacitance of the buses in these cases of the various phases. Only a single frequency appears in the case of the first phase to clear of a grounded short circuit or for any phase of an ungrounded short circuit.

Some surprise may be felt at the fact that the arc lengths on grounded short circuits were greater than the arc lengths on ungrounded short circuits, in spite of the fact that the magnitude of the recovery voltage peak is more than 50 per cent greater on the first phase to clear in the ungrounded case than for any phase in the grounded case. Three factors combined to cause this situation:

1. In the case of ungrounded short circuits, the recovery voltage is split, two-thirds being positive with respect to ground, and one-third negative, or vice versa, whereas in the case of grounded short circuits, on all phases, the entire recovery voltage has the same polarity with respect to ground.

2. The time required to reach the peak was somewhat less in the case of grounded than in the case of the ungrounded short circuits.

3. Each of the three poles of the breaker is attempting to clear its phase in the case of the first phase to clear of an ungrounded short circuit, whereas in the case of the last phase of a grounded short circuit, only one pole of the breaker is operating. While this does not make it any easier for the individual poles to clear in the ungrounded case, purely on the basis of probability, other things being equal, one of the three poles will usually clear before the one pole of the last phase to clear of the grounded case.

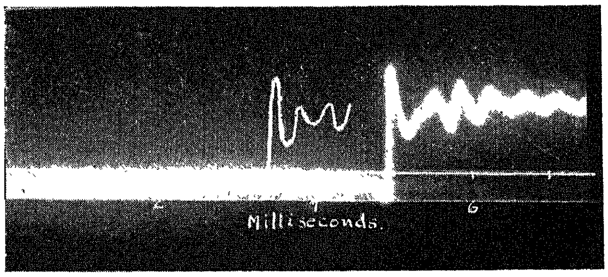

Fig. 3—Recovery Voltage Characteristic for Last Phase to Clear of a Grounded Short Circuit with No Lines Connected. 680,000-Kva. Short Circuit

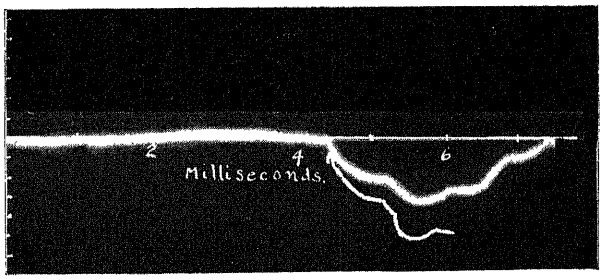

Fig. 1—Recovery Voltage Characteristic for Last Phase to Clear of a Grounded Short Circuit with Three Transmission Lines Connected. 1,030,000-Kva. Short Circuit

One of the principal sources of the demand for high-speed breakers has been the requirement for keeping the generating station in synchronism with its load. It is therefore of interest to system operators to know whether the longer or shorter arc durations will obtain when the question of stability is involved. Under such conditions, there will always be at least one line connected to the bus, so that recovery voltage conditions will tend to be lenient, and the shorter arc lengths may usually be expected.

Reviewing the test as a whole, there are several features which distinguish them from previous field tests on circuit breakers. These features are:

1. Field interrupting capacity tests have been made on a high capacity breaker designed for normal operation with the recovery voltage characteristic of the system under control.
2. Cathode-ray oscillograms have been recorded for at least one phase under each recovery voltage condition.
3. These oscillograms check calculated recovery voltage characteristics.
4. The breaker has been found to respond to recovery voltage manipulation substantially in the manner expected.

In view of these facts it is not too much to expect that the lack of confidence which has existed as to the significance of field tests at one location with regard to performance of the breaker at any other point on the same or another system may cease to worry system operators.

G. T. Tretjak: The development of the oil-blast circuit breaker marks a new advancing step in the technical solution of the problem of arc extinction in oil circuit breakers.

The test data given by D. C. Prince and W. F. Skeats and the tests described by K. M. Spurck and H. E. Strang yield an excellent proof of a considerable improvement in the oil circuit breaker by introducing the principle brought out in the papers.

However, the test data undeniably show (see Fig. 9 of the Spurck and Strang paper) that the arc extinction proceeds slightly otherwise than described by Messrs. D. C. Prince and W. F. Skeats, *viz.*, the arc in most cases is extinguished outside the explosion chamber, *i. e.*, at the time when the end of the hollow tubular contact has already left the limits of the chamber.

Taking into consideration that the most advantageous conditions for the arc extinction are created when the end of the tubular contact is in the throat of the chamber and the current passes its zero value I conceive that the height of the throat of the explosion chamber might be chosen so that at the given speeds in breaking the oil circuit breaker, the time of the passing of the hollow tubular contact end through the throat of the explosion chamber should not be less than half a cycle. Thus the passing of the current through zero would inevitably meet with the movement of the tubular contact end through the throat of the explosion chamber, and the length of the arc would not extend beyond the limits of the explosion chamber. It is very possible, that a 2- to 3-in. height of the throat would answer the purpose.

Philip Sporn: One of the principal motives behind our participation in the Philo tests was the hope that the tests would give us sufficient data to demonstrate the feasibility of the multiduty cycle. The present OCO, 2 minutes, OCO breaker duty cycle of the Institute is admittedly archaic and does not meet actual operating conditions or practise. Where it takes a good part of a day to empty the oil of a breaker to take a good look at it and then refill it with oil, it is a physical impossibility to inspect a breaker after two openings, even if the expense were not objectionable. The result is, therefore, that engineers buy a breaker guaranteed on a 2-shot duty cycle and then let it handle from 10 to 50 shots before inspecting it. It would seem time to bring Institute standards and service practise into closer conformity.

This was difficult at one time. The test experience with the deion grid breaker and with the oil-blast explosion breaker has removed most of these difficulties however. A standard duty cycle (on high-voltage outdoor breakers at any rate) calling for from 20 to 30 shots would seem in order and would be welcomed by every operating man.

I submit this for the careful consideration of the Institute members and of the Protective Devices Committee.

D. C. Prince: Regardless of the conclusion which anyone may reach concerning the relative merits of the various theories presented, the fact must not be lost sight of that the papers presented have described a new form of circuit breaker which has given an excellent account of itself, both in factory and in field tests.

Mr. R. H. Park has raised the question of whether the influence of displaced waves is being properly considered in the determination of circuit breaker duty. At the present time, the actual variation in pressures obtained is rather greater than the range of factors presented by Mr. Park so that it would be difficult to make a differentiation of the sort that he has indicated. We are proceeding to refine our test methods with the hope that we will be able to separate out the variables influencing pressure. If these endeavors are successful, a refinement may result in which such factors as mentioned by Mr. Park may be identified.

Dr. Joseph Slepian has presented a large number of both general and specific criticisms of our papers. The general criticisms may be summed up by saying that our theory of the interruption of an arc under oil materially differs from his own and therefore tends to detract from the simple picture which would be obtained were only one theory in the field. We believe that we have quantitative proofs substantiating the accuracy of our theory and therefore submit that if this quantitative theory be substituted for the qualitative theories heretofore advanced, the resultant picture will be more clear and precise than the one which he has presented.

I will endeavor to take up in order the specific criticisms which have been raised. If any are overlooked or slighted it will be through inadvertence as it is believed that adequate and straightforward answers can be made to all the points raised.

Fig. 4 has been criticised on the ground that the various views shown are at time intervals too great before and after the current zero. This was not our intention in drawing this figure. The third diagram should be interpreted to apply to time about 100 microseconds before the current zero and the last to a time say between 10 and 50 microseconds after the current zero. We are endeavoring to secure a photographic record of these events which will be published in the near future. Such records will, of course, settle this part of this question.

Regarding the requirement that the film of oil thrown across the arc space in the first few microseconds must be uniform, it will be apparent with a moment's thought that it need not be at all uniform but must simply increase at the weakest point at a rate to provide dielectric strength sufficient to withstand the recovering voltage. The form of recovering voltage is such that the rate of rise is not large in the first few microseconds so that the initial formation of film is not required to be at as great a rate as would be indicated by the average rate of voltage rise. The oil blast velocities required to provide this growing film of oil are not such as to require "extraordinary hydromechanics" but are easily obtained with the pressure known to exist in these explosion chambers by the application of the well-known laws of hydraulics. For instance, a 132,000-volt system might have the abnormal recovery rate of 1,000 volts per microsecond requiring an oil velocity of 150 ft. per second corresponding to a pressure of 183 lb. A further explanation of this is given later in this discussion.

In the variation of arc length with circuit voltages cited against the validity of the theory which we have presented, there are several very good reasons why a greater arc length is obtained in a high-voltage circuit interruption than in a low-voltage interruption. Four of these are:

1. The oil film will not necessarily increase to ten times its original thickness in ten times the time.
2. At higher voltages, a thicker oil film is required in which the voltage distribution may be sufficiently uneven to overstress considerably a part of the dielectric although the average stress is below the breakdown point.
3. When a voltage wave having a steep front is impressed on an oil film, the puncture occurs at a value considerably higher than when a 60-cycle wave is used. As the time required to reach the crest voltage increases, the strength of the oil per

unit thickness falls progressively from the impulse value to the 60-cycle value. Even with the same rate of rise of voltage, the crest of a 150,000-volt wave will not be reached until ten times later than the crest of a 15,000-volt wave. During this lapse of time, the effective strength of the oil will have fallen much farther from the impulse value.

4. Recovery rates may run several times as high on high-voltage as on low-voltage circuits as illustrated by our own 1,200-volt per second maximum at 14,500 volts and 2,800 volt per microsecond at 110,000 volts.

With the lengthening of the arc, the effectiveness of the oil blast increases progressively. The pressure in the explosion chamber increases with time thus giving an increase in the oil velocity as the arc is lengthened. A given velocity of oil flow will produce correspondingly greater increase in the rate at which the oil film thickness grows. This will be apparent if we consider first an arc occupying a conical space 1 in. long and 1 in. in diameter. The oil must flow ½ in. from the circumference to reach the center and so establish 1 in. of dielectric between the contacts. If the arc is drawn to a length of 3 in., oil flowing ½ in. from the surface of the cone to the axis will establish an oil path 3 in. between contacts. Thus the oil-blast action is progressively more effective as the length increases and this increase of effectiveness is maintained even after the moving contact has passed through the explosion-chamber throat.

In using the cone figure, it must not be assumed that the gases surrounding the arc must assume a regular cone shape. There will be a good deal of solid oil inside the cone boundaries prior to the current zero but this does not detract from the usefulness of the figure in explaining what occurs.

Regarding the question as to whether the arc is columnar in structure, there are plenty of illustrations showing the arc both ways. Even when the arc drawn under oil appears diffused, the line of demarcation between the gas bubble and the oil refracts the light emitted by the arc considerably and may magnify the arc appearance. In addition to this, the carbon resulting from decomposition to the surface of the oil makes the bubble to some extent opaque and may prevent the actual arc structure from showing clearly in the illustration. There is nothing about the theory which would stand or fall depending on whether the arc is columnar or diffuse in structure.

All of the arguments which have been presented on the value of blasting gas through the arc stream apply with even greater force when oil is considered as the dielectric blasted across the arc stream. It is a matter of common knowledge that oil has a dielectric strength many times that of the gases formed by an arc under oil even when the latter are cooled. There appears to be no real evidence showing that the action of various forms of explosion chambers is due to gas effects while there is considerable evidence that it can be explained entirely by oil-blast effects.

The use of a hollow moving electrode is not new, having been introduced in a patent[1] granted in 1908. The authors have naturally not overlooked the possibility that such hollow contact rods might be nearly as effective as the preferred oil-blast arrangement. Tests were conducted in which the floating contacts *(h)* in the oil-blast structure were omitted while other conditions remained the same. The operation was markedly inferior. It has been General Electric practise for a number of years to use hollow contacts on explosion-chamber circuit breakers of certain ratings. Improvement is sometimes obtained in this way although it is not of the same order of magnitude as that obtained by the oil-blast explosion chamber finally developed.

A number of "loose and inaccurate statements" are then listed by Dr. Slepian. Regarding the rate of recovery of dielectric strength by gas, little has been published. The data given by Dr. Slepian in his paper, "Theory of the Deion Circuit Breaker,"[2] is referred back to Fig. 6 of his paper, "Extinction of an A-C. Arc."[3] The data of Fig. 6 was obtained in tests in which a very short arc between plate electrodes was studied under alternating-current conditions. It is well known that this configuration with the plates reasonably cool presents a pronounced rectifying effect. The ability of such a gap to withstand voltage in the reverse direction immediately after a current zero is undoubtedly due to this rectification effect and is not a measure of rate of recovery of dielectric strength by the gas. This accounts for the fact that in Fig. 1 of the latter paper, the curve of recovery does not commence at zero but instead at approximately 260 volts per gap. The slope of the curve thereafter may be a partial measure of the gas deionization. However, attention is directed to the fact that in 10 microseconds it has recovered less than 100 volts out of about 3,500 required for a completely deionized gap of this size. Such a rate of increase is too small to be of interest.

The use of the words "oil spray" seems to have confused Dr. Slepian. Anyone who has watched the play of an ordinary garden hose will recall that the water first emerges in a sheet which is only later broken up into discrete drops.

The question is raised as to where the gases go when displaced by a film of oil. Obviously they have to go only far enough to permit the interposition of a thin film of oil. After the circuit is once interrupted, the contacts move much farther apart and a great deal more oil is interposed so that there is no danger of the slowly moving oil bubbles causing any serious weakening of the dielectric strength.

Regarding the difficulty of introducing "solid" oil so as to actually break the continuity of the gas path, we have found that the methods which we have employed overcome this difficulty very acceptably. At the current zero, the cessation of gas generation allows the oil walls to close in and the pressure from the gap insures that the oil will effectively cut the continuity of the gas path between the lower contacts.

In his next to the last paragraph, Dr. Slepian reiterates his wonder that an oil film can break up the continuity of the gas path in such a short time. We can only submit that the evidence indicates that, remarkable as this may seem to Dr. Slepian, it nevertheless occurs.

In answer to the question, "Where to, in the few microseconds available?". The arc products need be swept only far enough away to permit interposition of a very thin oil film.

From the fact that the arc is most often extinguished in the throat of an explosion chamber, we see no reason to conclude that gas is the active medium rather than oil. We merely have Dr. Slepian's unsupported statement that it is gas and not oil, to which we take exception.

There may be some disturbance between the oil and the arc while current is high but as the current approaches zero, the rate of formation of gas also decreases rapidly so that just prior to the time current zero occurs, there is no very pronounced gas formation to prevent the interposition of oil in solid form.

It is not magic that the oil is unable to invade the gas space during the time of considerable current. Such oil as does invade the arc is gasified immediately and after the current zero, the oil coming in finds no energy to gasify it and therefore remains oil. This oil displaces gaseous arc products at first in only a thin sheet breaking the continuity of the gas path but this oil barrier quickly grows as the gaseous products escape by way of the hollow contact.

The first arc drawn in each explosion chamber in the 44-kv. tests referred to in our paper was approximately ¼ in. long and was ignored in arriving at the figure of 44,000 volts per inch of arc. In giving this figure, we had in mind a value which represented the effectiveness of the actual break. If the preliminary arc is included, the average values of volts per inch of break reduce from 44,000 volts to 29,000 volts. From a purely statistical point of view, however, this preliminary arc should have been included even though the authors are sure that the preliminary arc contributes practically nothing to the recovered dielectric strength of the breaker.

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[1] Patent No. 887,723. J. D. Hilliard, May 12, 1908.

[2] A. I. E. E. Trans., April 1929, p. 545.

[3] A. I. E. E. Trans., Oct. 1928, p. 1407.

Dr. Slepian's concluding question of whether and when are the arc products swept away is merely a repetition of similar questions which have gone before.

Mr. Wilcox has made some very natural mistakes in interpreting the data presented. In the tests at Philo the pressure forming gap was only ¾ in. The actual travel before this arc is drawn is ½ in. making a total of 1¼ in. which he apparently read as 2½ in. In the second place, arc lengths drawn from tests at 600 volts per microsecond rate of rise are not based on "average operating conditions." Such a rate of rise corresponds to but 21.6 per miles of line attached to the bus which is much less than will be present under average operating conditions and probably a great deal less than connected to breakers during tests which have been reported during the last year. For the full system tests which more nearly represent average operating conditions, the arc lengths were materially shorter.

In discussing the paper, Mr. MacNeill has been confused by the lack of differentiation in Fig. 13 between the actual arc stream and the gas evolved by it. The arc stream is shown as a narrow column passed from one point on the intermediate electrode to a point on the rim of the lower contact, just as Mr. MacNeill believes it should be. The gas bubble should undoubtedly be somewhat less regular than it has been shown and would probably be indented more on the side opposite to that being occupied by the actual arc stream. Such condition would obviously make easier rather than more difficult interpolation of solid oil barrier between the electrodes.

Fig. 10 in the Spurck and Strang paper referred to by Mr. MacNeill does not distinguish between the arc lengths obtained with the different rates of voltage recovery. In the full system tests the arc length varies from 2½ to 6.4 in. In the tests with maximum rate of voltage rise, the variation was from 6 to 14½ in. The performance at any given rate of recovery is therefore quite consistent and the comparison of 2½ in. and 14½ in. obviously unfair. The oil-blast action does not break down as the moving contact leaves the explosion chamber throat but rather increases in its effect.

In deducing from Table II in the Spurck and Strang paper that the explosion-chamber breaker was more effective with one chamber broken, Mr. MacNeill is mislead by the arrangement of the test data in the tables. The arrangement used was selected in order to group tests in the same rate of recovery voltage rise.

On the subject of extra breaks, it will be noted that all are of the butt type, are heavy and will withstand a large number of operations without serious burning. This is borne out by the number and severity of the tests performed without interruption at Philo.

In closing, the point which I would like to leave with you is that this paper is not presenting just one more oil circuit breaker theory. It is not merely a qualitative theory which has inspired improvements in circuit breaker structure although it certainly has done that. It is a quantitative theory for the oil circuit breaker which is suceptible of definite proof. The theory can be expressed by a simple mathematical relation. The rate at which voltage appears across the terminals of an oil circuit breaker after a current zero is subject to calculation.[4]

Let it be represented by $\frac{v}{t}$ volts per second. The dielectric strength of oil can be measured independently to be of the order of 30 kv. per 1/10 in. for a one minute high-potential test and perhaps 60 kv. per 1/10 in. for a momentary impulse. Let it be represented by $\frac{v}{1}$ *i. e.* volts per inch. Dividing the first by the second of these fractions we secure the relation

$$\frac{\frac{v}{t}}{\frac{v}{1}} = \frac{1}{t}$$

The fraction $\frac{1}{t}$ represents inches per second and is the velocity at which oil must be introduced into the space between the contacts in order to present a dielectric strength equal to the rate of increase of voltage across the gap. If the voltage rises at a higher rate than provided by this fraction, the gap will be broken down. If at a lower rate, the current will be successfully interrupted.

We have a quantitative proof of this proposition and are preparing to submit it at the earliest possible moment. (This proof has been published since the Institute meeting "Oil-Blast Breaker Theory Proved Experimentally," by D. C. Prince and E. J. Poitras, published in the February 28, 1931 issue of the *Electrical World*.

R. M. Spurck and H. E. Strang: The major part of the very interesting discussion that resulted from our paper divides itself logically into three subjects, *i. e.*, (1) volts per inch of break distance referred to by Messrs. MacNeill, Slepian, and Wilcox; (2) the interrupting characteristics of the plain and oil-blast explosion chamber referred to by Messrs. MacNeill, Slepian, Wilcox, Mahoney and Tretjak; (3) the effect of the new developments on the interrupting ratings and duty cycles of commercial oil circuit breakers mentioned by Messrs. Sporn and Treat.

Although volts per inch of arc has been used to compare the interrupting qualities of circuit breakers, it is not believed that too much emphasis should be placed on that figure alone as a criterion of breaker performance. In any breaker structure, the volts per inch of arc obtained is affected by many variables, among the most important of which are the magnitude and rate of recovery voltage rise and the speed of the moving contacts. As the rate of recovery voltage rise or the magnitude of recovery voltage increase, the volts per inch of arc decreases. Also, when other conditions remain the same, increasing the speed of the moving contacts will decrease the volts per inch of arc even though the actual arc duration may be shorter. Therefore, in order to obtain even general comparisons of the volts per inch of arc in the performance of a breaker, it is considered necessary to state the magnitude of rate of recovery voltage rise and also the speed of the moving contacts or its equivalent inches of arc and arc duration. Inasmuch as values of volts per inch of arc have been brought out in the discussion, it seems well to list herein the following tabulation showing the values obtained during the Philo tests on the oil-blast explosion chamber.

Recovery voltage rate volts per microsecond	Volts per inch of arc		
	Maximum	Average	Minimum
270	12,400	7,800	5,700
600	6,700	5,100	4,100
2,400	6,000	4,500	2,700

These values include the length of the auxiliary gap, which was ¾ in. and not 2½ in., even though the actual voltage drop across it is relatively small. Also, in order to permit discussion of the values submitted by Mr. Wilcox, the values given above are calculated on the assumption that the value of the recovery voltage is normal voltage. It is, however, forcibly emphasized that such an assumption is erroneous as will be seen from an examination of the cathode-ray oscillograms shown in Fig. 1 of our paper. A study of the oscillograms in that figure shows that the peak recovery voltage varied from about 170,000 volts at a recovery voltage rise rate of 270 volts per microsecond to 400,000 volts at a recovery voltage rise rate of 2,400 volts per microsecond. The speed of the moving contacts, as may readily be obtained from the tabulation in the paper, varied from 6 ft. to 18 ft. per second, depending on the severity of the duty on the breaker as indicated in general by the current interrupted and the rate of recovery voltage rise. In this connection, it is interesting to note that the two longest arc lengths obtained in the entire series of tests, namely 14.5 in. and 13.3 in., occurred at values of kva. interrupted of approximately 1,000,000 kva., at a recovery rate of 2,400 volts per microsecond and with a recovery voltage peak of about 400 kv. The speed of the moving contacts in these two tests was 18 and 15 ft. per second respectively, and the arc duration only 4.4 and 4.6 cycles respectively. In these two tests, the speed of the moving contacts was also greater than in any of the other tests.

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[4] *Circuit Breaker Recovery Voltages*, by R. H. Park and W. F. Skeats, A. I. E. E. TRANS., March 1931, p. 204.

The values of volts per inch of arc given by Mr. Wilcox do not give the speed of the moving contacts or the rate of rise or magnitude of the recovery voltage which makes it impossible to draw any conclusions from a comparison of his values with those obtained during the Philo tests. It may be assumed, however, that inasmuch as they were all system tests, the rate of recovery voltage rise would probably not exceed 150 or 200 volts per microsecond.

Referring to the comparison of the operation of the plain explosion chamber with that of the oil-blast explosion chamber, it is reasonable to expect that in the plain explosion chamber, the most favorable location of the moving contacts when the arc is broken is at or near the throat opening because it is not until the throat opening is approached that the blast of oil produced by the chamber becomes effective in contacting with the arc. There is also no reason to assume that the chamber is not an effective interrupting element even though the moving contact has moved out of the chamber throat. It is believed that the correctness of that statement will be obvious when it is realized that after the moving contact has left the throat opening without interrupting the circuit, the arc is playing through the throat opening to the contact inside the chamber. The arc inside the chamber is decomposing oil and forming gas which is forcing oil and gas out through the throat of the chamber. Therefore, when the current passes through zero, oil from the chamber displaces the gaseous arc stream in the throat opening and so introduces good insulation into the arc stream.

With the oil-blast explosion chamber, the hollow contact gives the effect of pushing the throat opening upward so that the oil blast produced by the auxiliary gap may become effective in contacting with the arc stream as soon as the second gap parts. It is our conception of the operation of this chamber that the oil and some gas flows from the chamber through the hollow contact rod to the lower part of the floating contact. As current zero approaches, however, the oil surrounding the arc gas bubble closes in, as explained by Mr. Prince, until a solid wall of oil is interposed between the moving and floating contacts.

Mr. Tretjak proposes moving the throat opening of the standard chamber upward several inches. Numerous schemes of that kind, including baffles, have been tried. Some of them have given improved performance but none have given the performance obtained with the oil-blast chamber which, as has been brought out above, gives something of the equivalent of a moving throat and does not require the end of the moving contact to come to the throat before the oil-blast is effective.

Commenting further on Dr. Slepian's comment on the improvement that might be obtained by omitting the primary gap in the oil-blast chamber, attention is directed to the fact that the butt contacts, the tests of which were reported in our paper, had hollow contact rods open on both ends. They did not give the results obtained with the oil-blast contacts.

Mr. MacNeill comments on the performance of the breaker with butt contacts after one explosion chamber on one pole had ruptured. The arrangement of the test data in the tables does not follow the actual sequence of testing due to our desire to group the tests in accordance with recovery voltage rise and progressive values of current interrupted. Also, it is not clear, as Messrs. Sporn and St. Clair stated, on exactly what test the chamber broke. It is significant, however, that even with one explosion chamber out of commission, the breaker performed well even under severe conditions of recovery voltage rise and high currents interrupted.

Mr. MacNeill questions the consistency of operation of the oil-blast explosion chamber as indicated by the values of arc lengths given. In this connection, attention is directed to the fact that the tests reported in reality cover three separate and distinct test conditions between which, as we have brought out, the severity of interrupting duty varies widely. For that reason, it is not considered proper to compare a minimum value taken from the series of least severity with a maximum value taken from the series of greatest severity. However, if values of arc duration—which in the final analysis are the values most pertinent in discussing rapid operation of breakers—are compared in the various groups, we have the following:

Recovery voltage rate volts per microsecond	Arc duration cycles		
	Average	Maximum	Minimum
270	2.6	3.0	1.6
600	3.7	4.7	2.7
2,400	4.1	5.0	2.7

We submit these as being consistent not only in their own group but relatively so over the range of conditions listed.

Messrs. Treat and Sporn have brought forward the suggestion that inasmuch as the interrupting performance of breakers had been greatly improved not only in the time of operation but also on the duty imposed on the breaker itself during interruption and the number of times that it would operate under high duty, the operation cycle now standardized might be liberalized. Such modifications are now under consideration.

Traveling Waves on Transmission Systems

BY L. V. BEWLEY*

Associate, A. I. E. E

Synopsis.—*The purpose of this paper is two-fold: First; to present the theory of traveling waves on multi-conductor systems, and second, to compile a brief compendium on the general subject of traveling waves on transmission systems. While the application of the multi-conductor theory is more laborious than that of the single-wire theory, yet it does not involve much greater complication, and it becomes necessary to go to the more general theory when mutual effects are important, as in the study of ground wires, or when discontinuities exist in paralleled circuits carrying traveling waves. The origin, shape, and general characteristics of traveling waves are discussed. The equivalent circuits of terminal equipment and the corresponding reflections and refractions from such junctions are given for a large number of cases. The methods of computing a multiplicity of successive reflections by means of lattices are described. The effect of line losses in equalizing the subsidence of traveling waves and on their attenuation and distortion is also discussed.*

* * * * *

ORIGIN OF TRAVELING WAVES

SURGES on transmission lines owe their origin to four different causes; induced from lightning discharges, direct lightning stroke, arcing grounds, and switching.

The maximum potential at the point of origin, and the crest of induced traveling waves are respectively:

$$V = \alpha G h$$

$$V' = \alpha' G h$$

where h is the height of the line conductor, G the field gradient, and α and α' factors depending on the law of cloud discharge and the distribution of bound charge.[1] For an instantaneous cloud discharge (an impossible assumption) $\alpha = 1.0$ and $\alpha' = 0.5$. (Fig. 1) As the time of cloud discharge increases both α and α' decrease, and rapidly approach equality; so that for a cloud discharge slower than 10 microseconds they are practically equal. The equation for α' based on the assumption of an exponential law of cloud discharge is:

$$\alpha' = \frac{1}{2}\left(1 - \epsilon^{-\frac{3x}{t}}\right)$$

where x is the length of the bound charge in thousands of feet, and t is the time of cloud discharge in microseconds. The length of the wave is:

$$L = x + t$$

and the front of the wave is x or t depending upon which is the smaller. It is evident that induced strokes become harmless for long periods of cloud discharge, and that high potential induced strokes are possible only with very short waves. For the accepted maximum value of $G = 100$ kv/ft., an average line height of $h = 60$ ft., a 3,000-ft. rectangular bound charge, and 10 microseconds for the cloud discharge, the maximum induced traveling wave that can occur on the line is:

$$V' = \alpha' G h = 0.30 \times 100 \times 60 = 1{,}800 \text{ kv.}$$

and this wave has a total length of only 13 microseconds. Had the time of cloud discharge been 30 microseconds then the induced voltage would have been only

$$V' = 0.13 \times 100 \times 60 = 780 \text{ kv.}$$

The induced voltages decrease in importance as the line insulation is increased, and the proportion of outages due to direct strokes increases. The induced voltages can be roughly halved by the use of one or more ground wires.

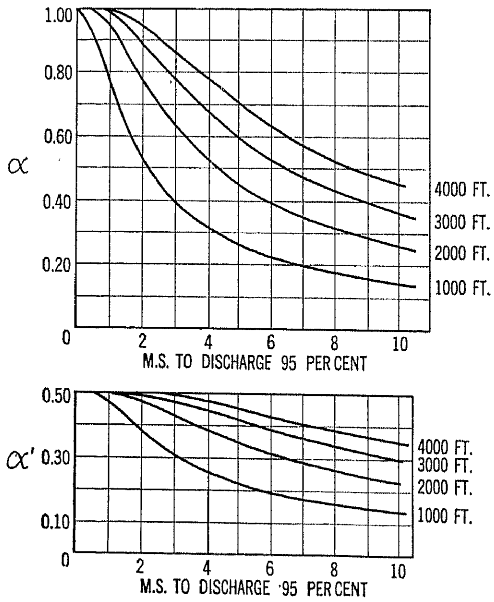

FIG. 1—REDUCTION FACTORS FOR INDUCED LIGHTNING POTENTIALS

The most feasible way of correlating the direct versus induced stroke arguments is through the simple theoretical relationships between the traveling waves originating from these two causes. It was found in a previous investigation[1] that the law of cloud discharge has a greater effect on the shape of the traveling wave than the distribution of bound charge, and therefore that the assumption of a rectangular bound charge may be made without involving any marked departure from the true shape of the traveling wave. Then the determination of the wave shape reduces to the problem

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[1] For references see Bibliography.

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of finding the law of cloud discharge. If the wave measured by the cathode-ray oscillograph station was due to a direct stroke, then the shape of the current wave in the lightning bolt is known. But if Q is the charge on the cloud, this current is

$$i = \frac{\partial Q}{\partial t} = Q_o \frac{\partial}{\partial t} F(t)$$

where Q_o is the total charge at the beginning of cloud discharge, and $F(t)$ is the law of cloud discharge. Therefore

$$F(t) = \frac{1}{Q_o} \int_0^t i \, dt$$

The actual numerical values of Q_o and i are unimportant. All that is necessary is to arbitrarily choose Q_o of such value that $F(t)$ reaches a final value of unity at the completion of cloud discharge. And the cloud discharge is complete when i has ceased. Suppose that the direct stroke wave has the typical shape, Fig. 3D,

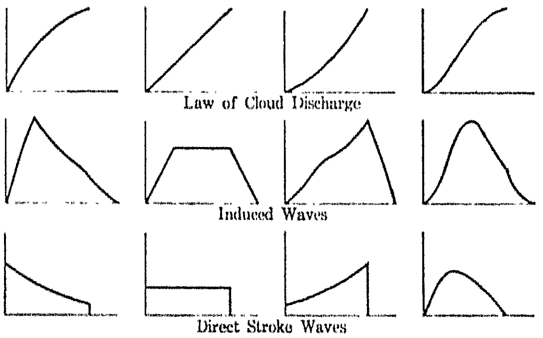


FIG. 2—INDUCED AND DIRECT STROKE WAVES CORRESPONDING TO DIFFERENT LAWS OF CLOUD DISCHARGE

$$i = I(\epsilon^{-at} - \epsilon^{-bt})$$

Then the law of cloud discharge is

$$F(t) = \frac{I}{Q_o} \int_0^t (\epsilon^{-at} - \epsilon^{-bt}) \, dt$$

$$= \frac{I}{Q_o} \left\{ \frac{1 - \epsilon^{-at}}{a} - \frac{1 - \epsilon^{-bt}}{b} \right\}$$

In Fig. 2 are shown a few simple direct stroke waves, and the corresponding laws of cloud discharge and induced waves. Thus if a cathode-ray oscillogram is definitely known to be that of either a direct or induced stroke, then the character of the other may be derived. Of course the length and shape of the bound charge distribution will influence the shape of the induced traveling wave, but not nearly to the same extent as the law of cloud discharge. The above expressions follow from the premise that the electrostatic field of the cloud collapses *uniformly*. Possibly there is considerable departure from this assumption. Nevertheless, the above equations do correlate the two types of lightning waves as near as our present knowledge of the mechanism of cloud discharge will permit.

The third cause of abnormal voltage oscillations on a transmission line is arcing grounds.[7] Theoretically, voltages as high as 7½ times normal line-to-neutral voltage are possible. But many assumptions underlie the present theories of arcing grounds. When the results arrived at by theory are judiciously shaded to compensate for the assumptions introduced to simplify the analysis, it is doubtful if an arcing ground can actually cause a voltage in excess of 5 times normal line-to-neutral voltage on an isolated neutral system. Arcing ground surges are oscillatory in nature, and consist of a normal frequency oscillation which gradually builds up to several times normal, and a superimposed high frequency oscillation, whose frequency depends chiefly upon the length of line between the station and the arcing ground.

Switching surges exhibit the same class of characteristics as arcing grounds, although they are more irregular in shape. Their proper consideration is outside the scope of this paper.

SHAPES AND SPECIFICATIONS OF TRAVELING WAVES

The principal shapes of most natural waves are included in Fig. 3, that is, may be represented by the difference of two exponentials, but of course actual waves are usually serrated by minor irregularities.

As far as mathematical simplicity is concerned the most simple wave to calculate the effects of, is the infinite rectangular, shown in Fig. 3A. Also, as a rule, such a wave is the most dangerous to terminal equipment, and therefore calculations based on it are apt to err on the side of safety. Still other reasons that have favored its use in analysis are that it is by far the easiest to study pictorially, and that until recently the actual shapes of lightning surges were not known. However, during the past few years a great many cathode-ray oscillograms of natural lightning waves have been obtained under many different conditions, so that fairly definite information as to their general shape and characteristics is available. It is therefore essential that calculations be made with these characteristic

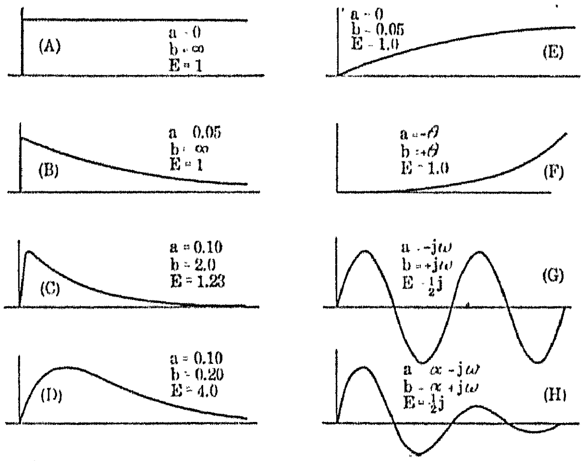


FIG. 3—EMPIRICAL WAVE SHAPES GIVEN BY $e = E(\epsilon^{-at} - \epsilon^{-bt})$.

lightning waves, in order that the influence of the fronts, tails, and lengths of the waves may be evaluated.

Three different methods for calculating waves of arbitrary shape are given in the following:

1. Express in operational notation, combine with the function representing the reflection or refraction operator, and solve the resulting operational equations.
2. Consider the wave as made up of a series of infinite rectangular waves, and add up the solutions corresponding to each component rectangular wave. This superposition may be done graphically as shown in Fig. 5 or mathematically by means of Duhamel's theorem.
3. Express the wave as the sum of a number of functions for which the individual solutions are known or can be found, and add these solutions. Fig. 4 illustrates a few simple examples.

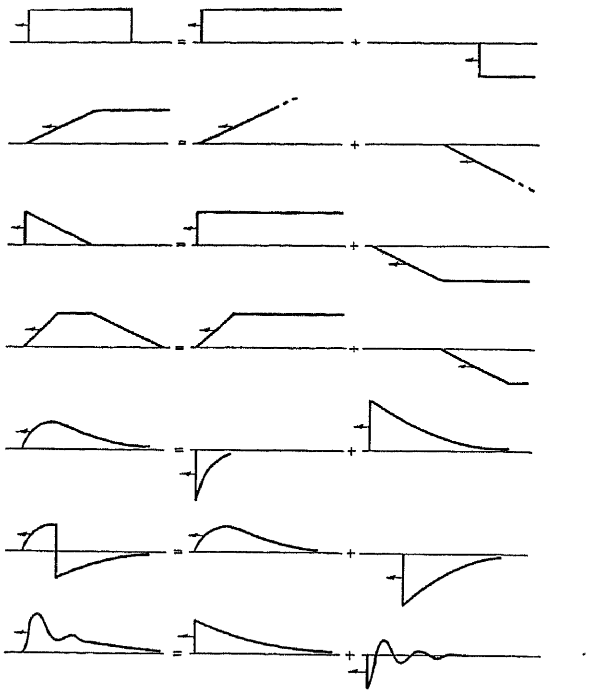


FIG. 4—COMPOUNDING OF SIMPLE WAVES TO OBTAIN COMPLEX WAVES

Which of these methods to use is entirely a matter of convenience in any specific case, each method having certain advantages and limitations. The difficulty usually encountered with the first method is that it complicates the operational equations so that a solution either cannot be obtained at all, or else only by the most laborious and complex process. As a rule, in dealing with the behavior of traveling waves at a transition point, it will be found that the reflection (or other) operator acting on the simple unit function (an infinite rectangular wave) is just about as complicated a proposition as can be handled with engineering expedience, and that any further complications in the operational equations are prohibitive of a solution. The application of Duhamel's theorem suffers from substantially the same defects, except that the trouble with it is in making a difficult integration. It is

$$f(t) = E(o) \cdot \phi(t) + \int_0^t \phi(\tau) \frac{\partial}{\partial t} E(t - \tau)\, d\tau$$

where

$\phi(t)$ = solution corresponding to unit function

$E(t)$ = applied wave of arbitrary shape

$f(t)$ = solution corresponding to $E(t)$

The graphical representation of a wave of arbitrary shape as a set of rectangular components, is of course only an approximation, but in a great many cases of engineering importance it is quite sufficient, and has the advantage of simplicity. In any event it can always be made as accurate as required, merely by subdividing the wave into a sufficiently large number of small rectangular components. In many cases the incident wave may be so complicated as to defy analytic expression; and then a graphical break-up into rectangular components is the only way out of the difficulty.

The third method, that of representing the wave as a sum of functions for which the solutions are known, is very powerful and practicable. There are a few simple functions for which the response of a network can usually be computed with reasonable ease, and by compounding such functions almost any desired wave shape can be reproduced to a good approximation. These elementary waves are:

a. Infinite rectangular
b. Simple exponential
c. Uniformly rising front
d. Damped sinusoid
e. Difference of two exponentials

As a matter of fact, by a suitable choice of the parameters in the difference of two exponentials, all of these elementary waves may be considered as special cases of *e*, as illustrated in Fig. 3. It so happens that the solution corresponding to

$$e_1 = E_1 \epsilon^{-at}$$

is easily effected by means of Heaviside's shifting theorem.

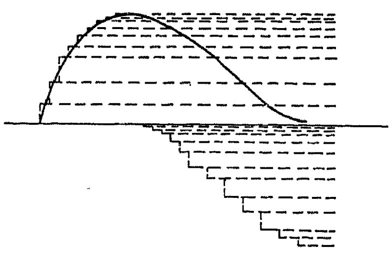


FIG. 5—APPROXIMATION BY RECTANGULAR COMPONENTS

$$f(p)\, \epsilon^{-at} = \epsilon^{-at} f(p - a)$$

and in many cases with no greater mathematical difficulty than attains with the unit function alone.

The references should be consulted for more details concerning the specification of traveling waves.[1,2,6]

ATTENUATION OF TRAVELING WAVES

It is usually justifiable to calculate traveling waves on the assumption of no losses, and then to compensate for the attenuation by an exponential decrement factor, arrived at experimentally.

Corona is the chief factor in causing attenuation and distortion. It levels off the top and elongates the wave, but its effect varies with the weather and other conditions, so that there is no such thing as a definite attenuation on a given line.

The effect of the line losses on traveling waves are three-fold: (1), the waves of voltage and current are attenuated, (2), the shapes are distorted with time, (3), the current and voltage waves depart from exact similarity, so that they are no longer connected by the simple linear proportionality factors called the surge impedances.

Fig. 8 shows the effect of attenuation in the charging of an open-ended line from an infinite voltage source. Without losses, the cycle of oscillations repeats indefinitely, but when line losses are present the oscillations gradually diminish until the line eventually reaches a steady state condition. However, a distortionless line can never become fully charged to the terminal potential, throughout its length, for the distortionless feature requires the presence of both leakage conductance and series resistance. The flow of current in the leakage conductance results in a progressive voltage drop along the line. Therefore, the ultimate level charging of a line requires that there be no leakage currents. Referring to Fig. 8, which has been drawn for a linear rather than exponential attenuation, the voltage at various instances of time for an attenuation $(1-\alpha)$ per trip is

Sending end	Units of time	Receiving end
1	0	0
1	1	α
$1+\alpha^2$	2	2α
1	3	$2\alpha-\alpha^3$
$1-\alpha^4$	4	$2\alpha-2\alpha^3$
1	5	$2\alpha-2\alpha^3+\alpha^5$
$1+\alpha^6$	6	$2\alpha-2\alpha^3+2\alpha^5$
1	7	$2\alpha-2\alpha^3+2\alpha^5-\alpha^7$
$1-\alpha^8$	8	$2\alpha-2\alpha^3+2\alpha^5-2\alpha^7$
etc.		etc.

The voltage at the receiving end after 4 n units of time is

$$e = 2(\alpha-\alpha^3+\alpha^5-\alpha^7+\ldots\ldots)$$
$$= 2(\alpha-\alpha^3)(1+\alpha^4+\alpha^8+\ldots\ldots)$$
$$= 2(\alpha)(1-\alpha^2)\sum_{0}^{n}\alpha^{4r} = 2\alpha\,\frac{1-\alpha^{4(n+1)}}{1+\alpha^2}$$

After an infinite number of oscillations

$$e = \frac{2\alpha}{1+\alpha^2}$$

Thus if the attenuation is $(1-\alpha) = 0.5$ as in Fig. 8, and the line is distortionless, the open end of the line finally stabilizes at a voltage of

$$e = \frac{2\times 0.5}{1+0.25} = 0.8$$

GENERAL PROPERTIES OF FREE TRAVELING WAVES

In Appendix I the general differential equations for potentials and currents of a multi-conductor transmission line, (Fig. 6), are formulated. Each conductor, (Fig. 7), is assumed to have its own resistance to ground and to each of the other conductors, self and mutual inductance between all wires, and leakage conductance from each wire to earth and to all the other wires. For an n-wire system there results a set of n simultaneous partial differential equations of the second order in both the time and distance derivatives. When these n equations are solved for any potential there results a linear partial differential equation with constant coefficients of the 2 n order in both the time and distance derivatives. This general differential equation is given as a determinate whose formal expansion, according to algebraic rules, yields the polynominal form. The differential equation and its coefficients is identical for the potential on any wire of the n-wire system; but the boundary conditions, and therefore the integration constants of the solution, may be different for each wire. The general linear partial differential equation with constant coefficients has a formal solution, but it has not been included in this paper (see "Treatise on Differential Equations" by Forsyth). But the solutions are given corresponding to the following special cases:

The No-Loss Line
The Completely Transposed Line
Solution for Alternating Currents

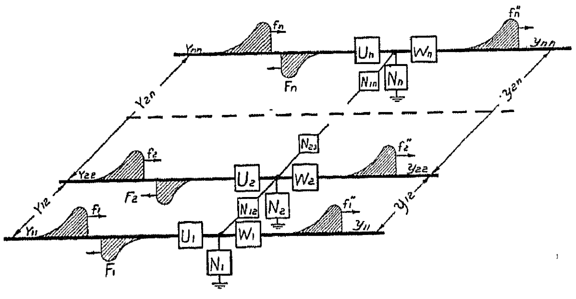


FIG. 6—GENERAL MULTI-CONDUCTOR SYSTEM

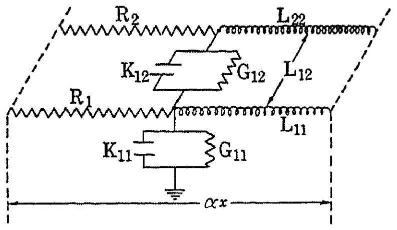


FIG. 7—CIRCUIT CONSTANTS OF MUTUALLY COUPLED CIRCUITS

If a line is free from losses the differential equations of the system are satisfied by pure wave functions. These waves travel without attenuation or distortion until a transition point on the circuit is reached, where they reflect and refract in accordance with the general laws given in Appendix II. In an n-wire no-loss system there are n possible velocities of wave propagation, but in the case of overhead conductors these n velocities all become equal to the velocity of light.

If the line is completely transposed, so that every conductor occupies the same relative position as every other conductor, and for the same distance, then the possible average velocities between terminals are reduced to two in number. If the line is made up of g groups, and the conductors of each group are completely transposed with respect to their own group, then there are $(g + 1)$ possible average velocities.

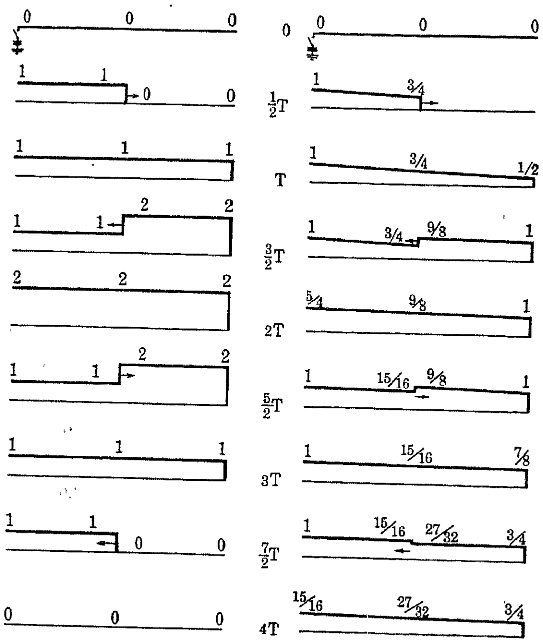


FIG. 8—EFFECT OF ATTENUATION ON CHARGING A LINE

The conventional method of calculating the steady state alternating-current behavior of a transmission line by using "constants to neutral" is a very special case, but is rigorous for a single-circuit, three-phase, completely transposed line. To obtain these simplified equations directly from the general alternating current solution is a matter of considerable manipulation. The reduction is carried out in detail in Appendix I.

In Appendix I it is shown that in an n-wire circuit the energy of free traveling waves is equally divided between the electrostatic and the electromagnetic fields. However, when these waves reach a transition point, or when waves traveling in opposite directions pass through one another, the energy balance is upset. Whether the energy resides mostly in one field or the other during a transition period depends entirely upon the nature of the transition.

Pure traveling waves of voltage and current on an n-wire system, are related to each other by a set of simultaneous linear equations having constant coefficients called the self and mutual surge impedances of the line. These equations[3] are

$$\begin{aligned} \pm e_1 &= z_{11} i_1 + z_{21} i_2 + \dots\dots\dots\dots + z_{n1} i_n \\ \pm e_2 &= z_{12} i_1 + z_{22} i_2 + \dots\dots\dots\dots + z_{n2} i_n \\ &\dots\dots\dots\dots\dots\dots\dots\dots \\ \pm e_n &= z_{1n} i_1 + z_{2n} i_2 + \dots\dots\dots\dots + z_{nn} i_n \end{aligned}$$

where the (+) sign is used for waves traveling in the forward direction, and the (−) sign for waves moving in the reverse direction. Also

$$z_{rr} = 60 \log \left(\frac{2h}{\rho} \right) = \text{self surge impedance of conductor } r$$

$$z_{rs} = 60 \log \left(\frac{a}{b} \right) = \text{mutual surge impedance between } r \text{ and } s$$

h = height of r above ground plane
a = distance between r and image of s
b = distance between r and s
ρ = radius of conductor

These equations, in conjunction with the principle of superposition and the conditions of voltage and current continuity at a junction or transition point, define the behavior of pure traveling waves on transmission systems. The principle of superposition is involved in calculating the effect of more than one independent wave at the same point simultaneously, such as occur for successive reflections, meeting of circuits, etc.

Instead of writing the voltages in terms of the currents and the surge impedances, the arrangement may be reversed and the currents written in terms of the voltages and surge admittances as follows:

$$\begin{aligned} \pm i_1 &= y_{11} e_1 + y_{21} e_2 + \dots\dots\dots + y_{n1} e_n \\ \pm i_2 &= y_{12} e_1 + y_{22} e_2 + \dots\dots\dots + y_{n2} e_n \\ &\dots\dots\dots\dots\dots\dots\dots\dots \\ \pm i_n &= y_{1n} e_1 + y_{2n} e_2 + \dots\dots\dots + y_{nn} e_n \end{aligned}$$

If

$$D = \begin{vmatrix} z_{11} & z_{21} & \dots\dots & z_{n1} \\ z_{12} & z_{22} & \dots\dots & z_{n2} \\ \dots & \dots & \dots\dots & \dots \\ z_{1n} & z_{2n} & \dots\dots & z_{nn} \end{vmatrix}$$

and

$$D' = \begin{vmatrix} y_{11} & y_{21} & \dots\dots & y_{n1} \\ y_{12} & y_{22} & \dots\dots & y_{n2} \\ \dots & \dots & \dots\dots & \dots \\ y_{1n} & y_{2n} & \dots\dots & y_{nn} \end{vmatrix}$$

then the y's and z's are related to each other as

$$y_{rs} = \frac{(\text{minor of } D \text{ for which the cofactor is } z_{rs})}{D}$$

$$z_{rs} = \frac{(\text{minor of } D' \text{ for which the cofactor is } y_{rs})}{D'}$$

For instance, in the case of two conductors

$$z_{12} = \left| \frac{y_{21}}{\begin{matrix} y_{11} & y_{21} \\ y_{12} & y_{22} \end{matrix}} \right| = \frac{-y_{21}}{y_{11}\, y_{22} - y_{12}\, y_{21}}$$

$$z_{11} = \left| \frac{y_{22}}{\begin{matrix} y_{11} & y_{21} \\ y_{12} & y_{22} \end{matrix}} \right| = \frac{y_{22}}{y_{11}\, y_{22} - y_{12}\, y_{21}}$$

TRAVELING WAVES ON A SINGLE-CONDUCTOR CIRCUIT

In the majority of problems dealing with traveling waves, it is sufficient to make the calculations on the basis of a single-wire circuit. The return part of the circuit may be either a second conductor or the ground.

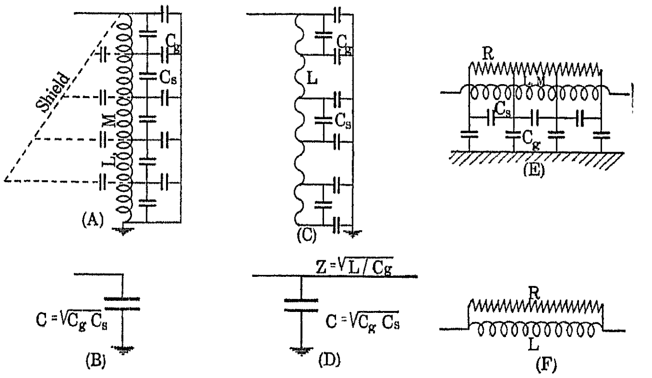


FIG. 9—EQUIVALENT CIRCUITS OF TERMINAL APPARATUS TO LIGHTNING SURGES

(A) Transformer, ideal
(B) Transformer, approx.
(C) Rotating machine, ideal
(D) Rotating machine, approx.
(E) Reactor with shunt resistor, ideal
(F) Reactor with shunt resistor, approx.

When the single-circuit theory is applicable, the potential and current waves are proportional to each other by the surge impedance of the circuit;

$e\,(x - v\,t) = Z\,i\,(x - v\,t)$ for forward moving waves

$e\,(x + v\,t) = -\,Z\,i\,(x + v\,t)$ for backward moving waves

where

$Z = \sqrt{L/C}$ = surge impedance in ohms
$Y = 1/Z$ = surge admittance in mhos
L = inductance in henrys
C = capacitance in farads

It is to be noticed that for waves traveling in the positive or forward direction that e and i have the same sign; but that for waves traveling in the reverse or negative direction they have opposite signs. However, it is immaterial which direction along the line be chosen as positive or forward; provided that the positive sense be strictly adhered to throughout the calculations.

The reflection and refraction operators, by equations (**18**) and (**19**) of Appendix II are respectively

$$\frac{Z_o - Z}{Z_o + Z} \text{ and } \left(\frac{1}{1 + N\,(W + z)}\right)\left(\frac{2z}{Z_o + Z}\right)$$

where Z_o is the total impedance, expressed in operational form, at the transition point as viewed from the approaching incident wave, z is the surge impedance of the line, and N and W are impedances having the same meaning as in Fig. 6. When these operators are applied to any incident wave e, expressed as a function of time t counted from the instant that e arrives at the transition point, they derive the reflected and transmitted waves e' and e'' respectively.

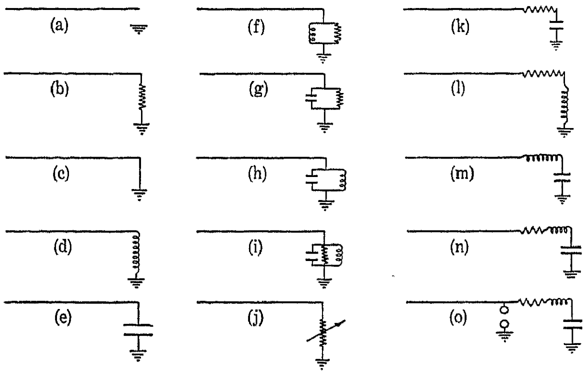


FIG. 10—TERMINAL CONDITIONS ON SINGLE-CONDUCTOR CIRCUITS

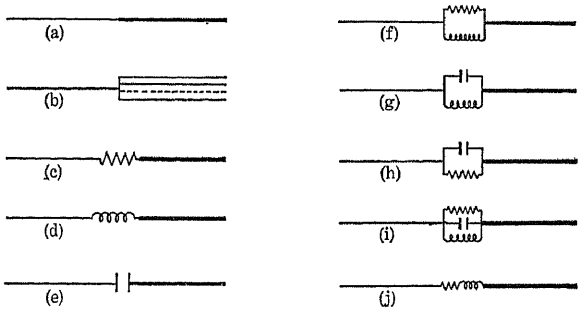


FIG. 11—JUNCTIONS BETWEEN SINGLE-CONDUCTOR CIRCUITS

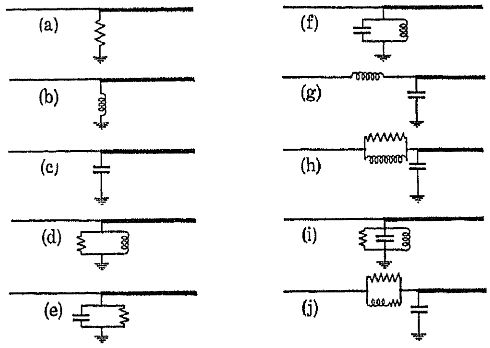


FIG. 12—JUNCTIONS BETWEEN SINGLE-CONDUCTOR CIRCUITS

The solutions and graphs of most of the thirty-five circuits shown in Figs. 10, 11, and 12 have been given in previous papers but the equations will be repeated here for ready reference. By a fortunate coincidence, nearly all of these circuits are included by two general equations, through an adjustment of the coefficients thereof. Therefore, all that is necessary here is to give these equations, and a table with the proper coefficients for each case. The equations are

$$A \left[\frac{a + \alpha}{a - \beta} \, \epsilon^{-at} - \frac{\alpha + \beta}{a - \beta} \, \epsilon^{-\beta t} \right] E \qquad (1)$$

$$A \left[\frac{\omega_o^2 - 2 \, a \, \alpha + a^2}{\omega_o^2 - 2 \, a \, \beta + a^2} \, \epsilon^{-at} + \frac{2 \, (\alpha - \beta) \, \epsilon^{-\beta t}}{\omega \, (\omega_o^2 - 2 \, a \, \beta + a^2)} \{ (\omega_o^2 - a \, \beta) \sin \omega \, t + a \, \omega \cos \omega \, t \} \right] E \qquad (2)$$

$$\omega_o^2 = 1/L\,C \text{ and } \omega^2 = (\omega_o^2 - \beta^2), \; e = E \, \epsilon^{-at}$$

In the following tables the reflected wave e' and the transmitted wave e'' are given. In all cases the total voltage at the transition point is

$$e_o = e + e'$$

The reflected and transmitted current waves are respectively

$$i' = - \frac{e'}{z_1} \text{ and } i'' = \frac{e''}{z_2}$$

and the total current at the transition point is

$$i_o = i + i'$$

TABLE I

Fig.	Equation	α	β	A
10-a (1)*	$e' = e$			
10-b (1)	$e' = \frac{R - Z}{R + Z} \, e$			
10-c (1)	$e' = - e$			
10-d (1)	e' = (I.)	$\frac{Z}{L}$	$\frac{Z}{L}$	1
10-e (1)	e' = (I.)	$\frac{1}{C\,Z}$	$\frac{1}{C\,Z}$	− 1
10-f (1)	e' = (I.)	$\frac{Z\,R}{L\,(R - Z)}$	$\frac{Z\,R}{L\,(R + Z)}$	$\frac{R - Z}{R + Z}$
10-g (1)	e' = (I.)	$\frac{R - Z}{Z\,R\,C}$	$\frac{R + Z}{Z\,R\,C}$	−1
10-h (1)	e' = (II.)	$- \frac{1}{2\,C\,Z}$	$\frac{1}{2\,C\,Z}$	−1
10-i (1)	e' = (II.)	$\frac{Z - R}{2\,R\,C\,Z}$	$\frac{Z + R}{2\,R\,C\,Z}$	−1
10-j (10)	$e' = e - z\,i_0$			
10-k (6)	e' = (I.)	$\frac{1}{C\,(Z - R)}$	$\frac{1}{C\,(Z + R)}$	$\frac{R - Z}{R + Z}$
10-l	e' = (I.)	$\frac{Z - R}{L}$	$\frac{Z + R}{L}$	1
10-m (5)	e' = (II.)	$\frac{- Z}{2\,L}$	$\frac{Z}{2\,L}$	1
10-n (5) (6)	e' = (II.)	$\frac{R - Z}{2\,L}$	$\frac{R + Z}{2\,L}$	1
10-o (5)	Same as 10-n before gap sparkover Same as 10-c after sparkover			

*See Bibliography references.

TABLE II

Fig.	Equation	α	β	A
11-a (1)	$e' = \frac{Z_2 - Z_1}{Z_2 + Z_1} e$ $e'' = \frac{2 Z_2}{Z_2 + Z_1} e$			
11-b (1)	$e' = \frac{1 - Z_1 Y_0}{1 + Z_1 Y_0} e$ $e'' = \frac{2}{1 + Z_1 Y_0} e$	Y_0 = total admittance of all outgoing lines in parallel		
11-c (1)	$e' = \frac{Z_2 - Z_1 + R}{Z_2 + Z_1 + R} e$ $e'' = \frac{2 Z_2}{Z_2 + Z_1 + R} e$			
11-d (1)	$e' = $ (I.) $e'' = \frac{\alpha - \beta}{a - \beta} (\epsilon^{-at} - \epsilon^{-\beta t})$	$\frac{Z_1 - Z_2}{L}$ “	$\frac{Z_1 + Z_2}{L}$ “	1
11-e	$e' = $ (I.) $e'' = $ (I.)	$\frac{1}{C (Z_1 - Z_2)}$ 0	$\frac{1}{C (Z_1 + Z_2)}$ $\frac{1}{C (Z_1 + Z_2)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$ $\frac{2 Z_2}{Z_2 + Z_1}$
11-f	$e' = $ (I.) $e'' = $ (I.)	$\frac{R (Z_1 - Z_2)}{L (R - Z_1 + Z_2)}$ $-\frac{R Z_2}{L (R + Z_2)}$	$\frac{R (Z_1 + Z_2)}{L (R + Z_1 + Z_2)}$ $\frac{R (Z_1 + Z_2)}{L (R + Z_1 + Z_2)}$	$\frac{R + Z_2 - Z_1}{R + Z_2 + Z_1}$ $\frac{2 (Z_2 + R)}{R + Z_2 + Z_1}$
11-g	$e' = $ (II.) $e'' = $ (II.)	$\frac{1}{2 C (Z_2 - Z_1)}$ 0	$\frac{1}{2 C (Z_2 - Z_1)}$ $\frac{1}{2 C (Z_2 + Z_1)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$ $\frac{2 Z_2}{Z_2 + Z_1}$
11-h	$e' = $ (I.) $e'' = $ (I.)	$\frac{R - Z_1 + Z_2}{R C (Z_1 - Z_2)}$ $-\frac{1}{R C}$	$\frac{R + Z_1 + Z_2}{R C (Z_1 + Z_2)}$ $\frac{R + Z_1 + Z_2}{R C (Z_1 + Z_2)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$ $\frac{2 Z_2}{Z_2 + Z_1}$
11-i (1)	$e' = $ (II.) $e'' = $ (II.)	$\frac{Z_2 - Z_1 + R}{2 R C (Z_2 - Z_1)}$ $\frac{1}{2 R C}$	$\frac{Z_2 + Z_1 + R}{2 R C (Z_2 + Z_1)}$ $\frac{Z_2 + Z_1 + R}{2 R C (Z_2 + Z_1)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$ $\frac{2 Z_2}{Z_2 + Z_1}$
11-j	$e' = $ (I.) $e'' = \frac{-2 Z_2}{L (a - \beta)} (\epsilon^{-at} - \epsilon^{-\beta t})$	$\frac{Z_1 - Z_2 - R}{L}$	$\frac{Z_1 + Z_2 + R}{L}$ $\frac{Z_1 + Z_2 + R}{L}$	1

Traveling Waves on Multi-Conductor Circuits

Equations (**9**) and (**10**) of Appendix II are the two general equations which must be satisfied by the incident, reflected, and transmitted waves on all wires at the transition points of the general system represented in Fig. 6. Therefore the solution of these simultaneous equations yields the reflected and transmitted waves in terms of the incident waves and the circuit constants of the lines and the transition points. These equations are considerably simplified when the networks N_{12}, N_{23}, etc., connecting the different lines at the junctions are equal to zero. In that case they become

$$(1 + N_r U_r) [Y_{r1} (e_1 - e_1') + \dots \dots + Y_{rn} (e_n - e_n')] - N_r (e_r + e_r') = (y_{r1} e_1'' + \dots \dots + y_{rn} e_n'')$$

$$(e_r + e_r') - U_r [Y_{r1} (e_1 - e_1') + \ldots\ldots + Y_{rn} (e_n - e_n')] = e_r'' + W_r (y_{r1} e_1'' + \ldots\ldots + Y_{rn} e_n'')$$

where n is the total number of conductors of the system, and r is any particular conductor. Each of the above equations must be written for each r so as to provide the necessary $2n$ simultaneous equations from which the $2n$ unknowns $(e_1' \ldots\ldots e_n', e_1'' \ldots\ldots e_n'')$ can be found. A few examples will be given to illustrate the use of these equations. In the interests of brevity, the illustrations are for a two-wire and ground circuit.

Fig. 13a. One of two lines suddenly terminates.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$y_{11} = 1/z_{11}, \; y_{22} = y_{12} = 0$$

Substituting these values in the general equation there is

$$\left.\begin{aligned} Y_{11} (e_1 - e_1') + Y_{12} (e_2 - e_2') &= y_{11} e_1'' \\ Y_{21} (e_1 - e_1') + Y_{22} (e_2 - e_2') &= 0 \\ (e_1 + e_1') &= e_1'' \\ (e_2 + e_2') &= e_2'' \end{aligned}\right\}$$

The solution of these simultaneous equations gives

$$e_1' = \frac{z_{11} - Z_{11}}{z_{11} + Z_{11}} e_1$$

$$e_2' = e_2 - \frac{2 Z_{12}}{z_{11} + Z_{11}} e_1$$

$$e_1'' = \frac{2 z_{11}}{z_{11} + Z_{11}} e_1$$

where

$$Z_{11} = \frac{Y_{22}}{Y_{11} Y_{22} - Y_{12}^2}$$

TABLE III

Fig.	Equation	α	β	A
12-a	$e' = \frac{Z_2 R - Z_1 R - Z_1 Z_2}{Z_2 R + Z_1 R + Z_1 Z_2}$ $e'' = \frac{2 Z_2 R}{Z_2 R + Z_1 R + Z_1 Z_2}$			
12-b (1)	$e' = $ (I.) $e'' = \frac{\alpha + \beta}{a - \beta} \frac{E}{\alpha} (a \epsilon^{-at} - \beta \epsilon^{-\beta t})$	$\frac{Z_1 Z_2}{L (Z_2 - Z_1)}$	$\frac{Z_1 Z_2}{L (Z_2 + Z_1)}$	$\frac{Z_2 - Z_1}{Z_2 + Z_1}$
12-c (1)	$e' = $ (I.) $e'' = E \frac{\alpha + \beta}{a - \beta} (\epsilon^{-at} - \epsilon^{-\beta t})$	$\frac{Z_2 - Z_1}{Z_1 Z_2 C}$ «	$\frac{Z_2 + Z_1}{Z_1 Z_2 C}$ «	-1
12-d	$e' = $ (I.) $e'' = e + e'$	$\frac{R Z_1 Z_2}{L (R Z_2 - R Z_1 - Z_1 Z_2)}$	$\frac{R Z_1 Z_2}{L (R Z_2 + R Z_1 + Z_1 Z_2)}$	β / α
12-e	$e' = $ (I.) $e'' = - E \frac{\alpha + \beta}{a - \beta} (\epsilon^{-at} - \epsilon^{-\beta t})$	$\frac{R Z_2 - R Z_1 - Z_1 Z_2}{Z_1 Z_2 R C}$	$\frac{R Z_2 + R Z_1 + Z_1 Z_2}{Z_1 Z_2 R C}$	-1
12-f	$e' = $ (II.) $e'' = e + e'$	$\frac{Z_1 - Z_2}{2 Z_1 Z_2 C}$	$\frac{Z_1 + Z_2}{2 Z_1 Z_2 C}$	-1
12-i (1)	$e' = $ (II.) $e'' = e + e'$	$\frac{Z_1 R - Z_2 R + Z_1 Z_2}{2 Z_1 Z_2 R C}$	$\frac{Z_1 R + Z_2 R + Z_1 Z_2}{2 Z_1 Z_2 R C}$	-1
12-g (2)	$e' = $ (II.) $e'' = $ (II.)	See reference	See reference	See reference
12-h (2)	$e' = $ (II.) $e'' = $ (II.)	See reference	See reference	See reference
12-j (2)	$e' = $ (II.) $e'' = $ (II.)	See reference	See reference	See reference

$$Z_{12} = \frac{-Y_{12}}{Y_{11}Y_{22} - Y_{12}^2}$$

If e_2 was induced on line No. 2 by e_1 on line No. 1 then

$$e_2 = \frac{Z_{12}}{Z_{11}} e_1$$

Or conversely, if e_1 was induced by e_2, then

$$e_1 = \frac{Z_{12}}{Z_{22}} e_2$$

If, as would likely be the case, the line to the right is simply a continuation of No. 1 wire, then

$$z_{11} = Z_{11}$$

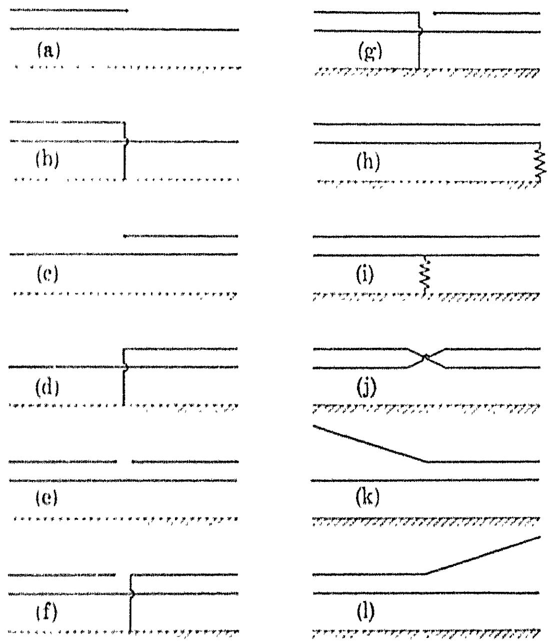


FIG. 13—TRANSITION POINTS OF A DOUBLE CIRCUIT

and the equations become

$$e_1' = 0$$

$$e_2' = e_2 - \frac{Z_{12}}{Z_{11}} e_1$$

$$e_1'' = e_1$$

that is, there is no reflection on line No. 1, and the full wave is transmitted. In this case, had e_2 been induced by e_1 there would be no reflection on No. 2 conductor either.

Fig. 13b. One of two lines is terminated and grounded.

$$N_1 = U_1 = U_2 = W_1 = W_2 = 0, \qquad N_2 = \infty$$

$$y_{11} = 1/z_{11}, \quad y_{22} = y_{12} = 0$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' \\ (e_2 + e_2') &= 0 \\ (e_1 + e_1') &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Solving these simultaneous equations there is

$$e_1' = \frac{Y_{11} - y_{11}}{Y_{11} + y_{11}} e_1 + \frac{2\,Y_{12}}{Y_{11} + y_{11}} e_2$$

$$e_2' = -e_2$$

$$e_1'' = \frac{2\,Y_{11} e_1}{Y_{11} + y_{11}} + \frac{2\,Y_{12}}{Y_{11} + y_{11}} e_2$$

Fig. 13c. Isolated conductor introduced.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$Y_{11} = 1/Z_{11}, \quad Y_{12} = Y_{22} = 0$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') &= y_{11} e_1'' + y_{12} e_2'' \\ 0 &= y_{21} e_1'' + y_{22} e_2'' \\ (e_1 + e_1') &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{z_{11} - Z_{11}}{z_{11} + Z_{11}} e_1$$

$$e_1'' = \frac{2\,z_{11}}{z_{11} + Z_{11}} e_1$$

$$e_2'' = \frac{z_{12}}{z_{11}} e_1'' = \frac{2\,z_{12}}{z_{11} + Z_{11}} e_1$$

Thus if No. 1 is a through conductor, so that $z_{11} = Z_{11}$, there is no reflection.

Fig. 13d. Grounded conductor introduced.

$$N_1 = U_1 = U_2 = W_1 = W_2 = 0, \quad N_2 = \infty$$

$$Y_{11} = 1/Z_{11}, \; Y_{12} = Y_{22} = 0$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') &= y_{11} e_1'' + y_{12} e_2'' \\ e_2 + e_2' &= 0 \\ (e_1 + e_1') &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{Y_{11} - y_{11}}{Y_{11} + y_{11}} e_1$$

$$e_1'' = \frac{2\,Y_{11}}{Y_{11} + y_{11}} e_1$$

$$e_2'' = 0$$

Fig. 13e. Break in one conductor.

$$N_1 = N_2 = U_1 = U_2 = W_1 = 0, \quad W_2 = \infty$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ (e_1 + e_1') &= e_1'' \\ 0 &= y_{21} e_1'' + y_{22} e_2'' \end{aligned}\right\}$$

Therefore, taking

$$Y_{11} = y_{11}, \; Y_{22} = y_{22}, \text{ and } Y_{12} = Y_{21} = y_{12} = y_{21}$$

$$e_1' = 0$$

$$e_2' = e_2 - \frac{z_{12}}{z_{11}} e_1$$

$$e_1'' = e_1$$

$$e_2'' = \frac{z_{12}}{z_{11}} e_1$$

Fig. 13f. Broken line—far section grounded.

$$U_1 = W_1 = W_2 = N_1 = 0, \quad U_2 = N_2 = \infty$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= 0 \\ (e_1 + e_1') &= e_1'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= 0 \end{aligned}\right\}$$

Therefore, since $e_2'' = 0$

$$e_1' = \frac{1 - Z_{11} y_{11}}{1 + Z_{11} y_{11}} e_1$$

$$e_2' = e_2 + \frac{2 Z_{12} y_{11}}{1 + Z_{11} y_{11}} e_1$$

$$e_1'' = \frac{2}{1 + Z_{11} y_{11}} e_1$$

$$e_2'' = 0$$

Fig. 13g. Broken line—near section grounded.

$$U_1 = U_2 = W_1 = N_1 = 0, \quad W_2 = N_2 = \infty$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ e_2 + e_2' &= 0 \\ e_1 + e_1' &= e_1'' \\ 0 &= y_{21} e_1'' + y_{22} e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{z_{11} Y_{11} - 1}{z_{11} Y_{11} + 1} e_1 + \frac{2 Y_{12} z_{11}}{z_{11} Y_{11} + 1} e_2$$

$$e_2' = - e_2$$

$$e_1'' = \frac{2 z_{11}}{z_{11} Y_{11} + 1} (Y_{11} e_1 + Y_{12} e_2)$$

$$e_2'' = \frac{z_{12}}{z_{11}} e_1'' = \frac{2 z_{12}}{z_{11} Y_{11} + 1} (Y_{11} e_1 + Y_{12} e_2)$$

Fig. 13h. One line grounded through a resistor at end of line.

$$U_1 = U_2 = N_2 = 0, \quad N_1 = 1/R, \quad y_{11} = y_{22} = y_{22} = 0$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') - (e_1 + e_1')/R &= 0 \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= 0 \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{R - Z_{11}}{R + Z_{11}} e_1$$

$$e_2' = e_2 - \frac{2 Z_{12}}{R + Z_{11}} e_1$$

If $R = Z_{11}$, then $e_1' = 0$ and there is no reflected wave on No. 1 wire. However, a wave

$$e_2' = e_2 - \frac{Z_{12}}{Z_{11}} e_1$$

is reflected on No. 2 wire.

Fig. 13i. Resistance ground on one wire.

$$U_1 = U_2 = W_1 = W_2 = N_2 = 0, \quad N_1 = 1/R$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' + (e_1 + e'_1)/R \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ e_1 + e_1' &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{- Z_{11}}{2 R + Z_{11}} e_1$$

$$e_2' = \frac{- Z_{12}}{2 R + Z_{11}} e_1$$

$$e_1'' = \frac{2 R}{2 R + Z_{11}} e_1$$

$$e_2'' = e_2 - \frac{Z_{12}}{2 R + Z_{11}} e_1$$

These equations are of importance in connection with the theory of ground wires.[3]

Fig. 13j. Transposition of a line.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$y_{11} = Y_{22}, y_{22} = Y_{11}, y_{12} = y_{21} = Y_{12} = Y_{21}$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ e_1 + e_1' &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{(Y_{11} - Y_{22})}{(Y_{11} + Y_{22})^2 - 4 Y_{12}^2} [(Y_{11} + Y_{22}) e_1 + 2 Y_{12} e_2]$$

$$e_2' = \frac{-(Y_{11} - Y_{22})}{(Y_{11} + Y_{22})^2 - 4 Y_{12}^2} [2 Y_{12} e_1 + (Y_{11} + Y_{22}) e_2]$$

$$e_1'' = e_1' + e_1$$

$$e_2'' = e_2' + e_2$$

If the two conductors are in the same horizontal plane so that $Y_{11} = Y_{22}$, then there are no reflections.

If the two incident waves are alike, that is $e_1 = e_2 = e$ then

$$e_1' = - e_2' = \frac{(Y_{11} - Y_{22}) e}{Y_{11} + Y_{22} - 2 Y_{12}}$$

Fig. 13k. Line entering a section parallel to another line.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$Y_{12} = 0, \quad Z_{11} = z_{11}, \quad Z_{22} = z_{22}$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ (e_1 + e_1') &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{[(Y_{11} - y_{11})(Y_{22} + y_{22}) + y_{12}^2] e_1 + 2 y_{12} Y_{22} e_2}{(Y_{11} + y_{11})(Y_{22} + y_{22}) - y_{12}^2}$$

$$e_2' = \frac{[(Y_{11} + y_{11})(Y_{22} - y_{22}) + y_{12}^2] e_2 - 2 y_{12} Y_{11} e_1}{(Y_{11} + y_{11})(Y_{22} + y_{22}) - y_{12}^2}$$

$$\frac{2\,Y_{11}\,(Y_{22} + y_{22})\,e_1 - 2\,y_{12}\,Y_{22}\,e_2}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - y_{12}^2}$$

$$\frac{2\,Y_{22}\,(Y_{11} + y_{11})\,e_2 - 2\,y_{12}\,Y_{11}\,e_1}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - y_{12}^2}$$

ase of this kind it is highly improbable that both
e_2 would exist simultaneously, so that the equa-
ould be simplified to that extent.

3l. Line leaving a section parallel to another line.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$y_{12} = 0, \quad Z_{11} = z_{11}, \quad Z_{22} = z_{22}$$

$$\left.\begin{array}{rl} Y_{11}\,(e_1 - e_1') + Y_{12}\,(e_2 - e_2') = & y_{11}\,e_1'' \\ Y_{21}\,(e_1 - e_1') + Y_{22}\,(e_2 - e_2') = & y_{22}\,e_2'' \\ e_1 + e_1' \qquad\qquad = & e_1'' \\ e_2 + e_2' = & e_2'' \end{array}\right\}$$

fore

$$\frac{[(Y_{11} - y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2]\,e_1 + 2\,Y_{12}\,y_{22}\,e_2}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

$$\frac{[(Y_{11} + y_{11})\,(Y_{22} - y_{22}) - Y_{12}^2]\,e_2 + 2\,Y_{12}\,y_{11}\,e_1}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

$$\frac{2\,[Y_{11}\,(Y_{22} + y_{22}) - Y_{12}^2]\,e_1 + 2\,Y_{12}\,y_{22}\,e_2}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

$$\frac{2\,[Y_{22}\,(Y_{11} + y_{11}) - Y_{12}^2]\,e_2 + 2\,Y_{12}\,y_{11}\,e_1}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

Successive Reflections

re are many important problems, such as in the
of ground wires,[3] the effect of short lengths of
[1] trunk lines tapped at short intervals, etc.,[4]
it is necessary to consider a number of successive
ions of traveling waves. Sometimes it is exceed-
difficult to keep track of the multiplicity of these
ions. A lattice has therefore been devised which
at a glance the position and direction of motion
ry incident, reflected and refracted wave on the
n at every instant of time. In addition, this
provides the means for calculating the shape
reflected and transmitted waves and gives a
ete history of their past experience. Even the
of attenuation and wave distortion can be
d on the lattice, if the defining functions are
n.

principle of the reflection lattice is illustrated in
4. Three junctions, Nos. 1, 2, and 3 placed at
n intervals along the line are shown. These
ons may consist of any combinations of im-
ces in series with the line or shunted to ground.
rcuits between junctions may be either overhead
or cables; having in general, different surge im-
ces, velocities of wave propagation, and attenua-
actors. To construct the lattice, lay off the
ons to scale at intervals equal to the times of passage of the wave on each section between junctions. Now choose a suitable vertical time scale, shown in Fig. 14 at the left of the lattice, and draw in the diagonals. At the top of the lattice, at any convenient place centered on the junctions, place indicators with the reflection and refraction operators marked on them. In the notation of this paper these indicators are shown as little double-headed arrows marked as follows:

a = reflection operator for waves approaching from the left.

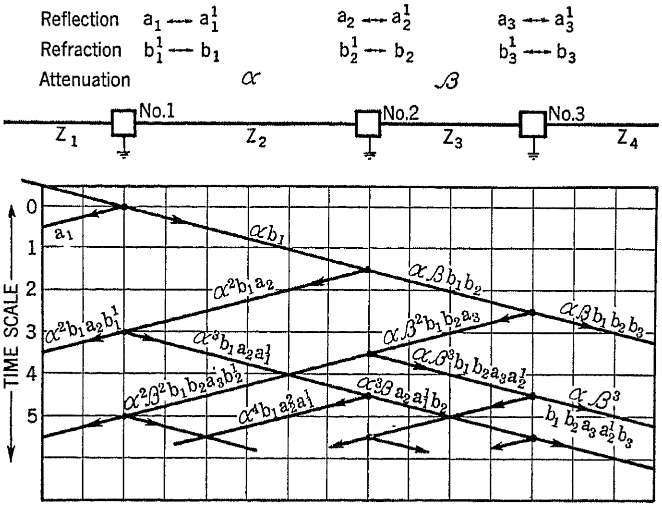
a' = reflection operator for waves approaching from the right.

b = refraction operator for waves approaching from the left.

b' = refraction operator for waves approaching from the right.

β, α = attenuation factors for section between junctions

It is understood, of course, that these operators are operational expressions involving the impedance functions of the junctions, and no restrictions are placed on their generality. Now, starting at the origin of the initial incident wave at the top of the lattice, obtain the reflected and refracted wave at each junction by applying the operators of that junction to the incident wave arriving there, and proceed until the lattice has been completed. It will be observed that:



Reflection
$a_1 \leftrightarrow a_1^1$
$a_2 \leftrightarrow a_2^1$
$a_3 \leftrightarrow a_3^1$
Refraction
$b_1^1 \leftrightarrow b_1$
$b_2^1 \leftrightarrow b_2$
$b_3^1 \leftrightarrow b_3$
Attenuation
α
β
No.1
No.2
No.3
Z_1
Z_2
Z_3
Z_4
TIME SCALE
0
1
2
3
4
5
a_1
αb_1
$\alpha\beta b_1 b_2$
$\alpha^2 b_1 a_2$
$\alpha\beta b_1 b_2 b_3$
$\alpha^2 b_1 a_2 b_1^1$
$\alpha^3 b_1 a_2 a_1^1$
$\alpha\beta^2 b_1 b_2 a_3$
$\alpha\beta^3 b_1 b_2 a_3 a_2^1$
$\alpha^2\beta^2 b_1 b_2 a_3 b_2^1$
$\alpha^3\beta a_2 a_1^1 b_2$
$\alpha^4 b_1 a_2^2 a_1^1$
$\alpha\beta^3 b_1 b_2 a_3 a_2^1 b_3$

Fig. 14—Lattice for Computing Successive Reflections

1. All waves travel downhill.
2. The position of any wave at any time is determined from the vertical time scale at the left of the lattice.
3. The total potential at any point at any instant of time is the superposition of all the waves which have arrived at that point up until that instant of time, displaced in position from each other by intervals equal to the instants of their time of arrival.
4. The previous history of any wave is easily traced,

Fig. 13f. *Broken line—far section grounded.*

$$U_1 = W_1 = W_2 = N_1 = 0, \quad U_2 = N_2 = \infty$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= 0 \\ (e_1 + e_1') &= e_1'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= 0 \end{aligned}\right\}$$

Therefore, since $e_2'' = 0$

$$e_1' = \frac{1 - Z_{11} y_{11}}{1 + Z_{11} y_{11}} e_1$$

$$e_2' = e_2 + \frac{2 Z_{12} y_{11}}{1 + Z_{11} y_{11}} e_1$$

$$e_1'' = \frac{2}{1 + Z_{11} y_{11}} e_1$$

$$e_2'' = 0$$

Fig. 13g. *Broken line—near section grounded.*

$$U_1 = U_2 = W_1 = N_1 = 0, \quad W_2 = N_2 = \infty$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ e_2 + e_2' &= 0 \\ e_1 + e_1' &= e_1'' \\ 0 &= y_{21} e_1'' + y_{22} e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{z_{11} Y_{11} - 1}{z_{11} Y_{11} + 1} e_1 + \frac{2 Y_{12} z_{11}}{z_{11} Y_{11} + 1} e_2$$

$$e_2' = - e_2$$

$$e_1'' = \frac{2 z_{11}}{z_{11} Y_{11} + 1} (Y_{11} e_1 + Y_{12} e_2)$$

$$e_2'' = \frac{z_{12}}{z_{11}} e_1'' = \frac{2 z_{12}}{z_{11} Y_{11} + 1} (Y_{11} e_1 + Y_{12} e_2)$$

Fig. 13h. *One line grounded through a resistor at end of line.*

$$U_1 = U_2 = N_2 = 0, \quad N_1 = 1/R, \quad y_{11} = y_{22} = y_{22} = 0$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') - (e_1 + e_1')/R &= 0 \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= 0 \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{R - Z_{11}}{R + Z_{11}} e_1$$

$$e_2' = e_2 - \frac{2 Z_{12}}{R + Z_{11}} e_1$$

If $R = Z_{11}$, then $e_1' = 0$ and there is no reflected wave on No. 1 wire. However, a wave

$$e_2' = e_2 - \frac{Z_{12}}{Z_{11}} e_1$$

is reflected on No. 2 wire.

Fig. 13i. *Resistance ground on one wire.*

$$U_1 = U_2 = W_1 = W_2 = N_2 = 0, \quad N_1 = 1/R$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' + (e_1 + e'_1)/R \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ e_1 + e_1' &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{- Z_{11}}{2 R + Z_{11}} e_1$$

$$e_2' = \frac{- Z_{12}}{2 R + Z_{11}} e_1$$

$$e_1'' = \frac{2 R}{2 R + Z_{11}} e_1$$

$$e_2'' = e_2 - \frac{Z_{12}}{2 R + Z_{11}} e_1$$

These equations are of importance in connection with the theory of ground wires.[3]

Fig. 13j. *Transposition of a line.*

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$y_{11} = Y_{22}, \; y_{22} = Y_{11}, \; y_{12} = y_{21} = Y_{12} = Y_{21}$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') + Y_{12}(e_2 - e_2') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{21}(e_1 - e_1') + Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ e_1 + e_1' &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{(Y_{11} - Y_{22})}{(Y_{11} + Y_{22})^2 - 4 Y_{12}^2} [(Y_{11} + Y_{22}) e_1 + 2 Y_{12} e_2]$$

$$e_2' = \frac{- (Y_{11} - Y_{22})}{(Y_{11} + Y_{22})^2 - 4 Y_{12}^2} [2 Y_{12} e_1 + (Y_{11} + Y_{22}) e_2]$$

$$e_1'' = e_1' + e_1$$

$$e_2'' = e_2' + e_2$$

If the two conductors are in the same horizontal plane so that $Y_{11} = Y_{22}$, then there are no reflections.

If the two incident waves are alike, that is $e_1 = e_2 = e$ then

$$e_1' = - e_2' = \frac{(Y_{11} - Y_{22}) e}{Y_{11} + Y_{22} - 2 Y_{12}}$$

Fig. 13k. *Line entering a section parallel to another line.*

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$Y_{12} = 0, \quad Z_{11} = z_{11}, \quad Z_{22} = z_{22}$$

$$\left.\begin{aligned} Y_{11}(e_1 - e_1') &= y_{11} e_1'' + y_{12} e_2'' \\ Y_{22}(e_2 - e_2') &= y_{21} e_1'' + y_{22} e_2'' \\ (e_1 + e_1') &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{[(Y_{11} - y_{11})(Y_{22} + y_{22}) + y_{12}^2] e_1 + 2 y_{12} Y_{22} e_2}{(Y_{11} + y_{11})(Y_{22} + y_{22}) - y_{12}^2}$$

$$e_2' = \frac{[(Y_{11} + y_{11})(Y_{22} - y_{22}) + y_{12}^2] e_2 - 2 y_{12} Y_{11} e_1}{(Y_{11} + y_{11})(Y_{22} + y_{22}) - y_{12}^2}$$

$$e_1'' = \frac{2\,Y_{11}\,(Y_{22} + y_{22})\,e_1 - 2\,y_{12}\,Y_{22}\,e_2}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - y_{12}^2}$$

$$e_2'' = \frac{2\,Y_{22}\,(Y_{11} + y_{11})\,e_2 - 2\,y_{12}\,Y_{11}\,e_1}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - y_{12}^2}$$

In a case of this kind it is highly improbable that both e_1 and e_2 would exist simultaneously, so that the equation could be simplified to that extent.

Fig. 13l. Line leaving a section parallel to another line.

$$N_1 = N_2 = U_1 = U_2 = W_1 = W_2 = 0$$

$$y_{12} = 0, \quad Z_{11} = z_{11}, \quad Z_{22} = z_{22}$$

$$\left.\begin{aligned} Y_{11}\,(e_1 - e_1') + Y_{12}\,(e_2 - e_2') &= y_{11}\,e_1'' \\ Y_{21}\,(e_1 - e_1') + Y_{22}\,(e_2 - e_2') &= y_{22}\,e_2'' \\ e_1 + e_1' &= e_1'' \\ e_2 + e_2' &= e_2'' \end{aligned}\right\}$$

Therefore

$$e_1' = \frac{[(Y_{11} - y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2]\,e_1 + 2\,Y_{12}\,y_{22}\,e_2}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

$$e_2' = \frac{[(Y_{11} + y_{11})\,(Y_{22} - y_{22}) - Y_{12}^2]\,e_2 + 2\,Y_{12}\,y_{11}\,e_1}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

$$e_1'' = \frac{2\,[Y_{11}\,(Y_{22} + y_{22}) - Y_{12}^2]\,e_1 + 2\,Y_{12}\,y_{22}\,e_2}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

$$e_2'' = \frac{2\,[Y_{22}\,(Y_{11} + y_{11}) - Y_{12}^2]\,e_2 + 2\,Y_{12}\,y_{11}\,e_1}{(Y_{11} + y_{11})\,(Y_{22} + y_{22}) - Y_{12}^2}$$

SUCCESSIVE REFLECTIONS

There are many important problems, such as in the theory of ground wires,[3] the effect of short lengths of cable,[11] trunk lines tapped at short intervals, etc.,[4] where it is necessary to consider a number of successive reflections of traveling waves. Sometimes it is exceedingly difficult to keep track of the multiplicity of these reflections. A lattice has therefore been devised which shows at a glance the position and direction of motion of every incident, reflected and refracted wave on the system at every instant of time. In addition, this lattice provides the means for calculating the shape of all reflected and transmitted waves and gives a complete history of their past experience. Even the effects of attenuation and wave distortion can be entered on the lattice, if the defining functions are known.

The principle of the reflection lattice is illustrated in Fig. 14. Three junctions, Nos. 1, 2, and 3 placed at uneven intervals along the line are shown. These junctions may consist of any combinations of impedances in series with the line or shunted to ground. The circuits between junctions may be either overhead lines or cables; having in general, different surge impedances, velocities of wave propagation, and attenuation factors. To construct the lattice, lay off the junctions to scale at intervals equal to the times of passage of the wave on each section between junctions. Now choose a suitable vertical time scale, shown in Fig. 14 at the left of the lattice, and draw in the diagonals. At the top of the lattice, at any convenient place centered on the junctions, place indicators with the reflection and refraction operators marked on them. In the notation of this paper these indicators are shown as little double-headed arrows marked as follows:

a = reflection operator for waves approaching from the left.

a' = reflection operator for waves approaching from the right.

b = refraction operator for waves approaching from the left.

b' = refraction operator for waves approaching from the right.

β, α = attenuation factors for section between junctions

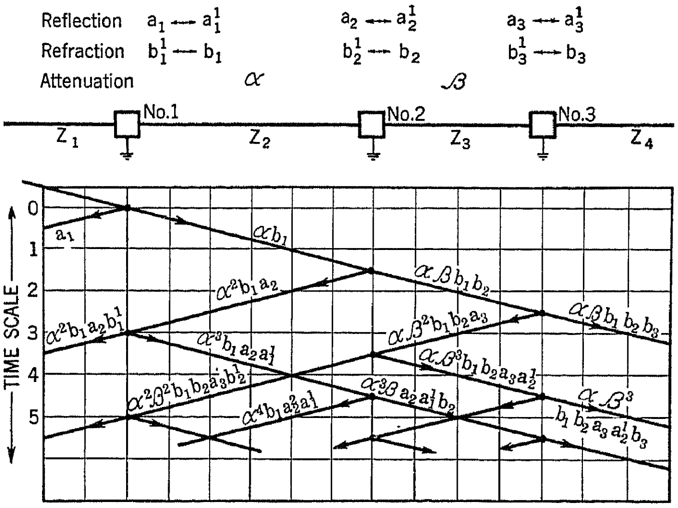

Reflection $a_1 \leftarrow a_1^1$
$a_2 \leftarrow a_2^1$
$a_3 \leftrightarrow a_3^1$
Refraction $b_1^1 \leftarrow b_1$
$b_2^1 \leftarrow b_2$
$b_3^1 \leftrightarrow b_3$
Attenuation
α
β
No.1
No.2
No.3
Z_1
Z_2
Z_3
Z_4
TIME SCALE
0
1
2
3
4
5
a_1
αb_1
$\alpha\beta b_1 b_2$
$\alpha^2 b_1 a_2$
$\alpha\beta b_1 b_2 b_3$
$\alpha^2 b_1 a_2 b_1^1$
$\alpha\beta^2 b_1 b_2 a_3$
$\alpha^3 b_1 a_2 a_1^1$
$\alpha\beta^3 b_1 b_2 a_3 a_2^1$
$\alpha^2\beta^2 b_1 b_2 a_3 b_2^1$
$\alpha^3\beta a_2 a_1^1 b_2$
$\alpha\beta^3$
$\alpha^4 b_1 a_2^2 a_1^1$
$b_1 b_2 a_3 a_2^1 b_3$

FIG. 14—LATTICE FOR COMPUTING SUCCESSIVE REFLECTIONS

It is understood, of course, that these operators are operational expressions involving the impedance functions of the junctions, and no restrictions are placed on their generality. Now, starting at the origin of the initial incident wave at the top of the lattice, obtain the reflected and refracted wave at each junction by applying the operators of that junction to the incident wave arriving there, and proceed until the lattice has been completed. It will be observed that:

1. All waves travel downhill.
2. The position of any wave at any time is determined from the vertical time scale at the left of the lattice.
3. The total potential at any point at any instant of time is the superposition of all the waves which have arrived at that point up until that instant of time, displaced in position from each other by intervals equal to the instants of their time of arrival.
4. The previous history of any wave is easily traced,

that is, where it came from and just what other waves went into its composition.

5. Attenuation is included.

6. If it is desired to carry the computations to a point where it is not practical to place the various operators directly on the lattice itself, then the arms may be numbered and the operational expressions tabulated in a suitable table. No difficulty is encountered in this practise, and sometimes it is possible to devise purely tabular methods which can be filled in automatically.

Appendix I

GENERAL EQUATIONS OF A MULTI-CONDUCTOR SYSTEM

The conventional treatment of transmission line transients is based on consideration of a single wire and its return, and ignores the presence of other conductors. However, there are many cases in the study of traveling waves where the effect of the other conductors cannot be neglected. Sometimes their influence is so vital as to completely change the characteristics of the phenomenon, and entirely erroneous results are obtained if they are not considered. Problems of this type are of special engineering interest in connection with the design of ground wires and other protective schemes, and in general, in the study of mutual effects due to traveling waves. This appendix is a generalization and extention of the method of attack given in a previous paper,[3] in that line-to-line, line-to-ground, and series resistances are included in the analysis.

The General Differential Equation. Fig. 6 shows a system of n transmission line conductors, parallel to each other and to the ground plane, and mutually coupled electromagnetically, so that the effects of currents and potentials on any wire are felt on all the other wires. The circuit constants involved are shown in Fig. 7. Associated with each unit length of line and conductors r and s there is

L_{rr} = self inductance coefficient of conductor r
L_{rs} = mutual inductance coefficient between r and s
K_{rr} = self capacitance coefficient of conductor r
K_{rs} = mutual capacitance coefficient between r and s
R_r = series resistance of conductor r
g_{rr} = leakage conductance to ground of conductor r
g_{rs} = leakage conductance between r and s

It will also be convenient to introduce the notation

$$\begin{aligned}
G_{rr} &= g_{r1} + g_{r2} + g_{r3} + \cdots\cdots + g_{rn}\\
G_{rs} &= G_{sr} = -\,g_{rs} = -\,g_{sr}\\
Z_{rr} &= (R_r + p\,L_{rr})\\
Z_{rs} &= p\,L_{rs}\\
Y_{rr} &= (G_{rr} + p\,K_{rr})\\
Y_{rs} &= (G_{rs} + p\,K_{rs})\\
p &= \partial/\partial t
\end{aligned}$$

The charges per unit length on the conductors in terms of Maxwell's electrostatic coefficients and the potentials $e_1, e_2, \ldots, e_n$ are

$$\left.\begin{aligned}
Q_1 &= K_{11}\,e_1 + K_{12}\,e_2 + \cdots\cdots + K_{1n}\,e_n\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
Q_n &= K_{n1}\,e_1 + K_{n2}\,e_2 + \cdots\cdots + K_{nn}\,e_n
\end{aligned}\right\} \quad (1)$$

The magnetic fluxes linking the conductors per unit length in terms of the inductance coefficients and the currents $i_1, i_2, \ldots, i_n$ are

$$\left.\begin{aligned}
\phi_1 &= L_{11}\,i_1 + L_{12}\,i_2 + \cdots\cdots + L_{1n}\,i_n\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
\phi_n &= L_{n1}\,i_1 + L_{n2}\,i_2 + \cdots\cdots + L_{nn}\,i_n
\end{aligned}\right\} \quad (2)$$

The leakage currents flowing to ground and to the other conductors are

$$\left.\begin{aligned}
i_1' &= g_{11}\,e_1 + g_{12}\,(e_1 - e_2) + \cdots + g_{1n}\,(e_1 - e_n)\\
&= G_{11}\,e_1 + G_{12}\,e_2 + \cdots + G_{1n}\,e_n\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
i_n' &= g_{n1}\,(e_n - e_1) + g_{n2}\,(e_n - e_2) + \cdots + g_{nn}\,e_n\\
&= G_{n1}\,e_1 + G_{n2}\,e_2 + \cdots + G_{nn}\,e_n
\end{aligned}\right\} \quad (3)$$

The differential equations of the first conductor are

$$-\frac{\partial e_1}{\partial x} = \frac{\partial \phi_1}{\partial t} + R_1\,i_1 = Z_{11}\,i_1 + Z_{12}\,i_2 + \cdots\cdots + Z_{1n}\,i_n \quad (4)$$

$$-\frac{\partial i_1}{\partial x} = \frac{\partial Q_1}{\partial t} + i_1' = Y_{11}\,e_1 + Y_{12}\,e_2 + \cdots\cdots + Y_{1n}\,e_n \quad (5)$$

Differentiating equation (4) with respect to x and substituting the equations of type (5) there is

$$\begin{aligned}
\frac{\partial^2 e_1}{\partial x^2} &= Z_{11}\,(Y_{11}\,e_1 + Y_{12}\,e_2 + \cdots + Y_{1n}\,e_n)\\
&\quad + Z_{12}\,(Y_{21}\,e_1 + Y_{22}\,e_2 + \cdots + Y_{2n}\,e_n)\\
&\quad \cdots\cdots\cdots\cdots\cdots\cdots\\
&\quad + Z_{1n}\,(Y_{n1}\,e_1 + Y_{n2}\,e_2 + \cdots + Y_{nn}\,e_n)\\
&= (Z_{11}\,Y_{11} + Z_{12}\,Y_{21} + \cdots + Z_{1n}\,Y_{n1})\,e_1\\
&\quad + (Z_{11}\,Y_{12} + Z_{12}\,Y_{22} + \cdots + Z_{1n}\,Y_{n2})\,e_2\\
&\quad \cdots\cdots\cdots\cdots\cdots\cdots\\
&\quad + (Z_{11}\,Y_{n1} + Z_{12}\,Y_{2n} + \cdots + Z_{1n}\,Y_{nn})\,e_n\\
&= J_{11}\,e_1 + J_{12}\,e_2 + \cdots + J_{1n}\,e_n
\end{aligned} \quad (6)$$

where

$$\begin{aligned}
J_{rs} &= Z_{r1}\,Y_{1s} + Z_{r2}\,Y_{2s} + Z_{r3}\,Y_{3s} + \cdots\cdots + Z_{rn}\,Y_{ns}\\
&= Z_{1r}\,Y_{1s} + Z_{2r}\,Y_{2s} + Z_{3r}\,Y_{3s} + \cdots\cdots + Z_{nr}\,Y_{ns}
\end{aligned} \quad (7)$$

Let

$$A_{rr} = \left(J_{rr} - \frac{\partial^2}{\partial x^2}\right) \quad (8)$$

Then the complete set of differential equations for the n conductors may be written in the symbolic form

$$\left.\begin{aligned}
0 &= A_{11}\,e_1 + J_{12}\,e_2 + \cdots\cdots + J_{1n}\,e_n\\
0 &= J_{21}\,e_1 + A_{22}\,e_2 + \cdots\cdots + J_{2n}\,e_n\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
0 &= J_{n1}\,e_1 + J_{n2}\,e_2 + \cdots\cdots + A_{nn}\,e_n
\end{aligned}\right\} \quad (9)$$

where the J's are operators in the time derivative $p = d/dt$, and the A's are operators in both the time and space derivatives. Solving these n simultaneous equations for any e, there results a determinate of which the numerator is zero on account of having a column of zeros. In order, therefore, that a solution other than zero can exist, it is necessary that the denominator also be zero (on the assumption that the indeterminate so formed will evaluate to a finite value). Therefore, there must be

$$\begin{vmatrix} A_{11} & J_{12} & \cdots & J_{1n} \\ J_{21} & A_{22} & \cdots & J_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ J_{n1} & J_{n2} & \cdots & A_{nn} \end{vmatrix} e = 0 \qquad (\mathbf{10})$$

Now, dropping subscripts

$$Z\,Y = (R + p\,L)(G + p\,K) = L\,K\,p^2 + (L\,G + R\,K)p + R\,G \qquad (\mathbf{11})$$

so that ultimately, the expansion of (**10**) will lead to a polynominal of degree n in $(\partial^2/\partial x^2)$ and degree n in $p^2 = \partial^2/\partial t^2$. The solution of this partial differential equation is the most general solution for a system of parallel conductors.

There are three conditions under which equation (**10**) may be considerably simplified in obtaining a solution. These are I, *The No-Loss Line*, II, *The Completely Transposed Line*, and III, *The Alternating Current Solution.*

Case I. The No-Loss Line. The solution for this case has been given in a recent paper,[3] and will not be repeated here. However, the following discussion of the energy relationships was not included in the previous paper.

Consider a system of n potential and current waves $(e_1 \ldots . e_n,\ i_1 \ldots . i_n)$. From electrostatics (see "Electricity and Magnetism" by J. H. Jeans, page 94) and equations (1) of Appendix I of this paper, the total energy residing in the electrostatic field is

$$W_e = \frac{1}{2} \int (Q_1\,e_1 + Q_2\,e_2 + \ldots\ldots + Q_n\,e_n)\,d\,x$$

$$= \frac{1}{2} \int (K_{11}\,e_1^2 + K_{12}\,e_2\,e_1 + \ldots\ldots + K_{1n}\,e_n\,e_1)\,d\,x$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$+ \frac{1}{2} \int (K_{n1}\,e_1\,e_n + K_{n2}\,e_2\,e_n + \ldots\ldots + K_{nn}\,e_n^2)\,d\,x \qquad (\mathbf{12})$$

where the integration is to extend over the lengths of the waves.

The electromagnetic energy (Jeans page 443) by equations (2) of Appendix I is

$$W_r = \frac{1}{2} \int (\phi_1\,i_1 + \phi_2\,i_2 + \ldots\ldots + \phi_n\,i_n)\,d\,x$$

$$= \frac{1}{2} \int (L_{11}\,i_1^2 + L_{12}\,i_2\,i_1 + \ldots\ldots + L_{1n}\,i_n\,i_1)\,d\,x$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$+ \frac{1}{2} \int (L_{n1}\,i_1\,i_n + L_{n2}\,i_2\,i_n + \ldots\ldots + L_{nn}\,i_n^2)\,d\,x \qquad (\mathbf{13})$$

The total energy of the waves is (considering only the waves moving in one direction, and calling the current waves i)

$$\begin{aligned} W &= \int (e_1\,i_1 + e_2\,i_2 + \ldots\ldots + e_n\,i_n)\,dt \\ &= \int (z_{11}\,i_1^2 + z_{12}\,i_2\,i_1 + \ldots\ldots + z_{1n}\,i_n\,i_1)\,dt \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &+ \int (z_{n1}\,i_1\,i_n + z_{n2}\,i_2\,i_n + \ldots\ldots + z_{nn}\,i_n^2)\,dt \\ &= \int (y_{11}\,e_1^2 + y_{12}\,e_2\,e_1 + \ldots\ldots + y_{1n}\,e_n\,e_1)\,dt \qquad (\mathbf{14}) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &+ \int (y_{n1}\,e_1\,e_n + y_{n2}\,e_2\,e_n + \ldots\ldots + y_{nn}\,e_n^2)\,dt \end{aligned}$$

But for free traveling waves on overhead lines,[3]

$$z = c\,L \qquad (\mathbf{15})$$

and

$$d\,x = v\,d\,t = c\,dt \qquad (\mathbf{16})$$

Therefore, equation (**14**) may be written

$$\begin{aligned} W &= \int (L_{11}\,i_1^2 + L_{12}\,i_2\,i_1 + \ldots\ldots + L_{1n}\,i_n\,i_1)\,d\,x \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &+ \int (L_{n1}\,i_1\,i_n + L_{n2}\,i_2\,i_1 + \ldots\ldots + L_{nn}\,i_n^2)\,d\,x \\ &= 2\,W_i \qquad (\mathbf{17}) \end{aligned}$$

thus proving that the energy of the system is divided equally between the electrostatic and magnetic fields for free waves traveling in one direction. While waves moving in opposite directions are passing through each other the total energy is not equally divided, but may be distributed in any proportion between the two fields. This is also true at a transition point, where the incident waves give rise to reflected waves. In such cases the energies must be computed from equations (12) and (13) and added to find the total. Equation (**14**) applies only to waves moving in the same direction, and while it serves to determine the total energy by computing the energies in each system of oppositely moving waves and adding them, it does not hold for resultant potentials and currents.

Case II. The Completely Transposed Line. If all n conductors are completely transposed with respect to each other and to the ground, and if the conductors have the same resistance, then in effect

$$L_{rr} = L\,, \qquad K_{rr} = K\,, \qquad G_{rr} = G$$

$$L_{rs} = L'\,, \qquad K_{rs} = K'\,, \qquad G_{rs} = G'$$

$$\begin{aligned} Z_{rr}\,Y_{rr} &= (R + p\,L)\,(G + p\,K) = Z\,Y \\ Z_{rr}\,Y_{rs} &= (R + p\,L)\,(G' + p\,K') = Z\,Y' \\ Z_{rs}\,Y_{rr} &= p\,L'\,(G + p\,K) = Z'\,Y \\ Z_{rs}\,Y_{rs} &= p\,L'\,(G' + p\,K') = Z'\,Y' \end{aligned}$$

$$\begin{aligned} J_{rs} &= Z_{1r}\,Y_{1s} + Z_{2r}\,Y_{2s} + \ldots\ldots + Z_{rr}\,Y_{rs} \\ &\quad + \ldots\ldots + Z_{sr}\,Y_{ss} + \ldots\ldots + Z_{nr}\,Y_{ns} \\ &= Z\,Y' + Z'\,Y + (n-2)\,Z'\,Y' = J \qquad (\mathbf{18}) \end{aligned}$$

$$\begin{aligned} J_{rr} &= Z_{1r}\,Y_{1r} + Z_{2r}\,Y_{2r} + \ldots\ldots + Z_{rr}\,Y_{rr} \\ &\quad + \ldots\ldots + Z_{nr}\,Y_{nr} \\ &= Z\,Y + (n-1)\,Z'\,Y' \qquad (\mathbf{19}) \end{aligned}$$

$$A_{rr} = \left(J_{rr} - \frac{\partial^2}{\partial x^2}\right) = \left[Z\,Y + (n-1)\,Z'\,Y' - \frac{\partial^2}{\partial x^2}\right] = A \tag{20}$$

$$(A - J) = \left[\,(Z - Z')\,(Y - Y') - \frac{\partial^2}{\partial x^2}\,\right] \tag{21}$$

Hereby the determinate of equation (**10**) becomes, upon dividing each column through by J and calling $A/J = a$, subtracting adjacent columns from each other, dividing out $(a - 1)$, adding all rows to the last, and finally expanding the remaining determinate in terms of the minors of which the lower right hand elements are the cofactors

$$\begin{vmatrix} A & J & \dots & J \\ J & A & \dots & J \\ \dots & \dots & \dots & \dots \\ J & J & \dots & A \end{vmatrix} e = (A-J)^{n-1}\,(A-J+n\,J) \tag{22}$$

This is the operational equation of the completely transposed line in terms of the operators A and J. Upon substituting the values of A and J from equations (**18**) and (**20**) it takes the form

$$\left(v_1^2\,p^2 + w_1\,p + u_1 - \frac{\partial^2}{\partial x^2}\right)^{n-1}\left(v_2^2\,p^2 + w_2\,p + u_2 - \frac{\partial^2}{\partial x^2}\right) e = 0 \tag{23}$$

If the line is free of losses, as well as completely transposed, then (**22**) becomes

$$\left(v_1^2\,p^2 - \frac{\partial^2}{\partial x^2}\right)^{n-1}\left(v_2^2\,p^2 - \frac{\partial^2}{\partial x^2}\right) e = 0 \tag{24}$$

This is satisfied by the equation of wave motion

$$e = f\,(x + v\,t) \tag{25}$$

which substituted in (**43**) gives

$$v = \pm\,v_1 \text{ and } v = \pm\,v_2 \tag{26}$$

thus showing that there are only two possible velocities of wave propagation on the completely transposed no-loss line. In the case of overhead conductors in air both of these velocities approach equality with the velocity of light, in agreement with the findings of the previous section.

Case III. Solution for Alternating Currents. Suppose that the line is operating under steady-state alternating-current conditions, so that the potentials on the n conductors at coordinate x are given by

$$\left.\begin{aligned} e_1 &= E_1 \sin\,(\omega\,t + \theta_1) = \text{imaginary part of } E_1\,\epsilon^{j(\omega t + \theta_1)} \\ &\dots\dots\dots\dots\dots\dots \\ e_n &= E_n \sin\,(\omega\,t + \theta_n) = \text{imaginary part of } E_n\,\epsilon^{j(\omega t + \theta_n)} \end{aligned}\right\} \tag{27}$$

where the amplitudes $(E_1 \;.\;.\;.\;.\;.\; E_n)$ are functions of x. Substituting (**27**) in (**6**) there is

$$\frac{\partial^2 \dot{E}_1}{\partial x^2} = \dot{J}_{11}\,\dot{E}_1 + J_{12}\,\dot{E}_2 + \;.\;.\;.\;.\;.\; + \dot{J}_{1n}\,\dot{E} \tag{28}$$

where

$$\left.\begin{aligned} \dot{E}_1 &= E_1\,\epsilon^{j\theta_1} \\ &\dots\dots\dots \\ \dot{E}_n &= E_n\,\epsilon^{j\theta_n} \end{aligned}\right\} \tag{29}$$

$$\left.\begin{aligned} \dot{J}_{11} &= J_{11} \text{ when } p = j\;\omega \\ &\dots\dots\dots\dots \\ \dot{J}_{1n} &= J_{1n} \text{ when } p = j\;\omega \end{aligned}\right\} \tag{30}$$

The common factor $\epsilon^{j\omega t}$ has been canceled out on both sides of equation (**28**).

Now according to equation (**10**), the general differential equation is the same for every conductor of the n wire system, and therefore each $\dot{E}$ of (**28**) must follow the same function of x, but the integration constants are, in general, different for each $\dot{E}$. Since the differential equations are ordinary linear differential equations with constant coefficients of order $2\,n$ and homogeneous in $d^2/d\,x^2$ it follows that the solutions are of the form

$$\left.\begin{aligned} \dot{E}_1 &= \sum_1^n \left(\dot{C}_{1r}\,\epsilon^{\lambda_r x} + \dot{C}_{1r}'\,\epsilon^{-\lambda_r x}\right) \\ &\dots\dots\dots\dots\dots\dots \\ \dot{E}_n &= \sum_1^n \left(\dot{C}_{nr}\,\epsilon^{\lambda_r x} + \dot{C}_{nr}'\,\epsilon^{-\lambda_r x}\right) \end{aligned}\right\} \tag{31}$$

where the $\dot{C}$'s are complex integration constants and the λ's are the roots of equation (**10**). There are $2\,n^2$ of these integration constants, of which all except $2\,n$ are redundant. To prove this, substitute equations (**31**) into the original equations of type (**28**), obtaining n equations of the type

$$\begin{aligned} \sum_1^n \lambda_r^2 \left(\dot{C}_{1r}\,\epsilon^{\lambda_r x} + \dot{C}_{1r}'\,\epsilon^{-\lambda_r x}\right) &= \sum_1^n \dot{J}_{11}\left(\dot{C}_{1r}\,\epsilon^{\lambda_r x} + \dot{C}_{1r}'\,\epsilon^{-\lambda_r x}\right) \\ &\dots\dots\dots\dots\dots\dots \\ &+ \sum_1^n \dot{J}_{1n}\left(\dot{C}_{nr}\,\epsilon^{\lambda_r x} + \dot{C}_{1r}'\,\epsilon^{-\lambda_r x}\right) \end{aligned} \tag{32}$$

Collecting terms

$$\begin{aligned} \sum_1^n \Big\{&\left[(\lambda_r^2 - \dot{J}_{11})\,\dot{C}_{1r} - \dot{J}_{12}\,\dot{C}_{2r} - \;.\;.\;.\;.\; - \dot{J}_{1n}\,\dot{C}_{nr}\right]\epsilon^{\lambda_r x} \\ &+ \left[(\lambda_r^2 - \dot{J}_{11})\,\dot{C}_{1r}' - \dot{J}_{12}\,\dot{C}_{2r}' - \;.\;.\;.\;. - \dot{J}\,\dot{C}_{nr}'\right]\epsilon^{-\lambda_r x}\Big\} = 0 \end{aligned} \tag{33}$$

It is now necessary to digress long enough to prove (what is probably self-evident to many) that each of the coefficients in (**33**) is individually equal to zero. In the general case let

$$A\,f_1\,(x) + B\,f_2\,(x) + \;.\;.\;.\;.\;. + N\,f_n\,(x) = 0 \tag{34}$$

where the functions $f\,(x)$ are all different. Assuming that each of these admits of expansion as a power series in x, by Maclaurin's theorem, there is

$$\begin{aligned} A\,(a_1 + b_1\,x + c_1\,x^2 + \;.\;.\;.\;.\;) + \;.\;.\;.\;. \\ + N\,(a_n + b_n\,x + c_n\,x^2 + \;.\;.\;.\;.\;) = 0 \end{aligned} \tag{35}$$

Collecting terms

$$\begin{aligned}&(a_1 A + a_2 B + \ldots + a_n N) \\ &\quad + (b_1 A + b_2 B + \ldots + b_n N)\, x + \ldots = 0\end{aligned} \tag{36}$$

But by the method of indeterminate coefficients, each term of this power series in x must individually equal zero, so that there are the simultaneous equations

$$\left.\begin{aligned} a_1 A + a_2 B + \ldots + a_n N &= 0 \\ b_1 A + b_2 B + \ldots + b_n N &= 0 \\ c_1 A + c_2 B + \ldots + c_n N &= 0 \\ \text{etc.} \end{aligned}\right\} \tag{37}$$

Since the coefficients $(a, b, c \ldots)$ are entirely arbitrary, the solution of these equations leads to a determinate of which the denominator is finite and the numerator is zero (by virtue of a column of zeros), and therefore

$$A = B = \ldots = N = 0 \tag{38}$$

Thus in any equation of type (**34**) the individual coefficients are separately equal to zero.

Returning now to the equations of type (**33**), and considering all n of these equations, there is

$$\left.\begin{aligned} (\dot{J}_{11} - \lambda_r^2)\,\dot{C}_{1r} + \dot{J}_{12}\,\dot{C}_{2r} + \ldots + \dot{J}_{1n}\,\dot{C}_{nr} &= 0 \\ \dot{J}_{21}\,\dot{C}_{1r} + (\dot{J}_{22} - \lambda_r^2)\,\dot{C}_{2r} + \ldots + \dot{J}_{2n}\,\dot{C}_{nr} &= 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{J}_{n1}\,\dot{C}_{1r} + \dot{J}_{n2}\,\dot{C}_{2r} + \ldots + (\dot{J}_{nn} - \lambda_r^2)\,\dot{C}_{nr} &= 0 \end{aligned}\right\} \tag{39}$$

and exactly the same relationships hold between the $\dot{C}'$ coefficients. Now in order that (**39**) may be satisfied by values of the $\dot{C}$'s other than zero, the denominator of the determinate must be equal to zero, that is

$$\begin{vmatrix} (\dot{J}_{11} - \lambda_r^2) & \dot{J}_{12} & \ldots\ldots & \dot{J}_{1n} \\ \dot{J}_{21} & (\dot{J}_{22} - \lambda_r^2) & \ldots\ldots & \dot{J}_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \dot{J}_{n1} & \dot{J}_{n2} & \ldots\ldots & (\dot{J}_{nn} - \lambda_r^2) \end{vmatrix} = 0 \tag{40}$$

Therefore, if (**40**) holds, there are $(n - 1)$ independent relationships between the $\dot{C}$'s in equation (**39**), so that any $(n - 1)$ of them may be eliminated. But since there are n values of r, there will remain n integration constants that must be determined from the terminal conditions. Likewise, there will remain n arbitrary integration constants among the $\dot{C}'$ coefficients.

Thus, the n-wire transmission system has associated with it n propagation constants (λ_r) and $2\,n$ integration constants $\dot{C}_r$ and $\dot{C}_r'$.

If the line is completely transposed, then by equation (**22**) there are only two independent roots to the differential equations, and therefore only four integration constants.

If the line is a completely transposed three-wire line, there is, by equations (**22**), (**18**), and (**20**)

$$\left[(Z - Z')\,(Y - Y') - \frac{\partial^2}{\partial x^2} \right]^2 \left[(Z + 2\,Z')\,(Y + 2\,Y') - \frac{\partial^2}{\partial x^2} \right] \dot{E} = 0 \tag{41}$$

Therefore the propagation constants are

$$\left.\begin{aligned} \lambda_1^2 &= (Z - Z')\,(Y - Y') \\ \lambda_2^2 &= (Z + 2\,Z')\,(Y + 2\,Y') \end{aligned}\right\} \tag{42}$$

But (λ_2^2) does not satisfy (**40**), so that there are no $r = 2$ constants. However, (λ_1^2) does satisfy (**40**) and therefore the solution for a completely transposed three-wire line is

$$\left.\begin{aligned} \dot{E}_1 &= \dot{C}_{11}\,\epsilon^{\lambda_1 x} + \dot{C}_{11}'\,\epsilon^{-\lambda_1 x} \\ \dot{E}_2 &= \dot{C}_{21}\,\epsilon^{\lambda_1 x} + \dot{C}_{21}'\,\epsilon^{-\lambda_1 x} \\ \dot{E}_3 &= \dot{C}_{31}\,\epsilon^{\lambda_1 x} + \dot{C}_{31}'\,\epsilon^{-\lambda_1 x} \end{aligned}\right\} \tag{43}$$

If for the complex number (λ_1) there be substituted

$$\lambda_1 = \alpha + j\,\beta \tag{44}$$

then equations (**44**) may be expressed in any of the following familiar forms

$$\begin{aligned} \dot{E} &= A\,\epsilon^{\lambda x} + B\,\epsilon^{-\lambda x} \\ &= A\,\epsilon^{\alpha x}\,(\cos\beta\,x + j\sin\beta\,x) + B\,\epsilon^{-\alpha x}\,(\cos\beta\,x - j\sin\beta\,x) \\ &= (A + B)\cosh\lambda\,x + (A - B)\sinh\lambda\,x \\ &= (A + B)\,(\cosh\alpha\,x\,.\,\cos\beta\,x + j\sinh\alpha\,x\,.\,\sin\beta\,x) \\ &\quad + (A - B)\,(\sinh\alpha\,x\,.\,\cos\beta\,x + j\cosh\alpha\,x\,.\,\sin\beta\,x) \end{aligned} \tag{45}$$

This is the so-called vector solution. The actual potential as function of x and t is

$$\begin{aligned} e &= \text{imaginary part of } \dot{E}\,\epsilon^{j\omega t} \\ &= \text{imaginary part of } (A\,\epsilon^{\lambda x} + B\,\epsilon^{-\lambda x})\,\epsilon^{j\omega t} \end{aligned} \tag{46}$$

It is worth noticing from (**42**) that λ_1 is in terms of the so-called "constants to neutral" used in practical transmission line calculations. For if

h = geometric mean height above ground
s = geometric mean spacing between conductors
r = radius of conductors

then

$$Z = R + j\left(\frac{1}{2} + 2\log\frac{2\,h}{r}\right)10^{-9}$$

$$Z' = \quad + j\left(2\log\frac{2\,h}{s}\right)10^{-9}$$

$$(Z - Z') = R + j\left(\frac{1}{2} + 2\log\frac{s}{r}\right)10^{-9}\text{ ohms}$$

$$(Y - Y') = (G - G') + \frac{j}{\left(18\log\dfrac{s}{r}\right)10^{11}}\text{ mhos}$$

Appendix II

BEHAVIOR OF WAVES AT A TRANSITION POINT

In a previous paper[3] equations were given for the reflected and transmitted waves on an n-wire transmission system similar to that shown in Fig. 6, but lacking the mutual connecting admittances, N_{12}, N_{13}, etc. In this appendix these admittances are included in the analysis, and the more general equations obtained. The application of these general equations is illustrated in the paper by a number of practical examples which have come up from time to time in the investigation of artificial lightning surges. Even the complicated circuit of Fig. 6 will not serve for every conceivable case, but it hardly seems profitable to generalize any further. If a particular transition point cannot be made a special case of Fig. 6, it will probably be as easy to solve it directly as to reduce it from anything more general. In any event, the procedure followed in setting up the equations and solving them is the same regardless of how complex the transition points may be.

Referring to Fig. 6 let

$Y_{11}, Y_{22}, \ldots, Y_{nn}$ = self surge admittances of lines on the left
Y_{12}, Y_{13}, etc. = mutual surge admittances of lines on the left
$y_{11}, y_{22}, \ldots, y_{nn}$ = self surge admittances of lines on the right
y_{12}, y_{13}, etc. = mutual surge admittances of lines on the right
$U_1, U_2, \ldots, U_n$ = series impedance network on the left
$W_1, W_2, \ldots, W_n$ = series impedance network on the right
$N_1, N_2, \ldots, N_n$ = admittances to ground
N_{12}, N_{23}, etc. = admittances from junction to junction
e, i = potential and current incident waves
e', i' = potential and current reflected waves
e'', i'' = potential and current transmitted waves

When the incident waves arrive at the transition points, they give rise to reflected and transmitted waves which satisfy the general equations of the transmission line, and are in accord with Kirchhoff's laws and the conditions of current and voltage continuity at the junctions.

The total potential at the junction of any incoming line r is the sum of the incident and reflected waves on that line

$$(e_r + e_r') \tag{1}$$

and the total current is[3]

$$(i_r + i_r') = Y_{r1}(e_1 - e_1') + \ldots + Y_{rn}(e_n - e_n') \tag{2}$$

The potential across the admittance N_r is

$$E_r = (e_r + e_r') - U_r(i_r + i_r') \tag{3}$$

and the current through N_r therefore is

$$I_r = N_r E_r \tag{4}$$

The current transmitted to the outgoing line is

$$i_r'' = y_{r1} e_1'' + y_{r2} e_2'' + \ldots + y_{rn} e_n'' \tag{5}$$

and the current transferred to the other junction is

$$I_r' = N_{1r}(E_r - E_1) + N_{2r}(E_r - E_2) + \ldots + N_{nr}(E_r - E_n) \tag{6}$$

The condition of current continuity is

$$i_r + i_r' = i_r'' + I_r + I_r' \tag{7}$$

The potential wave transmitted to the outgoing line is

$$e_r'' = E_r - W_r i_r'' \tag{8}$$

Substituting (**2**), (**3**), (**4**), (**5**) and (**6**) in (**7**) and rearranging, there is

$$\begin{aligned}
&[Y_{r1} + Y_{r1} U_r (N_r + N_{1r} + \ldots + N_{nr}) - Y_{11} N_{1r} U_1 \\
&\qquad - \ldots - Y_{n1} N_{nr} U_n](e_1 - e_1') \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&+[Y_{rn} + Y_{rn} U_r (N_r + N_{1r} + \ldots + N_{nr}) - Y_{1n} N_{1r} U_1 \\
&\qquad - \ldots - Y_{nn} N_{nr} U_n](e_n - e_n') \\
&+N_{1r}(e_1 + e_1') + \ldots + N_{nr}(e_n + e_n') \\
&\qquad - (N_r + N_{1r} + \ldots + N_{nr})(e_r + e_r') \\
&= (y_{r1} e_1'' + \ldots + y_{rn} e_n'')
\end{aligned} \tag{9}$$

Substituting (**2**), (**3**), (**4**), (**5**) and (**6**) in (**8**) and rearranging, there is

$$\begin{aligned}
&(e_r + e_r') - U_r[Y_{r1}(e_1 - e_1') + \ldots + Y_{rn}(e_n - e_n')] \\
&\qquad = e_r'' + W_r(y_{r1} e_1'' + \ldots + y_{rn} e_n'')
\end{aligned} \tag{10}$$

For an n-wire system n equations of type (**9**) and n equations of type (**10**) can be written, and these $2n$ simultaneous equations suffice for the determination of the $2n$ unknowns $(e_1' \ldots e_n', e_1'' \ldots e_n'')$. The other quantities may then be found from equations (**1**) to (**8**). These equations are therefore sufficient to completely formulate the behavior of the incident, reflected, and transmitted waves at a general transition point. Some simplifications and examples are given below.

Mutual Connecting Networks Removed. Suppose that N_{12}, N_{23}, etc., are all zero. Then equations (**9**) and (**10**) reduce to

$$\begin{aligned}
&(1 + N_r U_r)[Y_{r1}(e_1 - e_1') + \ldots + Y_{rn}(e_n - e_n')] - N_r(e_r + e_r') \\
&\qquad = (y_{r1} e_1'' + \ldots + y_{rn} e_n'')
\end{aligned} \tag{11}$$

$$\begin{aligned}
&(e_r + e_r') - U_r[Y_{r1}(e_1 - e_1') + \ldots + Y_{rn}(e_n - e_n')] \\
&\qquad = e_r'' + W_r(y_{r1} e_1'' + \ldots + y_{rn} e_n'')
\end{aligned} \tag{12}$$

These are the general equations derived in a previous paper.[3]

Single Wire Line. In this case only e_1, e_1' and e_1'' exist, and equations (**11**) and (**12**) become

$$(1 + N_1 U_1) Y_{11}(e_1 - e_1') - N_1(e_1 + e_1') = y_{11} e_1'' \tag{13}$$

$$- U_1 Y_{11}(e_1 - e_1') + (e + e_1') = (1 + W_1 y_{11}) e_1'' \tag{14}$$

Solving these two simultaneous equations for the reflected and transmitted waves, substituting $Z_{11} = 1/Y_{11}$ and $z_{11} = 1/y_{11}$, and dropping subscripts, there is

$$e' = \frac{(z + W)(1 + N U) + U - Z - Z N (z + W)}{(z + W)(1 + N U) + U + Z + Z N (z + W)}\, e \tag{15}$$

$$e'' = \frac{2z}{(z + W)(1 + N U) + U + Z + Z N (z + W)}\, e \tag{16}$$

The conventional traveling wave theory is based on a single-wire line and is expressed by the above equations. In terms of the total impedance at the transition point,

$$Z_o = U + \frac{1}{\left(N + \frac{1}{W + z}\right)} = \frac{U + (1 + N\,U)\,(W + z)}{1 + N\,(W + z)} \tag{17}$$

the above equations take the more familiar form

$$e' = \frac{Z_o - Z}{Z_o + Z}\,e \tag{18}$$

$$e'' = \frac{1}{1 + N\,(W + z)} \quad \frac{2\,z}{Z_o + Z}\,e \tag{19}$$

Equations and curves of a great many combinations have been worked out as special cases of these expressions. Those shown in Figs. 10, 11, and 12 are in the readily accessible literature. It will be noticed that many of the combinations illustrated are special cases of the more complicated circuits, merely by substituting limiting values (zero or infinite) for the constants R, L, C, and Z as required. Moreover, each of these cases is of practical importance in the study of high-voltage surges.

Energy Relationships at the Junctions. The energy of a free traveling wave is given by equations (12) to (17) of Appendix I. During the time that the incident waves are at the junction a redistribution of energy is taking place. The division of energy during this transition period furnishes a valid check on the reflection, refraction, and transfer operators, and is of interest on its own account. At any time t, counting from the instant when the system of incident waves $(e_1 \; . \; . \; . \; . \; e_n)$ arrive at the junction, there is

$$\int_t^\infty (e_1\,i_1 + \; . \; . \; . \; . + e_n\,i_n)\,dt$$

$$= \text{energy remaining in the incident waves} \tag{21}$$

$$-\int_o^t (e_1'\,i_1' + \; . \; . \; . \; . + e_n'\,i_n')\,dt$$

$$= \text{energy in the reflected waves} \tag{22}$$

$$\int_o^t (e_1''\,u_1'' + \; . \; . \; . \; . + e_n''\,i_n'')\,dt$$

$$= \text{energy in the transmitted waves} \tag{23}$$

$$\int_o^t (E_1\,I_1 + \; . \; . \; . \; . + E_n\,I_n)\,dt$$

$$= \text{energy absorbed by the networks } (N_1 \; . \; . \; . \; . \; N_n) \tag{24}$$

$$\int_o^t [(e_1 + e_1' - E_1)\,(i_1' + i_1) + \ldots + (e_n + e_n' - E_n)(i_n' + i_n)]\,dt$$

$$= \text{energy absorbed by the networks } (U_1 \; . \; . \; . \; U_n) \tag{25}$$

$$\int_o^t [(E_1 - e_1'')\,i_1'' + \; . \; . \; . + (E_n - e_n'')\,i_n]\,dt$$

$$= \text{energy absorbed by the networks } (W_1 \; . \; . \; . \; W_n) \tag{26}$$

$$\int_o^t \sum_1^n \sum_s (E_r - E_s) \; . \; N_{rs}\,(E_r - E_s)\,dt$$

$$= \text{energy absorbed in connecting networks } N_{rs} \tag{27}$$

The $\sum\limits_s$ summation in (27) ranges from $s = 1$ to $s = n$, excepting $s = r$.

Equating the sum of these energies to the energy in the original incident waves, by the conservation of energy,

$$\begin{aligned} \int_o^\infty \sum_1^n e_r\,i_r\,dt = & \int_t^\infty \sum_1^n e_r\,i_r\,dt - \int_o^t \sum_1^n e_r'\,i_r'\,dt \\ & + \int_o^t \sum_1^n e_r''\,i_r''\,dt + \int_o^t \sum_1^n E_r\,I_r\,dt \\ & + \int_o^t \sum_1^n (e_r + e_r' - E_r)\,(i_r + i_r')\,dt \\ & + \int_o^t \sum_1^n (E_r - e_r'')\,i_r''\,dt \\ & + \int_o^t \sum_1^n \sum_s (E_r - E_s) \; . \; N_{rs}\,(E_r - E_s)\,dt \end{aligned} \tag{28}$$

Combining the first term on the right with the term on the left, according to the rule

$$\int_o^\infty - \int_t^\infty = \int_o^t \tag{29}$$

and discarding the integrals, there is

$$\begin{aligned} \sum_{r=1}^n [e_r\,i_r + e_r'\,i_r' - e_r''\,i_r'' - E_r\,I_r - (e_r + e_r' - E_r)(i_r + i_r') \\ - (E_r - e_r'')\,i_r'' - \sum_s (E_r - E_s) \; . \; N_{rs}\,(E_r - E_s)] = 0 \end{aligned} \tag{30}$$

The currents and voltages at a transition as determined by the reflection, refraction, and transfer operators, must satisfy equation (30).

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Discussion

For discussion of this paper see page 557.

Attenuation and Successive Reflections of Traveling Waves

BY JAMES C. DOWELL*
Associate, A. I. E. E.

Synopsis.—*This paper describes the method followed in separating out the individual waves from the oscillographic record of the voltage at a transition point due to the superposition of several successive reflections. The separation is necessary in determining the attenuation of the waves on the transmission line. In the past it has not been unusual to find the reduction in the crest value of each successive reflection attributed to attenuation, whereas the major cause of such reduction was reflection at the terminal equipment. The paper discusses a class of lattices which offer a simple and convenient means for keeping track of successive reflections, so that the position, direction, and previous history of each separate wave may be ascertained at any instant of time almost at a glance. The reflection and refraction operators are defined and applied to specific examples. A few oscillograms of successive reflections are analyzed and a number of attenuation curves has been plotted.*

* * * * *

INTRODUCTION

IN the summer of 1929 a million-volt impulse generator and a cathode-ray oscillograph laboratory were installed on the double circuit 132-kv. Philo-Canton line of the Ohio Power Co. for the purpose of studying natural and artificial lightning surges. Details regarding this installation have been published before,[1] and will not be repeated here.

Conditions at this location were not favorable to the accurate determination of attenuation for artificial surges owing to the short periods of time that either circuit could be taken out of service for experimental purposes. It was, therefore, necessary to depend upon measurements of successive reflections at the oscillograph station. This method involves a change of wave shape at each reflection, and therefore introduces inaccuracies and complications in the segregation of the attenuation constants. The object of this paper is to describe how this problem was solved, since both the method and the results are believed to be of interest.

Tests were made with three different waves of less than one microsecond front and exponential tails decreasing to half value in four, seven, and twenty microseconds respectively. Waves of this shape were used in order to facilitate calculation. The crest voltages of the applied waves were obtained by sphere gap measurements, and the voltage of successive reflections by comparative deflections on the oscillograms. All impulses applied to the line were of negative polarity, but of course changed sign completely when reflected from a grounded end, and changed sign partially when reflected by the capacitance of the impulse generator.

All the waves on which attenuation data are included in this paper were applied on a single line, either grounded or open circuited at the far ends. After reflection from the end of the line the wave returned and charged the capacitance of the impulse generator. A capacitor acts like a short circuit at the first instant, but finally, if the wave is sufficiently long, the capacitor becomes fully charged, and thereafter acts like an open circuit. In other words, the reflected wave at a capacitor is entirely different in magnitude and shape than the incident wave which caused it. Moreover, the total voltage measured at the capacitor due to a single incident wave is the sum of the incident and reflected waves. If in addition, one or more successive reflections from the end of the line have reached the impulse generator before it has completely discharged, then the total voltage measured at its terminal is the sum of all incident and reflected waves which are present simultaneously at its terminals. This may happen if the initial wave is of the order of twice or more the length of the transmission line. In such cases it is impossible to obtain the attenuation constants until the individual waves have been segregated out from the total potential measurements as given by the oscillograms. This means that the oscillogram record must be separated into its component waves and the amplitudes of these waves compared.

This problem has been worked in the following stages:

1. Construction of a reflection diagram or lattice.
2. Derivation of the reflection operators.
3. Calculation of the traveling wave components.
4. Calculation of the total voltage at the generator.
5. Determination of the attenuation constants by comparison with the oscillograms.

REFLECTION LATTICES

In analyzing the problem of successive reflections of traveling waves use has been made of the reflection lattices as described in the paper by L. V. Bewley, on page 532.

Consider Fig. 1 representing an impulse generator connected to an open end transmission line. As the capacitance of the impulse generator discharges, a wave e_1 travels out on the line, and when it reaches

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1. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, January 26-30, 1931.

the open end reflects without change of shape or sign. The reflection returns over the line to the impulse generator, and causes another reflection. But this second reflection is quite different in shape from the first reflection, depending upon the constants of the impulse generator circuit. The behavior of traveling waves for a considerable variety of terminal conditions has been worked out,[4] and general methods developed for their calculation. Theoretically, it is always possible to express the terminal equipment as generalized impedances in terms of Heaviside's operators; and it is also theoretically possible to calculate the reflection from any such generalized impedance for incident waves of arbitrary shape.[5] Suppose that the operator, which when applied to the wave arriving at the impulse generator derives the reflected wave, be designated by α. Then the zig-zag geometric scheme (lattice) in Fig. 1 shows how each wave arriving from the end of the line experiences a reflection at the impulse generator, and that each successive reflection is α times the preceding or incident wave which caused it. Of course α is not a simple constant multiplier, but a function involving the constants of the circuit and the time derivative, $p = d/dt$. According to this scheme all waves travel "down hill" and it is therefore convenient to place a time scale along the side of the lattice so that vertical distances between points represent the differences in time that it takes a traveling wave to reach those points. Thus the lattice presents a clear time-distance picture of the phenomenon of successive reflections and shows at a glance just what is happening at all points along the circuit at all instants of time. For example, at the end of 1¼ units of time (counting one unit of time equal to the time that it takes a wave to travel twice the length of the line) a wave αe_1 reflected from the impulse generator has just reached the midpoint of the line. Prior to its arrival, two waves of equal size e_1 traveling in opposite directions had passed this point. The total resultant voltage as function of time at any point is the sum of all waves which have arrived up to the given instant, properly displaced, of course, by the intervals of their relative position on the line.

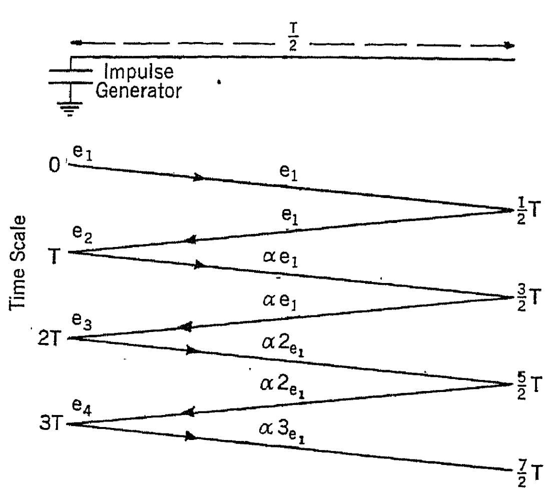

FIG. 1—SUCCESSIVE REFLECTIONS FROM IMPULSE GENERATOR

Fig. 2 shows a slightly more complicated situation in which the impulse generator is a distance $x_1/2$ from one end of a line and a distance $x_2/2$ from the other end. In this case a wave e arriving at the impulse generator gives rise to a reflected wave αe and a refracted or transmitted wave βe where α is called the *reflection operator* and β the *refraction operator*. An interesting feature of this condition is that at definite intervals (as at e_5 on the lattice) waves arrive simultaneously from the right and left and add up by superposition. Verification of this is shown on the oscillogram in Fig. 3. It will be noticed that the fifth wave has practically doubled up, due to the superposition of two waves arriving simultaneously from the right and the left branches of the line. It is clear from the lattice that each of these time waves has traveled a distance equal to twice the length of the line, that is $x_1 + x_2$.

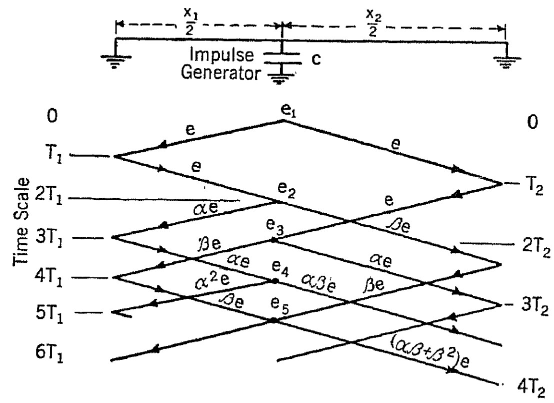

FIG. 2—SUCCESSIVE REFLECTIONS FROM IMPULSE GENERATOR AT INTERMEDIATE POINT ON LINE

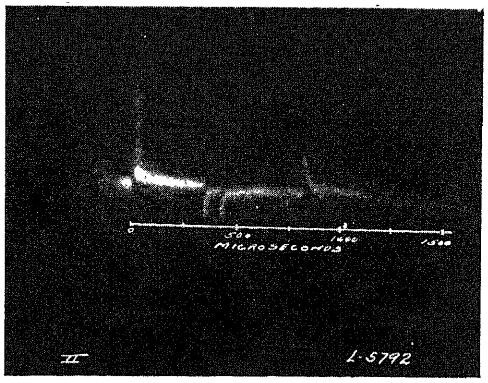

FIG. 3—OSCILLOGRAM CORRESPONDING TO FIG. 2

REFLECTION AND REFRACTION OPERATORS

A reflection operator is a general differential expression associated with a transition point of a transmission line circuit, such that when applied to the function of time representing an incident wave striking that transition point, yields a differential equation whose solution gives the reflected wave. Similarly, the refraction operator is defined as that operator which gives the refracted or transmitted wave which passes the transition point and enters another line.[5] In this paper the simple condition of one outgoing line, and no series impedance networks, is under consideration. The following notation is used in subsequent derivations.

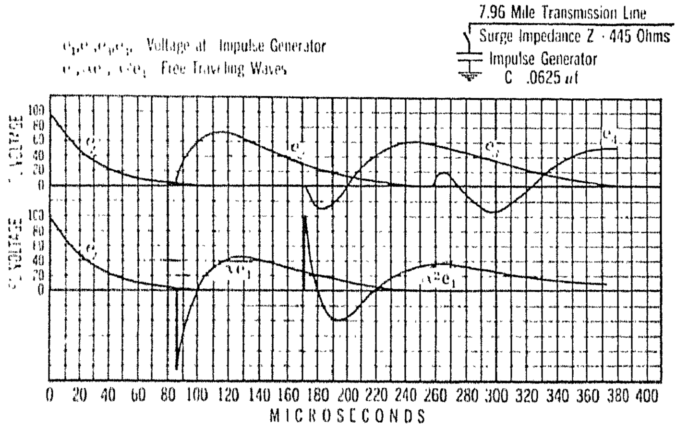

Fig. 4 Successive Reflections on a 7.96-Mile Line

Calculated free waves and total voltages

Z — surge impedance of the line.

$Z_o(p)$ — total impedance at the transition point.

e = $Z\,i$ — incident voltage wave.

i — incident current wave.

e' = $-Z\,i'$ — reflected voltage wave.

i' — reflected current wave.

e'' = $Z\,i''$ total voltage = transmitted voltage wave.

i'' — transmitted current wave.

From traveling wave theory and the condition that there cannot be a discontinuity of potential at the transition point, there is

$$e'' = e + e' = (i + i')\,Z_o(p) = (e - e')\,\frac{Z_o(p)}{Z}$$

Solving for e' and e'' respectively.

$$e' = \frac{Z_o(p) - Z}{Z_o(p) + Z}\,e = \textit{reflected wave.}$$

$$e'' = \frac{2\,Z_o(p)}{Z_o(p) + Z}\,e = \textit{transmitted wave} = \textit{total voltage.}$$

When the impulse generator is simply a capacitor C, connected to a single transmission line,

$$Z_o(p) = 1/p\,C$$

$$\alpha = \frac{1/p\,C - Z}{1/p\,C + Z} = \frac{a - p}{a + p} \quad \text{where } a = \frac{1}{C\,Z}$$

and the total voltage is obtained from the refraction operator

$$\beta = \frac{2/p\,C}{1/p\,C + Z} = \frac{2\,a}{a + p}$$

The first wave due to the initial discharge of the impulse generator is

$$e_1 = E\,\frac{p}{p + a}\quad 1 = E\,\epsilon^{-at}$$

Where 1 is Heaviside's unit function

The subsequent voltages at the impulse generator due to successive reflections then are

$$e_2 = \beta\, e_1 = E\,\frac{2\,a\,p}{(p + a)^2}\; 1 = 2\,E\,\text{at}\;\epsilon^{-at}$$

$$e_3 = \alpha\,\beta\,e_1 = E\,\frac{2\,a\,p\,(a - p)}{(p + a)^3}\; 1 = 2\,E\,\text{at}\,(a\,t - 1)\;\epsilon^{-at}$$

$$e_4 = \alpha^2\beta\,e_1 = E\,\frac{2\,a\,p\,(a - p)^2}{(p + a)^4}\; 1$$

$$= \frac{2}{3}\,E\;a\,t\,(2\,a^2\,t^2 - 6\,a\,t + 3)\;\epsilon^{-at}$$

The traveling waves are

$$\alpha\,e_1 = E\,\frac{p\,(a - p)}{(a + p)^2}\; 1 = E\,(2\,a\,t - 1)\;\epsilon^{-at}$$

$$\alpha^2\,e_1 = E\,\frac{p\,(a - p)^2}{(a + p)^3}\; 1 = E\,(2\,a^2\,t^2 - 4\,a\,t + 1)\;\epsilon^{-at}$$

$$\alpha^3\,e_1 = E\,\frac{p\,(a - p)^3}{(a + p)^4}\; 1 = E\left(\frac{4}{3}\,a^3t^3 - 6\,a^2t^2 + 6\,at - 1\right)\epsilon^{-at}$$

These voltages have been plotted in Fig. 4 for the case of a 0.0625 μ f. capacitor as impulse generator, and a 7.96 mi. open end transmission line of Z = 445 ohms surge impedance. Some of these voltages have more than one maximum. The greatest of each of these is (excepting the 100 per cent peaks of $\alpha\, e_1$ and $\alpha^2\, e_1$).

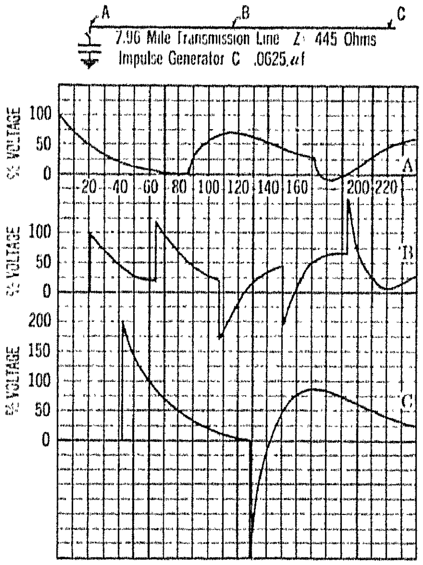

Fig. 5—Voltage as Function of Time at Points A, B and C

Line open at C

e_2/E	e_3/E	e_4/E	$\alpha\, e_1/E$	$\alpha^2\, e_1/E$
0.735	0.618	0.546	0.446	−0.414

In Figs. 5 and 6, corresponding to an open and grounded line respectively, the voltage at three points along the line has been plotted as function of time; and in Fig. 7 the distribution of potentials along the line at different instants of time are shown. It is evident from these curves that successive reflections exhibit the following characteristics:

a. Reduced maximum crests
b. Increased lengths
c. Oscillatory components

Due to the elongations experienced by the reflections there are three or more pairs of incident and reflected waves present simultaneously at the impulse generator after the second reflection; and while some of these components contribute very little to the total voltage, yet they cannot be neglected in computing the attenuation.

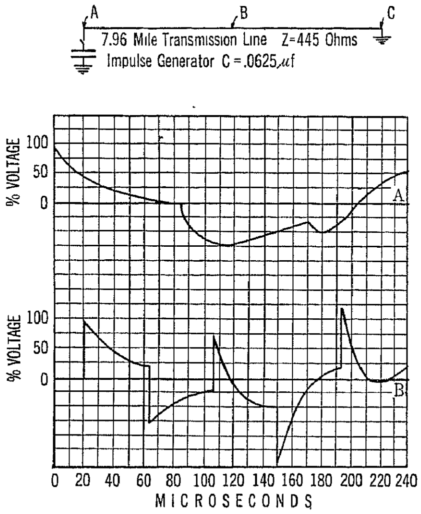

FIG. 6—VOLTAGE AS FUNCTION OF TIME AT POINTS *A* AND *B*

Line grounded at C

In the case of the impulse generator at an intermediate point along the line, there is

$$Z_o\,(p) = \frac{Z}{1 + Z\,C\,p}$$

$$\alpha = \frac{-\,p}{p + b} \text{ where } b = 2/C\,Z$$

$$\beta = \frac{b}{p + b}$$

Then according to the lattice of Fig. 2

$$e_1 = e \quad = E \quad \frac{p}{p + b} \quad 1 = E\,\epsilon^{-bt}$$

$$e_2 = \beta\, e \quad = E\,\frac{b\,p}{(p + b)^2} \quad 1 = E\,b\,t\,\epsilon^{-bt}$$

$$e_3 = \beta\, e \quad = E\,\frac{b\,p}{(p + b)^2} \quad 1 = E\,b\,t\,\epsilon^{-bt}$$

$$e_4 = \alpha\,\beta\, e = E\,\frac{-\,b\,p^2}{(p + b)^3} \quad 1 = \frac{E}{2}\,(b^2\,t^2 - 2\,b\,t)\,\epsilon^{-bt}$$

$$e_5 = (\beta + \alpha\,\beta + \beta^2)\, e = E\,\frac{2\,b^2\,p}{(p + b)^3} \quad 1 = E\,b^2\,t^2\,\epsilon^{-bt}$$

$$\alpha\, e = -\,E\,\frac{p^2}{(p + b)^2} \quad 1 = -\,E\,(1 - b\,t)\,\epsilon^{-bt}$$

$$\beta\, e = e_2 = E\,b\,t\,\epsilon^{-bt}$$

and the greatest maximum of each of these is:

e_1/E	e_2/E	e_3/E	e_4/E	e_5/E	e/E	$\alpha\, e/E$	$\beta\, e/E$
1.000	0.368	0.368	0.230	0.549	1.000	0.134	0.368

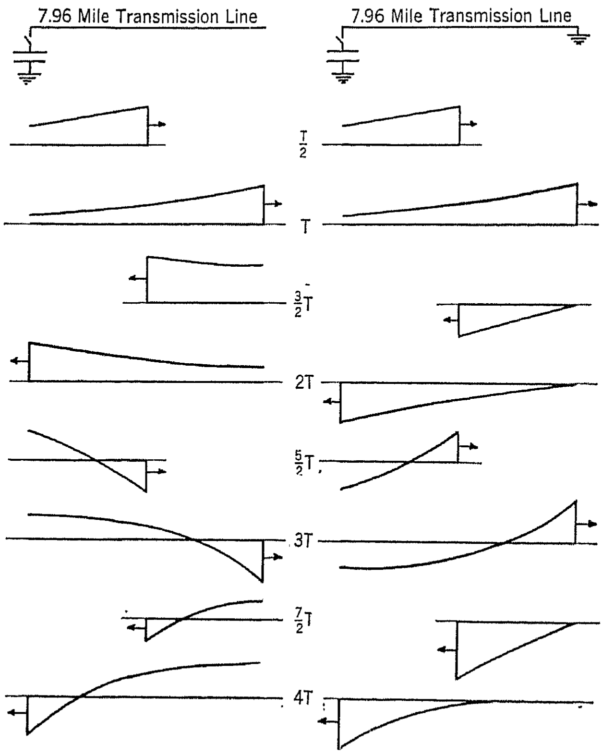

FIG. 7—VOLTAGE DISTRIBUTION ALONG OPEN AND GROUNDED LINES

ATTENUATION

As the waves travel along the line they suffer a decrease in amplitude and a distortion of shape due to the line losses, these being principally corona, and to a lesser extent skin effect. It has been fairly well established through numerous cathode-ray oscillograph studies of artificial lightning waves that they may attenuate considerably without an appreciable change in shape, other than some slowing down of the front. On the basis of this experimental evidence it is permissible to treat such traveling waves as though the transmission line were a "distortionless line," in which case the attenuation is accounted for by a simple exponential decrement factor. This attenuation constant is most accurately determined by a comparison of the crests of the wave at different intervals along the circuit. These crests should be measured by a sphere gap or cathode-ray oscillograph, although rough figures may be obtained with the surge recorder or klydonograph. However, if time or operating conditions do not permit measurements to be taken for the same wave at various distances along the line, then recourse may be had to oscillograms showing a number of successive reflections. Figs. 8 and 9 show the oscillograms of several successive reflections on a 7.96-mi. transmission line, both open-circuited and grounded at its far end.

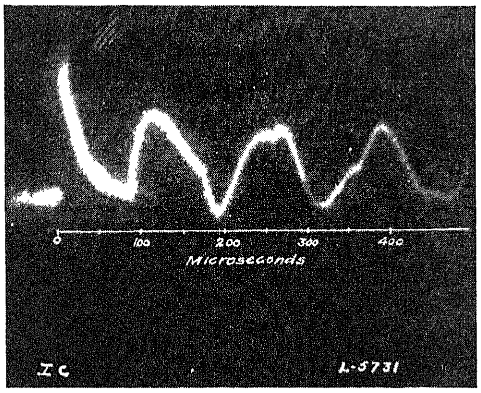

FIG. 8—OSCILLOGRAM OF SUCCESSIVE REFLECTIONS ON A 7.96-MILE TRANSMISSION LINE OPEN AT THE FAR END

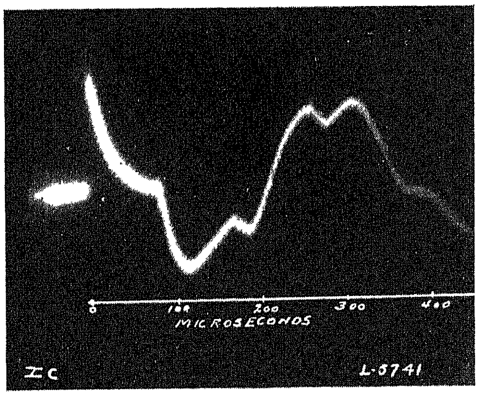

FIG. 9—OSCILLOGRAM OF SUCCESSIVE REFLECTIONS ON A 7.96-MILE TRANSMISSION LINE GROUNDED AT THE FAR END

Figs. 10 and 11 show the records transcribed from the oscillograms in Figs. 8 and 9 respectively, and the calculated voltage components at the impulse generator as given in Fig. 4. From a comparison of the oscillograms and the calculations the attenuation constants α_1, α_2, α_3, α_4, etc., are deduced. These factors are different for each wave, for although the distance traveled between reflections is the same, yet each wave has a different shape and crest due to the influence of the impulse generator impedance on the character of the reflection.

It will be noticed that after the third reflection there is considerable deviation between the measured and calculated values. This apparent distortion appeared to be due to limitations of the cathode ray oscillograph circuit, rather than to line loss distortion.

The attenuation curves for a number of typical cases are shown in Figs. 12 and 13. The (− −) curves correspond to the line open at its far end, while the (+ −) or (− +) curves correspond to the line grounded at its far end. Since a voltage wave reflects with reversed sign from a grounded point, a wave traveling over such a line is alternately positive and negative, as indicated by the (+ −) or (− +) sign. On the short 7.96-mi. line, the grounded line condition showed the least attenuation, as was to be expected, because the wave used was longer than the line, and consequently, as shown in Fig. 7, the average voltage over the line was less and the losses smaller. Moreover, the voltage at the end of the line is at ground potential for the grounded line, but double the wave crest for the open line. As the corona loss varies approximately as the square of the excess corona voltage, this doubling up of voltage at the open end of a short length of line may have considerable effect on the attenuation. Of course, if the line is long compared with the length of the waves,

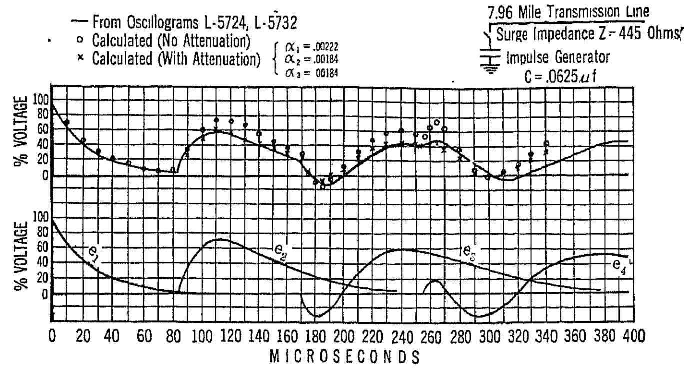

FIG. 10—SUCCESSIVE REFLECTIONS ON A 7.96-MILE LINE BETWEEN A CAPACITANCE AND AN OPEN END

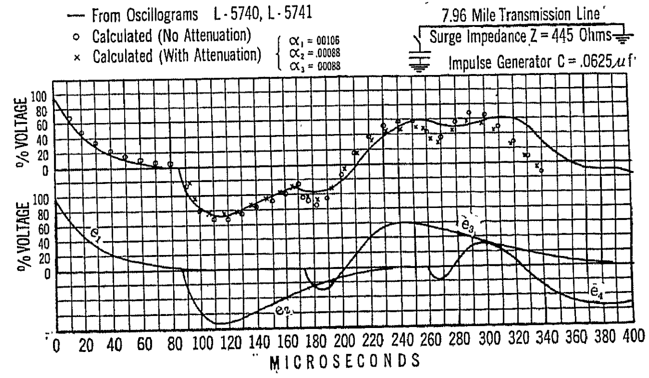

FIG. 11—SUCCESSIVE REFLECTIONS ON A 7.96-MILE LINE BETWEEN A CAPACITANCE AND A GROUNDED END

the importance of an open or grounded end is not so material.

As an example of how the attenuation constant is computed, take one of the cases given in Fig. 13 where the line is so long in comparison with the length of the wave that there is never more than a single incident wave and its reflection at the impulse generator at any one time. Let the measured voltages on the oscillograms be V_1, V_2, V_3, V_4, etc., and the corresponding calculated voltages, neglecting attenuation, be e_1, e_2, e_3, e_4, etc. Then on the assumption of simple exponential attenuation there is

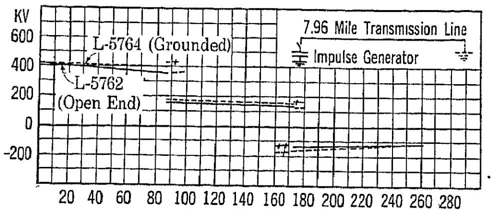

FIG. 12—ATTENUATION ON A 7.96-MILE LINE

Grounded and open ends

$$\frac{V_2}{V_1} = \frac{e_2}{e_1}\,\epsilon^{-\alpha_1\tau} \text{ or } \alpha_1 = \frac{1}{\tau}\log\left(\frac{e_2}{e_1}\;\frac{V_1}{V_2}\right)$$

$$\frac{V_3}{V_2} = \frac{e_3}{e_2}\,\epsilon^{-\alpha_2\tau} \text{ or } \alpha_2 = \frac{1}{\tau}\log\left(\frac{e_3}{e_2}\;\frac{V_2}{V_3}\right)$$

$$\frac{V_4}{V_3} = \frac{e_4}{e_3}\,\epsilon^{-\alpha_3\tau} \text{ or } \alpha_3 = \frac{1}{\tau}\log\left(\frac{e_4}{e_3}\;\frac{V_3}{V_4}\right)$$

etc. etc.

where τ is the time that it takes the wave to travel twice the distance of the line, and α_1, α_2, α_3, etc. are the attenuation constants.

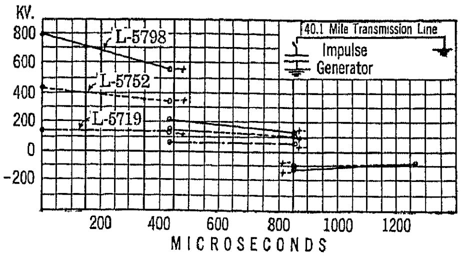

FIG. 13—ATTENUATION ON A 40.1-MILE LINE

Far end grounded

These attenuation constants are in general different, since each successive reflected wave is different in shape and magnitude.

If the line is so short that there is more than one pair of incident and reflected waves at the impulse generator at the same time, these waves may be added graphically as shown in Figs. 4, 5, and 6, and the comparison based on scaled values.

The attenuation curves shown in Figs. 12 and 13 are in reality curves of

$$V_{[k+1]}\left(\frac{e_k}{e_{k+1}}\right) = V_k\,\epsilon^{-\alpha_k\tau}$$

Some of the results obtained on attenuation are tabulated in Table I.

TABLE I

Kv. voltage range Initial	Final	Attenuation constant	Polarity	Total distance traveled in miles	Capacity of impulse generator (μ f.)
788	460	0.00127	− +	80.2	0.0125
204	135	0.00097	+ −	“	“
125	85	0.00091	+ −	“	“
427	346	0.00050	− +	“	0.0208
154	106	0.00088	+ −	“	“
98	81	0.00045	+ −	“	“
143	135	0.00014	− +	“	0.0625
60	56	0.00016	+ −	“	“
52	(?)	(?)	+ −	“	“
427	405	0.00065	− +	15.92	0.0208
180	165	0.00093	+ −	“	“
154	88	0.00644	+ −	“	“
427	364	0.00184	− −	“	“
161	141	0.00155	− −	“	“
131	126	0.00049	+ +	“	“
136	124	0.00106	− +	“	0.0625
55	60	(?)	+ −	“	“
55	(?)	(?)	+ −	“	“
122	101	0.00222	− −	“	“
45	39	0.00172	− −	“	“
36	(?)	(?)	+ +	“	“
775	345	0.00226	− +	66.4	0.0125
360	188	0.00154	− +	“	0.0208
427	260	0.00580	− −	15.92	“
427	187	0.00186	− +	82.3	“

CONCLUSIONS

The data secured on attenuation by this investigation cannot be regarded as accurate and definite. But the study was of value in demonstrating the limitations of trying to calculate attenuation from the measurements of successive reflections—limitations in both the mathematical work and the oscillograph circuit. Moreover, the calculation demonstrates once more that the behavior of artificial lightning surges on a transmission line may be computed with a reasonable degree of engineering accuracy by treating the line as distortionless.

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4. *Traveling Waves Due to Lightning*, by L. V. Bewley, A. I. E. E. Trans., July 1929, p. 1050.

5. *Shunt Resistors for Reactors*, by Kierstead, Rorden, and Bewley, A. I. E. E. Trans., July 1930, p. 1161.

6. "Reflection of Transmission Line Surges at a Terminal Impedance," by O. Brune, *G. E. Review*, May 1929.

7. "Calculation of Voltage Stresses Due to Traveling Waves with Special Reference to Choke Coils," by E. W. Boehne, *G. E. Review*, Dec. 1929.

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Discussion

TRAVELING WAVES ON TRANSMISSION SYSTEMS

(L. V. Bewley)

ATTENUATION AND SUCCESSIVE REFLECTIONS OF TRAVELING WAVES

(James C. Dowell)

C. L. Fortescue: With regard to some of the remarks that Mr. Bewley makes concerning waves due to lightning, I am inclined to agree with him that the characteristic wave of the direct stroke of lightning has an extremely steep front and an exponential tail, but caution is necessary in drawing inferences as to the general shape of the potential impressed by the lightning stroke from cathode-ray oscillograms because the surge that is propagated over the wire is entirely different from the surge impressed by the stroke since it is modified by reflections at tower points by flashover of the insulator string itself, by attenuation due to corona, and by several other modifying influences. For example, when a stroke hits a tower or ground wire there is no surge induced on the line wire as a result of the negative surge which flows over the ground wires. There is the usual induced positive wave caused by the release of the charge from the cloud which has been generally termed "the induced wave." This wave occurs whether the cloud is discharged by hitting the tower or ground wire or whether it is discharged by hitting earth at some neighboring point. On the line wire for a limited distance on either side of the stroke a negative potential will be induced, the value of which is reduced at each tower until finally it becomes negligible.

If the ground wire protection system fails as a result of an excessive difference of potential between ground wires and line wires then, and only then, will a negative surge be transmitted over the line but this surge will be chopped by the flashover of the insulator and will also be modified by the reflections at the towers adjacent to the point of stroke and therefore will be not at all representative of the actual impressed potential due to the stroke itself.

I was very much interested to find that Mr. Bewley uses the method of successive reflections in making computations. For lightning calculations to determine proper clearances and so forth, this method is simple and direct and is sufficiently accurate. The results can be worked out graphically with simple ways in a very short time.

H. L. Rorden: A purely mathematical treatment of a subject which might become extremely involved is often looked upon with skepticism. Experimental verification of such calculations is therefore valuable in that it strengthens our confidence in theoretical results. Moreover, where differences between the mathematical and experimental results are found, experimental data are particularly valuable in locating the cause of such deviation, and frequently suggests methods of obtaining better agreement.

In an extensive group of tests of traveling waves made with the cathode-ray oscillograph, the results obtained with line conditions as are given by Fig. 10D of Mr. Bewley's paper are illustrative of the general correlation found. A transmission line slightly over two miles in length was used, ($11\frac{1}{2}$ microseconds) and the waves applied to terminal impedances at the far end of the line were recorded with the cathode-ray oscillograph. In Fig. 1 the results obtained with the line as illustrated in Mr. Bewley's Fig. 10D are given, where the incident wave is assumed to be of the form $e = E\epsilon^{-at}$ for the purpose of calculation. No extraneous constants were introduced into the circuit to slow up the front of waves sent from the lightning generator. The oscillogram of the incident wave e illustrates how losses have caused it to deviate from the assumed form. The record of the total voltage at the terminal impedance e_o is plotted together with its calculated wave, in which it is seen that the maximum voltage obtained is but $57\frac{1}{2}$ per cent of the theoretical value. The reflected wave e' is plotted at the lower left.

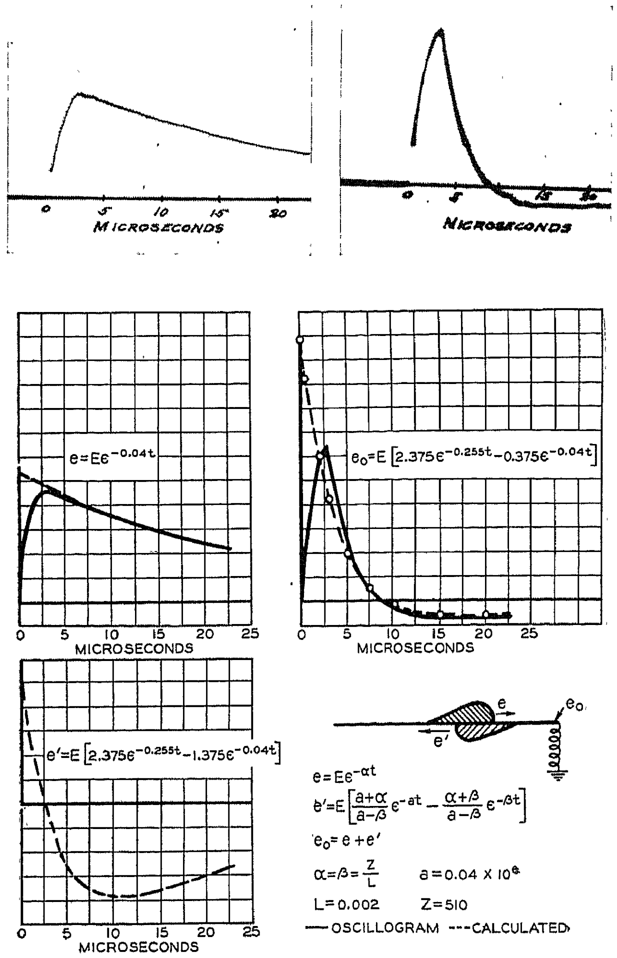

Fig. 1

From these results it appears obvious that the wave front cannot be neglected without introducing a serious error, since the wave used here has almost reached its crest in two microseconds. In order to obtain comparable results it becomes necessary to more exactly approximate the incident wave by employing the general form $e = E(\epsilon^{-at} - \epsilon^{-bt})$. In this instance the parameters a and b are determined from the oscillogram of the incident wave. By the method derived by Mr. Bewley in an earlier publication* the equations given in his present paper are very easily extended to apply to the case having the latter form of incident wave, and such as is illustrated in his Fig. 3C and D. Fig. 2 is a comparison of the same oscillograms shown in Fig. 1 with the waves calculated by more nearly approximating the incident wave. It is seen that the incident wave may be very exactly reproduced by the equation employing the difference of two exponentials. By Mr. Bewley's method, this could be extended to three exponentials if still closer approximation were found necessary. The agreement of the mathematical with the experimental results is obvious from a consideration of the plotted wave of the voltage at the terminal impedance e_0 where the circles represent calculated points and the curve is drawn from the oscillogram. At the lower left again is given the reflected wave as was obtained by both the mathematical and experimental methods.

**Shunt Resistors for Reactors*, by Kierstead, Rorden and Bewley, A. I. E. E. Trans., July 1930, p. 1161.

Ernst Weber: I would like to call attention to the first equation in the paper which gives the length of the traveling wave in the form

$$L = x + t$$

where x is the length of the bound charge in thousands of feet and t is the time of the cloud discharge in microseconds. This is a strange combination of two different dimensions and it is not apparent what the dimension of L itself is in this case. It is unfortunately an example of the non-scientific writing which often gives the physicists just cause to belittle or disparage the scientific work of engineers.

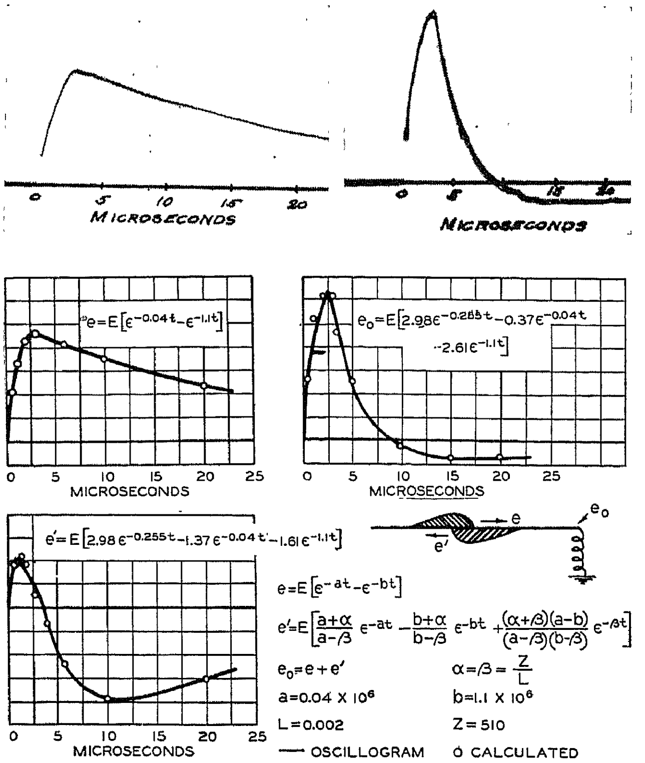

Fig. 2

I would like to call attention to a reference regarding the specification of traveling waves as indicated on the second page of the paper. The general expression of the given eight shapes of traveling waves by means of exponential functions with varying exponents is not new. Mr. W. Deutsch has given an ingenious variation of Heaviside's operational method, by expressing the applied voltages of different wave shapes in terms of exponential functions with a "determining exponent." In his paper (*Archiv. f. Electrotechnik*, 1918, volume 6) he uses the method of the complex function theory and obtains very elegant results, even in the case of transmission line problems.

F. W. Peek, Jr.: Mr. Bewley has presented a very important and valuable paper. In the early work on traveling waves it was generally necessary for the sake of simplicity to use a rectangular wave shape. Mr. Bewley has shown how to get results without much difficulty for any wave shape. He has presented a very important tool in the lattice. By means of this lattice any number of reflections may be taken care of readily. Mr. Dowell has shown in his paper just how this can be done and, by the examples he has given on measured traveling waves, shows the accuracy of the results.

O. Ackermann: The paper of Mr. Dowell is very interesting and remarkable as it shows how operational calculus methods, brought to increased attention and clarified by several papers of Mr. Bewley, can be applied to actual cases and result in a very desirable simplification in the calculating process.

The check with actual oscillograms (as demonstrated in Figs. 10 and 11) is so conspicuous that one is inclined to welcome the conclusions from these tests as reliable information with regard to attenuation characteristics of transmission lines. However, the results of this paper, as pertaining to attenuation, deviate from anything experienced before, in such alarming manner that it is imperative to look for the reasons of this wide discrepancy. The strange results are best illustrated by Figs. 12 and 13 which indicate that surges only decrease 10 to 20 per cent in a travel of 85 microseconds which corresponds to about 16 miles. All other tests of the General Electric Company and those conducted by the Westinghouse Company, as well as measurements made in Europe, show a very much higher rate of attenuation, indicating, in the average, that, over a 16-mile stretch, surges attenuate to about one-half their original value.

We believe that the error in Mr. Dowell's paper lies in an assumption as this: "The measured voltages at the surge generator check very well with those calculated; therefore, the free waves causing these voltages must have been as assumed in the calculation."

However, there is the fact that incident waves of quite different shape but conveying about equal charges may produce almost the same voltage at the surge generator because of the integrating action of the surge generator capacity.

We would like to ask Mr. Dowell whether a wave half as high but twice as long as the one taken as "reflected" wave would not produce oscillograms quite similar to the ones shown in the paper. We see the possibility of very great percentage variations in the attenuation decrement with very much smaller modifications in the shape of the reflected wave.

The attenuation data given in the paper postulate a resistance of line plus ground return of three to five ohms per mile and a leakage resistance of around 50,000 ohms per mile if a distortionless line is assumed as suggested in the paper.

Our own tests and those conducted by many others, restricted to direct measurements of attenuation, bring out values of the resistance of line plus ground return in the order of 40 ohms per mile, the largest part of which is contributed by the ground return, and from 500 to 2,000 ohms per mile leakage resistance, if the latter is pictured in series with some sort of a valve to present the characteristics of corona losses.

E. W. Boehne: There is quite a general agreement among various observers that the principle cause of attenuation is corona. Mr. Dowell is of the opinion that this attenuation is directly related to corona loss and suggests that since corona loss varies approximately as the square of the excess corona voltage, that the doubling up of voltage at an open end may have considerable effect on the attenuation.

Undoubtedly some part of the rapid attenuation in the presence of corona is due to this corona loss, however, it is the purpose here to point out in the light of the information gathered from field observations one additional way in which corona affects the attenuation of voltage surges.

Consider a simple transmission system on which a large lightning surge has originated, the crest value of the surge being well above the critical corona voltage as shown in Fig. 3.

That portion of the conductor subjected to voltages above the critical corona voltage, will due to formation of corona, experience a change in line constants with respect to the wave level at that point. Neglecting leakage and resistance, the principal changes will be of the capacitance and inductance of the conductor. For a simple demonstration of the distortion of such a wave we will assume that only the capacitance of the conductor is increased for each differential wave level in the presence of corona and that the inductance remains unaltered.

The fundamental relation which will be used is:

$$1/2\ L\ I^2 = 1/2\ C\ E^2 \qquad (1)$$

Defining E/I as the surge impedance, we have

$$\frac{E}{I} = Z = \sqrt{L/C} \qquad (2)$$

Similarly

$$v = \frac{1}{\sqrt{L\ C}} = \frac{1}{Z\ C} \qquad (3)$$

Now allowing only the capacitance in the above relations to increase for that portion of the wave above the critical corona voltage and allowing this capacitance to be greater for each higher wave level in the corona region, let us predict what will happen to the wave as it travels to the right.

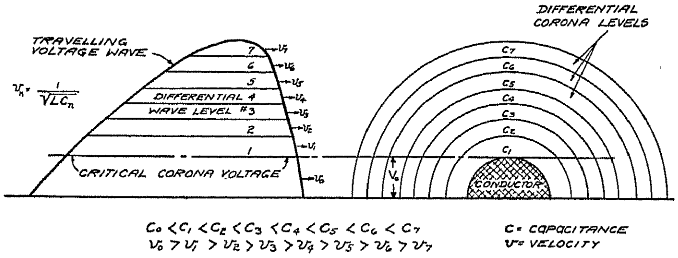
TRAVELLING VOLTAGE WAVE
$v_n = \frac{1}{\sqrt{LC_n}}$
DIFFERENTIAL WAVE LEVEL #3
7
6
5
4
2
1
v_7
v_6
v_5
v_4
v_3
v_2
v_1
v_0
CRITICAL CORONA VOLTAGE
DIFFERENTIAL CORONA LEVELS
C_7
C_6
C_5
C_4
C_3
C_2
C_1
V_0
CONDUCTOR
$C_0 < C_1 < C_2 < C_3 < C_4 < C_5 < C_6 < C_7$
$v_0 > v_1 > v_2 > v_3 > v_4 > v_5 > v_6 > v_7$
C = CAPACITANCE
v = VELOCITY

FIG. 3

1. The surge impedance of the conductor to the various wave levels in the corona region will be lowered. This follows from equation (2). Moreover the surge impedance will be minimum to the highest wave level and will hence follow the peak of the wave.

2. That portion of the wave above the critical corona voltage will travel at a slower speed than that portion below the critical corona voltage and the highest voltage level will travel at the slowest speed.

This is all we can predict physically, but let us see what apparent changes the above brings about. (See Fig. 4.)

a. We would expect that portion of the wave below the critical corona voltage to slip ahead of the rest of the wave at a constant rate depending upon the difference in velocity of the waves free from corona and in the presence of corona. Moreover we would expect this phenomenon to take place suddenly at the critical corona voltage. The actual oscillograph story of this physical fact has been observed by field studies made by Mr. McEachron in Michigan.

b. Each differential wave level above the critical corona voltage will recapitulate this slippage or shearing effect in that each differential voltage level will appear to slip ahead of the voltage level above it giving the effect of distorting the front of the wave in the corona range. This distortion will be to slope off the front of the wave above the critical corona voltage in much the same manner as does a lumped capacitor.

c. Because of the peaked nature of the usual lightning waves the top of the wave will be continually "toppling" over to the rear because of its slower velocity. This action will result in a rapid "apparent" attenuation of the wave and in any stage of the process the peak of the wave becomes flatter.

d. Because of this slippage to the rear there will be a building up of the tail of the wave. This fact has been actually observed and could not take place if the whole wave were considered to travel at one speed and attenuate due to losses.

e. The wave after traveling some distance will eventually resolve itself into a much longer wave traveling below the corona voltage.

f. These effects will be modified to some degree by what might well be termed "residual corona" or the corona which exists on the line immediately after the wave has passed.

This simple analysis of allowing only the capacitance in the presence of the various corona levels to change explains one possibility of the following observed facts.

1. The records of natural lightning show that most of the low-voltage surges are usually long surges while the high-voltage surges are comparatively short in length.

2. The fact that positive corona is much more severe than negative corona it is in agreement with the fact that the attenuation of positive surges is greater than that of negative surges, this being in agreement with the above.

3. Similarly the fact that the distortion of positive waves is greater than that of negative waves is readily explained.

4. It has been shown that the attenuation is a function of the wave shape in that long flat waves attenuate slowly and peaked waves attenuate rapidly. The above explanation shows why this is so.

5. The surge impedance in the presence of corona has been observed to be lower and is in agreement with this theory.

6. The slipping ahead of that part of the wave below the critical corona voltage has been measured with an oscillograph.

7. The building up of the tail of the wave has been observed oscillographically.

8. Positive voltage surges of *natural lightning* have been recorded which show a second increase in voltage. This second rise in voltage is more than likely that part of the wave held back by the corona. With a little study we will be able to measure this displacement and predict not only how far the wave has traveled but its proximate magnitude and shape at the point of origin and this in turn should give some clue as to how it originated.

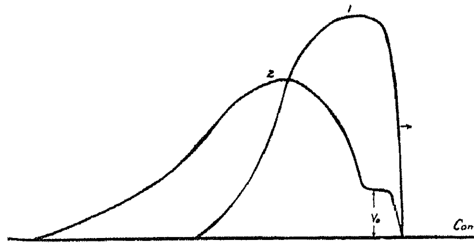

FIG. 4—(1) A POSITIVE SURGE SHORTLY AFTER IT ORIGINATED ON AN OVERHEAD LINE

(2) THE SAME SURGE AFTER TRAVELING SEVERAL MILES AND DISTORTED IN A MANNER DESCRIBED ABOVE

Such a study would be impossible without the study of artificial surges on actual transmission lines as Mr. McEachron has carried out in Michigan during the past few years.

The general conclusion is that a large component of the observed phenomena of traveling surges on overhead lines, including attenuation, distortion, etc., can be attributed to a change in line constants in the presence of corona, the latter merely causing various sections of the wave to travel at speeds corresponding to the constants at that point and time.

In this concept it is true that these changes will *accompany* a certain amount of corona loss but that these changes are not caused by this loss.

L. V. Bewley: Mr. Fortescue states that the surge propagated over the line wire is entirely different from the surge impressed by the stroke, since it is modified by reflections at the tower, flashover of insulators, attenuation due to corona, and several other modifying influences. In a previous paper (Bibliography No. 3) I discussed the effect of successive reflections at some length, and gave lattices, equations, and curves to illustrate their influence in the case of both direct and induced surges. Mr. Fortescue's conclusions are in agreement with that study.

Mr. Rorden has just completed an experimental verification of the cases given in Tables I, II, and III of the paper, and in every case has found excellent detailed agreement between the calculations and the cathode-ray oscillograms, of which his Fig. 2 is an example. The results of his study are to be published in the near future; and will then provide the most complete and conclusive evidence available as to the validity of the methods of calculation. It is a fortunate circumstance that transmission line terminal equipment may be replaced by comparatively simple equivalent circuits (to traveling waves) which are amendable to calculation. The errors introduced by the neglect of the distortion of the traveling waves are on the safe side.

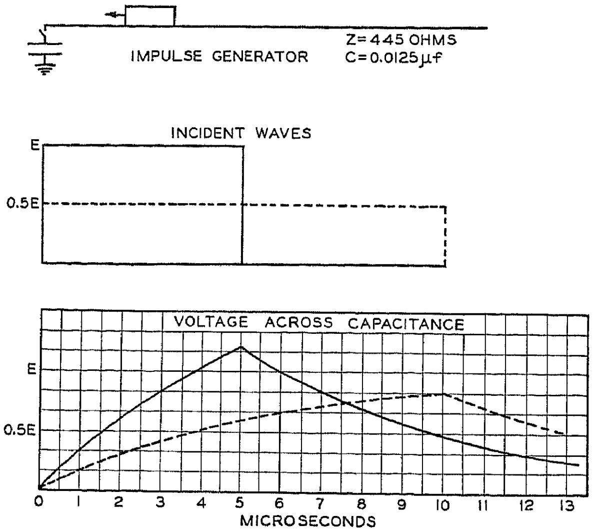

Fig. 5—Effect of Rectangular Waves of Different Shape

Mr. Weber takes me to task for writing a dimensionally unbalanced equation. It should have been "scientifically" written as

$$L = x + v\,t$$

where v is the velocity of propagation. However, if x is in thousands of feet and t is in microseconds, then

$$v = 1 \text{ (\textit{thousands of feet}) per (\textit{microsecond})}$$

and numerically

$$L = x + t$$

where L is in *thousands of feet* if measured on a *space* axis, or in *microseconds* if measured on a *time* axis.

Mr. Weber calls attention to an article by W. Deutsch (1918) as having been the first to represent different wave shapes by exponential functions. The idea of expressing any function (with certain restrictions relative to a finite number of discontinuities, etc.) over a finite range, as a series of exponentials is quite old. It is the basis of the *Euler-Maclaurin formula* (1730-1740). As a matter of fact, the Heaviside expansion theorem itself yields the solution in terms of a series of exponentials. The specification of an impulse (my Fig. 3D) as the difference of two exponentials appears in Steinmetz's books. Letting the parameters take on different limiting values in order to describe a few wave shapes is such an obvious thing to do that I did not consider it worth while to search the literature in order to anticipate any claim of priority.

I would like to remark that the exponential occupies a position of importance relative to operational calculus which is comparable to that of the sinusoid in Fourier analysis. The reason for this, of course, is because the exponential is the natural solution to the general differential equation with constant coefficients. It is therefore a great help to be able to express the applied forces as exponentials.

James C. Dowell: In explaining the low rate of attenuation, Mr. Ackermann stated that an error might be introduced by assuming the wrong shape of free traveling waves in the calculations. These free waves were calculated in each case, using the constants of the impulse generator and the transmission line and were not assumed in calculating the voltages across the impulse generator.

In regard to the question as to whether a wave twice as long and half as high would produce the same voltage across the impulse generator, it would seem that this could not be the case. The following equations were derived for two rectangular waves, one being half as high and twice as long as the other:

$$e = 2\,E\,(1 - \epsilon^{-t/zc}) \text{ for } t \leqq L = \text{length of wave}$$

$$e = 2\,E\,\epsilon^{-t/zc}\,(\epsilon^{L/zc} - 1) \quad \text{for } t \geqq L$$

Both the incident waves and the voltage waves across the impulse generator are plotted in Fig. 5. Waves of five- and ten-microsecond length respectively are compared. The ten-microsecond wave reached only 70 per cent of the crest value of the shorter wave, indicating an appreciable difference in the voltage across the impulse generator with this modification in wave shape. While the above is only a special case, the length of the waves and the constants used are comparable with those used in the tests and it would seem to be a fairly representative case. The possibility of getting the same voltage across the impulse generator with widely different wave shapes seems to be slight.

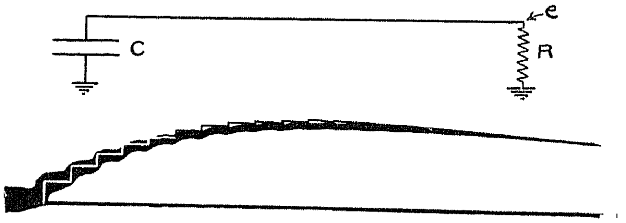

Fig. 6—Successive Reflections between a Capacitance and a Resistance

Since the above tests were made many oscillograms have been taken of traveling waves reflecting from various combinations of terminal impedance. Extensive calculations have been made for each condition and they check the oscillograms very closely in each case. An oscillogram of the voltage across a resistance placed at the end of a 600-ft. line with a long impulse wave applied is shown in Fig. 6. Note the agreement between the calculations and the oscillogram. It all goes to show that the behavior of traveling waves on transmission lines can be represented fairly simply by mathematical equations.

Impulse Tests on Substations

BY R. N. SOUTHGATE,* Associate, A. I. E. E.   A. S. BROOKES,* Associate, A. I. E. E.   E. R. WHITEHEAD† Associate, A. I. E. E. and W. G. ROMAN† Associate, A. I. E. E.

Synopsis.—*This paper presents a description and discussion of a series of impulse tests on two substations of the Public Service Electric and Gas Company's system.*

The 220-kv. substation of the Roseland switching station was surged through eight miles of overhead line. A study of the effects of various amounts of connected bus and station apparatus was made. The effect of different arrester locations on the protection afforded to the station insulation was determined.

The Elizabethport 26-kv. substation was surged through five miles of overhead line and two miles of cable. The effects of various arrester locations, both on the line and in the substation were determined. The attenuation in and surge impedance of the cable were measured.

* * * * *

DURING the last few years a great deal of experimental work has been accomplished in studying the nature of disturbances due to lightning, and in the determination of transmission line characteristics, in order to forecast the behavior of transients of this nature on the transmission system. The experimental work to date on terminal equipment has been largely confined to the study of individual units.[1,2] In the summer of 1930, the opportunity to study surge behavior on typical substation layouts presented itself, and the tests described in this paper are the result of cathode ray oscillograph work in this field.

Protection to terminal apparatus is even more important to the operating engineer than protection to line equipment, since higher installation and replacement costs are involved, and the hazard to service is in most cases greater. An effort was made to check graphically by means of these tests some of the theories generally accepted in regard to the action of a traveling wave striking typical substation structures, in the expectation that the results might serve to clarify the general protection problems. It was realized that actual layouts vary greatly, and that test results must depend largely upon the type of station under investigation and upon the chosen impulse wave. For these reasons, the results secured are largely qualitative in nature.

Two types of substation structures were employed. The first series of tests was made on the 220-kv. bus and transformers at the Roseland Switching Station of the Public Service Electric and Gas Company, a typical substation of the high-voltage class; and the second, on the structures of the Elizabethport 26.4-kv. substation, which is of the distribution type. The results are divided under these two locations.

TESTS AT 220-KV. SWITCHING STATION

The Roseland Switching Station is the terminal point of two 220-kv. interconnections. The station feeds power to a 132-kv. transmission ring which connects the other switching stations of the system. At the time the tests were made, the 132-kv. equipment was in service. The 220-kv. structures, while complete, had not yet been placed in operation, pending completion of the transmission lines. A schematic diagram of the bus layout is shown in Fig. 1. For the purpose of these experiments, only one line and one bank of transformers were available for connection to the station bus structure and oil circuit breakers.

Fig. 2 illustrates the type of station construction. The transmission line is terminated at an A frame. The conductor distance from this point to the line disconnecting switches is 290 ft., and that from the bus disconnecting switches to the transformer 330 ft. Main and transfer buses are aluminum tubing 270 ft. in length. The transformer bank consists of three single-phase 30,000-kva. oil immersed self-cooled units. These were connected wye-wye with solidly grounded neutral on both 220-kv. and 132-kv. windings.

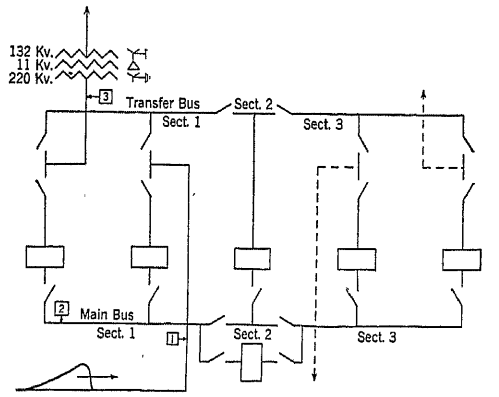

FIG. 1—ROSELAND 220-KV. SWITCHING STATION

The high-voltage surge generator and the cathode ray oscillograph used in the tests have been described in previous papers. The equipment is essentially the same as that used at Stillwater, New Jersey in 1929.[3,5]

The voltage impressed upon connected apparatus in a station depends upon the magnitude of the incoming traveling wave; the surge impedance of the transmission line; and the complicated impedance network presented by the station bus, the apparatus itself, and other lines common to the terminal. This voltage is expressed by the equation:

$$E_t = \frac{2\, Z_t\,(p)\, E_1}{Z_t\,(p) + Z_1} \qquad (1)$$

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*Public Service Electric and Gas Company, Newark, N. J.

†Westinghouse Electric and Mfg. Company, East Pittsburgh, Pa.

1, 2. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

where E_t = terminal voltage
E_1 = traveling wave voltage
Z_1 = line surge impedance
$Z_t(p)$ = generalized terminal impedance

Maximum voltage is measured at the terminal when the line is disconnected from all apparatus, bus, and other lines; $Z_t(p)$ then being simply that of the measuring system employed to record the transient. If this be a potentiometer of sufficiently high resistance, the error involved is in most cases negligible. Where necessary, the formula may be used to determine the correction. Using the open-end voltage as a basis for comparison, the effect of the bus and transformer upon the terminal voltage was investigated.

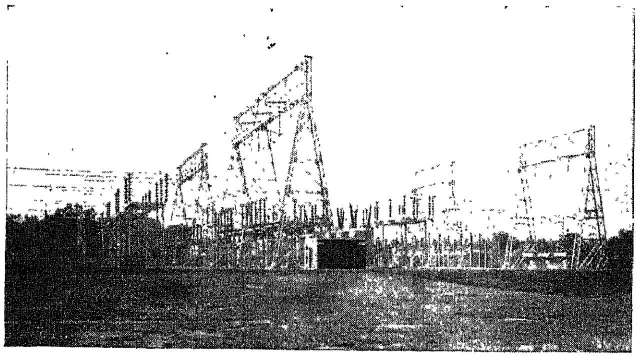

FIG. 2—ROSELAND 220-KV. SWITCHING STATION

The Roseland 220-kv. switching station was surged through an eight-mile loop of the transmission line, the surge generator being adjusted to produce a traveling wave at the station having a crest of 169 kv., a front of 5.5 microseconds and a time to half value of 19 microseconds. A very short surge of 4-microsecond length and 140-kv. crest was obtained by chopping this wave with sphere gaps near the surge generator.

TABLE I

Station conditions	Per cent reduction of crest kv. 19 microsecond wave	4 microsecond wave
Section 1 transfer bus	4.5	
Transformer connected	11.2	32.0
Section 1 main bus line and transformer breakers	5.6	
Transformer connected	14.0	45.0
Complete main and transfer buses all breakers	24.0	
Transformer connected	35.0	65.0

Volt-time oscillograms were taken at the line end (1) of Fig. 1 with the transformer disconnected and at the transformer bushing (3) for each of the following station conditions:

(a) Line connected to section 1 transfer bus.

(b) Line connected to section 1 main bus, line and transformer circuit breakers.

(c) Line connected to complete main and transfer bus, and all circuit breakers.

Figs. 3 and 4 show oscillograms of the terminal voltage for the station conditions indicated. In general, the transient is lengthened and its crest lowered by the station network. Table I gives the reduction in crest voltage in per cent of the open end voltage. It will be noted that this effect is considerably greater for the short wave than for the long wave. Conversely, the effect would be less for longer waves.

Measurements made at positions (1) and (3) showed no difference in crest voltage for the same station conditions.

ARRESTER LOCATION

The correct position for lightning arresters in large substations has been a matter of much discussion. If only one arrester be connected to a single bus leading directly to a transformer, the voltage at the arrester location will be a minimum; while that at any other point will depend upon the arrester voltage, the rate of rise of the traveling wave, and the distance from that point to the arrester. Assuming full reflection, the transformer voltage is expressed by the following relation when $\frac{d\,e_1}{d\,t}$ can be taken as uniform up to the arrester breakdown voltage, and $e_t \mathrel{\overset{=}{<}} 2\,e_a$.

$$e_t = e_a + \frac{2\,d\,e_1}{d\,t}\,\frac{D}{1000} \qquad (2)$$

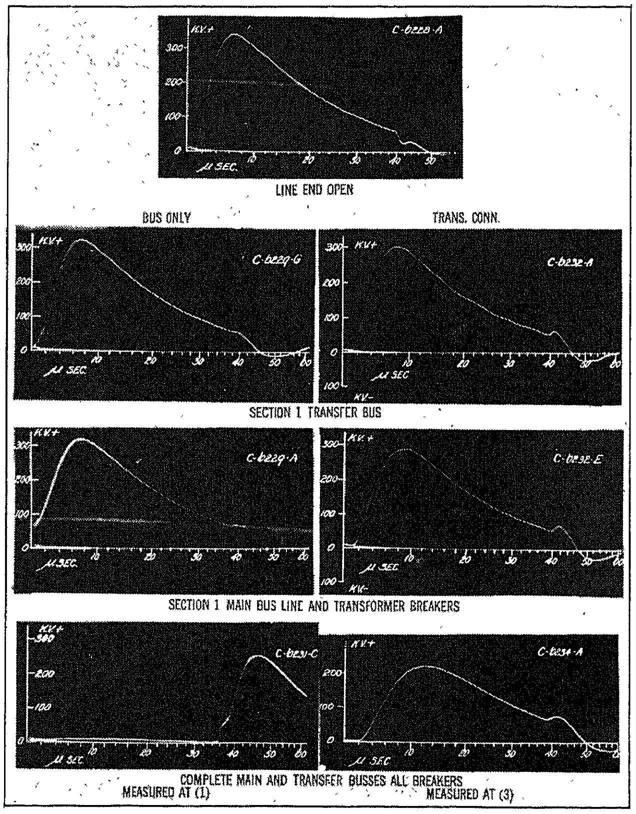

FIG. 3—OSCILLOGRAMS SHOWING THE EFFECT OF TERMINAL EQUIPMENT ON THE STATION VOLTAGE

where e_t = voltage at the transformer in kv.
e_a = arrester voltage in kv.
e_1 = traveling wave on the bus in kv.
D = distance from arrester to transformer in ft.

$$\frac{d\,e_1}{d\,t} = \text{kilovolts per microsecond}$$

Fig. 5 shows oscillograms of the transformer voltage for three different locations of the arrester. The arrester used in these tests was a 25-kv. L V porous block unit, located at 0, 595, and 920 ft. from the transformer, positions (3), (2), and (1) Fig. 1.

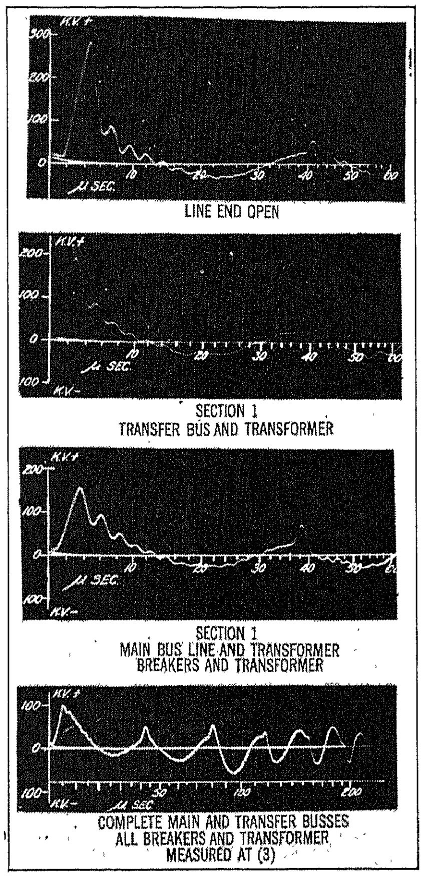

Fig. 4—Oscillograms Showing Effect of Terminal Equipment on Short Wave

The curve of transformer voltage plotted against the distance in feet from the transformer to the arrester is shown in Fig. 6. With $\frac{d\,e_1}{d\,t}$ and e_a taken from the oscillograms, the calculated values check the oscillograph measurements.

If D is increased to the value

$$D' = \frac{1000\,e_a}{2\,\frac{d\,e_1}{d\,t}} \tag{3}$$

$$e_t = e_a + \frac{2\,d\,e_1}{d\,t} \cdot \frac{1}{1000} \cdot \frac{1000\,e_a}{2\,\frac{d\,e_1}{d\,t}}$$

$$= 2\,e_a$$

that is, the transformer voltage will reach twice arrester voltage. This is the maximum for e_t regardless of the position of the arrester, and the relation fails when $D > D'$.

From Fig. 6, the rise in transformer voltage over arrester voltage for a distance of 350 ft. is 25 kv. This is based upon the front of the test traveling wave which rose at the rate of 43 kv. per microsecond. If the front of the lightning transient be taken as 150 kv. per microsecond, and the 25 kv. rise arbitrarily considered allowable, the arrester must be placed at only

$$D = \frac{25 \times 1000}{2 \times 150} = 83 \text{ ft.}$$

from the transformer.

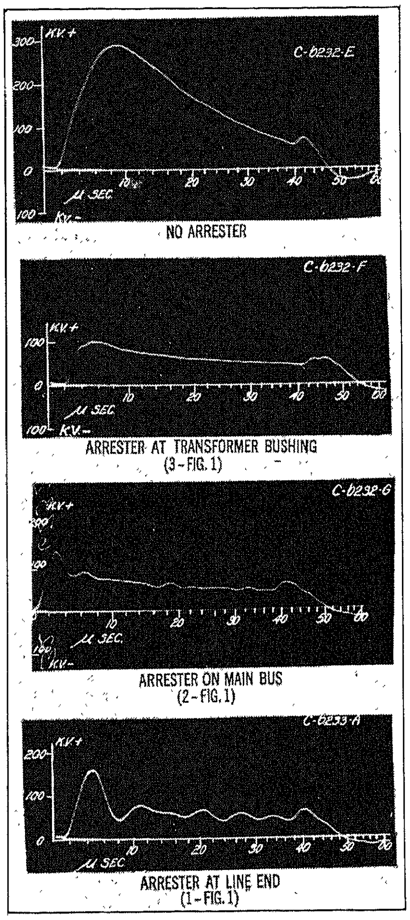

Fig. 5—Oscillograms Showing Effect of Arrester Location on Voltage at Transformer Terminal

As e_a decreases corresponding to the particular application, the per cent increase of transformer voltage over arrester voltage rises for any given arrester position. As an illustration we may take the following cases:

220-kv. application

$$D = 600 \text{ ft.}$$

$$\frac{d\,e_1}{d\,t} = 150 \text{ kv. per microsecond}$$

$$e_a = 710 \text{ kv.}$$

$$\text{Kv. rise} = \frac{2 \times 600}{1000}\,150$$

$$= 180 \text{ kv.}$$

$$= 25.4 \text{ per cent rise over } e_a$$

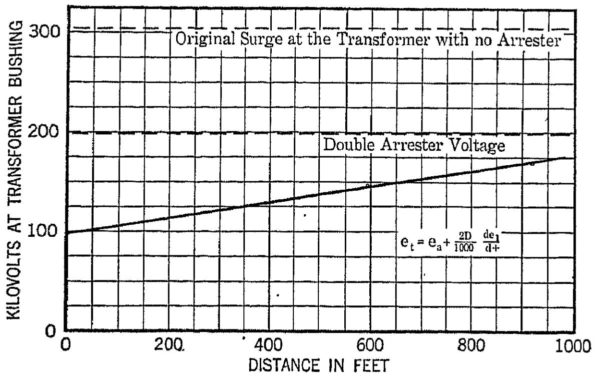

Fig. 6—Curve of Switching Station Transformer Voltage Against Distance of Arrester from Transformer

25-kv. application

$$D = 600 \text{ ft.}$$

$$\frac{d\,e_1}{d\,t} = 150 \text{ kv. per microsecond}$$

$$e_a = 100 \text{ kv.}$$

but

$$D' = \frac{1000 \times 100}{2 \times 150}$$

$$= 333 \text{ ft.}$$

Here the distance is evidently greater than D'; hence the transformer terminal voltage will reach twice arrester voltage.

Tests at 26.4-Kv. Substation

The Elizabethport Substation is designed to receive energy at 26,400 volts, for distribution at 4,150 volts. The type of construction is illustrated in Fig. 7, and a schematic diagram of the 26.4-kv. bus layout in Fig. 8. The completed substation will provide for eight 26-kv. feeders entering in cable. Two of these were available for test purposes. Two banks of transformers were used, the first consisting of two 4,000-kva., three-phase oil cooled units, the second a single 4,000-kva., three-phase unit. The 26-kv. substation bus is 150 ft. in length.

The line used in the impulse tests extends a distance of 31,370 ft. in aerial conductor on wood pole construction from the power station to a point 9,600 ft. from the substation, where it enters underground cable leading to the substation bus. The cable is of the belted type, 350,000 cm. with $\frac{10 \times 5}{32}$ paper insulation. The surge generator was located at the far terminal of the open wire in order to secure sufficient distance to establish a traveling wave effect at the cable pothead; that is, the point of junction of open wire and cable.

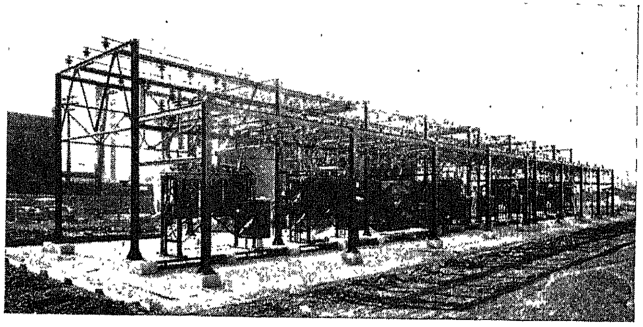

Fig. 7—Elizabethport 26.4-Kv. Substation

The cathode ray oscillograph was located at the substation where it could be connected to any point in the outdoor structures.

The inaccessibility of the cable pothead location prevented a measurement of the wave entering the cable by means of the oscillograph. In order to determine the wave front at this point the cable pothead was grounded and sphere gap measurements were made at points several hundred feet apart and increasingly distant from the pothead. This procedure was continued until two equivalent readings indicated that the crest value of the wave had been reached. By this method the breakdown voltage of the gap plotted against twice the distance from the gap to the pothead, expressed in microseconds or time of wave travel, gives a point on the approximate curve of wave front.

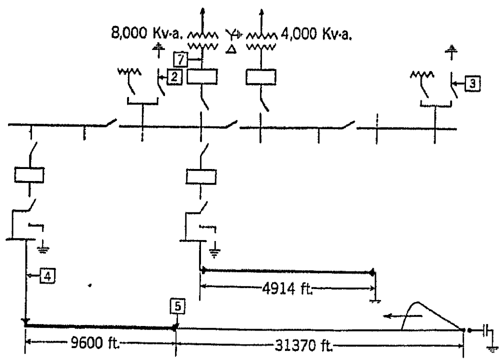

Fig. 8—Elizabethport 26.4-Kv. Substation and Lines

Fig. 9 shows the front of the incident wave on the open wire plotted from the sphere-gap measurements. This wave has a crest value of 136 kv. and a time to crest of 2.7 microseconds. The tail of the wave could not be determined except by comparison with the wave after arrival at the station end, where it was recorded by the oscillograph. Fig. 10, oscillogram C-6250-D, shows the wave measured at the transformer terminal.

It was next desired to determine the attenuation of the traveling wave in the 9,600-ft. length of cable. The incoming test feeder was connected through the station bus to the second cable. Thus measured, the crest voltage of the traveling wave in the substation was 16.4 kv. The three conductors of one cable were connected in series and a measurement made of the attenuation in twice its length. This method showed a reduction in crest of 3.4 kv. for 19,200 ft.

To find the surge impedance of the cable, the feeder was connected to ground through a resistance of known value. An oscillogram of the voltage across the resistor was taken. From this measurement and that of the traveling wave the surge impedance of the cable was calculated to be 30.2 ohms.

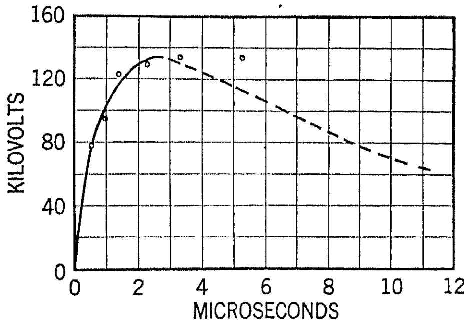

FIG. 9—TRAVELING WAVE ON THE LINE, PLOTTED FROM SPHERE GAP MEASUREMENTS (CURVE)

Knowing this value, the surge impedance of the open wire under test conditions was calculated to be 447 ohms.

TERMINAL CONDITIONS

The effect of connecting bus and transformers to the cable is to reduce the crest voltage of the incident wave below the open end value. This reduction is a function of both the station network impedance and the surge impedance of the entering conductor, as noted in the discussion of the Roseland tests. By reference to (1) it may be seen that for a given terminal impedance, the ratio of the terminal voltage, E_t, to the open end voltage, $2\,E_1$, will increase as the conductor impedance, Z_1, decreases in magnitude. A less pronounced modification at Elizabethport would therefore be expected due to the lower impedance of the cable as compared with that of the line at Roseland.

Fig. 10, oscillogram C-6250-D, shows the wave measured at the transformer bushing. By comparison with the measurement made at an open end, the crest voltage is found to have been reduced from 32 kv. to 30.5 kv. The difference of 1.5 kv. represents a modification of about 5 per cent, as compared to the 35 per cent reduction at the Roseland 220-kv. switching station with the entire bus and transformers connected.

Measurements made at the transformer bushing and at the cable terminal showed no noticeable difference in wave shape between these two points.

Under actual operating conditions several 26 kv. cables will be connected to the station bus. The effect of these added impedance paths in parallel will be to lower the equivalent total impedance of the station, thus reducing the impressed voltage on the station equipment. Fig. 11, oscillogram C-6251-B, shows that the second cable connected to the bus reduces the crest value of the wave at the station to 16.4 kv., from its original crest of 30.5 kv. This second cable was grounded at the far end.

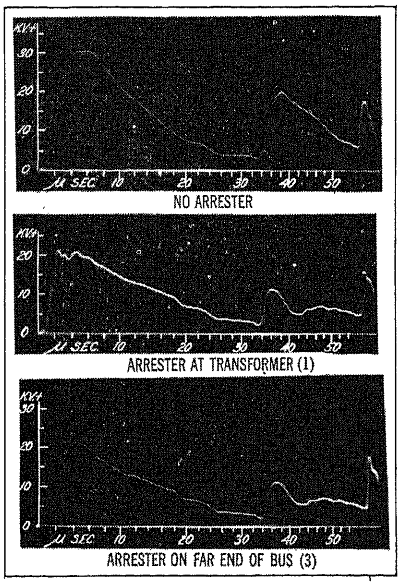

FIG. 10—OSCILLOGRAMS SHOWING THE EFFECT OF ARRESTER LOCATION ON THE VOLTAGE AT THE TRANSFORMER, ELIZABETHPORT SUBSTATION

STATION PROTECTION

In the study of protection of station equipment, lightning arresters were placed at several different locations within the station and the voltage at the transformer bushing measured for each location. Due to limitations on voltage imposed by the insulation of the open wire line, the maximum test voltage obtainable at the station was about 30 kv. For this reason the arresters used were standard 6-kv. station type, which

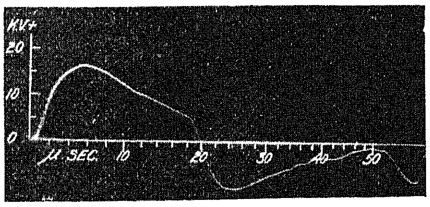

FIG. 11—OSCILLOGRAM SHOWING THE EFFECT OF CONNECTING A SECOND CABLE TO THE ELIZABETHPORT SUBSTATION BUS

had a breakdown voltage sufficiently under the incident crest voltage to secure comparative results.

Referring to Fig. 8, the arrester locations chosen were, as marked on the diagram; (1) the transformer bushing, (2) the main bus 90 ft. distant from the transformer, (3) the main bus 150 ft. distant from the transformer, and (4) the cable pothead 160 ft. distant from the transformer. The maximum voltages recorded at the transformer with the arresters located at positions (1), (2), and (3), were 21, 23.5, and 25 kv. respectively. The oscillogram obtained at position (4) showed a rise in transformer voltage too great for the distance involved, and cannot be explained except by the possibility of an erratic breakdown of the arrester due to the slight margin of incident voltage over the arrester breakdown value. The results for this position are therefore omitted.

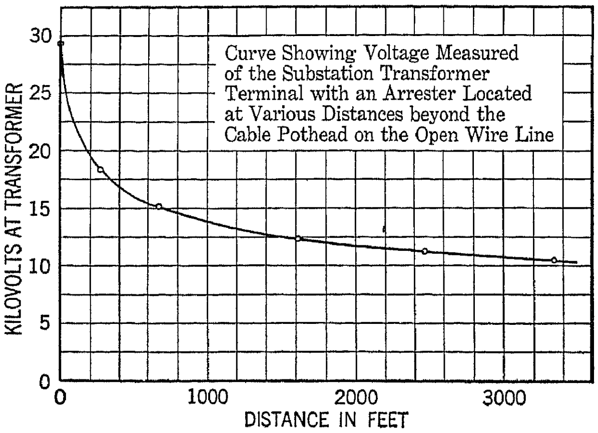

FIG. 12

Fig. 10 shows oscillograms of the voltage with no arrester and of the resulting voltage with the arrester located at position (1) and at position (3), which are the positions nearest to, and farthest from the transformer, respectively.

The difference in crest of four kv. is, then, the maximum to be expected with a wave of front similar to that of the test wave, having a rate of rise of 10 kv. per microsecond. Under extreme conditions the rate of rise to be expected on a lightning transient would be in the order of 150 kv. per microsecond. The greatest conductor distance probable in this type of substation is 200 ft. Referring to (2) an approximation of maximum station voltage variation due to arrester location may be obtained. For the conditions described this value becomes 60 kv. As shown by Fig. 10, oscillogram C-6251-F, such rise in voltage over the arrester breakdown value takes a shape similar to the first ripple on the wave crest.

The foregoing test results were obtained with only one cable tied to the bus. A second cable was available, but when this was connected, the voltage on the station was reduced below the point of arrester breakdown. Under normal operating conditions, the several cables feeding the bus will form a junction of parallel impedances; equivalent to a lumped impedance of considerably lower value than that of an incoming surge path. For this reason, a surge due to a lightning disturbance outside the station would, upon arriving at the station bus, be considerably reduced in magnitude because of the lower impedance at this point.

STATION PROTECTION BY LINE ARRESTERS

Another problem under investigation at Elizabethport was the determination of an effective location of an arrester at or beyond the junction of cable and open wire. The locations chosen for test were at (5), Fig. 8, the cable pothead, and at distances of 267, 668, 1,610, 2,470, and 3,340 ft. from this point.

For these tests the arresters used were the 6-kv. line type. Fortunately ground conditions along the open wire line were good; arrester ground resistances varying in a narrow range of 0.5 to 1.5 ohms.

Fig. 12 is a curve showing crest voltage at the transformer plotted against distance in feet from the cable pothead to the arrester location. Due to the decreased conductor surge impedance at the cable pothead, the voltage at this point was insufficient to cause breakdown of the arrester. The station voltage decreases as the arrester is moved away from the pothead, the slope of the curve decreasing considerably at about the 1,000-ft. point.

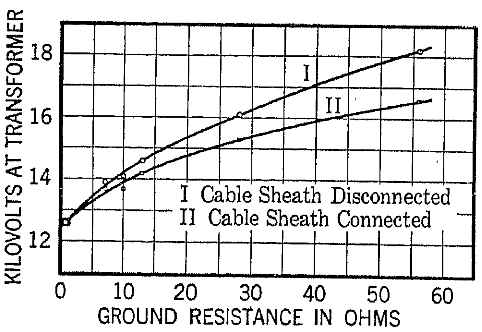

FIG. 13—COMPARATIVE STATION VOLTAGES WITH VARYING GROUND RESISTANCES, WITH AND WITHOUT A CONNECTION FROM THE CABLE SHEATH TO THE ARRESTER

Arrester placed 1,610 ft. from pothead

Maximum voltage at the station occurred when the arrester was located at position (5) Fig. 8, the cable pothead, since the voltage at this point was less than the arrester breakdown voltage. In this case the maximum voltage at the pothead may be expressed by (1), where $Z_t\ (p)$ is replaced by the surge impedance of the cable. Considering the station bus as an open end, and neglecting attenuation in the cable, the maximum station voltage is twice the voltage at the line pothead. Had the crest voltage of the incident wave been high enough to cause the arrester to break down, the maximum station voltage would approach twice the arrester terminal voltage.

Minimum station voltage occurs when the arrester is moved a distance from the pothead, expressed in time of wave travel, equivalent to one-half the duration of the incident wave from zero to the point of arrester cutoff on the tail. The station voltage then approaches double that appearing in the cable as a result of a traveling wave on the line equal to the arrester voltage. Substituting the test values, a minimum station voltage of 5.3 kv. would be expected with the arrester placed about 11,000 ft. from the pothead.

Effect of Ground Resistance on Line Arrester Operation

The grounds used for the arresters in these tests were of exceptionally low resistance. Since such conditions are not always obtainable in practise, measurements were made to determine the effect of increasing ground resistance on arrester performance, and to compare these results with those obtained when the arrester was also connected to the cable sheath. Curve I of Fig. 13 shows the crest voltages measured at the substation transformer with the arrester on the line 1,610 ft. from the cable pothead, for resistances in the ground lead up to 55 ohms. Curve II, shows the voltage under the same conditions with an additional ground connection consisting of a wire laid on the ground from the arrester to the cable sheath.

Summary

1. The capacity to ground of the bus and circuit breakers in large substations tends to reduce the terminal voltage. In general, the effect of connecting the transformer to the bus system is further to slope off the front and reduce the crest. The total effect is more pronounced in the case of very short waves, but appears appreciable for those of moderate length.
2. When lightning arresters are placed on the bus, the voltage at the transformer will be greater than that at the arrester by an amount depending upon the steepness of the lightning transient, the arrester voltage, and the circuit distance from the arrester to the transformer. Oscillograms show results in agreement with calculations. The seriousness of such overvoltages decreases with increasing arrester voltage rating for any given arrester location.
3. The data indicate that the attenuation in cable of surges of the voltages used in these tests is of relatively small magnitude.
4. In small substations fed by cables the capacity of the bus and connected apparatus causes a relatively small decrease in the terminal voltage. This effect is slight because of the lesser terminal capacity and the low surge impedance of the entering feeders.
5. In small substations, no appreciable difference in surge wave shape is found at various points within the station.
6. A large measure of protection is afforded substations by a number of connected high-voltage cables.
7. In the case of a lightning arrester placed on a line joining a long cable leading to a substation, the station protection is increased as the arrester is moved away from the junction along the open wire. This is true only for a surge originating beyond the arrester. The greatest increase in protection occurred in the first thousand feet with the wave shape used. The exact location must be selected after weighing the possibility of a stroke between the pothead and arrester.
8. High ground resistance is found to decrease materially the protective value of line arresters. In the case of very high resistance grounds, arrester performance can be improved by running a connection from the high resistance ground to the cable sheath. Economic considerations will decide whether such a measure should be adopted.

Acknowledgment

The authors wish to express their appreciation to Mr. J. J. Torok for his suggestions in planning these tests, to Mr. H. K. Sels for his help with the paper, and to Messrs. J. M. Reel and W. A. Merriam for their assistance during the tests and in the analysis of the data.

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Discussion

For discussion of this paper see page 583.

Lightning Investigation on a Wood Pole Transmission Line

BY R. R. PITTMAN* and J. J. TOROK†
Associate, A. I. E. E. Associate, A. I. E. E.

***Synopsis.**—The purpose of this paper is to report the results of an investigation made on a wood pole line. The entire line was originally very highly insulated. The splintering of the structures by lightning indicated that excessively high voltages were appearing quite frequently. Because of the high insulation a cathode ray oscillograph station was installed on the line. A total of 38 significant surges was recorded, two of these being a five million and a four and one-half million-volt surge. Analysis shows that these high surges were the results of lightning bolts striking the lines causing faults between conductors but not to ground. The data also indicate that harmful effects are caused only by direct strokes. These and current measuring records indicate that currents varying from very low values to 100,000 amperes may readily be obtained.*

Two methods of protection are dealt with, one utilizes a fuse, the other an expulsion lightning discharge device. The new device operated eight times, all of these operations being successful.

INTRODUCTION

THAT wood possesses a certain insulating quality to impulse voltage has been recognized by a number of investigators,[1,2] and a considerable amount of laboratory work has been done to determine quantitatively this value.

The use of this insulating property of the wood to reduce transmission line outages resulting from lightning flashovers has been attempted. A number of transmission lines so designed has failed under actual operating conditions to give the results desired. That the amount of insulation inherently present in a wood structure is insufficient to prevent lightning flashovers has been repeatedly demonstrated.

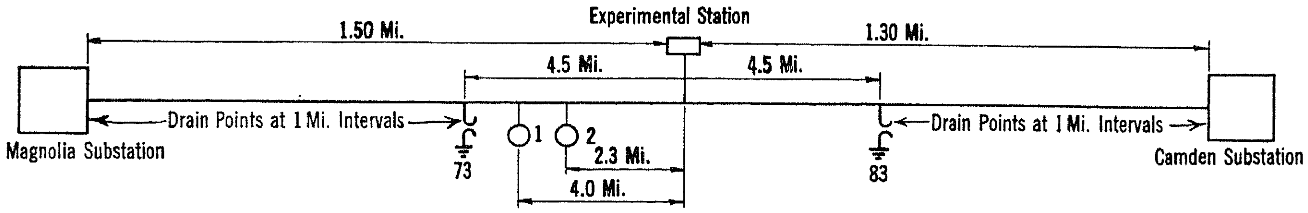

FIG. 1—DIAGRAMMATIC ARRANGEMENT OF LINE AND OSCILLOGRAPH STATION

1. Structure No. 167 2. Structure No. 153

As a result, investigation of the character of voltages causing the flashover of the entire length of wood supporting structures was considered of great importance and interest, entailing the measurement of voltages considerably above the flashover value of the usual type of transmission line. Previous investigations,[3,4] of a similar nature have been made on steel tower lines, where the magnitude of the voltages to be measured has been limited to the flashover value of the insulator strings.

DESCRIPTION OF LINE

A 110-kv. line, 28 miles in length, extending from Camden to Magnolia in southern Arkansas, was selected for the investigation. No guyed structures were installed except at angles in the line. The customary storm guying of tangent structures at frequent intervals along the line was entirely omitted.

The oscillograph station was located at about the midpoint of the line as shown in Fig. 1 to avoid the recording of reflected voltage waves. For a distance of four and one-half miles on either side of the station no guying of any kind was installed, the angles in this section of the line being supported by bracing the structure with wood poles. Other angle structures beyond this section were insulated from ground by means of an 18-ft. length of wood in each guy wire.

So-called drain points, equipped with fused horn gaps from each phase to ground, (Figs. 2 and 3) were installed at one mile intervals along the line, except in the nine-mile section in which the oscillograph station was located. All guyed structures were made drain points and the fused horn gaps on the poles (Fig. 2) performed the additional function of protecting the wood guy insulators from damage. The performance of these drain points assisted somewhat in coordinating the flashovers and circuit breaker operations with the oscillograph records. The line conforms to the following brief specifications:

Conductor	4/0 A. C. S. R.
Poles	50 ft. creosoted southern yellow pine
Arms	6 in. by 8 in. by 29 ft. creosoted southern yellow pine
Height of conductor at structure	37 ft. (approx.)
Average height of conductor	32 ft. (approx.)
Conductor spacing	14 ft.
Configuration	Flat horizontal
Insulators	6—10-in. suspension disks
Average span	600 ft.

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*Arkansas Power & Light Co., Pine Bluff, Ark.

†Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

1, 2. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

LINE HISTORY

As originally constructed, the 18-ft. length of wood in the guys necessary on angle structures was shunted by a 13 ft. 6 in. gap in each pole ground wire. This length of wood pole was damaged very severely by the lightning currents in a number of cases with little apparent decrease in line outages as compared with similar lines of conventional wood pole design. The horn gap arrangement was installed on these angle structures primarily to prevent damage to wood poles and the fuses added in an attempt to interrupt dynamic follow up current.

Such an arrangement prevented further damage to the guyed angle structures, but flashovers of the intermediate tangent structures were still rather frequent. These flashovers caused considerable damage to the structures in some cases by removing sections of the treated sap wood from the full length of the pole, and at the same time caused interruptions to service due to circuit breaker operation.

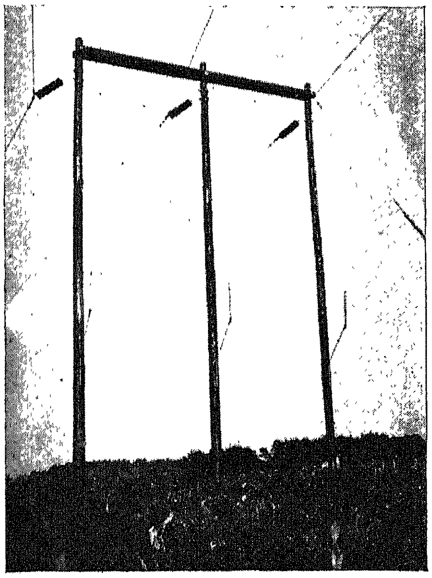

FIG. 2—ANGLE STRUCTURE ARRANGED WITH FUSED HORN GAP

In an effort to improve the operating performance of the line, as well as make possible better coordination of line flashovers with the oscillograph and circuit breaker operations, the number of these fused structures was increased by equipping certain tangent structures. As operating records indicated that the amount of damage to tangent structures had been considerably greater than on similar lines with uninsulated guys at one mile intervals along the line, it was hoped that the additional fused structures would prevent or reduce the damage to adjacent poles.

The substation protection at each end of the line consisted of a 20-in. gap between rings on the suspension insulators, with the top ring grounded to the station ground network through a 73-kv. five-ampere fuse. Such an assembly differs from the fused structure used on the line in that the line structures are provided with a five-foot air gap in series with the fuse and ground wire.

OSCILLOGRAPH STATION

The oscillograph station is in general very similar to those described in other papers,[3,5] and is shown in Fig. 4.

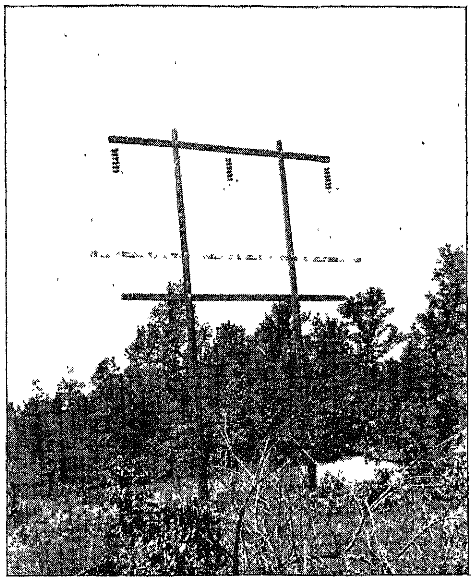

FIG. 3—MULTIPLE-FUSE HORN GAP ARRANGEMENT

The oscillograph equipment is of the latest type comprising a modified Norinder oscillograph, pumping equipment, timing system, oscillator, high-voltage supply and capacity potentiometer. The potentiometer is of special interest as it is designed to withstand potentials in excess of seven million volts. The high-voltage electrode is built up of fine copper screening stretched over a 20-ft. square pipe frame. It is believed that with an electrode of such size the corona formation on the edges will produce a relatively small increase in area, thus minimizing corona distortion. This upper plate is supported by four 60-ft. poles and four strings of insula-

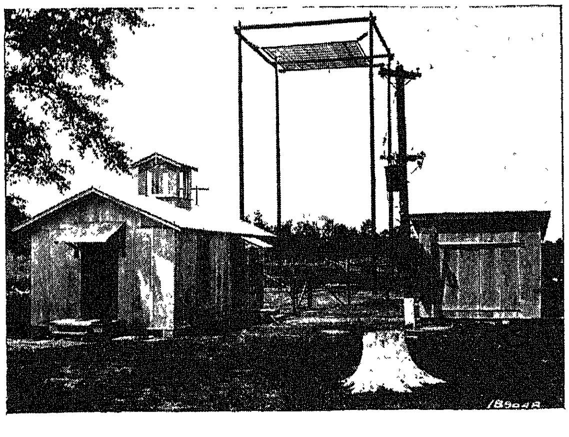

FIG. 4—VIEW OF OSCILLOGRAPH STATION

tors each comprising seven 10-in. suspension units. More recently this potentiometer has been revamped into a resistance potentiometer so that the advantages of a delaying cable can be realized.[6] The resistors and cable are coupled to the high potential electrode by a special porous block arrester which by its valve action permits only transients exceeding 125-kv. in crest to appear across the resistors.

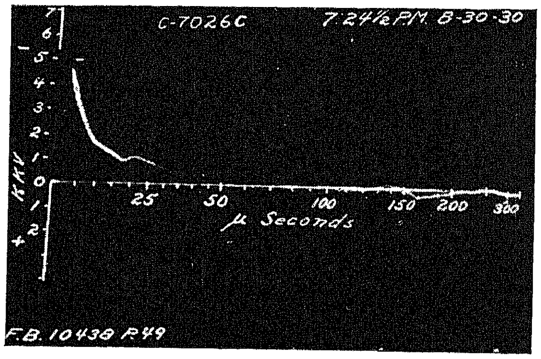

Fig. 5A—Oscillogram of Lightning Surge

Inasmuch as the potentiometer is connected to only one conductor it was deemed necessary to bond the cross arm hardware on the line structure adjacent to the station, making it possible for the high potentials appearing on either of the other two conductors to flash over the insulators and so appear on the potentiometer. To prevent an outage due to this bonding and for the purpose of records the connections were made through fuses.

Results Obtained at the Oscillograph Station

During the season a total of 38 oscillograms of voltages imposed on the line by natural lightning was obtained, of these only six were of special interest, the others being relatively low in magnitude. The highest voltage recorded was 5,000 kv. as shown on Fig. 5A and Fig. 5B. It is of interest to note that this voltage was transmitted along the line for a distance of four miles, this being the length of line between the oscillograph station and the point on the line subjected to the lightning stroke. This was a negative surge reaching its crest in something less than two microseconds, the maximum rate of rise shown by the record is in the order of 4,000 kv. per microsecond. Since this potential is well above the corona voltage of the line, it is to be expected from the results reported by Conwell and Fortescue that the rate of rise at the point of origin was much greater, perhaps as much as 10,000 kv. per microsecond. The surge originated near structure No. 167 which is four miles from the oscillograph station and one-half mile from structure No. 173, the nearest drain point. This location is designated in Fig. 1 as (1). The entire length of the cross arm was marked and both poles slightly splintered, indicating that all three conductors flashed over to ground, causing a circuit breaker operation at the same time that the oscillograph was tripped. The three fuses at structure No. 173 were blown. A flashover also occurred from the middle conductor to a telephone line 22 ft. below. The telephone line crossed the transmission line at right angles at a point 150 ft. from structure No. 167. The fuses on the structure adjacent to the oscillograph station were not blown and this, together with the splintered arm on structure No. 167, indicates that the potential of all three conductors was approximately the same.

The current determinations in the above surge must be limited to the crest of the wave measured by the oscillograph as flashover to ground occurred at several different points, and it is to be understood that the results will indicate only a fraction of the total current. Experimental results and calculations[9,10] indicate the approximate surge impedance of three conductors in parallel as 200 ohms. Therefore, the current of the surge as it passed the experimental station was $\frac{5,000,000}{200}$ or 25,000 amperes. Since this current is only one branch of the traveling wave set up on the line the total current, neglecting attenuation, would be 50,000 amperes. If attenuation is considered in a manner suggested by O. Ackermann the total current flowing on the conductors in both directions prior to flashover would range from 100,000 to 160,000 amperes.

Another interesting oscillogram is shown in Fig. 6A and a replot in Fig. 6B, also a negative surge reaching 4,500 kv. in about one microsecond. The voltage was transmitted from a point designated in Fig. 1 as (2) which is about 2.3 miles from the station. An entire structure was flashed over, and other conditions in general were the same as those under which the record shown in Fig. 5A was obtained.

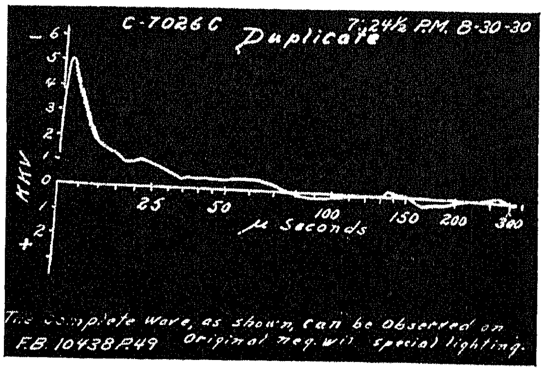

Fig. 5B—Replot of Fig. 5A

That there are certain voltages requiring considerable time for flashover is shown by the oscillogram in Fig. 7. Attention is also called to the positive reflection of about 1,000 kv. The oscillogram in Fig. 8 was about a 20-microsecond front, and a crest value of 1,000 kv.

There was no interruption to service on the line when the oscillogram in Fig. 9 was obtained, although the voltage at the oscillograph station was about 2,000 kv. The voltage responsible for this oscillogram probably flashed over the line at some point without establishing a 60-cycle arc. The surge shown in Fig. 10, though of exceedingly long duration, was not accompanied with circuit breaker operation. The oscillogram shown in Fig. 11 cannot be definitely explained or tied in with blown fuses or damaged structures as flashover did not occur. No damaged structures could be found which could be correlated with these latter five records.

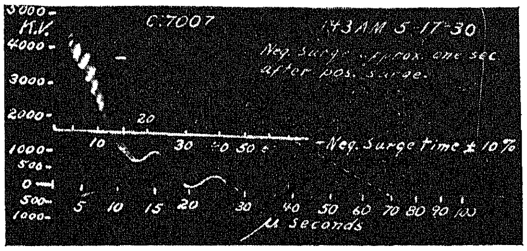

Fig. 6A—Oscillogram of Lightning Surge

All of the higher voltage surges were negative in polarity. Such positive surges as were obtained were small in magnitude, and probably without effect on the operation of the line.

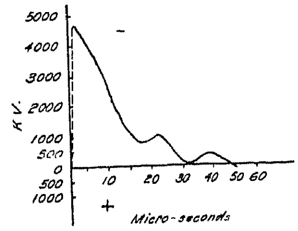

Fig. 6B—Replot of Fig. 6A

Direct Strokes

A number of lightning strokes to ground near the transmission line was observed by the laboratory operators during the season. In no case did such strokes affect the operation of the line, or induce sufficient voltage on the conductors to cause the operation of the oscillograph. A direct stroke 700 ft. from the oscillograph potentiometer flashed over several poles on a 33-kv. line. The station was in operation at the time, but the induced voltage was insufficient to trip the oscillograph circuit. At this time the oscillograph was adjusted to initiate operation at values slightly above normal crest line voltage.

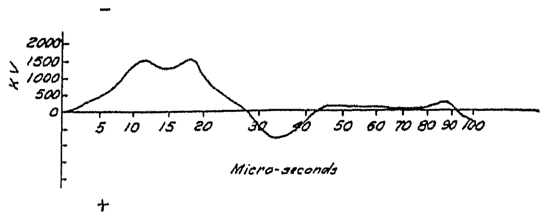

Fig. 7—Replot of Lightning Surge Oscillogram

Failure to obtain records on the oscillograph from these strokes near the line leads to the conclusion that any induced voltages from nearby lightning strokes are small in magnitude and of a harmless character to this type of transmission line.

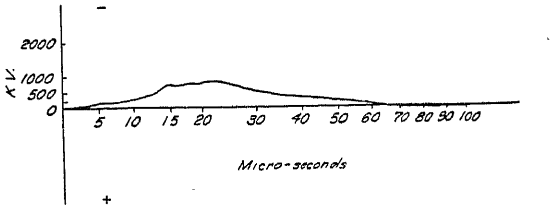

Fig. 8—Replot of Lightning Surge Oscillogram

Lightning Current Measurements

During the latter part of the lightning season klydonographs were placed on the wood poles to determine the current during flashover. About 10 ft. of the wood poles are shunted by a copper conductor in series with a zircon or carbon resistor having a resistance of about 0.15 ohms. The klydonograph is connected across the terminals of the resistor. This arrangement is shown in Fig. 12. The klydonograph which takes the structural form of a telephone receiver is mounted on a wooden box containing the resistor. By this arrange-

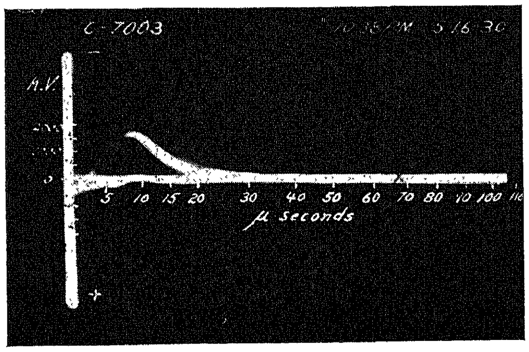

Fig. 9—Oscillogram of Lightning Surge

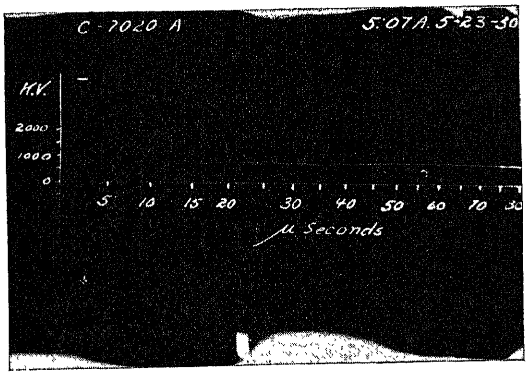

Fig. 10—Oscillogram of Lightning Surge

ment the current can be determined from the size of the klydonograph figure and the resistance of the zircon rod. Only one record was obtained and that at a drain point, when only one fuse was blown, indicating that the surge was very light. The klydonograph figure indicated a current value of 25,000 amperes. To this should be added the current flowing in each direction over the line conductor to obtain the total.

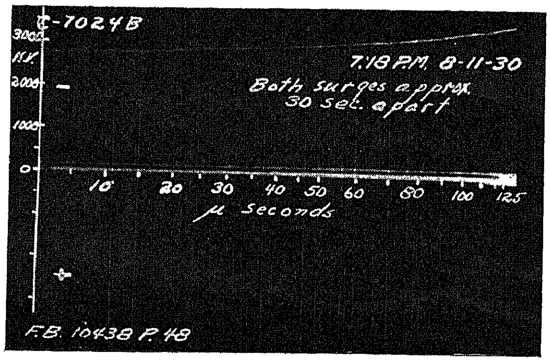

FIG. 11—OSCILLOGRAM OF LIGHTNING SURGE

OPERATING RESULTS

It was anticipated that the installation of the fused horn gaps in the ground circuit might act under certain conditions to discharge the current due to the lightning voltage and then quickly cut off the 60-cycle arc current, thus avoiding the operation of the oil circuit breaker with the attendant service interruption.

FIG. 12—KLYDONOGRAPH ARRANGEMENT FOR SURGE CURRENT MEASUREMENTS

The following tabulation shows the results obtained with 20 of these so-called drain points in the 28 miles of line.

Date	Fuses blown	Flashover on line not causing switch operation	Flashover on line causing switch operation
Aug. 4	2	2	0
Aug. 30	6	1	1
Sept. 3	2	1	0
Sept. 15	2	1	0
Sept. 26	1	1	0
Total	13	6	1

The flashover on the line incurring switch operation was caused by a stroke to the line near structure No. 167. The voltage on the line due to this stroke was measured at the oscillograph station as a 5,000-kv. surge. Circuit breaker operation was caused by a three-phase short circuit at structure No. 167. The same lightning stroke blew all three fuses at drain point No. 173. These results indicate that the protective effect of a drain point does not extend to one-half mile even with the high insulation used on this line.

The following tabulation shows where blown fuses were located with reference to structure numbers and conductors. It should be remembered that the average span on this line is 600 ft. The numbering begins at the Camden substation, and extends south. Structures 1 and 273 have no air gap in the ground circuit.

Struction No.	West Conductor	Middle Conductor	East Conductor
1	0	0	0
7	0	0	0
13	0	0	0
27	0	0	0
35	0	0	0
46	0	0	0
55	0	0	0
64	0	0	0
76	0	1	0
87	0	0	0
173	1	1	1
186	0	0	0
197	1	1	0
210	0	0	0
219	1	0	0
231	0	0	0
242	0	0	0
253	0	0	1
270	0	0	0
273	2	1	2
Total	5	4	4

The fuses used for this service were of the expulsion type, and rated by the manufacturer at five amperes, 73,000 volts. The maximum short-circuit kva. along the line investigated is about 175,000.

A single fuse in the ground circuit of a structure seemed inadequate, first, because many flashes of lightning are made up of a number of successive discharges which follow each other with very short time intervals between them, and second, because after the single fuse has blown no protection is available to prevent the operation of the oil circuit breaker in case the line is subjected to additional strokes.

In an effort to obtain some idea of the importance of the foregoing considerations, the multiple fuse arrangement shown in Fig. 3 was installed at six locations. This arrangement was intended to provide three fuses so disposed under each line wire that they would successively blow whenever the character of the lightning voltage was such as to cause the gap to flash over. Only four fuses were blown on this type of structure during the season, and these fuses were in each case the middle fuse of the group.

The results obtained from the fuses installed at drain points were quite encouraging, however the continuous patrolling and replacements became a large factor in the economic considerations. Accordingly, an expulsion type of lightning arrester was later devised independently by Pittman consisting of a fiber tube 38 in. long, into each end of which was inserted a wire electrode reducing the effective arcing length to 30 in., thus insuring that the breakdown would take place within the tube. The device was inserted without additional gaps in the hardware grounding lead. Thus the path of the surge would be over the line insulator to the cross arm hardware and from thence by copper wire to the expulsion arrester which would break down inside and conduct the surge to ground. The subsequent 60-cycle arc initiated by the surge is then quenched out by the expulsion effect of the fiber tube.

The operating record of this experimental expulsion form of arrester is quite encouraging. During the period of time they have been in service on another similar 110-kv. line, eight successful operations were recorded.

During the season a number of flashovers occurred but the total physical damage to the line has consisted in one broken insulator unit, three slightly splintered cross arms, and two slightly splintered poles. No conductor has fallen, and service has been immediately restored following each lightning flashover.

CONCLUSIONS

1. The frequent flashovers of crossarms and poles accompanied by circuit breaker operations indicate that insulation alone is not a solution to the lightning problem.
2. Voltages of high magnitudes may be transmitted for several miles along highly insulated transmission lines to terminal apparatus.
3. The experimental results of this investigation show that voltages of harmful character to high-voltage lines appear only when direct strokes or their branches terminate on the conductors.
4. The fronts of surges at their points of origin may have a rate of rise of 4,000 kv. or more per microsecond.
5. Tripouts on ungrounded wood pole construction of the type described in this paper are usually the result of phase-to-phase flashovers.
6. In this investigation lightning currents of more than 25,000 amperes are indicated.
7. Transients appearing on transmission lines can be limited to values within the strengths of terminal equipment by the use of suitable protective devices.
8. Limited field investigations indicate that an arrester of the expulsion form will successfully function to interrupt dynamic current follow up.

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Discussion

For discussion of this paper see page 583.

1929 Lightning Experience
on the 132-Kv. Transmission Lines of the American Gas and Electric Company

BY PHILIP SPORN*
Fellow, A. I. E. E.

Synopsis.—*A record of the 1929 lightning performance of a 1,200-mile 132-kv. transmission net workis given, and compared with the record of the system during the previous three years. The system is almost entirely of steel tower construction with conductors in vertical configuration; and on most lines one ground wire is employed.*

It is shown that some 75 per cent of high-voltage transmission line outages on this system are due to lightning. Lightning performance of lines being dependent on the severity of lightning conditions, a record of these conditions should be kept from year to year if records are to be compared.

Data are presented on relative yearly lightning intensity, line outages, damage found to insulators and hardware, performance of double circuit lines, lines with and without ground wire, and with two ground wires, and also with and without grading shields. The apparent part tower footing ground resistance plays in line outages due to lightning is also given.

The conclusions are drawn that

1. Two circuit lines are far more reliable, from a lightning point of view, than single circuit lines.

2. One ground wire offers considerable protection against lightning, and two ground wires considerably more.

3. Grading shields properly applied do not reduce line outages, but decrease the damage to insulators, hardware, and conductors.

4. Both direct and induced lightning strokes have to be considered on these lines.

5. Flashovers do not always concentrate on high tower footing resistance towers.

6. If a ground wire is to offer a high degree of protection against lightning flashover at the tower, the tower footing resistance must be relatively low—in the order of five ohms or less.

* * * * *

INTRODUCTION

THIS paper, dealing with the 1929 lightning experience on the 132-kv. transmission lines on the system of the American Gas and Electric Company, is a continuation of a record, begun in 1926, which has been presented as a series of papers† before the Institute during the past five years.

The lightning problem, in so far as it affects the operation of electric light and power systems logically, divides itself into three parts:

(*a*) *Theoretical and Laboratory Investigations.* This includes, obviously, mathematical studies and analysis of various lightning phenomena under various system conditions: laboratory studies with artificial lightning on lightning phenomena, and on the performance of various insulating structures and of various portions of the electric light and power system under lightning conditions.

(*b*) *Field Research with Natural and Artificial Lightning.* The development to a high state of reliability of the klydonograph, or surge recorder, of the Dufour oscillograph, of the direct hit recorder, of flashover indicators, of field intensity recorders and the practical development of portable impulse generators has placed in the hands of the engineer and the investigator a large number of tools which makes possible extensive and important researches on actual apparatus that were not dreamed of a few years ago. This field of investigation is obviously a very fertile one in the search for a complete solution to the lightning problem.

(*c*) *Operating Records.* These comprise the recording of actual operating experience with various devices and remedies for the mitigation or elimination of lightning damage that may have been used, and the analysis of actual system performance under lightning conditions. While this would at first appear to be the least scientific and easiest avenue of approach to the solution of the problem, it is a fact, nevertheless, that it is actually the most difficult operation. Obviously, too, and this cannot be too strongly emphasized, it is the field experience, properly analyzed and interpreted that gives the final answer as to the tenability of the many

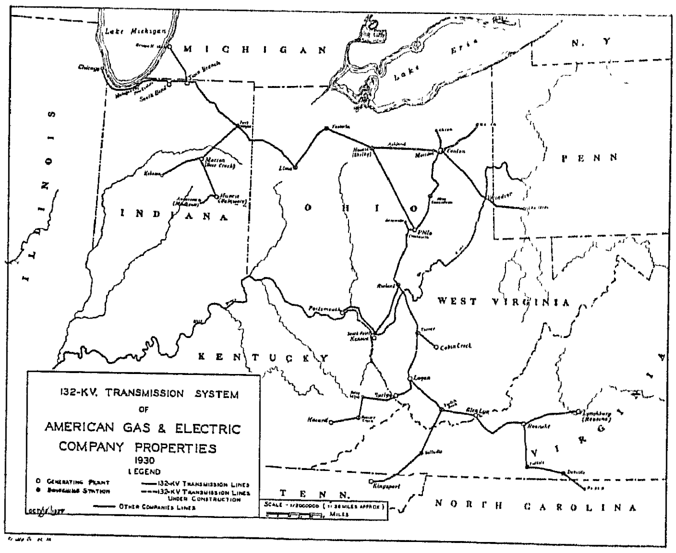

FIG. 1

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†See Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

TABLE I

Column No.	1	2 Crooksville So. Point	3 Fort Wayne Marion Muncie	4 Glen Lyn Roanoke	5 Glen Lyn Switchback	6 Lima Fort Wayne	7 Lima Fostoria	8 Logan Sprigg	9 Marion Kokomo	10 Philo Canton	11 Philo Crooksville	12 Philo So. Point Turner
Length of line (miles)		102.3	80.6	65.0	30.0	63.2	45.6	21.0	28.9	73.0	15.4	118.7[1]
Line placed in service (date)	No. 1	2/26/28	9/23/28	6/13/26	8/13/27	9/18/25	9/3/25	6/28/26	11/4/28	9/28/24	9/28/24	9/18/28
	No. 2		10/14/28	6/13/26	8/13/27				11/4/28	9/28/24		4/22/26
Number of circuits	1929		2	2	2	1	1	1	2	2	1	1
Number of ground wires	1929		1	1	1	1	1	0	1	1	1	1
Grading shields	Line No. 1		R & R[2]	R & R	R & H[3]	R & H	None	None	R & R	R & H	None	R & H
Grading shields	Line No. 2		R & R	R & R	R & H				R & R	R & H		
Grading shields installed (date)	Line No. 1		9/23/28	5/16/28	8/13/27	5/29/26			11/4/28	5/13/26		9/18/28
Grading shields installed (date	Line No. 2		10/14/28	4/30/28	8/13/27				11/4/28	5/13/26		
1926 Lightning Performance										1925-26		
Number of circuits				2		[4]	1	1		2–2	1	1
One circuit only tripped out				16			9	3		–13	3	11
Both circuits tripped out				4						–2		
Total outages				20			9	3		88–15	3	11
Circuit outages/100 miles of line/year				48[5]			19.8	31[5]		120–20.5	19.5	10.6
Circuit outages/100 miles of circuit/year				24[5]			19.8	31[5]		60–10.3	19.5	10.6
1927 Lightning Performance												
Number of circuits				2	2	[4]	1	1		2	1	1
One circuit only tripped out				21[6]			4	7		3[6]	1	1
Both circuits tripped out				3						1		
Total outages				24	0		4	7		4	1	7
Circuit outages/100 miles of line/year				37	0		8.8	33.3		5.5	6.5	5.9
Circuit outages/100 miles of circuit/year				18.5	0		8.8	33.3		2.8	6.5	5.9
Damage to insulators (no towers affected)				17	1		7[8]	0[7]		0	0	2
Damage to conds. and hardware (no towers affected)				14	3		5	0[7]		3	0	1
Total cases of damage				20	3		9	0[7]		3	0	2
Cases of damage per line outage				0.83	0		2.25[8]	0		0.75	0	0.29
1928 Lightning Performance												
Number of circuits		1	2	2	2	[4]	1	1	2	2	1	1
One circuit only tripped out		4		5	1		1	9		10	2	12
Both circuits tripped out		0		2	1					2		
Total outages		4		7	2		1	9		12	2	12
Circuit outages/100 miles of line/year		3.9		10.8	6.7		2.2	42.9		16.4	13	7.1[9]
Circuit outages/100 miles of circuit/year		3.9		5.4	3.4		2.2	42.9		8.2	13	7.1[9]
Damage to insulators (no towers affected)		0		2	1		2	5		10.	2	10
Damage to conductors		0		2	2		1	0		6	1	6
Arcing rings burned (no towers affected)		3		5	3					10		0
Total cases of damage		3		6	4		2	5		11	2	11
Cases of damage/line outage		0.75		0.86	2		2.0	0.55		0.92	1.0	0.92
1929 Lightning Performance												
Number of circuits	Operated as part of Philo-Turner Line.		2	2	2	1	1	1	2	2	1	2[12]
One circuit only tripped out			5	19	6	0	4	16	1	9	1	24
Both circuits tripped out			1	11	3				0	2		3
Total outages			6	30	9	0	4	16	1	11	1	27
Circuit outages/100 miles of line/year			40.3	46	33.3	0	8.9	76	3.5	15.0	6.5	16.3
Circuit outages/100 miles of circuit/year			20.2	23	16.7	0	8.9	76	1.7	7.5	6.5	8.4
Damage to insulators (no towers affected)			1	9	4	2	5	5		9	1	22
Damage to conductors			1	16(1S)[13]	4(1S)	0	0	6(1S)		4	1	8(2S)
Arcing rings burned (no towers affected)			1	27	13	0				12		17
Total cases of damage			1	36	14	2	5	7	Records	12	2	29
Cases of damage per line outage			0.12	1.2	1.55		1.25	0.44	incomplete	1.09	2	1.04

[1] Philo to Rutland only.

[2] Ring and Ring.

[3] Ring and Horn.

[4] Operated as Lima-Twin Branch

[5] Corrected for 1 calendar year.

[6] One circuit out of service at time of I. F. O.

[7] Superficial inspection—towers not climbed.

[8] Probably partly accumulation of 2 previous years.

[9] Rutland So. Point No. 2 (50.3 mi.) also on this circuit.

[10] Chargeable to 3 years' operation.

[11] Includes Howard Fostoria Line.

[12] Philo-So. Point-Turner (168 mi.) taken as 2 circ. line 161 mi. long.

[13] Indicates no. of total cases with severed strands.

[14] Installed Sept. 24, 1928.

TABLE I—*Continued*

Column No.	1	13	14	15	16	17	18	19	20	21	22	23	24
		Roanoke Danville	Roanoke Reusens	Saltville Kingsport	So. Bend Mich. City	Switchback Logan	Switchback Saltville	Turner Cabin Creek	Turner Logan	Twin Branch Fort Wayne	Twin Branch So. Bend	Windsor Canton	Philo Howard Fostoria
Length of line (miles)		65.0	43.0	56.0	40.0	50.0	46.0	23.6	40.2	65.4	4.9	55.0	125.5
Line placed in service (date)	No. 1	9/16/26	5/5/26	8/25/27	6/24/26	11/27/27	10/16/27	7/29/28	1/30/26	9/8/25	4/5/25	9/ /17	8/2/29
	No. 2		9/ 9/27			11/27/27		7/31/28	1/30/26		4/5/25	9/ /17	
Number of circuits	1929	1	2	1	1	2	1	2	2	1	2	2	1
Number of ground wires	1929	1	1	1	1	1	1	1	1^{14}	1	1	2	1
Grading shields	Line No. 1	R & R	R & H	R & H	None	R & H	R & H	R & R	R & H	R & H	R & H	None	R & R
Grading shields	Line No. 2		R & H			R & H		R & R	R & H		R & H	None	
Grading shields installed (date)	Line No. 1	9/16/26	5/22/26	8/25/27		11/27/27	10/16/27	7/29/28	7/20/26	6/7/26	6/8/26		8/2/29
Grading shields installed (date)	Line No. 2		9/ 9/27			11/27/27		7/31/28	7/20/26		6/8/26		
1926 Lightning Performance													
Number of circuits		1	1		1				2	^{4}	2	2	
One circuit only tripped out		0	13						16		0	9	
Both circuits tripped out									2		0	2	
Total outages		0	13						18		0	11	
Circuit outages/100 miles of line/year		0(A)	36.6^{5}						44.8		0	20	
Circuit outages/100 miles of circuit/year		0(A)	36.6^{5}						22.4		0	10	
1927 Lightning Performance													
Number of circuits		1	1	1	1	2	1		2	^{4}	2	2	
One circuit only tripped out		16	16		4				20		1	2^{6}	
Both circuits tripped out									4		1	0	
Total outages		16	16		4				24		2	2	
Circuit outages/100 miles of line/year		24.6	37.2	0	10				57.2		41	3.6	
Circuit outages/100 miles of circuit/year		24.6	37.2	0	10				28.6		20.5	1.8	
Damage to insulators (no towers affected)		14	2	0	1				2		1	0	
Damage to conds. and hardware (no towers affected)		5	7	0	0				4		0	0	
Total cases of damage		14	7	0	1				5		1	0	
Cases of damage per line outage		0.87	0.44		0.25				0.21		0.50	0	
1928 Lightning Performance													
Number of circuits		1	2	1	1	2	1	2	2	^{4}	2	2	
One circuit only tripped out		13	6	5	7	16	5		12		0	2	
Both circuits tripped out			4			2			1		0	0	
Total outages		13	10	5	7	18	5		13		0	2	
Circuit outages/100 miles of line/year		20	23.3	8.9	17.5	36	10.9		32.3		0	3.6	
Circuit outages/100 miles of circuit/year		20	11.7	8.9	17.5	18	10.9		16.2		0	1.8	
Damage to insulators (no towers affected)		18	7	1	4	5	1		23^{10}		0	0	
Damage to conductors		5	1	1	1	2			22^{10}		0	1	
Arcing rings burned (no towers affected)			25	6		12	3		25^{10}		0		
Total cases of damage		18	28	6	4	14	3		38^{10}		0	1	
Cases of damage/line outage		1.38	2.8	1.2	0.57	0.78	0.6		1.07		0	0.50	
1929 Lightning Performance													
Number of circuits	Operated as part of Philo-Turner Line.	1	2	1	1	2	1	2	2	1	2	2	1
One circuit only tripped out		30	11	13	9	11	16	4	11	7	1	2	3
Both circuits tripped out			5			9		3	4	0	2	0	
Total outages		32	16	13	9	20	16	7	15	7	3	2	3
Circuit outages/100 miles of line/year		49.3	37.2	23.2	22.5	40	34.8	29.6	37.4	10.7	61.2	3.6	8^{11}
Circuit outages/100 miles of circuit/year		49.3	18.6	23.2	22.5	20	34.8	14.8	18.7	10.7	30.6	1.8	8^{11}
Damage to insulators (no towers affected)		5	4	1	3	10	5	6	3	5	2	0	1
Damage to conductors		15	5(1S)	5	3	6	4(1S)	3(1S)	3(3S)	4	2	0	2
Arcing rings burned (no towers affected)		32	16	17		17	16	12	16	0	0	0	3
Total cases of damage		34	18	17	3	20	17	12	17	8	3	0	3
Cases of damage per line outage		1.06	1.12	1.31	0.33	1.0	1.06	1.71	1.13	1.15	1.0	0	1.00

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[1] Philo to Rutland only.
[2] Ring and Ring.
[3] Ring and Horn.
[4] Operated as Lima-Twin Branch.
[5] Corrected for 1 calendar year.
[6] One circuit out of service at time of I. F. O.
[7] Superficial inspection—towers not climbed.
[8] Probably partly accumulation of 2 previous years.
[9] Rutland So. Point No. 2 (50.3 mi.) also on this circuit.
[10] Chargeable to 3 years' operation.
[11] Includes Howard Fostoria Line.
[12] Philo-So. Point-Turner (168 mi.) taken as 2 circ. line 161 mi. long.
[13] Indicates No. of total cases with severed strands.
[14] Installed Sept. 24, 1928.

theories developed in connection with lightning and as to the practicability of any methods or structures that may be developed for the elimination of the harmful effects of lightning.

The above has been offered for what is hoped is the final *raison d'etre* for this particular type of approach to the lightning problem, since its permanent value has been questioned at times although, it is true, not by those intimately associated or acquainted with the problem.

GENERAL

The transmission network, shown in Fig. 1, comprised in 1929—1,198 miles of line; the total circuit miles of line in service during the same period was 1,825 miles. The system, it will be noted, has been enlarged by the addition of the Fort-Wayne-Marion-Muncie line, of the Marion-Kokomo line, of the Turner-Cabin line, and of the Philo-Howard-Fostoria line. No changes of a fundamental nature outside of those described in the last paper were made during the year 1929, except the addition of one ground wire in the fall of 1928 on the Turner-Logan line.

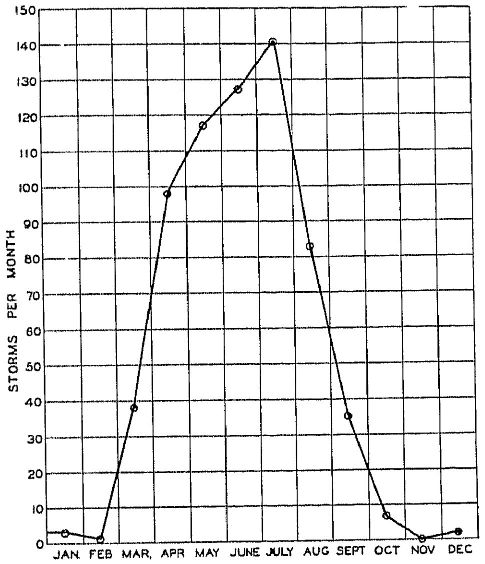

FIG. 2—1929 LIGHTNING STORMS RECORDED AT 24 STATIONS ON 132-KV. SYSTEM

Total number of storms—652

While complete details with regard to the tower structures, configuration of conductors, etc., have been previously given, it might be well to summarize here a few salient points. All lines coming under the range of observation, as described in this paper, are of double-circuit steel-tower construction (except the Logan-Sprigg wood pole line) with the conductors in a vertical configuration, all lines with a single exception using an A. C. S. R. conductor and a single ground wire of the same material. The average insulation employed consists at suspension points of 10 disk units, their length varying from 4¾ to 5⅛ in., and employing two additional units on dead ends. The majority of all lines was, during 1929, equipped with some form of arcing protection consisting either of a ring and horn combination, the horn in that case being located at the ground end, or of a ring and ring combination.

1929 PERFORMANCE

Table I gives a further résumé of the principal characteristics of the various circuits together with the 1926, 1927, 1928, and 1929 lightning performance. No attempt will be made to completely dissect the data given there, since it is believed that these data are given in a concise and readily enough comparable form to make comparison and analysis a simple matter.

The record of storms started for the first time in 1928 was maintained through 1929. Since no real knowledge of the lightning performance of a line can be obtained without a knowledge of the number of times the line was exposed to lightning, an attempt was further made this year to obtain even more thorough records of lightning storms in the vicinity of the lines than was obtained in 1928 when this feature of observation was inaugurated. The system of recording storms which was set up required the operators at all switching points on the 132-kv. system to obtain and send in records of all lightning-storms observed at their respective stations. A curve integrating the storms at all stations for each month is shown in Fig. 2. The records show a total of 652 storms reported over the 132-kv. system during the entire year. It must be realized, of course, that this represents a lower figure than the actual number to which the system was subjected, since it is possible for some storms to pass over the system at points where no observations were recorded, and further that it is not absolutely certain that all lightning storms were observed and reported at operating points on the 132-kv. system.

TABLE II—LIGHTING OUTAGES 1929

	Line Outages			
	Total	Sleet	Ltng.	% Ltng.
Fort Wayne-Marion Muncie	26	19	6	23.1
Glen Lyn-Roanoke	32		30	93.6
Glen Lyn-Switchback	12		9	75.0
Lima-Fort Wayne	0		0	..
Lime-Fostoria	7		4	57.2
Logan-Sprigg	20		16	80.0
Marion-Kokomo	3		1	33.3
Philo-Canton	27		11	40.7
Philo-Crooksville	8	1	1	12.5
Philo-Turner	40		27	67.5
Roanoke-Danville	36		32	89.0
Roanoke-Reusens	16		16	100.0
Saltville-Kingsport	13		13	100.0
South Bend-Michigan City	13		9	69.2
Switchback-Logan	22		20	90.8
Switchback-Saltville	17		16	94.0
Turner-Cabin Creek	9		7	77.8
Turner-Logan	18		15	83.3
Twin Branch-Fort Wayne	13	2	7	53.8
Twin Branch-South Bend	6	3	3	50.0
Windsor-Canton	4		2	50.0
Philo-Howard	18	11*	3	16.6
Totals	360	36	248	69.

*Dancing conductors.

Per cent lightning outages 69%

Per cent lightning outages (omitting sleet and dancing conductors)76.6%

The lightning outages over the entire system as shown in Table I, amount to 248. The total outages on the system from all causes was 360 and of this number, outages directly attributable to sleet and dancing conductors were 36. Lightning therefore accounted for 69 per cent of the total outages, or if sleet and dancing conductor troubles are eliminated, (the general experience being that troubles of this nature occur at rather infrequent and long spaced intervals) then lightning accounted for 76.6 per cent of the total outages. This checks data previously given to the effect that on a well-constructed and operated system, lightning today accounts for about 75 per cent of the total outages.

The 1929 lightning outages for the entire system considerably exceeded those recorded in 1928, the outages being 133 in 1928 and 248 in 1929. This increase is partly due to the added miles of transmission line since 1928, but chiefly to the more severe lightning conditions which existed in 1929. This statement is borne out, first from actual observations and comments from the operating people in the field; and second from the record of storms reported at six operating points on the system for 1928 and 1929 which were as follows:

TABLE A—LIGHTNING STORMS

	1928	1929	1929 % of 1928
Canton	21	38	180
Kingsport	23	24	104
Newcomerstown	23	34	148
Philo	31	40	129
South Bend	21	33	157
Turner	42	53	126
Per cent Storms	100	138	141

During 1929 the southern section of the system reported unusually severe lightning conditions which will account to a certain extent for the rather poor performance of such lines as the Glen Lyn-Switchback, Roanoke-Danville, and Switchback-Saltville lines.

Comparative line outages due to lightning during 1928 and 1929 are given below and indicate in every case the more severe lightning storm conditions over the system as a whole.

TABLE B—LINE OUTAGES PER 100 MI. OF LINE PER YEAR

	1929	1928	1929 % of 1928
All lines	21.6	13.5	160
Two circuit lines	23.3	17.7	136
One circuit line	17.8	10.0	178
With grading shields	20.8	13.8	151
Without grading shields	13.3*	8.1	165
Average			158

Line outages per 100 mi. of line per year on 4 new lines in operation in 1929......7.3.

*Exclusive of Windsor-Canton and Logan-Sprigg lines.

The average ratio of lightning outages in 1929 to 1928 for the entire system is 1.58; and the lightning storm record for five stations where records were kept for both years showed a ratio of 1.38 for the two years which is in the same general order as the outages. This shows conclusively that in analyzing the relative lightning performance of lines from year to year lightning conditions to which the lines are subjected must also be considered, or else erroneous and misleading conclusions may be drawn This is again clearly demonstrated by an analysis of outages on lines with and without grading shields discussed below.

DETAIL DISCUSSION OF RESULTS

Effectiveness of Grading Shield Protection. During 1929 the Philo-South Point-Turner line operated for the entire lightning year with one circuit completely equipped with grading shields (ring and horn arcing combinations) while the other circuit had no protection of any sort, with the exception of a short horn that was employed on dead end strings. The performance of this line during the year is shown in Table IV. For a very short distance this line was but single circuit and it was the unprotected circuit that had this additional length. It will be noted from Table IV that there were 13 outages on one side and 14 on the other, a figure that gives substantially the same number of outages per 100 mi. of circuit per year. Apparently, therefore, the difference in protective devices used had no effect on the number of outages. On the other hand there were five cascaded insulator strings on the protected or shielded line as against sixteen on the unshielded line, four cases of burned strands on the unshielded line against none on the shielded line, twenty towers flashed on the unshielded circuit against eleven on the shielded and four cases of blistered conductor strands on the unshielded circuit against only two in the shielded circuit. The above was determined from careful inspection consisting of a complete climbing of each tower on the line.

TABLE III—OUTAGES ON TWO-CIRCUIT LINES

	Miles	Outages: One line	Outages: Two line	Outages: % Two line
Fort Wayne-Marion-Muncie	80.6	5	1	16.7
Glen Lyn-Roanoke	65.0	19	11	36.7
Glen Lyn-Switchback	30.0	6	3	33.3
Philo-Canton	73.0	9	2	18.2
Philo-Turner	161.	24	3	11.1
Roanoke-Reusens	43.0	11	5	31.2
Switchback-Logan	50.0	11	9	45.0
Turner-Cabin Creek	23.6	4	3	42.8
Turner-Logan	40.2	11	4	26.7
Twin Branch-South Bend	4.9	1	2	66.7
Windsor-Canton	55.0	2	0	0.0
	626.3	103	43	
Per cent outages		70.5%	29.5%	

Performance of Double Circuit Lines. In Table III there is given an analysis of the outages on the two circuit lines. It will be seen that out of a total of 146 outages 103 or 70½ per cent involved but one circuit. This shows a higher percentage where both lines were affected than was the case in 1928, where the figure was 19 per cent and considerably higher than the 1927 figure of 16 per cent. It is possible that the growth of the system and the concentration of additional capacity, and therefore short-circuit current on the system without a concurrent development along the lines of faster switch operation, has brought about a situation where a greater percentage of what were originally single-line flashovers are developed by arc communication into two circuit flashovers, but of course, no data are available on that score. Some system changes that are under discussion with a view of speeding up switch operation may throw further light on this question in the future.

From this record it is clear that the line outages are not materially different with grading shields from what they are when no grading shields are used. A study of Table B, however, would tend to indicate some 60 per cent greater outages with grading shields than without. The explanation for this apparent discrepancy lies in the fact that grading shields are installed on lines located in the worst lightning territory, where the greatest protection against damage is needed; and lines without grading shield protection are in the district where lightning conditions are less severe, and where less outages would be expected.

TABLE IV—COMPARISON OF DAMAGE TO LINE WITH AND WITHOUT RING AND HORN PHILO-SOUTH POINT-TURNER LINE (From Tower Climbing Inspection)

Tower location with	With ring and horn	Without ring and horn
Cascaded insulators	5	16*
Strands burned in two in wire	0	4‡
Number of flashovers	11	20
Blistered strands	2	4†
Lightning outages	13	14
Length of line (miles)	152.6	168
Outages/100 mi. of cir./year	8.5	8.3

*5 cases of badly stripped insulators.

†In addition to cases with strands burned in two.

‡Replaced about 50 ft. of conductor each side of tower, in one case.

Conductor Damage. The damage to the conductors was observed from tower climbing inspections and is given in detail in Table I. A total of 90 cases is recorded and in only 11 cases was the burning severe enough to sever one or more strands in the conductor. Analyzed on the basis of 100 mi. of circuit per year, there are 0.6 cases of severed strands against a total number of cases of five burned or blistered conductors. In no case was the damage to conductors severe enough to prevent any line being placed back into service, and practically all damage to conductors was observed from a tower climbing inspection on the semi-yearly patrol. In no case was burning of the conductor found within the limits of the arcing protection shield at the live side of the insulator string.

Grading Shield Damage. The damage observed on grading shields due to flashover was in all cases quite negligible, being confined to slight burning at the tips of the horns and similar burning on the ends of the rings. In no case has it been necessary this year, or in past years, to replace or repair a grading shield assembly due to damage from flashover. The burning on the grading shields is clear cut evidence that the shield is in most cases carrying and holding the 60-cycle arc instead of permitting it to concentrate on the conductor and hardware.

Location of Damage by Physical Inspection. In an attempt to determine the location where lightning first affected the line, that is on top, middle or bottom conductor, the results of the physical inspection have been tabulated in Table V, where the location of burning or other evidence of flashover have been recorded. From this table it appears that cases of trouble involving top or top and middle conductors comprise 44.2 per cent of the total cases. Those involving the middle conductor or bottom and middle conductor comprise 42.8 per cent.

TABLE V—LOCATION OF TROUBLE BY PHYSICAL INSPECTION

Line	T-M-B	Top	Mid.	Bot.	T&M	T&B	M&B
Fort Wayne-Marion-Muncie				1			
Glen Lyn-Roanoke	3	8	5	8		8	2
Glen Lyn-Switchback	2	5	3	3			1
Lima-Fort Wayne			1				
Lima-Fostoria		4		1			
Marion-Kokomo							
Philo-Canton	1	9		1			1
Philo-Crooksville		2					
Philo-Turner	1	6		1			
Roanoke-Danville	3	7	5	6	6	1	6
Roanoke-Reusens	2	4	2	7			3
Saltville-Kingsport		9	4	3			1
South Bend-Michigan City	1					1	1
Switchback-Logan		11	3	3	1	1	1
Switchback-Saltville		6	4	3		1	3
Turner-Cabin Creek		4	2	2	1	1	1
Turner-Logan	1	10	1			1	
Twin Branch-Fort Wayne		3	2	1	1	1	
Twin Branch-South Bend		1	1			1	
Windsor-Canton							
Philo-Howard		1					2
Howard-Fostoria							
Totals	13	90	33	41	9	16	22
Total locations . . 224							
Per cent of totals	5.8	40.2	14.7	18.3	4.0	7.2	9.8

Cases involving top or top & mid. only	99	= 44.2%
Cases involving bot. mid. or top & mid. only	96	= 42.8%
Involving one phase only	164	= 73.2%
Involving more than one phase	60	= 26.8%

Assuming that those cases that involved the top conductor only or the top and middle conductor exclusively were all caused by direct strokes (although it is possible that some of these cases may have been caused by induced voltages), and assuming further that those cases that did not involve the top conductor at all were all caused by induced voltages, it would appear that, on this system at any rate, direct and induced lightning strokes are of practically equal importance in so far as they contribute to the system outages.

Another interesting conclusion drawn from this table is that 73.2 per cent of the cases of trouble involved only one phase of the circuit and only 26.8 per cent involved more than one phase. It is also shown that in only 5.8 per cent of the cases were top, middle, and bottom conductors affected (that is, a complete three-phase short circuit developed). Conclusions drawn from this type of data are not infallible, since lightning flashover may occur on the line without a power follow-up arc; and also if such an arc does exist, it is not certain that the evidence of the 60-cycle follow-up will be left, in all cases, to be observed by physical inspection. Another method, however, has been used to check these results. This method is the determination of the phase wire affected, by the record of relay targets at the time the circuits tripped out.

Location of Damage by Relay Targets. A great many of the relays on the 132-kv. circuits are equipped with a target which indicates when the relay has operated. A record of these target indications is kept at each station and an analysis has been made to indicate which phases of a circuit have been in trouble. A summary of these data is given in Table VI. The analysis of this table shows that in 45 per cent of the cases the top or top and middle conductors have been affected and in 44.3 per cent of the cases the bottom or bottom and middle conductors have been involved. This checks very closely with the figures obtained by the method of physical inspection where the corresponding figures were 44.2 and 42.8 per cent. The table further shows that one phase was involved in only 64.1 per cent of the cases against 73.2 per cent as shown by physical inspection. It also shows 4.7 per cent of the cases involved all three conductors of a circuit against 5.2 per cent given in Table V.

TABLE VI—LOCATION OF TROUBLE BY RELAY TARGET INDICATION

Line	Conductors affected						
	Top	Mid.	Bot.	T&M	T&B	B&M	T-B-M
Ft. Wayne-Marion Muncie	3						
Glen Lyn-Roanoke	5	6	3	5	1	3	1
Glen Lyn-Switchback	2	2					
Philo-Canton	1		1	1	1		
Philo-Turner	6	1	4	3	1	1	
Roanoke-Danville	2	2	5	5		2	2
Roanoke-Reusens	1	1	3		1		1
Saltville-Kingsport	3		1	2			
Switchback-Logan	4			1	1		1
Switchback-Saltville		1			1	1	
Turner-Logan	4	3	3			3	
Philo-Fostoria		1					
Totals	31	17	20	17	6	10	5
Per cent of total	29.2	16.0	18.9	16	5.7	9.5	4.7
Total indications							106
Per cent involving top or top and mid. only					45.2		
Per cent involving bottom and mid. but not top					44.3		
Per cent involving top, mid. and bottom					4.7		
Per cent involving one phase only					64.1		
Per cent involving more than one phase					35.9		

Reasoning again on the assumption that all cases involving the top conductor were of direct stroke origin, and that cases not involving the top conductor were of induced stroke origin, the evidence seems to be very positive that on this system at any rate induced strokes as well as direct strokes must be considered in supplying protection to the system, and that the two are of about equal importance.

Tower Footing Resistances. On a great many of these lines the tower footing has been measured, in some cases with the ground wire in place, and in some cases with the ground wire electrically disconnected from the tower. Since it has been extensively pointed out of late that the tower footing resistance plays an important part in the lightning protection of a line, it is considered worth while to present operating data to throw some light on this aspect of lightning protection as found in service. Tables VII and VIII give the tower footing resistances on the Philo-Canton and Turner-Logan lines at those towers where flashovers were observed on inspection. There is also given the tower footing resistance of the tower on each side of the one where the flashover was observed. On both of these lines tower footing resistances were measured with the ground wire disconnected from the tower so that the ohmic values are true tower footing resistances and do not include any effect of the ground wire and adjacent towers.

TABLE VII—TOWER FOOTING RESISTANCE WHERE FLASHOVER OCCURRED ON PHILO-CANTON LINE 1929

Case No.	Tower No.	Grd. resist. ohms*	F. O. 1928	Resistance of preceding and succeeding towers	
1	30B	1.8	No	5.1	2.0
2	76A	2.2	No	10.3	2.0
3	89A	18.2	No	3.5	8.0
4	96A	21.2	No	8.2	27.0
5	126B	5.1	No	3.0	31.0
6	151C	2.4	Yes	10.3	5.0
7	152C	5.0	No	2.4	6.4
8	155B	2.8	No	5.8	10.1
9	191B	4.7	No	1.3	33.0
10	192B	33.0	No	4.7	1.5
11	225A	18.4	No	22.7	10.8
12	291A	2.3	No	7.8	1.9

Total number of towers in the line..............................299
Total number of towers with flashover 12
Average resist. per tower for whole line (ohms).................. 7.7
Average resist. per tower for those with flashover (ohms)......... 9.9

Ratio of resist. $\frac{\text{Flashed over towers}}{\text{Average of line}}$ 1.29

Per cent of cases where flashover occurred at tower having lowest resistance of three adjacent towers......................... 25%

*Tower resistance measured with ground wire disconnected.

There are two things to be noted in these tables: first, that the towers where flashovers occurred have a higher average footing resistance than the average for the entire line, the ratios being 1.29 on the Philo-Canton line and 1.67 on the Turner-Logan line, that is, the towers where flashovers occurred have a footing resistance averaging some 30 to 70 per cent higher than the average for the line. The other point which the operating data show is that in 25 per cent of the cases on the Philo-Canton line and in 47 per cent of the cases on the Turner-Logan line the tower where the flashover occurred has a lower resistance than either of the towers on

TABLE VIII—TOWER FOOTING RESISTANCE WHERE FLASHOVERS OCCURRED ON TURNER-LOGAN LINE 1929

Case No.	Tower No.	Gr. resist. ohms*	F. O. 1928	Resist. of preceding and succeeding towers	
1	18B	2.50	No	2.25	2.25
2	24C	2.50	No	1 25	1.50
3	25C	1.50	No	2.50	5 50
4	28C	3.50	No	5.50	1.25
5	40C	1.50	No	1.50	3.25
6	44C	8.50	No	6.25	7.50
7	45C	7.50	No	8.50	15.25
8	63C	6.00	No	60.0	6.25
9	72E	250.	Yes	11.0	250.
10	85B	250.	No	50.0	25 0
11	120E	28.0	No	20.0	8 75
12	122C	3.0	No	8.75	12.5
13	128C	6.5	No	7.5	18.5
14	140C	14.5	No	6.5	3.5
15	144E	2.75	No	3.0	13.0
16	149E	16.0	No	25.0	25.0
17	151E	21.0	No	25.0	22.5

Total number of towers in the line 153
Total number of towers with flashover 17
Average resist. per tower for whole line (ohms) 22.0
Average resist. per tower for those with flashover 36.8

Ratio of resist. $\frac{\text{Flashover towers}}{\text{Average of line}}$ 1.67

Per cent of cases where flashover occurred at tower having lowest resistance of three adjacent towers 47%

*Tower resistances measured with ground wire disconnected from towers

each side of it. On the theory that lightning flashovers will occur where the tower footing resistances are high, the condition just referred to is rather difficult to explain.

Tables IX and X give similar data on the Roanoke-Reusens and Glen Lyn-Roanoke lines. On both of these lines the tower footing resistances were measured with the ground wire attached to the towers. Here again the results show, first, that on the Roanoke-Reusens line the towers where flashover occurred have a footing resistance of 78 per cent of the average of the line and on the Glen Lyn-Roanoke line 1.84 times the average tower footing resistance for the line.

On the Roanoke-Reusens Line 27.8 per cent of the flashed-over towers had a lower tower footing resistance than the tower on either side; and on the Glen Lyn-Roanoke line in 13.9 per cent of the cases the same condition holds true.

The data in Tables VII, VIII, IX and X have been analyzed in a different way in Fig. 3. There it is shown that in 50 per cent of the cases where towers were flashed over the tower footing resistance was 12 ohms or less. In about 65 per cent of the cases the tower footing resistance was 20 ohms or less and in 80 per cent of the cases was below 50 ohms. On the other hand, in 17½ per cent of the cases the tower footing resistance was about 250 ohms, which was the upper limit of the ground ohmer used to obtain the measurements.

TABLE IX—TOWER FOOTING RESISTANCE WHERE FLASHOVERS OCCURRED ROANOKE-REUSENS LINE 1929

Case No.	Tower No.	Gr. resist. ohms*	F. O. 1928	Resist. of preceding and succeeding towers	
1	46B	20	No	40.0	20.0
2	67A	40	No	40.0	40.0
3	70B	58	Yes	25.0	14.0
4	79A	20	No	40.0	24.0
5	80B	24	No	20.0	25.0
6	89A	35	Yes	22.5	40.0
7	95B	20	No	40.0	20.0
8	96A	20	No	20.0	24.0
9	102A	50	No	4.0	22.0
10	103C	22	No	50.0	25.0
11	115A	12	No	50.0	19.0
12	116A	19	No	12.0	45.0
13	119A	11	No	90.0	21.0
14	124A	10	No	21.0	12.0
15	126A	12	No	12.0	6.0
16	132A	30	No	30.0	16.0
17	136A	20	No	30.0	14.0
18	140B	40	No	60.0	20.0

Total number of towers in the line 200
Total number of towers with flashover 18
Average resistance per tower for whole line .... 32.6 ohms
Average resistance per tower for those with flashover .... 25.8 "

Ratio of resistance $\frac{\text{Flashed over towers}}{\text{Average of line}}$ 0.78

Per cent of cases where flashover occurred at tower having lowest resistance of three adjacent towers 27.8%

*Tower resistances measured with ground wire connected to towers.

TABLE X—TOWER FOOTING RESISTANCE WHERE FLASHOVER OCCURRED ON GLEN LYN-ROANOKE LINE 1929

Case No.	Tower No.	Gr. res. ohms*	F. O. 1928	Resist. of preceding and succeeding towers	
1	12C	15.	No	8.25	250
2	13C	4.5	No	250	3.75
3	14C	3.75	No	4.5	2.25
4	17B	4	No	2.5	250
5	21C	250	No	250	250
6	22A	250	No	250	250
7	24C	250	No	250	250
8	26A	60	No	250	250
9	28C	250	No	250	250
10	31C	5.75	No	4 5	9.5
11	35C	250	No	13.0	7.5
12	40B	250	No	5 0	4.0
13	49B	17	No	7.75	10.0
14	53C	250	No	6.5	5.5
15	102B	8.5	No	1.5	6.5
16	135B	6.0	No	3.0	7.5
17	124A	3 5	No	2.5	9.0
18	123A	2.5	No	2.0	3.5
19	81C	2.0	No	1 75	4.25
20	80C	1.75	No	4.0	2.0
21	74A	8.5	No	4.0	4.3
22	72E	8	No	80	4.0
23	71E	80	No	250	8.0
24	69C	9	No	250	250
25	65B	250	No	250	250
26	64C	250	No	250	250
27	63B	250	No	250	250
28	51C	4.8	No	250	21.0
29	46B	3	No	17	3.0
30	22B	10	No	25	12.3
31	21B	25	No	25	10
32	20B	25	No	250	25
33	18A	250	Yes	13	250
34	14A	250	No	250	15
35	9A	10	No	17	13
36	6A	250	No	250	12.0

Average resist. of towers where measurements could be made .. 7.9 ohms
Average resist. of towers measured taking 250 ohms for all towers higher than this ....47 1 ohms
Average of tower resistances at above locations 86.5

Ratio of resistance $\frac{\text{Flashed over towers}}{\text{Average of line}}$ 1.84

Per cent of cases where flashover occurred at tower having lowest resistance of three adjacent towers. 13.9

*Tower footing resistances measured with ground wire connected to towers.

It would appear from the above that if a criterion for immunity from flashover is to be established on the basis of tower footing resistance, values considerably lower than have heretofore been considered necessary will have to be obtained. For example, a value of 20 ohms, which has been considered by some as a sufficiently low value, would appear to offer very little protection, the experience on the four lines given in Fig. 3 being that 65 per cent of the flashovers occurred at points where the resistance was 20 ohms or less. The order of limiting resistance will depend of course upon the relative immunity desired, but a resistance of 5 ohms would not appear an unreasonably low one to seek, and it may be that even lower values will, under certain conditions, be found desirable.

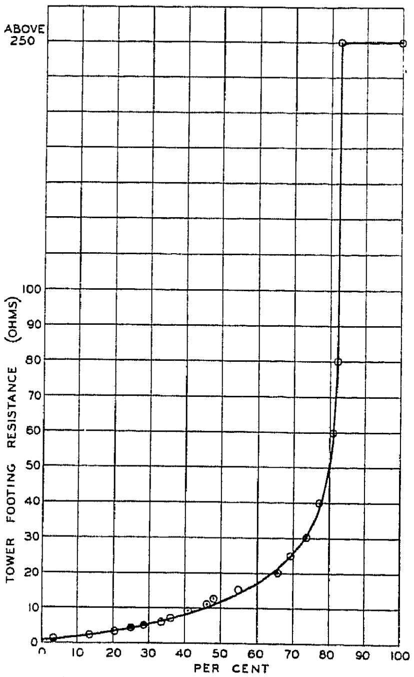

FIG. 3—TOWER FOOTING RESISTANCE WHERE FLASHOVER OCCURRED—1929

1. Philo-Canton
2. Turner-Logan
3. Glen Lyn-Roanoke
4. Roanoke-Reusens

Ground Wire Protection. The beneficial effects of the ground wire are demonstrated from the 1929 records in several ways. First, on the Logan-Sprigg line which, while being a wood pole line, nevertheless has steel crossarms grounded at every pole, the outages were 76 per 100 mi. of line per year. This is 1.8 times as many as recorded last year and checks fairly closely with the more severe lightning conditions experienced in 1929. On the other hand, the outages on this line are 3½ times the average on the system as a whole, the rest of the system being in all cases equipped with at least one ground wire.

In the case of the Windsor-Canton line where two ground wires are employed the outages were only 3.6 per 100 mi. of line per year which is exactly the same figure found in 1928 and 1927. It is believed, however, that the Windsor-Canton line is in a district where lightning conditions are not nearly as severe as in the Logan-Sprigg district; but the discrepancies in outages are far too great to attribute to this difference of location only.

A very direct comparison of the benefits of the ground wire is found on the Turner-Logan line where one ground wire was in operation during 1929 for the first time. In 1928 the outages per 100 mi. of line were 32.3 and in 1929 the outages were 37.4. Correcting the 1928 outages on this line by increasing them 40 per cent (see Table A) due to the more severe lightning conditions encountered in 1929, and further increasing them 31 per cent to allow for the time (some seven weeks) the line was out of service for installing the ground wire, there would have been 59.3 outages if this line had not been equipped with a ground wire. On the above basis the installation of one ground wire resulted, therefore, in a decrease in outages of at least 37 per cent. This is most assuredly a minimum figure since the general experience has been that during a year when there is a great frequency of lightning storms, there is experienced a proportionately greater number of heavy storms than one would be led to expect from the numerical ratio of all storms.

CONCLUSIONS

The lightning experience obtained during the year 1929, thoroughly analyzed and considered in the light of previous experience on the same lines, leads to the following conclusions:

1. *Reliability of Two-Circuit Lines.* Two-circuit lines when installed in vertical configuration on the same towers enhance the reliability of the system in a greater proportion than the mere doubling of the circuits. Experience definitely indicates that the likelihood of both circuits tripping out is comparatively small, only 16 to 29½ per cent of the cases of the trip-outs involving both circuits. There appears a likelihood that this percentage can be reduced by higher speed switching.

2. *Effectiveness of Ground Wires.* The only line equipped with two ground wires, the Windsor-Canton line, had 3.6 outages per 100 mi. of line per year. The only line not equipped with a ground wire, the Logan-Sprigg line, had an outage record of 76. The Turner-Logan line operating with one ground wire in 1929 had 37.4 outages per 100 mi. of line per year, and in 1928, when no ground wires had been installed, had an actual outage of 32.3; this figure corrected on the basis of the more severe lightning conditions in 1929 becomes 59.3 outages per hundred miles of line if the no ground wire condition had obtained in 1929.

This shows clearly the benefits of one ground wire, and indicates definitely that two ground wires are more beneficial than one. It is of course necessary to keep in mind that the Windsor-Canton line is built with a short span and therefore at a low height above ground, and is moreover located in a territory where lightning conditions are not as severe as on other parts of the system.

3. Effectiveness of Grading Shields. Grading shields have very little effect on the lightning outages of these 132-kv. lines, neither increasing nor decreasing the number of outages. On the other hand they definitely limit the damage to insulators and conductors; further the amount of cascading is reduced. The cascaded strings over the entire system (and most of the system is equipped with grading shields) were 23 per cent of the total number of towers flashed over. Those lines that had grading shields installed recorded only 21 per cent; but the lines that had no shields had a record of 70 per cent. This is a very striking indication of the benefit rendered by grading shields.

4. Lightning Storm Data. No reliable interpretation of performance of lines under lightning is possible, or is of real value, without an accompanying record of lightning storm conditions during the particular time discussed. Conclusions gravely misleading will otherwise be made.

5. Line Conductors Involved in Flashovers. Records of physical inspection and relay target indications show that in tripouts due to lightning, all three phases are involved, in from 4.7 to 5.8 per cent of the cases. In 44.2 to 45.2 per cent the top and middle conductors only are involved. In 42.8 to 44.3 per cent the bottom and middle conductors are involved. It is also shown that in from 73.2 to 64.2 per cent only one phase is involved. It is interesting to note that the number of cases where either the middle or bottom conductors only were involved is almost equal to the number of cases in which the top conductors only were involved.

This analysis indicates very strongly that both direct and induced strokes are of vital importance and play a great part in the performance of this 132-kv. network.

6. Effect of Tower Footing Resistance on Lightning Flashover. Detailed data on the tower footing resistances on four 132-kv. lines do not show any tendency for lightning flashovers to concentrate on towers of high-footing resistance in preference to those having low-footing resistance. In fact, whether lightning strikes a tower with low- or high-footing resistance seems to be, from these data, a matter of chance. Flashed-over towers were observed where the footing resistance varied from 1.5 to over 250 ohms.

ACKNOWLEDGMENTS

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Discussion

IMPULSE TESTS ON SUBSTATIONS

SOUTHGATE, BROOKES, WHITEHEAD AND ROMAN)

LIGHTNING INVESTIGATION ON A WOOD POLE TRANSMISSION LINE

(PITTMAN AND TOROK)

1929 LIGHTNING EXPERIENCES

(PHILIP SPORN)

R. N. Southgate: Mr. Sporn points out that cases of trouble involving top or top and middle conductors are about the same percentage of total flashovers as those involving middle or middle and bottom conductors. He assumes that those cases which did not involve the top conductor were caused by induced strokes. There seems to be a possibility here that with the single-ground wire and vertical configuration, streamers of the lightning stroke might come in contact with the middle or bottom conductor without involving the top conductor or ground wire.

In cases of direct stroke where the ground wire or tower is not involved, the possibility of insulator flashover would not depend on the tower footing resistance as flashover would occur due to over-voltage on the conductor rather than due to tower top voltage rise. This might account in part for the fact that a large number of flashovers occurred on towers of low footing resistance.

Results of 1930 lightning investigation with the cathode-ray oscillograph on 12 miles of unenergized 220-kv. line terminating at the Roseland Switching Station lend added weight to the opinion, drawn from previous experience, that induced surges on a line of this nature are of negligible magnitude.

This line has the same structural characteristics as that used in the 1929 investigation and described by Messrs. Conwell and Fortescue, A. I. E. E. TRANS., Vol. 49, with the exception that the two ground wires were in place. During the entire season, four positive surges were recorded, all below 100 kilovolts in crest and less than 10 microseconds in length. Two of these surges were definitely correlated with records of ground current in the tower structures, while two were of unknown and possibly induced origin. Tower footing resistances on the section under investigation were low; the average being 3.9 ohms and the maximum 20 ohms.

Several strokes were observed to strike very close to the tower line near the station without causing any measurable surges on the oscillograph. A considerable length of the 220-kv. line parallels on the same right-of-way, a 132-kv. transmission line which was in service. One heavy stroke was observed to strike the ground in almost direct line with the parallel 132- and 220-kv. lines, and was calculated by velocity of sound to have occurred at about 2,000 ft. from the station. A photograph was obtained of the stroke (see Fig. 1) and from this record the location of the stroke was placed within 350 ft. of the 220-kv. line to which the oscillograph was connected, and within 275 ft. of the live 132-kv. line. No surge was recorded on the oscillograph which was in operation and adjusted to record surges of low magnitude, nor did any trip-out occur on the 132-kv. line.

This fact and other evidence would indicate the immunity of these and similar lines to strokes of this nature and lend weight to the theory that direct strokes are responsible for the major number of flashovers.

J. J. Torok: One of the great difficulties of the lightning investigation programs has been the lack of organization on the part of the utility engineers in obtaining operating data of the right kind so that a complete analysis of a line or type of line can be made. This paper presents data of the type that is needed for such analyses.

In his evaluation of the data and conclusions he makes the following statement: "Assuming that these cases that involved the top conductor only or the top and middle conductors exclusively were all caused by direct strokes (although it is possible that some of these cases may have been caused by induced voltages) and assuming further that these cases that did not involve the top conductor at all were all caused by induced voltages, it would appear that, on this system at any rate, direct and induced lightning strokes are of practically equal importance in so far as they contribute to the system outages."

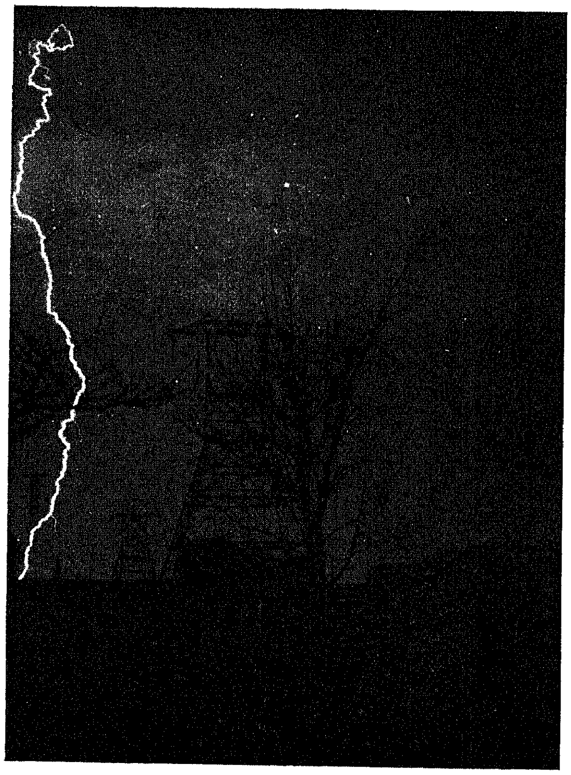

FIG. 1

The data presented by Mr. Sporn can be interpreted on another basis, purely that of the direct stroke. The difference between the following interpretations and that of Mr. Sporn's is that Mr. Sporn assumes that when ground wires are used that the lightning stroke in all or the majority of cases terminates on the ground wire or top conductor, and that the other flashovers are caused by induced voltage. On that assumption he is entirely justified in drawing the conclusions which he has, namely that flashovers have taken place under the conditions where the direct-stroke theory as it now stands will not prove adequate especially where very low tower footing resistances have been measured and a large number of flashovers has occurred. However, the analysis that has been made on several types of towers on which ground wire protection of various forms are used has shown that the presence of the ground wire does not necessarily indicate that the ground wire only was struck. In a large number of cases where only one ground wire is used the top conductor or middle conductor were struck directly. This evidence has been determined from the nature of the markings left on the porcelain insulator surfaces. If the data submitted by Mr. Sporn are analyzed on this premise entirely different conclusions may be obtained.

Our analysis will be confined to Tables V, VII, VIII, IX, and X. Table VII tabulates the tower footing resistance of the Philo-Canton line. These resistances are fairly low except for four cases. In Table V it is shown that on the Philo-Canton line the majority of the outages took place on the top conductor with only one or two on the bottom and middle conductor. This indicates that the majority of the outages were caused by direct strokes to the top conductor while strokes to the tower, the ground wire, or top conductor did not raise the tower potential to a value sufficient to cause flashover on the conductors. Table VIII which is a tabulation of the flashovers on the Turner-Logan line shows that the majority of the tower footing resistances are fairly low. Only five or six towers may be said to have tower footing resistances above safe values. Table V shows that this line had 10 flashovers on the top conductor and one on the top, middle, and bottom, one on middle and one on top and bottom. In this case the predominance of the flashover of the top conductor is evident, showing that direct strokes terminated on the top conductor. Table IX tabulates the flashovers occurring on the Roanoke-Reusens line. The tower footing resistance on this line is fairly high ranging between 11 and 58. These values are probably too high to prevent flashover when struck directly. Because of the lower coupling between the ground wires and the bottom conductor it is expected that the voltage across the insulators on the bottom conductor will be higher than on the others. This is borne out by the values taken from Table V showing that the bottom conductor flashed over seven times, the top four, the middle two, and all three, that is top, middle, and bottom, two, and middle and bottom three.

Table X lists the flashovers occurring on the Glen Lyn-Roanoke line. The tower footing resistance on this line is fairly evenly divided between high and low resistance values. The locations of flashover with respect to position of the conductor, as would be expected from analysis, would be that the flashovers occurring on the top and bottom conductors would be very nearly the same and in several instances flashover would take place on all three conductors. This is borne out by data given in Table V. There were eight flashovers on the top conductor, five on the middle and eight on the bottom conductor.

It would be interesting to analyze the above tables in more detail in the manner shown above. However, this would require the tying in of the flashovers on each tower with the tower footing resistance.

The data in this paper very markedly show the importance of adequate ground wire protection. For example the Turner-Logan Line had ten flashovers on the top conductor. It would be safe to predict that if two ground wires were to be used, and so situated that they will adequately shield the conductors from direct strokes, the number of flashovers would be reduced from thirteen to one or two. It is quite evident that one ground wire does not effectively shield the conductors. Furthermore, the additional ground wire increases the coupling between the ground wires and the conductors to such an extent that the effectiveness of the line insulation will be raised from 10 to 30 per cent.

F. W. Peek, Jr.: The oscillograms in Figs. 5 and 6 of the paper by Messrs. Torok and Pittman are especially interesting. These figures record voltages of the order of 5,000,000. The fronts are exceedingly steep and the effective duration short. In fact the effect of these waves on the sparkover of insulators, bushings and gaps would be quite similar to the ½/5 wave so long used in our laboratory to obtain steep wave front effects.

The highest measured line voltages as shown by these oscillograms have not yet exceeded the 5,000,000 volts first produced in the laboratory about two years ago and the effects correspond to the steep laboratory waves. Full size wood poles were also flashed over and split in the laboratory as they have been in the field.

Mr. Sporn's paper is a very valuable one and could well be used as a model for recording and presenting data on operating experiences. I am going to try to arrive at the conclusions from a different angle than Mr. Sporn in that part of the sparkovers on his 132-kv. system are due to direct hits and part to induced voltages. Referring to Table VII, in 8 out of 12 cases the tower footing resistance varied from 1.8 to 5 ohms. In the remaining 4 cases it covered a range of resistance from 18 to 33 ohms. If all sparkovers were caused by direct strokes, only 4 would be expected since the tower footing resistance on the remaining 8 is too low to make a direct-stroke sparkover likely except under the most extreme conditions. However, for these particular data the ground resistance is low enough so that the induced voltages would be practically the same for all towers. Thus there would be an equal chance of an induced voltage outage for any of these towers regardless of the footing resistance. The possible conclusions, therefore, are that all outages are due to induced voltages; or that 8 or more outages are due to induced voltages with the remaining 4 or less due to direct strokes. This is based on the assumption of all the direct strokes striking the tower. If, however, half of the direct strokes hit the center of the line it might be concluded that 50 per cent were due to induced voltages and 50 per cent to direct strokes. An analysis of some of the other tables will give the same conclusions.

L. V. Bewley: The paper by Southgate, Brooks, Whitehead and Roman gives a number of oscillograms showing the effect of bus structure, breakers and transformers on traveling waves. But beyond calling attention to the fact that the crest is lowered and the transient is lengthened, as additional station equipment is added to the transition network, they do not draw any conclusions as to the characteristic effect of station equipment on traveling waves. In other words, they do not attempt to determine $Z_t\,(p)$ of their equation (1) from the test results which they obtained. In the past it has been customary to regard buses, breakers and transformers as simple capacitances, as far as traveling waves are concerned, and to compute the behavior of traveling waves on that basis. It is therefore gratifying to notice from the oscillograms that the character of the transient is essentially that of a reflection from a lumped capacitance. I should like to ask the authors if they made any attempt to check the experimental work by calculations based on measured values of capacitances, or if they have determined by trial what values of capacitance check the oscillograms? It seems to me that such information would greatly increase the value of their paper; because after all, we are not so much interested in a specific group of tests as we are in the general conclusions that can be drawn from them.

The authors have also discussed the effect of the arrester location, and in equation (2), have given a useful formula for estimating purposes, based on complete reflection at the terminal equipment, and a uniformly rising front on the incident wave. If, however, the terminal equipment involves a large capacitance, or outgoing lines and cables causing a reduction of surge impedance, then the reflection is only partial, and the arrester will protect the equipment at greater distances than that given by the formula. In that event a lattice can be constructed and the successive reflections computed. When the arrester is connected to the system by an independent lead, (one of the cases described in the paper by McEachron and Wade), then a lattice is well nigh indispensable as a means for calculating the protection afforded by the arrester.

Messrs. Pittman and Torok in their paper speak of the "approximate surge impedance of three conductors in parallel." It may, therefore, be appropriate to include in this discussion the definitions and derivations of the self and mutual surge impedances of the equivalent conductor which replaces any number of conductors in parallel.

Let there be n conductors carrying equipotential waves e_0 and currents $(i_1, i_2 \;.\;.\;.\;.\;.\; i_n)$. Then the total current is

$$i_0 = i_1 + i_2 + \;.\;.\;.\;.\;.\; + i_n \tag{1}$$

and the self surge impedance of all wires in parallel is defined as

$$Z_{00} = \frac{e_0}{i_0} = \frac{e_0}{i_1 + i_2 + \;.\;.\;.\;.\;.\; + i_n} \tag{2}$$

But the fundamental equations for traveling waves are

$$\left.\begin{aligned} e_1 &= Z_{11}\,i_1 + Z_{12}\,i_2 + \;.\;.\;.\;.\;.\; + Z_{1n}\,i_n \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ e_n &= Z_{n1}\,i_1 + Z_{n2}\,i_2 + \;.\;.\;.\;.\;.\; + Z_{nn}\,i_n \end{aligned}\right\} \tag{3}$$

If $e_1 = e_2 = \;.\;.\;.\;.\;.\; = e_n = e_0$, then solving (3) for the currents, there is

$$i_1 = \frac{A_1}{D}\,e_0,\; i_2 = \frac{A_2}{D}\,e_0, \;.\;.\;.\;.\;.,\; i_n = \frac{A_n}{D}\,e_0 \tag{4}$$

where

$$D = \begin{vmatrix} Z_{11}\; Z_{12} \;.\;.\;.\;.\;.\; Z_{1n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ Z_{n1}\; Z_{n2} \;.\;.\;.\;.\;.\; Z_{nn} \end{vmatrix} \tag{5}$$

and

$$A = \begin{array}{c} (1) \;.\;.\;.\;.\;.\; (r) \;.\;.\;.\;.\;.\; (n) \\ \begin{vmatrix} Z_{11} \;.\;.\;.\;.\;.\; 1 \;.\;.\;.\;.\;.\; Z_{1n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Z_{n1} \;.\;.\;.\;.\;.\; 1 \;.\;.\;.\;.\;.\; Z_{nn} \end{vmatrix} \end{array} \tag{6}$$

that is, A_r is the same as determinate D with the rth column replaced by a column of *ones*. Then

$$i_0 = i_1 + i_2 + \;.\;.\;.\;.\;.\; + i_n = \frac{A_1 + A_2 + \;.\;.\;.\;.\;.\; + A_n}{D}\,e_0 \tag{7}$$

and therefore

$$Z_{00} = \frac{e_0}{i_0} = \frac{D}{A_1 + A_2 + \;.\;.\;.\;.\;.\; + A_n} \tag{8}$$

The mutual surge impedance between this group of wires and an independent wire k (not of the group) is defined as

$$Z_{0k} = \frac{e_k}{i_0} \tag{9}$$

where e_k is the potential induced in k by i_0. By the general equations of traveling waves this potential is

$$\begin{aligned} e_k &= Z_{1k}\,i_1 + Z_{2k}\,i_2 + \;.\;.\;.\;.\;.\; + Z_{nk}\,i_n \\ &= \left(\frac{Z_{1k}\,A_1 + Z_{2k}\,A_2 + \;.\;.\;.\;.\;.\; + Z_{nk}\,A_n}{D}\right) e_0 \end{aligned} \tag{10}$$

and therefore the mutual surge impedance between the group and the independent wire k is

$$\begin{aligned} Z_{0k} &= \left(\frac{Z_{1k}\,A_1 + Z_{2k}\,A_2 + \;.\;.\;.\;.\;.\; + Z_{nk}\,A_n}{D}\right) \frac{e_0}{i_0} \\ &= \left(\frac{Z_{1k}\,A_1 + Z_{2k}\,A_2 + \;.\;.\;.\;.\;.\; + Z_{nk}\,A_n}{A_1 + A_2 + \;.\;.\;.\;.\;.\; + A_n}\right) \end{aligned} \tag{11}$$

As an example of the application of these equations consider the line described by Pittman and Torok specified on page 568. There is

$$Z_{11} = Z_{22} = Z_{33} = 60 \log_\epsilon \left(\frac{2 \times 12 \times 32}{.23}\right) = 487$$

$$Z_{12} = Z_{23} = 60 \log_\epsilon \left(\frac{65.6}{14}\right) \qquad = \quad 93$$

$$Z_{31} = 60 \log_\epsilon \left(\frac{69.9}{28}\right) \qquad = \quad 55$$

Of course, $Z_{21} = Z_{12}$ and $Z_{32} = Z_{23}$ and $Z_{13} = Z_{31}$.

$$D = \begin{vmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{vmatrix} = \begin{vmatrix} 487 & 93 & 55 \\ 93 & 487 & 93 \\ 55 & 93 & 487 \end{vmatrix} = 115 \quad \times 10^6$$

$$A_1 = \begin{vmatrix} 1 & Z_{12} & Z_{13} \\ 1 & Z_{22} & Z_{23} \\ 1 & Z_{32} & Z_{33} \end{vmatrix} = \begin{vmatrix} 1 & 93 & 55 \\ 1 & 487 & 93 \\ 1 & 93 & 487 \end{vmatrix} = 17.01 \times 10^4$$

$$A_2 = \begin{vmatrix} Z_{11} & 1 & Z_{13} \\ Z_{21} & 1 & Z_{23} \\ Z_{31} & 1 & Z_{33} \end{vmatrix} = \begin{vmatrix} 487 & 1 & 55 \\ 93 & 1 & 93 \\ 55 & 1 & 487 \end{vmatrix} = 15.26 \times 10^4$$

$$A_3 = \begin{vmatrix} Z_{11} & Z_{12} & 1 \\ Z_{21} & Z_{22} & 1 \\ Z_{31} & Z_{32} & 1 \end{vmatrix} = \begin{vmatrix} 487 & 93 & 1 \\ 93 & 487 & 1 \\ 55 & 93 & 1 \end{vmatrix} = 17.01 \times 10^4$$

The equivalent surge impedance of all three wires in parallel therefore is

$$Z_{00} = \frac{D}{A_1 + A_2 + A_3} = \frac{106.5 \times 10^6}{(17.01 + 15.27 + 17.01)\,10^4} = 216$$

as compared with the 200 ohms given by Pittman and Torok.

There are three points in Mr. Sporn's paper which I wish to discuss—his inference that a flashover of the bottom conductor (of a vertical arrangement) indicates an induced stroke, his data on the effects of tower footing resistance and his conclusions about the extra benefits of two ground wires. The voltage which can appear on a line conductor as an induced surge is

$$e = \alpha\, G\, h\, (P.\,R.)$$

where

α = reduction factor (less than unity) depending upon law of cloud discharge and distribution of bound charge,

G = field gradient at the earth due to the cloud,

h = average height of the line wire,

$(P.\,R.)$ = protective ratio of the ground wires.

Now α and G are the same for all wires, and therefore the voltages appearing on different line wires depend upon the product $h\,(P.\,R.)$, and while the protective ratio is the highest for the lowest conductor, h is the smallest for that conductor, so that the product is usually actually less for the bottom conductor than for the top conductor. On the other hand, the voltage across the insulator due to a direct stroke at the tower is

$$e_{1r} = \left(1 - \frac{Z_{1r}}{Z_{11}}\right) e_1 = \frac{Z_{11} - Z_{1r}}{Z_{11}}\, e_1$$

where

e_1 = voltage on ground wire,

Z_{1r} = mutual surge impedance between the ground wire and the line wire r,

Z_{11} = self surge impedance of the ground wire.

But Z_{1r} decreases with the separation between the line and ground wires, so that the maximum voltage across the insulation occurs for the *lowest* conductor. Curves illustrating the effect of increasing the separation are given in Fig. 13 of my paper *Critique of Ground Wire Theory*, A. I. E. E. TRANS., March 1931, p. 1. Thus it is seen that a flashover on the bottom conductor may be due to a direct stroke on the ground wire rather than an induced surge. With regard to the value of low-tower footing resistances, the situation can be summed up as follows:

1. A lightning stroke will strike the nearest tower quite independently of the tower footing resistance.

2. If any tower of a continuous transmission line has an insulator flashover, it will be the tower that is struck, quite regardless of how its resistance compares with that of adjacent towers.

3. The lower the tower footing resistance the greater the reduction of voltage on the tower, ground wires, line conductors and across the insulators, and the more limited the danger zone.

These points were discussed in the above paper. The first point was illustrated in Fig. 7, the second and third points in Figs. 11 and 12. If the tower footing resistances of all the towers are not brought down to the same level, at least that of the more exposed towers should be reduced. Whether two ground wires are better than one depends on the location and arrangement of the ground wire, and the relative importance of induced and direct strokes. If a 132-kv. line with *any* kind of ground wire protection is immune from induced surges, then a single properly placed direct-hit wire with sufficiently low-tower footing resistances is as effective as any number of ground wires. But if the ground wires are placed at minimum elevations (as is dictated on the induced surge theory) then they will not be high enough to give the necessary shielding effect, nor will they have enough separation to prevent side flashes in case of a direct hit at mid-span.

It seems to me that the principal value of Mr. Dowell's paper is not so much the numerical data on attenuation which he secured, as in demonstrating how a difficult problem in traveling waves can be accurately calculated by a comparatively simple and straight-forward method. While it has been repeatedly shown that waves of relatively low potential may be treated as distortionless and excellent engineering agreement with tests obtained; yet for waves considerably above the corona limit the distortion cannot be ignored. There is a great need for a reasonably accurate and simple method for computing the distortion experienced by a traveling wave as it moves along the line.

H. L. Melvin: The results of the Arkansas Power & Light Company and Westinghouse Electric and Manufacturing Company cooperative lightning investigation has yielded two important contributions; first, the measured values of lightning voltage magnitudes have been extended and second, the practical possibility is indicated of draining lightning current from transmission conductors without permitting dynamic current follow-up for a sufficiently long interval of time to interfere with service or requiring relay and oil circuit breaker action to clear the fault.

At the present time two basic design principles present themselves for the improvement of line performance; first, diversion of strokes from the line conductors, requiring more complete knowledge concerning the characteristics and magnitudes of lightning currents and voltages, the impedance characteristics of the paths utilized for discharging the lightning as well as the nature and magnitudes of the voltages appearing across the line insulators when the lightning current is being discharged; second, the drainage scheme, the success of which will largely depend upon the development and proper application of the arc interrupting devices.

The investigation has also demonstrated that no practicable amount of insulation is effective for holding lightning voltages of the high magnitudes at least, though it may be that by utilizing the impulse insulation strength of the wood to a considerable degree, some flashovers are avoided that might have occurred with the conductors supported on normal amounts of porcelain insulation. The surges in Figs. 10 and 11 illustrate such possibilities. These oscillograms are extremely interesting and unusual as compared to the high-voltage surges causing flashover, which have been obtained from this and other lightning investigations. Many of the oscillograms of low voltages which have been held by line insulation are of long durations, however, and in my opinion the two shown in the paper may be more nearly representative of actual lightning than any which have been obtained to date. It is reasonable to expect that the transmission line would hold the one million-volt surge but the two million seven hundred thousand-volt surge would probably break to earth if it occurred anywhere on the line except in the nine-mile test section.

An explanation which seems reasonable is that a stroke of relatively low-current magnitude or a side flash from a main stroke occurred in the test section rather close to the oscillograph station. A slightly increasing current value in the lightning stroke during the first 125 microseconds of the discharge would account for the rising voltage characteristic. Attenuation from 3,000 to 1,500 kv. or less in the four miles to the lower insulated section and to 600-kv. thirteen miles away at the substations also seems entirely possible. It must, of course, be remembered that the actual magnitudes of these high voltages are more or less approximate due to the potentiometer coupling calibration, also that they are measurements of voltage with respect to time at a point on the line referred to the earth connection at the oscillograph station. Regarding time it seems very reasonable to me that durations of lightning current flow of this order and even greater are normal rather than unusual.

C. E. Ambelang and F. E. Andrews: It would be of considerable interest if Messrs. Pittman and Torok would give an estimate of the value of lightning voltages at the points where flashover occurred. For example, in the case where the 5,000-kv. oscillograph record was taken from the lightning flashover four miles from the oscillograph station it would be interesting to know what value of lightning voltage could be estimated at structure No. 167 where the surge originated.

We assume that flashovers referred to are in general lightning flashovers, as distinguished from power arc flashovers, except that the latter is mentioned in the case of the flashover at structure No. 167. We have found in our work on 33-kv. wood pole lines that the condition under which 60-cycle power arc between conductors, or from conductor to ground, will follow a lightning sparkover determines to what extent the service on the line is affected by lightning flashovers.

We have established quite definitely by operating experience that 9 ft. of wood between conductors provides sufficient insulation to prevent line interruption due to the power follow-up, and that wood insulation in the path from conductor to ground over the guys is also effective in this same way. We have had a large number of lightning flashovers to ground on our 40-ft. poles of all wood construction without a 60-cycle power follow-up. We had approximately 90 miles of this type of construction in service during 1930.

The authors conclusion No. 1, stating that pure insulation is not a solution to the lightning problem, we interpret as referring to insulation with respect to lightning and our experience as indicated above, ties in with this conclusion.

Reference is made in conclusion No. 2, to voltages of high magnitudes being transmitted for several miles along highly insulated lines. Our observations indicate that this may not always be the case. We have observed a number of instances in which structures of high insulation value, that is, our all wood pole construction, have flashed over to ground due to lightning while adjacent structures of very low insulation have not flashed over. In one instance the highly insulated pole was only 150 ft. from the start of the pole line of lower insulated value. In another case a 40-ft. pole, 500 ft. from a structure with grounded guys, was splintered by lightning which apparently followed this path in preference to the nearby low insulation path. In no case has the travel been noted over a quarter of a mile.

In conclusion No. 5, the authors state that trip-outs on ungrounded wood pole construction are the result of line-to-line flashovers. This checks with our experience and was covered in the paper *Transmission Research and Design with the Field as a Laboratory*, by F. E. Andrews and C. L. Stroup, A. I. E. E. Trans., July 1930, p. 959. This experience has been borne out by information obtained during 1930. We have found that the steel crossarm brace used with the conventional type of 33-kv. construction short circuits a portion of the wood crossarm and must be eliminated in order to prevent power arcs from occurring on the 33-kv. construction.

We recognize that the problem with respect to preventing line outages is somewhat different at the lower voltage with which we have had experience, than with the 110-kv. lines on which the authors have conducted their investigation. However, the underlying principles are the same and our experience substantiates that which they cite, and corresponds to their conclusions with respect to line operation.

C. L. Fortescue: As the paper by Messrs. Pittman and Torok states, the line was freed from any drain points for the length of nine miles so as to provide the necessary conditions for obtaining the highest records due to lightning. Two such records were obtained during the last lightning season.

Fig. 11 has not been adequately explained in the text of the paper and I think it should be made clear that it cannot possibly be a cathode-ray oscillograph record of what happened on the line itself as on either side of the lightning experimental station four and one-half miles distant, the line was provided with fused drain points having a flashover voltage of 800 to 1,000 kv. for long surges.

While it is quite possible in the particular section in which the lightning station is located to have potentials as high as that recorded in Fig. 11, the duration of such a surge could not last more than 40 microseconds before being reduced to a value of about 1,000 kv. or less. The fact that, as stated in the paper, there was no record of flashover or blown fuses, this oscillogram must be considered as a false record probably due to something influencing the cathode-ray oscillograph circuit directly and not through the line.

The cathode-ray oscillograph is a very sensitive instrument and is susceptible to extraneous influences. Although precautions are taken to reduce such possibilities to a minimum, nevertheless they should be kept in mind in interpreting the results obtained.

It should be pointed out that with a line of this type of construction provided with drain points of the type described in the paper at one mile intervals, the duration of any surge of the magnitude indicated in Fig. 11 would not exceed 10 microseconds before being reduced to a relatively small value.

H. C. Don Carlos: It would seem that the experience the Hydro-Electric Power Commission of Ontario has had with certain of its transmission lines would have some bearing on the points mentioned in these papers.

In 1920 a wood pole 110-kv. transmission line approximately 70 miles long and lying along the north west shore of Lake Superior, was placed in service operating at 60 cycles. The construction used embodied wooden crossarms and braces as shown in Fig. 2, the average span being about 350 ft. It will be noticed that the line is equipped with a grounded sky wire. This sky wire was grounded at approximately every fifth pole. This grounding wire was 9/32-in. steel, similar to the sky wire and after being wrapped around the bottom of the pole several times was carried up the pole and stapled to it, to within about 17 ft. of the top. At this point it was pulled off and clamped to the sky wire about three feet from the poles. It will be seen that since the standard pole was 45 ft. high, the average conductor height from the ground was about 29 ft. This line was located in the center of an 80-ft. right-of-way cut through the bush and while certain sections were in rock it is believed that about 50 per cent was through muskeg. It will be evident that this circuit is well shielded by its grounded sky wire and the nearby trees, etc. However, the district in which this transmission line route lies, is called the Thunder Bay District, which title is probably justified by the average of about 30 lightning storms per year (47 in 1925 and 20 in 1927) since the line was placed in service, with practically all of these occurring within five months, with July and August being, of course, the months when storms were most prevalent.

However, during the ten years service of this circuit, operating at approximately 120 kv. there is no record of an outage on it due to lightning. There have been several outages during this time on this circuit but in each case, these have been due either to interference with the circuit, or to other circuits which have been solidly connected to it. During the first three and one-half years when we were solely dependent on this circuit for the supply of power to the city of Port Arthur, we were fortunate inasmuch as there were no outages on this line from any cause.

This right-of-way was widened about 60 ft. and a circuit mounted on steel towers was placed in service late in the summer of 1924. No ground wire was placed on these towers. Two outages on this new circuit this year, seven in 1925 and five in 1926 indicated the necessity for some additional protection to this circuit and a sky wire was placed in service on this tower circuit in 1927. Since we have had one outage in 1927 on this circuit due to a lightning flashover, one in 1928, two in 1929 and two in 1930, placing this ground wire in service would indicate some benefit from the sky wire.

A second circuit was erected on the steel towers in January 1930 and three outages during lightning storms in 1930 occurred on this circuit. None of these outages on this circuit occurred simultaneously with outages on the other circuit on these towers.

One or more outages per year is also experienced on the circuits of a 9-mile steel tower line extending on from the Port Arthur station which is fed from the above mentioned circuits.

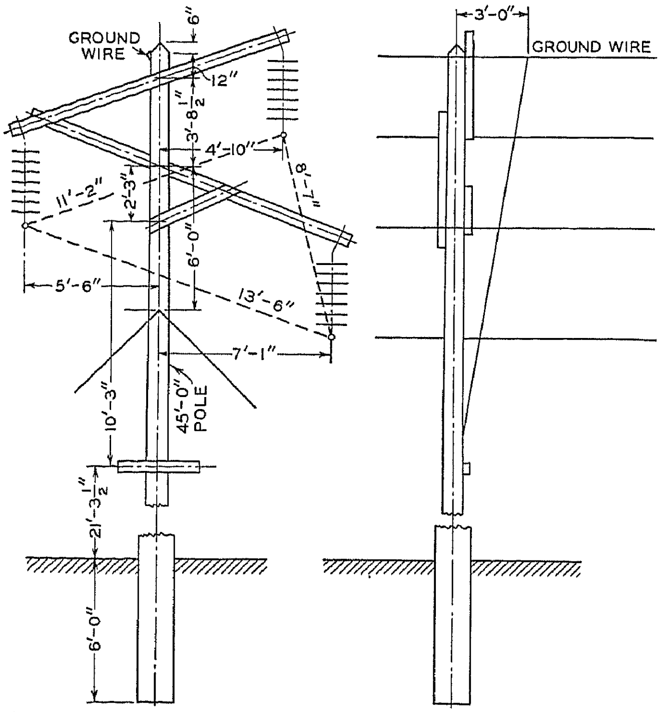

FIG. 2—STANDARD POLE CONSTRUCTION THUNDER BAY SYSTEM LINE

Our system superintendent suggests that the majority of the flashovers in this district occur in some high ground which is supposed to contain low-grade iron ore.

Contrasting very sharply, however, with the excellent performance of this wood pole circuit (from the Nipigon River to Port Arthur) is the experience we have had with a 66-mile wood pole circuit in Southern Ontario (from St. Thomas to Sarnia) operating at 25 cycles. This circuit was placed in service in January 1926. About three-quarters of this circuit used a standard two-pole H frame construction with horizontal configuration of the conductors and 500 ft. spacing between structures, while the remainder used single poles with triangular spacing similar to the Thunder Bay System line.

No ground wire was erected on these poles when the line was placed in service but 16 outages in 1926 due to lightning indicated the urgency for some additional protection on this circuit. A sky wire was erected on this circuit in Feb. 1927 with ground wires being carried down every fifth pole. However, seven outages due to lightning were experienced in 1927, two in 1928, eleven in 1929 which and eleven in 1930, which indicate that the problem of protecting this circuit is not yet solved.

During 1929 the resistance of the earth connections at the various poles on this circuit was measured and it was found that many of these were quite high and some effort was made during 1930 to reduce these resistances by driving additional pipe grounds. A more careful survey of this circuit also indicated somewhat inadequate clearances from power conductor to guy wires. In some cases insulators, bridged by one inch gaps have been used in these guy wires. The poor performance of this circuit, which runs practically due west from St. Thomas, is hard to understand in view of the fact that a steel tower line approximately 103 miles long carrying two circuits south west from St. Thomas at an angle of about 40 degrees from this wood pole line is practically immune from lightning trouble having only one or two outages per season. Further study will be made of this circuit with a view of applying additional protection to reduce the effect of lightning on it.

In this connection a brief mention might be made of the performance of the 230-mile 220-kv. transmission circuit from the Gatineau River in Quebec to Toronto in Ontario. This circuit went into service on October 1st, 1928. Up to the present time this circuit has had a total of nine outages, of which four were due to external interference and five were due to lightning flashovers. It is believed that one of these lightning flashovers went directly to ground from the middle of the span. The second circuit on this right-of-way was placed in final service on July 28, 1930 and has not yet had an outage. The only outage on the original circuit since August 23, 1929 occurred on September 17, 1930 and did not result in any interruption to service since the second circuit was not affected.

E. R. Whitehead: I quite agree with Mr. Bewley that a knowledge of the functional impedance used in the Roseland tests is highly desirable. It was not possible to give complete mathematical treatment of these various conditions in the limited time available for preparation of the paper.

I may add, however, that methods of calculation outlined by Mr. Bewley have been used in computing complex waves resulting from capacitance reflections from the towers themselves. In large substations bus structures may be of such a length that considerable error would be introduced into calculations by considering the station capacitance as lumped at the end of the transmission line.

It is also pointed out that the equation for estimating the effect of the lightning arrester location is approximate, and cannot be used where only partial reflection takes place at the equipment or point under consideration. The results obtained show that the equation can be used where a transformer constitutes the terminal load. It was felt that such an expression would furnish a simple guide to arrester location where conditions were comparable with those of the tests, and that any errors arising from its use must be on the safe side.

J. J. Torok: Mr. Peek in discussing the 5,000,000-volt surge simulated it to the ½-5 microsecond wave. In the nature of wave shape it is true the 5,000,000-volt surge is similar to the ½-5 microsecond wave but in making this comparison one must remember that this is a chopped wave. It was chopped by flashover of the wood pole and also by a flashover from one of the conductors to a telephone line running under the line. This oscillogram does not give sufficient information to enable us to reconstruct the original wave, that is the wave that would have traveled over the conductor had not the pole flashed over. For this reason we cannot make comparisons with any particular laboratory wave.

Mr. Bewley goes to considerable length to calculate the surge impedance of three conductors. His results check our calculations, however, we assumed the lower figure of 200 ohms simply because the mathematical calculation took no account whatsoever of corona. It is well known that corona increases the effective diameter of the conductor and consequently reduces its surge impedance. Through lack of definite data on corona we assume a somewhat lower value of 200 ohms,which we believe would be more nearly correct than calculating the value assuming only the diameter of the conductor.

Fig. 11 has been referred to and is under considerable discussion. It is to be remarked here that the oscillogram shown in Fig. 11 is somewhat questionable. There are two sources in which error might creep into this set up. Both of these sources are due to the fact that a capacity potentiometer is used. A lightning stroke terminating nearby might cause an induced potential on the intermediate electrostatic plate. Since this plate is connected directly to one of the deflecting plates on the oscillograph it requires only fairly low potentials to cause a relatively large deflection in the cathode-ray oscillograph. Another possibility is that the stroke terminating nearby caused considerable difference in potential between two points in the earth, that is between the ground electrode of the potentiometer and the ground of the cathode-ray oscillograph in the oscillograph building. This difference also may be sufficient to cause a deflection. To overcome this the potentiometer has been changed to a resistance type. It is hoped that with this new type of potentiometer all external influences will be completely eliminated.

Mr. Ambelang and Mr. Andrews have brought up a very interesting point. They have pointed out that by the use of wood, power follow arcs will not be established. It is this property of wood construction that makes it so desirable as a form of line support.

Wood in forming a path for lightning streamers acts as a restricting agent and prevents the formation of power follow arcs, however, it is necessary to have sufficient length of wood in a circuit to make it completely effective. Wood, in itself, is not sufficient to prevent a lightning flashover, however, the deionizing properties of wood will serve to suppress the arc and prevent an outage which after all is the essential factor in transmission line construction.

Philip Sporn: We have gathered the data presented in this paper, endeavoring to prove or disapprove in our own mind a few of the various theories floating around; that is, whether our troubles due to lightning are the result of direct strokes or induced surges. According to the data which we have accumulated, both are to be treated alike in regard to protection, etc. In the near future we hope that some definite conclusions will be reached in allocating the direct stroke to high-potential lines and induced surges to low-potential circuits. Our data which we have collected gives a true picture of what has actually occurred and has not been distorted or influenced in attempting to give any particular theory preference. We can only analyze the data which we obtained and have merely pointed out what it seems to indicate to us. If others can draw different conclusions, in some cases, from these data, it merely shows that records are not sufficiently in detail to definitely prove some of the points which are now in question.

Electron Tubes in Industry

BY W. R. KING[1]
Non-member

GENERAL INTRODUCTION

ELECTRON tubes have been in everyday use in radio and other communication fields for a number of years. It is only recently, however, that the field of applications has been extended to include industrial usages, and while the number of industrial installations employing electron tubes is not enormous, great progress has been made in applying these new electrical tools to the problems of the industries. It is not the intention to cover the entire field of industrial tube applications in this paper, but merely to present the fundamentals of operation and the practical aspect of the application of some of the more important of these new and interesting devices.

The applications and prospective applications of electron tube systems may be broadly divided into two general groups, (1) those in which the electron tube devices perform operations for which no reasonably suitable apparatus of a conventional nature is available, and (2) those for which conventional apparatus already exists. Obviously the dividing line between what we may call "competitive" and "non-competitive" applications cannot be clearly defined. Photoelectric devices for example, have no strict competition from existing apparatus, although they frequently fall in the "borderline" class. Electron tube equipment for supplying adjustable voltage to d-c. motors is, on the other hand, competitive with conventional apparatus and must be so considered from a standpoint of performances and economy.

In weighing the advantages and disadvantages of electron tube systems consideration must be given to the absence of major moving parts. The maintenance cost for an equipment is undoubtedly decreased if it contains no bearings to be oiled and no wearing parts to be frequently adjusted and occasionally replaced by skilled labor.

Another advantage that is generally important is that electron tube systems can respond to controlling functions of a very low energy level. In adapting electrical control to automatic machinery of various types the ability to secure control from small parts or from slight movements is often the difference between the failure and success of the machine.

The fragility of electron tubes has been a much discussed disadvantage in their application. It is obvious that the assemblage of glass and small metal parts must be handled and mounted with some care. For most applications, however, the fragility of the tubes is a psychological disadvantage rather than an actual one. The tubes are really more sturdy than they appear and it is felt that few potential applications of electron tubes will be found impractical because of the seemingly fragile nature of the tubes.

The question of tube life and tube replacement is largely one of economics. Tubes operated at rated values of current and voltage may be expected to have a certain average life. If they are operated somewhat under their rating this life expectation figure will be materially increased. By way of illustration, typical estimates of tube life at rated load run from 2,000 to 10,000 hours. When used well below rated values, the tube life may be increased to as much as double its expected life at rated load. It is a matter of economics, then, to determine whether to use a certain tube at its rated load, or to use a larger and more expensive tube operating well under its rated load and thus secure a greatly increased life.

THEORY AND CHARACTERISTICS OF PHOTOELECTRIC TUBE

Fundamentally, the photoelectric tube is a two-element light sensitive device which depends for its operation on the emission of electrons from a light sensitive surface. There are other light sensitive devices such as the selenium tube and the electrolytic cells, but this paper is confined to a consideration of light sensitive devices in which electrons are liberated from a sensitized cathode and flow to a positively charged anode.

Sodium, potassium, rubidium, caesium, and other alkali metals have been used to form the sensitive cathode surface. Caesium is commonly used at present because its color sensitivity in the red end of the spectrum is higher than that of other alkalis.

There are at present two general types of photoelectric tubes, the gas filled tubes and the vacuum type tubes. In both types light liberates electrons from the sensitive cathode. If a source of potential is applied to the tube to make the cathode negative and the anode positive, the negatively charged electrons will be attracted to the anode. In the vacuum tubes this flow of electrons from cathode to anode constitutes the entire current passed by the tube. In the gas filled tubes however, the maximum current which can flow is much greater than that represented by the number of electrons liberated from the cathode. The reason for this is that the electrons, in traveling from the cathode to the anode, collide with the atoms of the inert gas introduced in the tube and liberate from the atoms additional electrons which are also attracted to the anode. In going to the anode these electrons collide with other atoms releasing

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[1] Industrial Engg. Dept., General Electric Co., Schenectady, N. Y.

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additional electrons, etc. This phenomenon is called gas amplification and its use makes possible the construction of photoelectric tubes capable of passing approximately ten times the current per unit light flux that can be passed by the vacuum photoelectric tubes.

The number of electrons emitted from the sensitive cathode (and therefore the current which the tube passes) is dependent upon the amount of light on the cathode. For the vacuum photoelectric tube the relation between light flux and anode current (see Fig. 1) is linear to a high degree of accuracy. For the gas filled tubes this relation has a slight curvature (Fig. 1) but for most purposes may be considered substantially linear. This proportionality of current to light flux is the most important characteristic of the photoelectric tube and it is upon this characteristic that most of its applications are based.

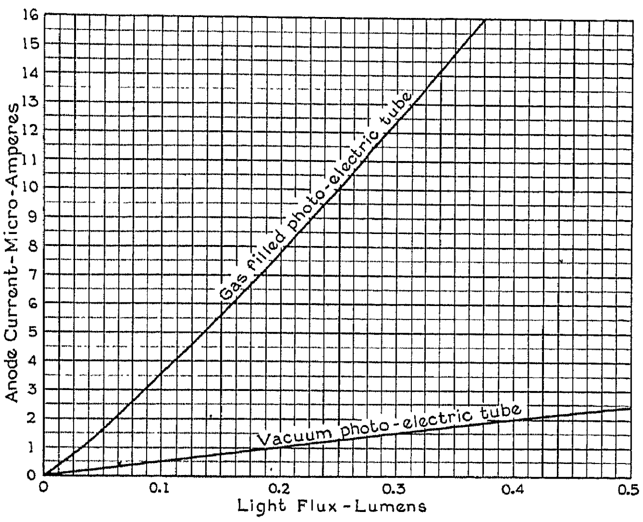

Fig. 1—Light Flux—Anode Current Characteristic for Gas-Filled and Vacuum Type Photoelectric Tubes

Amplification of Photoelectric Currents

To be readily useful, the minute photoelectric current (of the order of a few microamperes) must be amplified sufficiently to operate relays, ordinary recording and indicating instruments, or other devices. The most commonly used amplification systems employ a three-element thermionic amplifier tube similar to the type which has become familiar as a radio amplifier. This type of tube will be referred to as the pliotron.[2] The circuit is so arranged that a change in the photoelectric tube current changes the grid voltage of the pliotron. This results in a proportionate change of the pliotron plate current which is of the order of milliamperes and may be used to operate relays, etc. This type of circuit arrangement may be employed on alternating current or direct current. Because of the rectifying action of both the pliotron and the photoelectric tube the a-c. circuit is active only when the respective anodes are positive. The resultant output is a half-wave pulsating direct current. The effective value of this current depends chiefly on the illumination on the photoelectric tube but is affected to an extent by variations in the supply voltage.

A relay capable of operating on the few milliamperes output of the pliotron, however, cannot control power circuits directly, nor even the operating coil circuits of medium size contactors or solenoids. It is necessary, therefore, to employ an additional small contactor before currents of the order of amperes can be controlled. In some applications this is perfectly satisfactory but frequently applications are encountered in which the speed of response required is too great for this concatenation of relays. In such cases it has sometimes been found advantageous to employ a thyratron[3] in place of the relays. The operation of the thyratron in this capacity will be discussed more thoroughly later. For the present we may consider it as an electronic device which has an anode current of the order of amperes which may be controlled by certain functions of its grid voltage. In the system involving photoelectric tube, pliotron, and thyratron, the thyratron grid is controlled by the output of the pliotron in such a manner that when the illumination on the photoelectric tube reaches a predetermined value, the thyratron suddenly conducts and will carry currents of several amperes—sufficient to operate directly power size contactor coils, solenoids, etc. When the light decreases below a certain value, the thyratron stops conducting. In securing the amplification from microamperes to amperes by this system, the time lag is only a small fraction of that required by a series of relays to secure the same degree of amplification.

Color Analysis and Color Matching

A number of photoelectric equipments has been built for color analysis and color matching. The recording color analyzer and one type of color comparator is described here.

As its name implies the recording color analyzer is a photoelectric device which automatically draws for permanent record the color curve of a sample. The device is so constructed that the light reflected from the surface of a solid material or the light transmitted by a transparent substance may be the basis of the final curve drawn. By means of a flicker disk, the photoelectric tubes receive alternately monochromatic light reflected first from a standard then from the sample. A difference between these results in a pulsating photoelectric current which is amplified sufficiently to operate a motor and move a shutter in the standard light path. A recording pen indicates on a chart the position of the shutter at which the light from the standard equals that from the sample. As the chart is rotated the monochromatic band of light is shifted through the visible spectrum. The curve obtained indicates at each wavelength the balance point at which the light from the white standard equals that from the sample; the ordinates of the curve become the reflection coefficients of the sample in terms of standard white.

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[2] Trade name for electrostatically controlled high-vacuum discharge device.

[3] Trade name for electrostatically controlled arc discharge device.

The use of this device opens a new field to those working with color in view of the fact that the curve obtained from a certain sample is absolute identification for its color. It is not inconceivable that in the future numbers will be applied to the many curves and the arbitrary names such as royal blue, cardinal red, canary yellow, etc. will be discarded for purposes of specification.

The color comparator is a much simpler device than the color analyzer. Its most important use is in matching colors. The device consists of a simple optical system and an amplifier. A milliammeter indicates the difference between light reflected from two specimens placed successively in the sample stand. A tri-colored filter makes it possible to obtain comparisons in three parts of the spectrum in addition to white light. The accuracy seems to be as good as that of the eye for all samples and is better than that of the eye in the spectral regions where the eye is insensitive.

RECORDING OR INDICATING LIGHT CONDITIONS

Applications requiring the continuous recording or indicating of amounts of visible energy on the photoelectric tube may be divided into three general classes:

1. Light from a constant source passing through a substance of varying translucency or transparency to the photoelectric tube.
2. Light from a constant source reflected into the photoelectric tube from a surface of varying reflecting power.
3. Visible radiation from a varying source applied directly to the photoelectric tube.

In making such applications some inaccuracies are encountered due to voltage variations, changes in characteristics of photo-tubes, amplifying tubes, and light sources. Consideration must be given in each case to determine whether or not the accuracy secured will be acceptable, or if it is apparently not acceptable, whether or not the complication necessary to reduce the inaccuracies to acceptable values is warranted.

Photoelectric equipment of this nature has been used to make continuous records of smoke densities. In an application of this type a beam of light is transmitted through the duct carrying the smoke and onto the photoelectric tube. The variation in smoke density permits more or less light to fall on the photoelectric tube. When properly calibrated, then, the photoelectric current is a measure of the smoke density. A recording milliammeter measures the amplified photoelectric current and the resultant chart shows graphically the smoke conditions at all times.

Photoelectric tube and amplifying devices have been used for recording temperatures of hot bodies. At present, however, it is practically impossible for an electrical manufacturer to build a device which will be applicable to a number of different applications of this type. Each different application, then, will probably require some development, first to determine its feasibility, and second to determine the best operating scheme.

Other applications have been proposed in which a continuous record would be made of light transmitted through liquids or reflected from a moving web of paper or material. Variations in the photoelectric current would ensue from changes in color, shade, density or some other characteristic of the material affecting its light transmitting or reflecting properties. While it is felt that no insurmountable difficulties would be encountered in making such applications, it is known that the difficulties in working out a specific case would be considerable and the writer does not know of any such actual installations at present.

RELAYING APPLICATIONS

By far the greatest number of applications of photoelectric equipment which are operating at present are of the type in which an electric circuit is opened or closed in response to a certain change of illumination on the photoelectric tube.

In this manner street lights are being turned on in the evening and off in the morning in response to actual illumination requirements rather than according to a predetermined time schedule.

By similar devices electric signs are operated so that they are lighted when, because of low general illumination, their brilliance will attract attention, and are never lighted until illumination conditions are such that they will attract attention.

Room and factory lights can also be controlled in response to illumination requirements, with the result that sufficient illumination is maintained at a minimum cost for lighting energy.

Perhaps the simplest type of application of photoelectric equipment is that in which a beam of light falling on the photoelectric tube is intercepted by a practically opaque object and some electric circuit is closed or opened accordingly. By this means, for example, objects passing through the light beam may be accurately counted. The amplified photoelectric current operates a small relay which controls a magnetic counter. Or, if higher speed is required, the relay may be replaced by a thyratron which will operate the counter directly. Photoelectric counters are being used commercially in counting automobiles, people, steel bars and billets on rolling mill tables, drops of oil in an automatic oiling machine, etc., and are performing in such capacities in a thoroughly practical manner. Such a counting system offers distinct advantages in counting heavy objects which would soon destroy a mechanical counter, or small light objects which could not exert sufficient force to operate a mechanical

ften it is difficult or physically impossible
echanical counter in such a position that it
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a manner which has not heretofore been
Mechanical devices are not suited in some
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pace limitations. It requires no additional
moving object to intercept a light beam;
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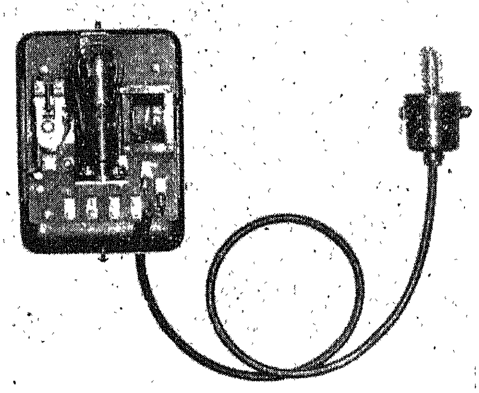

FIG. 2—PHOTOELECTRIC RELAY

The grid is placed between the filament and the anode and controls the flow of electrons from the filament to the anode. If the grid is made negative it repels electrons itself and reduces the flow from filament to anode. It is possible to stop the electron flow completely by making the grid sufficiently negative. The relation that exists between grid voltage and plate current for different values of plate voltage is shown in Fig. 3. This characteristic is utilized in almost every application of the pliotron and it is this characteristic that has made the pliotron and similar vacuum tubes almost invaluable as oscillators, amplifiers of weak signals, etc.

the monochromatic band of light is shifted through the visible spectrum. The curve obtained indicates at each wavelength the balance point at which the light from the white standard equals that from the sample; the ordinates of the curve become the reflection coefficients of the sample in terms of standard white.

The use of this device opens a new field to those working with color in view of the fact that the curve obtained from a certain sample is absolute identification for its color. It is not inconceivable that in the future numbers will be applied to the many curves and the arbitrary names such as royal blue, cardinal red, canary yellow, etc. will be discarded for purposes of specification.

The color comparator is a much simpler device than the color analyzer. Its most important use is in matching colors. The device consists of a simple optical system and an amplifier. A milliammeter indicates the difference between light reflected from two specimens placed successively in the sample stand. A tri-colored filter makes it possible to obtain comparisons in three parts of the spectrum in addition to white light. The accuracy seems to be as good as that of the eye for all samples and is better than that of the eye in the spectral regions where the eye is insensitive.

Recording or Indicating Light Conditions

Applications requiring the continuous recording or indicating of amounts of visible energy on the photoelectric tube may be divided into three general classes:

1. Light from a constant source passing through a substance of varying translucency or transparency to the photoelectric tube.
2. Light from a constant source reflected into the photoelectric tube from a surface of varying reflecting power.
3. Visible radiation from a varying source applied directly to the photoelectric tube.

In making such applications some inaccuracies are encountered due to voltage variations, changes in characteristics of photo-tubes, amplifying tubes, and light sources. Consideration must be given in each case to determine whether or not the accuracy secured will be acceptable, or if it is apparently not acceptable, whether or not the complication necessary to reduce the inaccuracies to acceptable values is warranted.

Photoelectric equipment of this nature has been used to make continuous records of smoke densities. In an application of this type a beam of light is transmitted through the duct carrying the smoke and onto the photoelectric tube. The variation in smoke density permits more or less light to fall on the photoelectric tube. When properly calibrated, then, the photoelectric current is a measure of the smoke density. A recording milliammeter measures the amplified photoelectric current and the resultant chart shows graphically the smoke conditions at all times.

Photoelectric tube and amplifying devices have been used for recording temperatures of hot bodies. At present, however, it is practically impossible for an electrical manufacturer to build a device which will be applicable to a number of different applications of this type. Each different application, then, will probably require some development, first to determine its feasibility, and second to determine the best operating scheme.

Other applications have been proposed in which a continuous record would be made of light transmitted through liquids or reflected from a moving web of paper or material. Variations in the photoelectric current would ensue from changes in color, shade, density or some other characteristic of the material affecting its light transmitting or reflecting properties. While it is felt that no insurmountable difficulties would be encountered in making such applications, it is known that the difficulties in working out a specific case would be considerable and the writer does not know of any such actual installations at present.

Relaying Applications

By far the greatest number of applications of photoelectric equipment which are operating at present are of the type in which an electric circuit is opened or closed in response to a certain change of illumination on the photoelectric tube.

In this manner street lights are being turned on in the evening and off in the morning in response to actual illumination requirements rather than according to a predetermined time schedule.

By similar devices electric signs are operated so that they are lighted when, because of low general illumination, their brilliance will attract attention, and are never lighted until illumination conditions are such that they will attract attention.

Room and factory lights can also be controlled in response to illumination requirements, with the result that sufficient illumination is maintained at a minimum cost for lighting energy.

Perhaps the simplest type of application of photoelectric equipment is that in which a beam of light falling on the photoelectric tube is intercepted by a practically opaque object and some electric circuit is closed or opened accordingly. By this means, for example, objects passing through the light beam may be accurately counted. The amplified photoelectric current operates a small relay which controls a magnetic counter. Or, if higher speed is required, the relay may be replaced by a thyratron which will operate the counter directly. Photoelectric counters are being used commercially in counting automobiles, people, steel bars and billets on rolling mill tables, drops of oil in an automatic oiling machine, etc., and are performing in such capacities in a thoroughly practical manner. Such a counting system offers distinct advantages in counting heavy objects which would soon destroy a mechanical counter, or small light objects which could not exert sufficient force to operate a mechanical counter. Often it is difficult or physically impossible to place a mechanical counter in such a position that it will be operated by the passing of objects. In using the photoelectric equipment, the devices may be placed at a considerable distance and the light beam alone projected across the path of the moving object. The possibilities of photoelectric counting equipment have only been scratched and it is believed that the next few years will find many industrial plants using photoelectric devices to secure an accurate count of their products.

This brings us to a consideration of photoelectric devices for limit switch operations. Their use in this capacity has made possible the automatic operation of machines in a manner which has not heretofore been practicable. Mechanical devices are not suited in some cases because of mechanical wear and tear, or because sufficient force is not available to operate them, or because of space limitations. It requires no additional force for a moving object to intercept a light beam; furthermore, the beam is in no way worn by the interception. In the manufacture of small cotton bags, photoelectric relays are being employed to initiate operations on the bags when they are in the proper position. Photoelectric relays are being installed in ore bins to indicate when the ore has reached the proper level. Several large pipe facing machines have been equipped and are in operation with photoelectric devices to stop the traversing motion and advance the cutting tool when the pipes reach the proper position. In a number of instances photoelectric relays have replaced mechanical limit switches on rolling mill tables.

Photoelectric devices can be made sensitive to smaller changes in illumination than those characterized by the applications involving the interception of a light beam by an opaque object. For example, photoelectric equipment is being used to indicate the proper operation of a filter in a sugar refining process. A beam of light is passed through the solution and if the solution runs cloudy because of improper filter operation the decreased translucency causes the photoelectric device to operate an alarm. With proper illumination black spots of sufficient size on paper or other material may be used to operate a photoelectric relay either by changing the amount of light transmitted through the paper to the photo-tube or by reflecting less light into it. Sorting devices can be made to operate from changes of reflected or transmitted light.

The application of photoelectric devices to the simpler types of problems has been facilitated by the standardization of photoelectric equipments. Several electrical manufacturers have introduced complete photoelectric equipments including photoelectric tube, some type of amplifying arrangement, and a small relay. A device of this type which also includes a contactor is shown in Fig. 2. The ability to procure such devices makes it no longer necessary for the plant engineer or production engineer to begin at the bottom and learn all about photoelectric tubes and amplifying systems when he sees the possibility of employing photoelectric devices to advantage.

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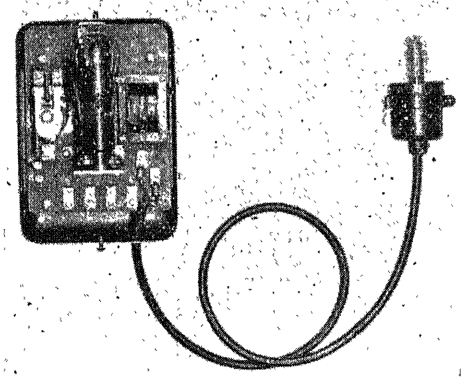

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THE PLIOTRON AS AN AMPLIFYING DEVICE

When the grid is maintained negative its current will be very small. Therefore the grid potential may be controlled with very little power. For this reason the tube may be used as an amplifier. Consider the amplification of photoelectric currents for example. The currents are of the order of a few microamperes and the effective impedances may be many megohms. Although the energy level is very low, a slight change in photoelectric current causes a change of several volts in the high impedance circuit. This voltage change, applied to the pliotron grid, is sufficient to change the anode current by several milliamperes, enough for example, to operate a small relay.

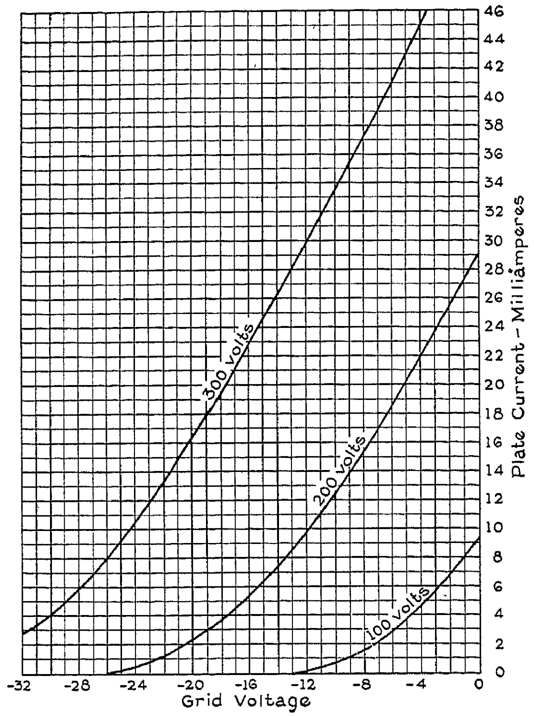

FIG. 3—GRID VOLTAGE—ANODE CURRENT CHARACTERISTIC FOR A TYPICAL PLIOTRON

This characteristic has been employed for several years in automatic train control. A small voltage is picked up from the tracks and amplified to operate signal devices in the engineer's cab.

The pliotron may be used as a very sensitive relay to respond to the opening and closing of delicate contacts or to contact between an electrode and a conducting liquid (such as water).

THE PLIOTRON AS AN OSCILLATING DEVICE

By a proper circuit arrangement in which the grid is inductively coupled to the plate circuit oscillations may be generated. Under certain conditions if the coupling between the plate coil and the grid coil is changed, as by placing an iron vane between the coils, the oscillations will be stopped. This characteristic of the oscillating circuit has been employed in the automatic leveling of elevators. An iron vane in the elevator hatchway passes between the plate and grid coil in an oscillating circuit on the car. When the iron vane is in the proper position the oscillations cease and a relay is closed, operating the control equipment which brings the elevator to a stop.

GENERAL THEORY AND CHARACTERISTICS OF THE THYRATRON

Practically speaking, the thyratron is a mercury-arc or mercury-vapor rectifier incorporating a control electrode or grid. This section is confined to a consideration of the hot cathode mercury vapor thyratron. The ionization of the mercury vapor results in an approximately constant internal voltage drop of about 15 volts.

The method of grid control is entirely different from that of the pliotron. For a given plate voltage there is a particular grid voltage at which ionization will just occur, thus allowing the tube to pass current. If the grid potential is below this critical potential, *i. e.*, more negative, no discharge will occur and no current will pass. As soon as the grid potential is increased above the critical voltage, ionization occurs and the tube passes current, providing the anode is positive with respect to the cathode. Once the anode current is started, the grid has no appreciable effect on it. The grid cannot limit or stop the flow of current, but can regain control to keep it from starting again if the flow ceases long enough for the mercury vapor to deionize.

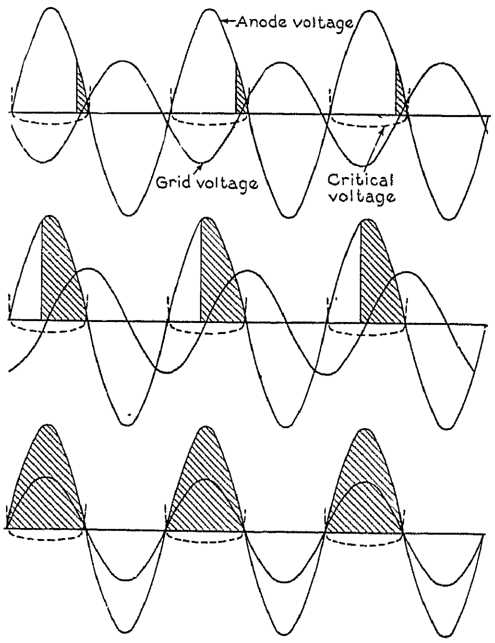

FIG. 4—CONTROL OF THYRATRON BY SHIFTING PHASE OF GRID VOLTAGE

If an a-c. voltage is applied to the plate, the grid has an opportunity of regaining control once each cycle and can delay the start of the arc for as long a period during the cycle as the grid is sufficiently negative. Therefore, if the grid as well as the plate is supplied with an alternating current, the phase relation between the grid and plate voltage determines the point in the wave at which current begins to pass in each cycle, hence the average amount of current passing through the tube (Fig. 4).

We may consider then, that there are two methods of controlling the grid of a thyratron supplied with alternating current; the voltage magnitude method and the phase shift method. The phase shift method makes it possible to vary the average current passed by the tube anywhere between zero and maximum while the voltage magnitude method can be readily adapted only to control between full on and full off.

Thyratrons are rated on an average and instantaneous current basis. For example, one size of thyratron has a maximum average plate current of 2.5 amperes averaged over a period not to exceed 15 seconds and a maximum instantaneous plate current of 10 amperes. This means that the tube will safely pass 10 amperes but that the duration of this current flow must not exceed a value which would make the average for 15 seconds greater than 2.5 amperes. Thus if a tube is operating as a half-wave rectifier on a 60-cycle or 25-cycle system it could have an average current of 2.5 amperes by passing 5 amperes during each positive half cycle, *i. e.*, half the time. Similarly two such thyratrons in a full wave rectifier system would have a maximum average d-c. output of five amperes as each tube could pass five amperes half the time, *i. e.*, one tube on one half cycle, the other on the next. One larger tube has a maximum average current rating of 12.5 amperes. Two such tubes in a full wave rectifier circuit could provide 25 amperes direct current. It is extremely probable that thyratrons of the future will be capable of passing current of the order of a hundred amperes or more.

The life of a thyratron will depend on the service in which it is placed. Typical life estimates run from 2,000 to 10,000 hours. If a very long life is required in a particular application it may be secured by using a tube whose current rating is higher than that actually required for the service. The question then is an economic consideration of first cost plus replacement costs of the small tube against first cost of the larger tube, allowing due consideration to the value of the greater continuity of service secured with the larger tube.

THE THYRATRON AS A RELAYING DEVICE

The operation of the thyratron as a relaying device differs greatly from the operation of other relaying devices. It may be used as a valve to start d-c. current in a circuit but cannot readily stop the current flow as long as the anode remains positive. It can be used, though, to start and stop a flow of current if alternating current is applied. The current which passes however is not alternating current, but is half-wave pulsating direct current if one thyratron is used. The other major difference between a thyratron relay and an ordinary relay is that the thyratron has an approximately constant internal voltage drop of 15 volts.

The field of the thyratron as a relaying device is chiefly in controlling solenoids, contactor coils, magnetic brakes and clutches, etc. Such devices may be wound to operate from the half-wave d-c. voltage resulting from the application of commercial a-c. voltages to the thyratron. Such a system involves no contacts and furthermore very high-speed operation can be secured. The thyratron grid circuit may be controlled by a change in either voltage or current directly; or by a change in reactance, capacitance, or resistance resulting in a change in voltage or a shift in phase of the grid voltage; or by the opening or closing of delicate contacts.

For example, the amplified current of a photoelectric tube may be used to control the thyratron. To accomplish this, a resistor is connected in the plate circuit of the pliotron and the voltage drop across the resistor is used to control the grid voltage of the thyratron and thus determine whether it shall conduct or not conduct. In this manner a change in illumination on a photoelectric tube may be employed to operate a solenoid or similar device without the aid of any contact making or moving part device whatever.

Similarly the change in reactance in a coil caused by moving an iron core in it may be used to shift the phase of the thyratron grid voltage and control the operation of a magnetic clutch or brake.

One relaying device has been developed in which thyratrons are employed to short circuit the high-voltage secondary of a series transformer whose primary is connected in series with a resistance welding transformer. The change in the impedance of the primary of the series transformer as the thyratrons are alternately permitted by their grids to pass current and not pass current, makes it possible to control at high rates of speed the intermittent flow of current to the welding machine without any mechanical wear and tear or burning of contact tips as experienced in mechanical interrupters and contactors.

THE THYRATRON AS A CONTROLLED RECTIFIER

Although the thyratron operates as a "controlled rectifier" in most applications this term will be used herein to refer to the thyratron system in which alternating current is rectified to provide a pulsating direct current, the voltage of which may be varied smoothly and without "steps" by properly controlling the thyratron grids.

The simplest system of this type is the single-phase, full-wave, grid-controlled rectifier, which will be discussed in detail. Fig. 5 shows a schematic wiring diagram of such an arrangement. Considered as a plain rectifier, when the anode of tube No. 1 is positive it will pass current from anode to cathode, through the load, and back to the neutral point of the transformer. This current will continue to flow as long as the No. 1 anode remains positive. At the end of that half cycle the anode of tube No. 2 becomes positive and tube No. 2 conducts, the current passing from anode to cathode, through the load, and back to the neutral point of the transformer. Each tube conducts on alternate half-cycles, but permits current to pass through the load in the same direction in each half cycle. The voltage across the load is shown in Fig. 6.

Now consider the effect of grid control upon such an arrangement with a resistance load. When the grid voltage is in phase with the anode voltage (*i. e.*, each grid in phase with its respective anode) the operation will be exactly the same as though the grids were not present because the grid of each tube will permit conduction as soon as the anode of that tube becomes positive. Furthermore, if the phase of the grid voltage is advanced to less than 180 deg. lead the operation will not be changed as the grid of each tube will be positive when its anode becomes positive, conduction will be permitted, and will continue for the remainder of that half-cycle despite the fact that the grid becomes negative meanwhile. If however, the grid phase is retarded from the "in phase" position, *i. e.*, made to lag the anode voltage, the grids will not be positive when their respective anodes become positive and conduction in each half-cycle will not commence until the grids have become less negative than the critical grid voltage. As the phase of the grid voltage is made to lag the anode voltage more and more the point at which conduction begins comes later and later in each half cycle, conduction occurs for shorter and shorter periods and consequently the average voltage across the load is decreased. Finally, when the g lag their anodes by 180 deg., no conduction oc because the grids are never positive when their res tive anodes are positive. Fig. 7a shows the rela between load voltage and grid-phase angle lag. inductance in the supply transformer accounts for slight deviation of the practical curve from the t retical.

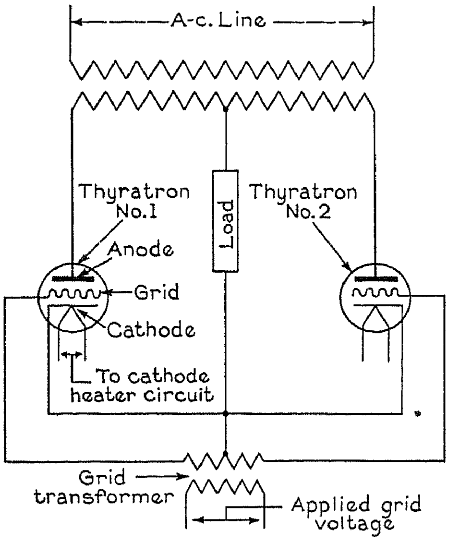

Fig. 5—Full-Wave Grid-Controlled Rectifier

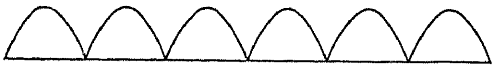

Fig. 6—Full-Wave Rectifier Output Voltage Wave Form for Resistance Load

In the case of an inductance load the operatio somewhat different because the energy storage in inductance tends to make the flow of plate current l over into the negative half cycle. A thorough dis sion of the phenomena involved in this type of opera would be somewhat complex and probably is beyond scope of this paper. Fig. 7b however, shows the t retical relation between load voltage and grid-ph angle lag for a theoretical pure inductance, and also example of typical deviation caused by the resista present in a practical inductance.

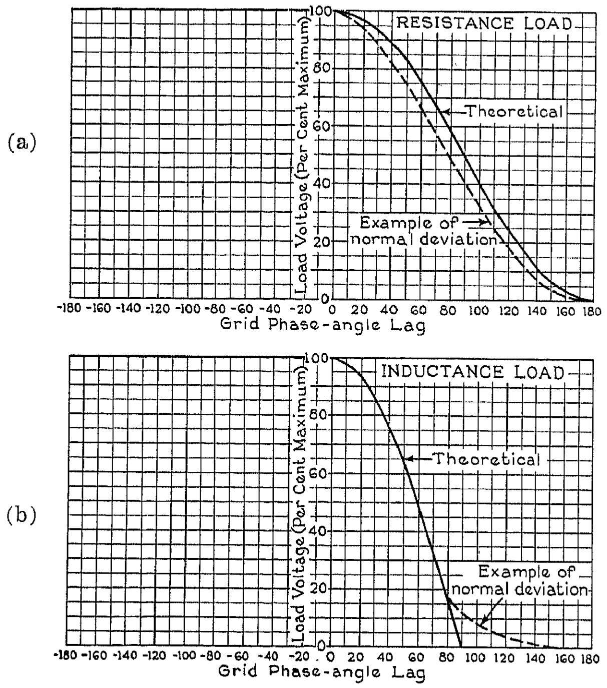

Fig. 7—Grid Phase—Load Voltage Characteristic fc Full-Wave Grid-Controlled Rectifier System

With (a) resistance and (b) inductance load

To make the system serve a useful purpose it necessary to devise means for shifting the phase of grid voltage of the thyratrons in response to electri functions or mechanical movements. One very sim method of securing such a phase shift is to set u reactance-resistance bridge as shown in Fig. 8a. change in the value of either the reactance or the re tance will cause the grid voltage phase to shift as sho in the vector diagram in Fig. 8b. If we make reactance the coil of an ordinary a-c. solenoid, reactance value may be varied by moving the plun in and out. Thus by moving the plunger it is possi to shift the grid voltage phase and change the output voltage of the thyratron system. A mechanical movement of an inch and a half can control the voltage from zero to maximum.

The a-c. coil of a small saturable core reactor may be connected in a circuit arrangement in such a manner that variation of the d-c. current through the d-c. coil of the reactor will shift the phase of the grid voltage. In this manner variations in a d-c. current of a few milliamperes may control the output voltage of the thyratron from zero to maximum.

A very small induction voltage regulator may be used in a circuit combination in which rotation of the regulator rotor causes the grid voltage to shift in phase and, consequently the thyratron output voltage to vary.

It is apparent, then, that there is a number of ways in which the phase shift of the grid voltage may be accomplished in response to changes in voltage or current, or in accordance with rotary or linear mechanical motions. Undoubtedly many new schemes will be devised to fit particular applications as the art progresses.

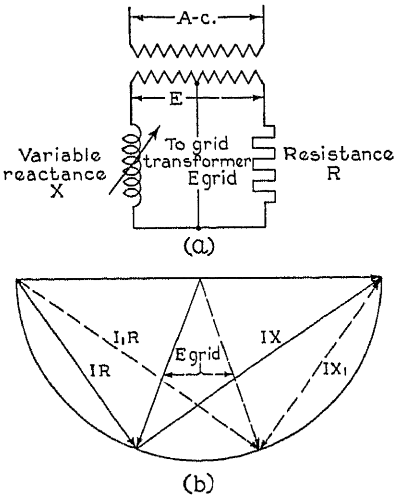

FIG. 8—TYPICAL CIRCUIT FOR SHIFTING PHASE OF GRID VOLTAGE AND VECTOR DIAGRAM FOR THIS CIRCUIT

The first important commercial installation of thyratron control was made in the Chicago Civic Opera House. In this installation full-wave grid-controlled rectifiers are employed to provide adjustable voltage direct current for saturable core reactors controlling the lighting effects for the stage and auditorium. The grids are controlled by means of small rotary regulator units. As the regulator unit is rotated the phase of the grid voltage is shifted, the thyratron output voltage changes, varying the degree of saturation in the saturable core reactors. The reactance of the a-c. coil of the reactor changes accordingly, permitting more or less current to flow through the lighting circuit, thus controlling the intensity of the lights. Thyratron control is also employed in this installation to control the speed of a d-c. motor by which it is possible, through a system of Selsyn[4] devices, to rotate all of the individual regulators for the various lighting circuits simultaneously, and thus control all of the lighting circuits from the rotation of one regulator.

Similar systems have been applied to other lighting control systems. The grid controlling regulators are so small that they may be continuously rotated by small synchronous motors similar to the kind used in electric clocks. This makes it possible to secure cyclic dimming effects without the use of motor operated rheostats.

The thyratron system may be used to supply small d-c. motor armature power, thus providing motor-speed control by armature voltage variation, somewhat after the manner of a non-reversing, non-overhauling Ward-Leonard system; or it may be used to supply motor field excitation to the conventional adjustable speed d-c. motors. Of course, a much greater speed range is secured by using armature voltage variation. The thyratron equipment can be controlled to deliver to the armature any voltage from zero to maximum and the motor speed will vary accordingly. Furthermore, the thyratron equipment may be controlled to vary the voltage delivered to the armature smoothly, *i. e.*, in infinitesimal steps. In applying the pulsating d-c. voltage to the motor armature, the ripple must be limited to secure good commutation. This is accomplished by connecting a suitable smoothing reactor in the d-c. circuit. As in other applications, the energy required to control the thyratron grids is very small.

The speed range which may be secured by using the thyratron equipment to supply field excitation is limited by the motor itself. The application of this system generally requires that both a-c. and d-c. power be available, since the thyratron equipment operates from alternating current and direct current must be supplied for the motor armature. The thyratron equipment in this case merely provides a d-c. source of excitation with unusual control features for conventional adjustable speed motors. The occasion that would warrant the installation of a motor-generator or rectifier to supply armature power, in addition to the thyratron equipment to supply controlled excitation, would be unusual although cases justifying such an arrangement may arise.

With the thyratrons at present available for industrial use it is possible to supply armature power for motors up to 5 hp., 230 volts, and field excitation up to 5 kw., 230 volts. Fig. 9 shows a thyratron panel for supplying armature power to a one-hp., 230-volt motor. Each of the thyratrons on this panel has an average current rating of 2.5 amperes so that the total average output may be up to 5 amperes, which is sufficient for a one-hp., 230-volt motor. Fig. 10 shows a 2.5 ampere tube with a 12.5 ampere tube. A panel similar to that shown in Fig. 9 but employing two of the larger tubes is capable of supplying up to 25 amperes direct current which is sufficient for a 5-hp., 230-volt motor. Power for motors larger than 5-hp. would have to be taken from a three-phase distribution and a three-phase grid-controlled rectifier circuit would have to be used.

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[4] Trade name for self-synchronous motion transmitting apparatus.

An installation of thyratron motor control is being employed to maintain constant tension on a wire drawing winder. To successfully wind fine wire after its final drawing it has been found necessary to maintain a practically constant tension in the wire as the reel diameter increases and as the speed of the drawing machine varies. The thyratron equipment has been found to meet these requirements admirably. In this installation (Fig. 11) a solenoid armature is attached to a floating pulley and moves up or down as the tension of the wire under the pulley increases or decreases. The winder motor speed is accordingly decreased or increased by the thyratron control which supplies its armature power. As this operation is continuous and the response to a change in tension is almost instantaneous, a practically constant tension results.

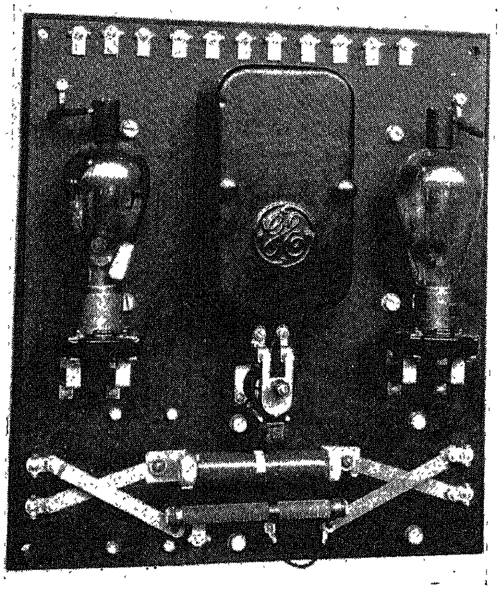

Fig. 9—Thyratron Panel for Supplying Variable D-C. Voltage to a One-Hp., 230-Volt Motor

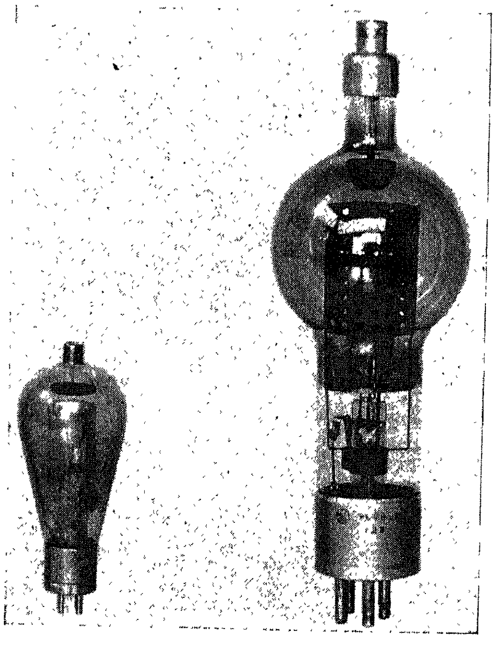

Fig. 10—The Relative Size of a 2.5-Ampere Thyratron and a 12.5-Ampere Thyratron

Another installation of thyratron motor control is being made for controlling the speed of a motor driving a reeling machine for rubberized belting. Here again the control is to function in response to change of tension in the belting as the reel diameter increases.

Many additional possibilities are open to consideration from an engineering and economic viewpoint. The thyratron should be given due consideration wherever relatively small powers of adjustable voltage direct current are required and should be given special consideration when automatic control of such voltage is desired in response to mechanical movements or electrical functions.

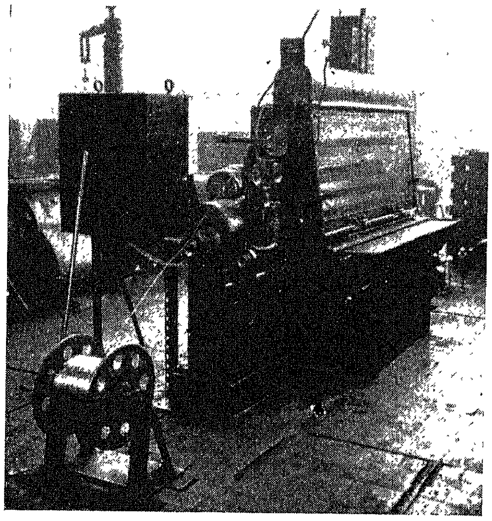

Fig. 11—Wire Drawing Winder Showing D-C. Motor Supplied with Variable Voltage from Thyratron Panel and Tension Regulating Device Controlling Small Reactor in Grid Circuit of Thyratrons

Thyratron panel is mounted in enclosing case to left of machine

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Discussion

J. V. Briesky: Mr. King's paper gives a very interesting review of a number of industrial tube applications which have been made in various industries during the past few years. The importance of this general subject is attested to by the large number of articles that has been published on various phases of it, particularly during the last two years.

Electrical manufacturers have come to recognize the importance of electron tubes in industry and are devoting much time and thought to the development of industrial tubes and tube apparatus. While of course much of this tube development work is an outgrowth of the earlier work on radio, it should be realized that new lines of tubes are being developed for industrial work because of the fact that different characteristics and sizes are required and also because of the fact that for this work long life, constancy, and sturdiness of tubes are of prime consideration.

The author deals in his paper with three general classes of tubes, *i. e.*,

1. Photoelectric tubes,
2. Grid-controlled vacuum tubes (referred to as pliotrons by the author).
3. Certain types of grid-controlled mercury vapor tubes (referred to as thyratrons in the paper).

Photoelectric tubes are fairly well known by now and many applications have already been made. Grid-controlled vacuum tubes are of course familiar to all of us because of their radio applications. Many new industrial uses are also being developed for these tubes. While their current capacity is quite small at this time, it is safe to predict that vacuum tubes handling larger currents will soon be available. This will bring about more widespread use of these tubes for a variety of applications.

The third type of tube described in Mr. King's paper, the grid-controlled mercury vapor tube employing a hot cathode, belongs to the family of grid-controlled, gas-filled tubes. This family may be subdivided into three groups:

1. Cold cathode tubes,
2. Hot cathode tubes,
3. Tubes employing a pool of mercury for a cathode.

The cold cathode tubes are the most sensitive, as far as grid control is concerned, but their current output is limited to milliamperes. The grid glow tube developed by D. D. Knowles of the Westinghouse Research Laboratories is a good example.

Hot cathode tubes may be subdivided into two types, those employing mercury vapor (this class is described in Mr. King's paper), and those using neon and similar gases. Each type has its advantages. For instance, the mercury vapor type has a somewhat lower voltage drop, while the neon type has a much smaller variation in characteristics with temperature. The hot cathode tubes are available in several sizes, from 0.1 ampere up to several amperes of continuous current carrying capacity.

Wherever large capacity tubes are required, the third group of tubes, the mercury pool type, should be considered, which can handle currents of several hundred amperes, and on which much research work is being expended at this time.

As pointed out by the author, quite a few applications of these grid-controlled gas-filled tubes have been made, but it is believed that during the next few years a tremendous increase in this activity will take place, particularly when the larger tubes become standardized and are generally available. Problems such as motor-speed control, speed matching, conversion of low to high frequency currents, will no doubt be solved by these tubes in many cases.

It is gratifying to note the increasing amount of interest taken by all industries in "industrial tubes," which interest will no doubt hasten the arrival of the day when applications of tubes will be as commonplace as those of the electric motor.

C. H. Sharp: The paper does not mention an important application of photoelectric cells and electron tubes which has been made in the electrical industry itself, namely, the photometry of incandescent lamps. At the present time the older method of photometry which depended upon the use of the eye has been to a very large extent superseded in the manufacture and testing of incandescent lamps by photoelectric photometry. The earlier photoelectric cells were not well adapted to this application because of their lack of red sensitivity. The caesium cell does not suffer from this disadvantage but has a spectral sensitivity distribution which is such that with a moderate amount of correction by color filters it is applicable to photometric purposes when the color differences between the lamps to be measured lie within the ordinary range of commercial lamps. An amplifier with any one of a number of hook-ups is added to the photoelectric cell so that only sturdy, easily-managed instruments are required. Measurements are made either by a deflection method or by a null method. The speed and precision of photometric measurements have been greatly enhanced by this application.

In one laboratory, at least, the photoelectric cell has been applied to the determination of the color temperature of lamps whereby a color match of a lamp against a standard is readily made to within five degrees. The procedure by which this is done has been described in the *Journal of the Optical Society of America*, volume 30, page 62.

H. S. Phelps: The proposal to use a photoelectric switch for control of street lighting circuits is of particular interest since:

1. Such an application should result in reduced capital investment and,
2. Municipal lighting would be scheduled in accordance with necessity for illumination in lieu of a schedule defined by the hours of the day.

Among the points that must be borne in mind in studying the possibilities of this application are:

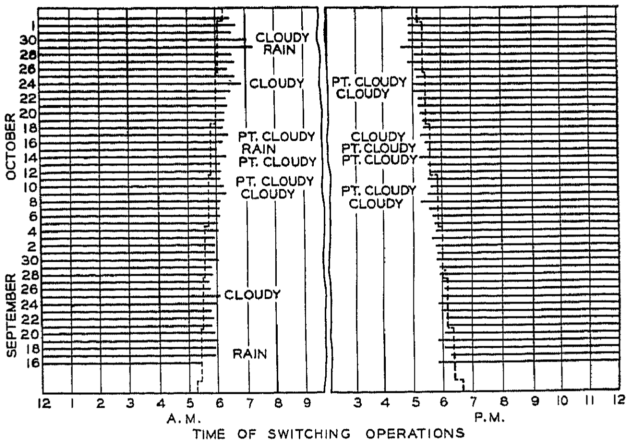

Fig. 1—Photoelectric Cell Street Light Control Unit

The broken lines indicate the Philadelphia street lighting schedule. Unit installed on east gable of radio shack on roof of 2301 Market Street with window exposed north.

a. Reliability.
b. Cost of Maintenance.

An operating test of such a device for a period of several months has disclosed:

1. Very close approximation to the municipal street lighting schedule in effect where the tests were made.
2. Extension, both night and morning, of the scheduled lighting period on cloudy or stormy days.
3. Rare operation as the result of storms that occur during the day except those in progress near sunrise and sunset.

It still remains to determine:

If the kw-hr. consumption allowed by this device during an average year differs from the consumption under fixed schedules sufficiently to require revision of present flat-rate contracts.

Is the deviation of individual years from the average year sufficient to indicate that contracts for municipal lighting service controlled by photoelectric switches should be on a kw-hr. consumption basis rather than on a flat rate per lamp year.

Fig. 1 shows the relation observed between the Philadelphia street lighting schedule and that maintained by a photoelectric switch for the period indicated.

Synchronous Motor with Phase-Connected Damper Winding for High-Torque Loads

BY M. A. HYDE, Jr.[1]
Associate, A. I. E. E.

Synopsis.—*Considerable progress has been made in recent years in the application of low-speed synchronous motors to industrial loads requiring both high starting torque and high pull-in torque with moderate starting current demand. The problems encountered are exemplified in the drive of grinding machinery in the cement and mining industries, to which several different adaptations of synchronous motors are discussed. A form of synchronous motor with phase-connected damper winding has been devised for this type of service, developing high-torque / kv-a. ratios in a simple mechanical construction.*

INTRODUCTION

Of the many directions in which the synchronous motor has within recent years extended its field of usefulness, one of considerable importance is that particular field of application involving low-speed ratings,[2] where the conditions of service impose low-starting current with a combination of high-starting and pull-in torque requirements.

While the permissible magnitude of starting current varies greatly in different installations, in general low-starting current is a desirable qualification and in many cases one of prime importance. By way of comparison take as a reasonable criterion of low current starting performance the usual engine type or direct-connected compressor motor, which is ordinarily started on full voltage drawing approximately 275 per cent full load current.[3] In this particular application due to the feasibility of starting the machine unloaded, the exerted starting and pull-in torques are low, ordinarily about 50 and 40 per cent respectively, and the pull-out torque about 150 per cent.

Most industrial drives do not present such a fortunate combination of requirements. There are many applications requiring either starting torque or pull-in torque to be emphasized and this is readily accomplished with motors of moderate inrush through the development by designers of improved methods for calculating starting performance and the employment of a variety of damper winding materials.

A more difficult problem is presented by the class of drives requiring development of full load pull-in torque and starting torques substantially in excess of this value, with moderate current inrush. As an example of this type of application take the ball mills used in the mining industry for reduction of ore, and the tube and compartment mills used in the cement industry for the grinding of raw materials and cement clinker.

These machines consist essentially of a cylindrical steel shell mounted horizontally, either on rollers supporting tires on the mill shell, or on hollow trunnions on which the shell is rotated and through which it is usual to pass the material under reduction. While passing through the interior of the rotating shell, the material is subjected to the grinding action of a charge of steel balls or slugs. The grinding charge is restrained in the mill by partitions containing slots or other openings to allow passage of the ground material. Rotation of the mill is accomplished through suitable gearing to the shell. The pinion shaft for tube and compartment mills operates usually at 150 to 180 r. p. m. with a single reduction to the shell which turns at 19 to 25 r. p. m. depending upon the diameter. Motor sizes range commonly from 200 to 1,000 horsepower. Ball mills of sizes requiring 200 horsepower or over usually employ single reduction gearing with pinion shaft speeds from 200 to 360 r. p. m. although sometimes as low as 150 r. p. m. A low-speed motor on the mill pinion shaft provides an excellent drive not only for mechanical simplicity but because the operating speed is constant and the magnitude of the load makes high power factor a desirable characteristic.

The load on such a mill for a given shell diameter and speed, is determined almost entirely by the weight of grinding charge, the weight of material under reduction being small in comparison. The grinding charge remains in the mill at all times, it being impracticable to start with the shell empty. It is apparent that after the mill has been accelerated to a speed approaching synchronous speed, the effective pull-in torque of the driving motor must be approximately the full-load running torque. The required starting torque is quite variable, depending upon the type and condition of the mill bearings and the condition of the mill charge, which latter must be lifted through a portion of a revolution before "spilling" occurs. While the required starting torque is often as low as 125 per cent of the normal running torque, it is usually desirable to provide for starting torques in the order of 200 per cent, to take care of the worst conditions.

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[1] Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

[2] The term low-speed ratings refers to the usual classification covering speeds of 450 r. p. m. and below.

[3] Unless otherwise stated herein, starting currents expressed in percentages are based on the current corresponding to operation at full mechanical load at unity power factor, regardless of the power-factor rating of the motor.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

Since these units, particularly in a cement plant, may be few in number and yet constitute an important proportion of the plant load, it is usually desirable that the current inrush at starting be moderate.

Another consideration—a mechanical one—results from the large masses involved in these mills. For example, a certain cement compartment mill driven by a 600-hp. motor, has a total weight of moving parts, including grinding charge and material under reduction, of approximately 120 tons. The inertia of such masses makes it desirable to start the mill with the initial exertion of a moderate torque, only sufficient to start the mill under favorable conditions. This takes up any play in the drive with less impact and imposes less strain on gearing and shell than would result from the sudden application, each start, of the maximum starting torque of which the motor is capable as determined by ability to start the mill under the most severe conditions.

REVIEW OF PREVIOUS PRACTISE ON APPLICATIONS REQUIRING HIGH-STARTING AND PULL-IN TORQUE

(*A*) *Motors of Standard Electrical and Mechanical Construction.* These have been used to some extent where the the high starting kva. is not objectionable. However the motor is physically oversized as compared with a motor of compressor service characteristics, which affects adversely its current inrush. Such a motor to operate at say 180 r. p. m., delivering 190 per cent starting torque and 110 per cent pull-in torque, will have a frame size corresponding to a compressor motor of almost twice its rating and an inrush of some 650 per cent, or starting torque / kva. ratio of about 0.30.

In 1927 Mr. H. V. Putman in his paper[4] examined the use of the double squirrel-cage type of damper winding in salient-pole machines, pointing out the greater suitability of this type of construction to applications requiring relatively high pull-in torques with low starting kva., the starting torque not materially exceeding the pull-in torque. The same paper brings out the combination of the torque of the bars of an ordinary damper winding with the torque of the field to produce high accelerating torques, the effect of the field being particularly important in low-speed machines. The double-cage form of damper winding is less effective in low-speed motors than in high-speed machines because of the limited space available in the low-speed pole head for submerging the high reactance element.

(*B*) *Mechanical Modifications of Standard Motors.* In the past the combination of starting characteristics required by tube mills and similar drives has been met largely by employing motors of ordinary low-torque electrical characteristics with various forms of clutches —magnetic, mechanical or the equivalent thereof— embodied sometimes in special mechanical construction, thus permitting the motor to be started and synchronized without load. The problems of starting and pull-in torque are thereby avoided entirely, it being necessary simply to make available adequate pull-out torque for starting and accelerating the load. Low-speed motors of this type are readily built with pull-out torques of 200 per cent and starting currents comparable to those of compressor motors. As an example of the extent to which this design may be carried, in a certain installation a 450-hp., 180-r. p. m., 80 per cent leading power-factor motor of this type delivers a maximum starting torque, or pull-out torque, of 216 per cent and starts on full voltage, drawing only 192 per cent of its full load running current. On the unity power-factor basis this percentage current would be 240 per cent.

This form of drive has the possibility of very smooth acceleration of the load without initial shock to the drive, when the loading device is properly constructed and controlled.

Although electrically advantageous, constructions of this type are likely to have mechanical drawbacks, the ideal arrangement being the simple two-bearing coupled type developing the necessary starting and pull-in characteristics electrically and without the need of any intermediate device for accelerating the load.

(*C*) *Motors of Ordinary Construction Applied with Special Control.* (1) Application of Over-voltage During Starting Interval: Temporary overvoltage may be applied to the motor stator throughout, or during any part of the starting interval. The induction motor torques will be increased as the square of the primary voltage, but the input kva. will also increase in the same ratio, so that there is no gain in the torque per kva. ratio. Due to the intermittent nature of overvoltage application, this scheme allows the motor to be built in a physical size more nearly normal for the running load than is the case with the straight full-voltage starting motor built for equally high torques. Depending upon the elaborateness of the voltage control, this system may or may not have the merit of smoothly handling the load during starting.

(2) Starting with Capacitor Across Motor Terminals: This improves the power factor and resultant input current of the combination. To reduce the cost of the capacitor it may be applied on an intermittent basis to operate at overvoltage, being disconnected at the end of the starting period, the capacitor kva. varying as the square of the voltage. This scheme improves the starting torque/kva. ratio by reducing the starting kva. rather than increasing the torque. It does not reduce the physical size of the motor and does not provide for an initial reduction in the starting torque for smooth handling of the load.

(3) Use of a Variable Resistor in Field Circuit: It is well known that the shape of the speed-torque curve is greatly affected by the amount of resistance in the field. The lower resistance values tend to flatten

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[4] *Starting Performance of Synchronous Motors*, A. I. E. E. TRANS., January 1927, p. 39.

the speed-torque curve near synchronism giving low slip, but at the same time tend to break the curve at a lower torque, and produce cusps. To take most advantage of the torque exerted by the field, several machines have been built in which the field is closed on a high resistance initially, in the order of 100 times the field winding resistance, which in combination with a high-resistance, low-reactance damper winding, produces a high-resistance machine, resulting in high starting torque per kva. As the motor accelerates the field resistance is reduced in steps by the control to a value of three or four times that of the field itself producing a low-resistance field circuit and thereby improving the torque near synchronism.

The use of a high resistance across the field at starting increases the extent to which the voltage induced in the field winding appears across the field terminals. This voltage, being produced by the transformer action of the field winding as a single-phase secondary to the polyphase primary winding, is proportional to the number of turns in series in the field winding. Hence the fields of these machines were designed with several sections in parallel thus reducing the total turns in series and the induced voltage, but likewise reducing the d-c. excitation voltage. A 600-hp., at 150-r. p. m., 48-pole, 60-cycle motor of this construction with 120 per cent pull-in torque and 205 per cent pull-out torque developed 240 per cent starting torque with 495 per cent starting kva., a starting torque/kva. ratio of 0.48. The voltage across the field at starting is about 3,500 volts, the motor exciting from a 50-volt supply.

Very high-starting torque as compared to pull-in torque was found to be typical of this construction. In fact the starting torque was so high that on tube mill applications a primary resistor was employed in the control equipment whereby the initial starting torque could be considerably reduced. A 300-hp., 450-r. p. m. machine of this type belted to a large jaw crusher in a quarry delivered a starting torque of 275 per cent with a somewhat better torque/kva. ratio, a pull-in torque of 100 per cent at 5 per cent slip and a pull-out torque of about 290 per cent, characteristics very desirable in an application of this sort.

THE SYNCHRONOUS MOTOR WITH PHASE-CONNECTED DAMPER

Experience gained in the application of motors of the type just described, emphasized the desirability of a synchronous motor with the starting characteristics of a slip-ring induction motor. The commercial solution of this problem has long been the desire of synchronous motor engineers. In undertaking this problem it was decided to adhere to the salient-pole construction rather than to use the synchronous-induction form of machine, in view of the natural advantages held by the salient-pole type in physical size, efficiency, and excitation characteristics.

Within the past two years several installations have been made employing such a low-speed motor, developing a pull-in torque in excess of full load torque, with adjustable starting torques up to and exceeding 200 per cent, and with remarkably high-torque/kva. ratios. This motor is of salient-pole construction with the very desirable operating features of that type. Starting characteristics resembling those of the slip-ring induction motor are attained through the use of a special form of damper winding, in which the bars are of low resistance and, instead of being joined to end rings to form the usual cage construction, are phase-connected and brought out through slip rings to an external starting resistor which is varied by suitable control. The idea forming the basis of this development was included in Mr. Putman's paper mentioned above.

To develop the induction motor torques to the greatest extent the construction throughout is directed toward low reactance. Thus the stator employs a larger number of slots with more coils and fewer turns per coil than commonly used in ordinary low-torque machines. The field is open-circuited at start thereby eliminating an undesirable low power-factor component in the primary current ordinarily reflected from the circulation of current in the highly reactive field circuit when closed upon a resistor of the proportions commonly used. The disadvantage generally resulting from open field starting is the high transformer voltage on the field winding at standstill as discussed above. On this account the first motors of this type were designed with fields for low-voltage excitation.

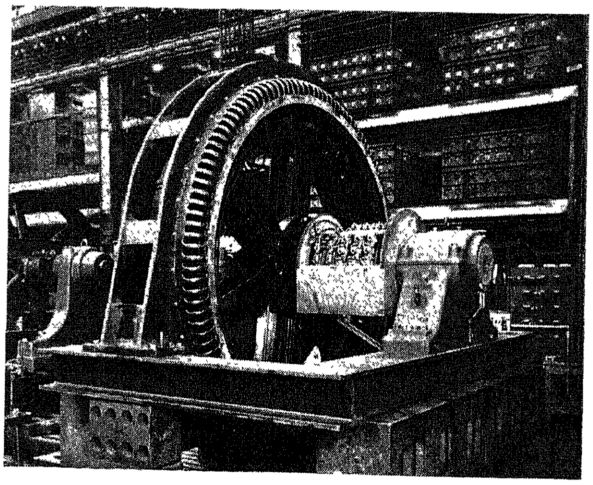

FIG. 1—A 1,000-HP., 40-POLE MOTOR WITH PHASE-CONNECTED DAMPER WINDING. COLLECTOR HOUSING OPEN

Fig. 1 shows one of two 1,000-hp., 180-r. p. m., 100 per cent power factor, 2,200-volt, three-phase, 60-cycle motors of this form, built in 1928 for cement compartment mill drive. The excitation of these units is at 45 volts, 8 kw. each. The induced voltage across field terminals at start is approximately 3,300 volts. The following data on starting performance apply:

Pull-in torque at 5 per cent slip	125 per cent
Maximum starting torque	210 per cent
Locked starting kva	430 per cent
Ratio starting torque/kva	0.49
Pull-out torque	180 per cent

Since the transformer voltage in the field is proportional to the number of field turns in series, a field sectionalizing device suggests itself as a means of reducing this voltage. This parallels the practise of using a field break-up switch on the synchronous converter, but with the very important mechanical difference that the synchronous motor field is conventionally on the rotor, where sectionalizing is less convenient. However, experiments were made with a 300-hp., 164-r. p. m. machine wound for 125-volt excitation and equipped with a centrifugally operated sectionalizing switch mounted in the rotor, and these experiments were so successful that after extended shop tests on the durability of such a switch and the practicality of its use, an installation of such a motor was made in 1929 to drive a tube mill in a cement plant, where it has since been in regular operation. Fig. 2 is a view of this installation.

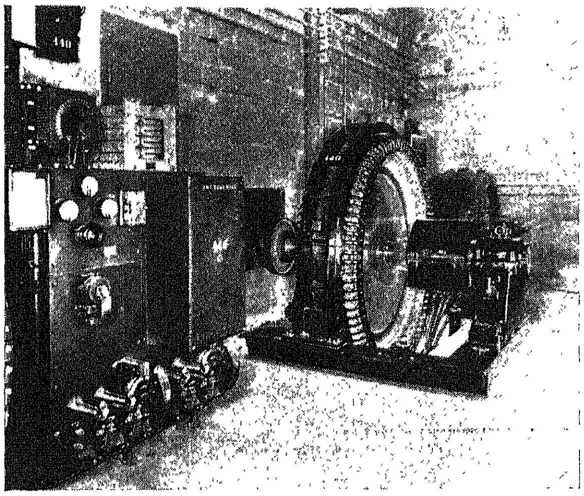

Fig. 2—A 300-Hp., 164-R. P. M., Motor of the Phase-Connected Damper Type, with Control as Installed in a Cement Plant

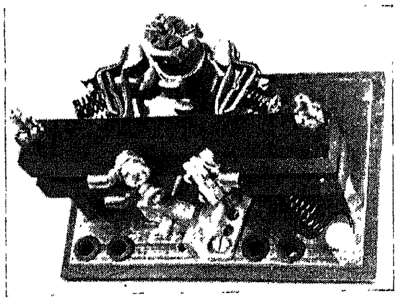

Fig. 3—Centrifugal Sectionalizing Switch-Cover Removed

The following test values on this 300-hp. machine are quite similar to the characteristics of the preceding 1,000-hp. motor, as was to be expected since the only radical change introduced was the adoption of the field sectionalizing switch.

Pull-in torque, 5 per cent slip	115 per cent
Maximum starting torque	210 per cent
Locked starting kva	460 per cent
Ratio starting torque/kva	0.46
Pull-out torque	234 per cent

The voltage induced on each section of the field of this machine at start is about 3,000 volts. This voltage diminishes as the rotor accelerates, being proportional to the rotor slip. The response of the sectionalizing switch to speed is very sharp and consistent, since the centrifugal force on the actuating member varies as the square of the speed. Closure of the switch solely in response to speed, or slip, means closure only upon the existence of a predetermined low percentage of induced field voltage.

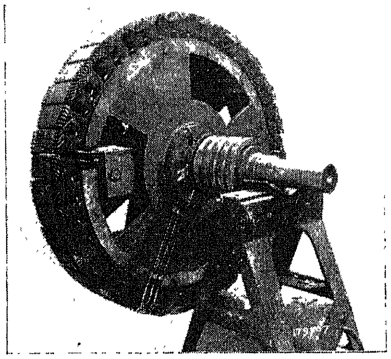

Fig. 4—Rotor of a 300-Hp., 164-R. P. M. Motor with Phase-Connected Damper Winding

Fig. 3 shows a switch designed for this service. The unit is enclosed in a tight welded steel housing so proportioned as to mount within a rotor as shown in Fig. 4 at suitable distance from the shaft center to develop the necessary actuating force at the desired rotor speed. As mounted on the rotor the switch is directly above the motor shaft when in the position shown in Fig. 3. At the closing speed the weight swings outward centrifugally against the restraining spring, closing the switch contacts. The mechanism is so arranged that the closing action, once initiated, proceeds very rapidly, and also that upon stopping the motor the switch maintains its closed position until a relatively low speed is reached, permitting the field to be connected to a discharge resistor and opened in the usual way by the field switch on the control panel. The sectionalizing switch is adjusted during the motor test, after which all moving parts are secured and, with the exception of the spring, made incapable of further adjustment.

The unusual appearance of the rotor shown in Fig. 4

results from a later improvement in the starting performance through the adoption of laminated rim construction to minimize the eddy currents in this member during starting.

Another feature apparent in Fig. 4 is the adoption of a comparatively broad pole face permitting the damper winding to occupy a greater pole embrace and to be more favorably distributed from the standpoint of phase balance.

This unit, rated 300 hp., 164 r. p. m., 100 per cent power factor, 2,200 volts, three phase, 60 cycles, developed the following starting performance:

Pull-in torque at 5 per cent slip	122 per cent
Starting torque with 15 per cent damper resistance	230 per cent
Corresponding locked kva	396 per cent
Ratio starting torque kva	0.58
Starting torque with 30 per cent damper resistance	169 per cent
Corresponding locked kva	280 per cent
Ratio starting torque kva	0.60

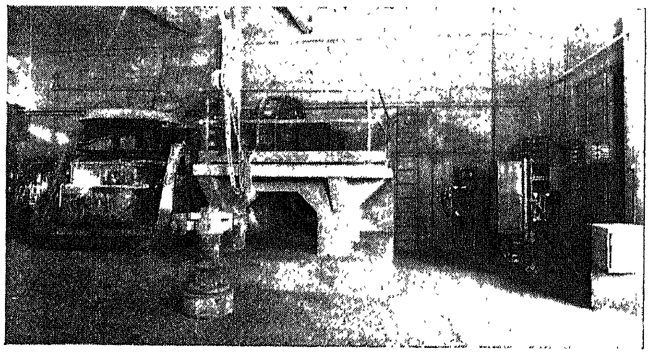

FIG. 5—350-HP., 400-R. P. M. INSTALLATION ON PRELIMINARY GRINDER IN A CEMENT MILL

The transformer field voltage at start tested about 3300 volts per section when split into three parts by the sectionalizing switch.

Fig. 5 shows a 350-hp., 400-r. p. m., 80 per cent power-factor motor of this construction driving a centrifugal type rock grinder. This installation operates on an excitation supply of 250 volts direct current and in this case the calculated transformer voltage on the field was such as to be undesirably high even when split into three sections, so two switches were mounted diametrically opposite on the rotor and the break-up process carried to five sections, the voltage per section at standstill being then about 4,000 volts.

This machine being of higher speed would naturally tend to exhibit a somewhat higher starting torque kva. ratio, and to further improve the starting performance it was provided with starting grids of non-magnetic alloy to minimize the reactance in the damper winding circuit.

The following starting performance was obtained:

Pull-in torque at 5 per cent slip	118 per cent
Starting torque with 25 per cent damper resistance	190 per cent
Starting kva. (based on 80 per cent power factor)	215 per cent
Ratio starting torque/kva., 80 per cent power factor	0.88
100 per cent power factor	0.71
Starting torque with 40 per cent damper resistance	152 per cent
Starting kva. (based on 80 per cent power factor)	166 per cent
Ratio starting torque/kva., 80 per cent power factor	0.91
100 per cent power factor	0.73
Pull-out torque	210 per cent

CONTROL

The results attained in this type of motor are to an unusual extent bound up in the control equipment.

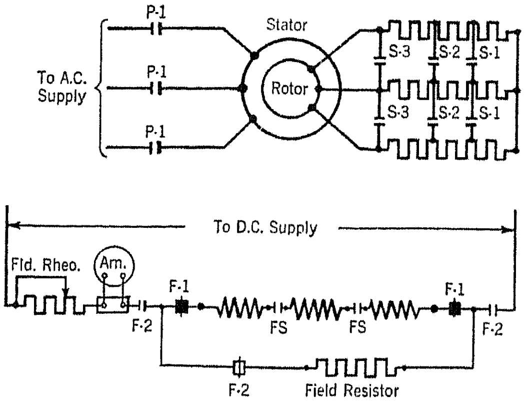

FIG. 6—(A) TYPICAL ELEMENTARY CONTROL DIAGRAM

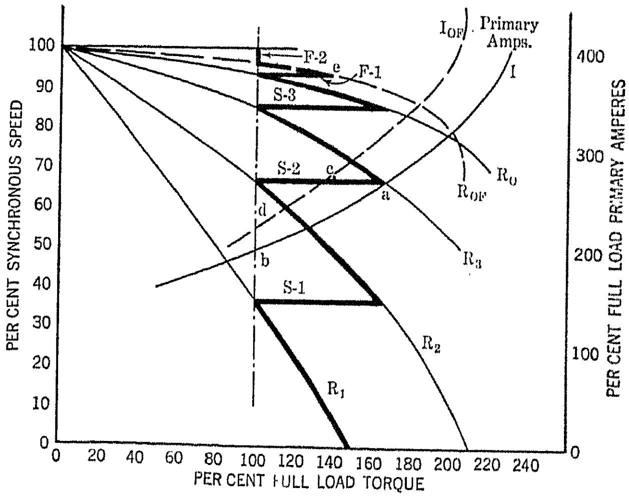

FIG. 6—(B) TYPICAL SPEED-TORQUE AND CURRENT-TORQUE CURVES

Curve R_O represents speed-torque with damper winding short-circuited, field open

Curve R_{OF} represents speed-torque with damper winding short-circuited, field closed on resistor. Current-torque curve I corresponds to speed-torque curves R_1, R_2, R_3, and R_O. Current-torque curve I_{OF} corresponds to speed-torque curve R_{OF}

This employs the primary switch, field switch, and resistor, and necessary auxiliary and protective relays found in the usual full-voltage synchronous motor starter; and in addition two or more accelerating contactors and starting resistor in the damper winding circuit, as well as a spring-actuated auxiliary field switch. The definite time sequence scheme of starting has been used thus far, in which the switching operations have been controlled by a motor-driven master drum. Fig. 2 shows a typical arrangement of control and also indicates the physical proportions of the starting resistor which in this case is mounted above the control panel.

The operation of the equipment can be readily understood by reference to the elementary control diagram shown in Fig. 6A, and the corresponding typical speed torque and speed-current curves shown in Fig. 6B. In starting, the first operation is to open the auxiliary field switch F-1, which, being a spring-closed contactor must have its coil energized to open. Next the primary switch P-1 is closed, thus starting the motor with such maximum resistance in the damper winding circuit as secures the desired initial torque, for example, 150 per cent. Assume that this torque is sufficient to break the static friction of the load and that the load torque then diminishes to 100 per cent. The motor accelerates along the speed-torque characteristic R_1 until a balancing speed is reached at the load torque. The motor continues to operate at this speed until the end of a predetermined interval when the first secondary contactor S-1 closed, transferring operation to the second speed-torque characteristic R_2. Now if the initial motor torque had not been sufficient to start the load, the motor would have remained at rest until closure of S-1, when a higher standstill torque (in this case 210 per cent) would have been exerted, and the starting current instead of being about 250 per cent as for 150 per cent torque, would then be about 350 per cent. If this torque is insufficient to start the load an overload relay can be arranged to remove the motor from the line. Assume acceleration along R_2 to a balancing torque of 100 per cent; then after the termination of a second predetermined interval S-2 closes and the motor accelerates along R_3 to the balancing speed, after which S-3 closes, short-circuiting all the external damper circuit resistance. The motor is now operating on a comparatively flat speed-torque curve R_o due to the low resistance of the copper damper winding. At the low slip now existing, the transformer voltage on the field has reduced to a small percentage of its initial value, and the centrifugal sectionalizing switch FS has closed its contacts. The field will now exert an appreciable torque if closed on a suitable resistor, and this is done by deenergizing the coil of F-1 whose spring-closed contacts then complete the circuit through the field resistor, and the motor now operates on a still higher speed-torque curve R_{oF}. As the induction motor torques vary between the extreme values indicated by heavy lines on curves R_1, R_2, R_3, and R_o, the primary current varies between points a and b on curve I. Upon transfer from speed-torque curve R_o to R_{oF}, the primary current transfers to curve I_{oF} point c and recedes to point d when the motor reaches the highest induction motor speed at 100 per cent torque. The motor is now near synchronism, and when the field switch F-2 is finally closed the motor synchronizes and operates thereafter as an ordinary synchronous motor. In line with usual control practise, the field resistor is removed by opening of the back contact shown on F-2, at the time the main contacts on this switch are closed. The auxiliary switch F-1 is made spring closed so as to be deenergized during running operation and thus normally in position to carry both the exciting and discharge circuits for the field, allowing the field excitation switch F-2 to complete these circuits in the customary manner. At the same time the auxiliary field switch performs the functions of separating the high voltage on the motor field terminals from the remainder of the circuit at start, and of closing the field on its resistor later in the starting sequence, when the field torque is a substantial factor in the high pull-in torque delivered by the machine.

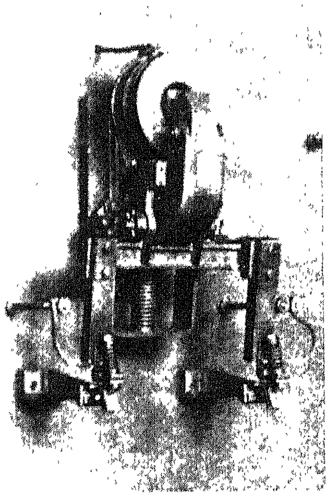

Fig. 7 Auxiliary Field Switch Specially Designed High-Voltage Service

Fig. 7 shows a switch constructed for this service. The lower stationary studs are connected to the motor-field terminals. An interlock on this switch carries the initial closing circuit of the primary switch, which arrangement makes it impossible to start the motor without first opening this switch. The switch may be mounted on a separate small panel at the rear of the main control panel, thus removing the field voltage at starting from the main control panel. The installation view in Fig. 2 shows the two starting resistance contactors at the bottom of the panel and the operating mechanism for the oil-immersed 2,200-volt primary switch, the remaining switches and relays being enclosed under a dust tight steel cover as a protection against the dust conditions incident to cement mill service.

The construction described above yields pull-in torques somewhat in excess of full-load torque and starting torques up to approximately twice full-load torque with starting torque/kva. ratios materially exceeding values heretofore published for low-speed motors without loading devices. The extent of improvement may be appreciated by comparison of the value 0.60 for the 300-hp., 164-r. p. m. motor described above, with the value 0.30 considered good for the usual standard machine of similar rating and torques. Or, in comparison with a compressor motor delivering 50 per cent starting torque and 40 per cent pull-in torque with 275 per cent kva., here is a motor delivering over three times these starting and pull-in torques with approximately the same kva. These high torques are developed in a manner similar to the starting characteristics of the slip-ring induction motor and are transmitted to the load through the simplest of mechanical parts. The salient-pole construction is used, with its advantageous operating characteristics including high efficiency, low excitation requirement, and commercial d-c. excitation voltage.

Discussion

For discussion of this paper see page 615.

Synchronous Motors

Design and Application to Meet Special Requirements

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and

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Synopsis. *Many present day applications of synchronous motors call for modifications of normal motor characteristics, as regards starting torques, thermal capacity of the starting winding, synchronizing ability, maximum synchronous torque, and mechanical design. The salient-pole motor with squirrel-cage amortisseur winding has advantages over other types which have led to the general adoption of this type as standard. Without abandoning this form, the starting and the synchronous characteristics can be adjusted to meet a variety of requirements by changing the proportions of the design and by using special forms of control equipment. Other forms, such as the synchronous induction motor, the phase-wound damper motor, and the supersynchronous motor, are sometimes justified for meeting strict starting current limitations or for accelerating extremely heavy inertia loads.*

In the mechanical design of machines for definite space requirements, fabricated construction allows changes in construction to be made at small expense, and often permits motors to be built integral with the driven machine.

INTRODUCTION

SYNCHRONOUS motors have been applied, in recent years, to drives for which only the induction motor was formerly considered suitable. This practise has resulted from the need for motors operating at unity or leading power factor and for motors having high efficiency at low speeds. Great improvements in starting characteristics and in mechanical design have made synchronous motors available for these applications. New methods of control and mechanical construction have also extended their use.

This paper describes the modern normal design and shows how characteristics may be varied through electrical design and control and how mechanical arrangements may be varied.

Normal Design Features

Before discussing the normal characteristics of conventional synchronous motor designing, it is well to consider why the salient-pole machine has been accepted as the conventional type, rather than one of the other possible designs.

To permit taking full advantage of separate excitation with the large number of field ampere turns required, it is desirable that the field copper space be a maximum. Therefore, very deep field slots are required, and for maximum space factor, only one slot per pole should be used. Hence, the salient-pole field winding is greatly preferable to the phase-wound field for a synchronous motor. For very high-speed machines, mechanical stresses prevent the use of salient poles, but otherwise they are almost universally employed.

Due to its greater excitation capacity, the salient-pole synchronous motor may employ a relatively large air gap, which is mechanically desirable, without impairment of electrical characteristics.

In order to reduce the magnetizing kva. induction motors, it is necessary to use small air gaps of the order of 0.25 per cent of the pole pitch. Mechanical limitations prevent the use of air gaps smaller than about 0.1 per cent of the diameter, so that induction motors with more than 8 poles are definitely handicapped. The normal design of the salient-pole synchronous motor permits a natural air gap of the order of 1½ to 2 per cent of the pole pitch and mechanical limitations are not reached until the motor has 60 poles or more. For this reason, the lower the speed, the more favorable are the synchronous motor characteristics in comparison with those of the induction motor.

In addition to its desirability from the mechanical standpoint the large air gap makes possible the use of larger stator slots with consequent improvement of space factor and efficiency. This comes about because the slot pitch is limited to a definite relation with the air gap to avoid high-tooth frequency iron losses. Furthermore, small high-frequency flux pulsations cannot cross a large air gap, hence thicker laminations can be used in the pole pieces.

While the rotating member may be either the field or the armature, the rotating field is the normal design with centrifugal stresses and simplicity dictating that it be inside the armature.

This normal design provides space in the pole face for an amortisseur winding with ample heat capacity to enable the motor to start most industrial loads. Actual starting torque and starting current characteristics vary with motor ratings as described later.

The conclusion is that the normal type of synchronous motor which is best for steady load operation has the following design characteristics:

1. Separate d-c. excitation. 2. Internal revolving field. 3. Laminated salient-field poles. 4. Air gap equal to about 1½ to 2 per cent of the pole pitch. 5. Amortisseur winding section of sufficient heat capacity to enable the motor to safely start the load. 6. Open armature slots, somewhat fewer in number than those of an equivalent induction motor.

Without abandoning this basic form, the electrical design may be proportioned to obtain the characteris-

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[1] Both of the General Electric Company, Schenectady, N. Y.

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tics required for many exacting applications; and where this is possible, it is preferable to use the normal type synchronous motor rather than to develop other types with a limited range of usefulness.

Normal Starting Characteristics

Starting and accelerating characteristics vary with speed and horsepower. A 100-hp., 60-pole motor will have high reactance, and consequently, low starting and pull-in torque while a 1,000-hp., 20-pole motor will have low reactance and high starting and pull-in torques. The empirical curves of Fig. 1 approximate the relationship between torques at synchronous and subsynchronous speeds for 60 cycles, 450 r. p. m. and lower. Table I shows typical torque requirements, often the major consideration, for various types of drives.

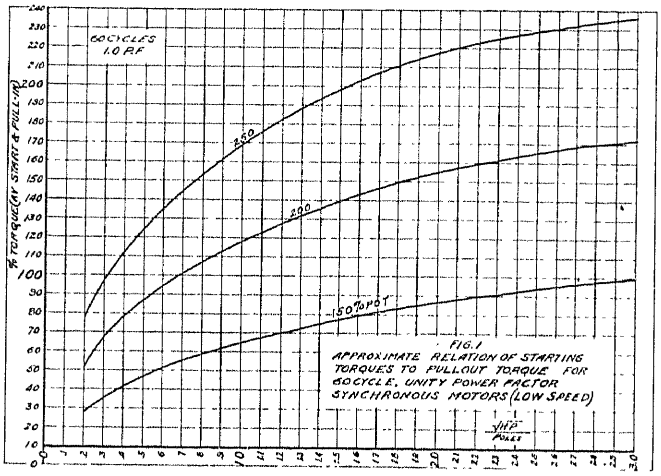

Fig. 1—Approximate Relation of Starting Torques to Pull-Out Torque for 60-Cycle, Unity Power-Factor Synchronous Motors. (Low Speed)

TABLE I

Application	Torque requirements Start Per cent	 Pull-in Per cent	 Pull-out Per cent
1. Reciprocating compressors (unloaded while starting)			
Air and ammonia (normal)	40	40	140
Low-pressure & vac. pumps	40–80	40–80	140–175
CO^2 compressors	40–90	40–60	150
2. Hydraulic pumps			
Reciprocating (starting unloaded)	40	40	140
Centrifugal	30–40	60–110	150
3. Fans-centrifugal compressors	15–40	50–120	150–175
4. Paper industries			
Pulp grinders	50	50	150
Jordans	50–60	50–60	150
Beaters	125	100	175
5. Rubber industries			
Mixers cracking mills	100–125	100	200–250
Plasticators	100–125	100	150
6. Rock Products			
Ball or Tube Mills	140–150	100–120	150–175
Hammer Mills	150–175	100–120	200–250
7. Coal pulverizers	150–175	100–120	200–250
8. Mining industry			
Ball or tube mills	175–200	100–125	175–200
Gyratory crushers (secondary cone type) (usually started light, with lower torque)	100–150	100–125	175–200
9. Steel			
Continuous rolling mills	50–100	100	250–350

(Subject to wide variations)

Modifications of Normal Starting Characteristics

It is evident that for many applications involving low-speed motors, it is necessary to find means of increasing the starting and accelerating torque: while for high-speed motors it is frequently desirable to reduce both the torque and the starting inrush. For the normal low-speed machine, the starting inrush is 250 to 350 per cent of the kva. rating at approximately 30 per cent power factor lagging; while for high-speed machines the inrush varies from 500 to 800 per cent of normal rating at 30 to 50 per cent power factor.

Reduction of Starting Current and Torque

Heretofore where reduction of starting current or torque has been required, autotransformers have been most widely used, while series reactors and resistors are sometimes used because of simplicity.

Increasing the reactance in the primary windings at starting reduces the inrush and torque without modifying the synchronous characteristics and without an external voltage reducing device. Fig. 2 shows the external connections for a motor having a double-circuit stator winding. One branch of each phase is energized at start, the other branch being connected in parallel as substantially full speed is attained.

The two windings are preferably arranged in alternate slots, although other coil groupings may be used. At starting, the reactance of the motor, with only half the coils energized in alternate slots, is much greater than with all the coils energized. The reduced current input creates less flux to act on the rotor and consequently less torque is developed. Fig. 3 compares normal and single-circuit, full voltage starting of a large high-speed salient-pole motor. As in the case with reactor starting, the torque characteristics approach normal during acceleration. A torque characteristic of this type is particularly suitable for blower and centrifugal pump drives. The second inrush when energizing the second branch is relatively small. As shown in Fig. 3 the starting torque is about 50 per cent and the starting kva. is about 70 per cent of the normal full voltage values but practically the full torque is available at pull-in.

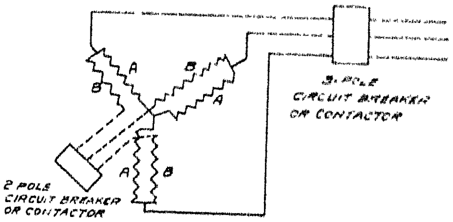
A
B
B
A
A
B
A
B
3-POLE CIRCUIT BREAKER OR CONTACTOR
2 POLE CIRCUIT BREAKER OR CONTACTOR

Fig. 2—Primary Connections for Double Winding Starting

High-torque loads such as ball mills, tube mills, and pulverizers can be started without shock by the double winding motor. When the first circuit is energized, the motor develops sufficient torque to take up all gear and coupling slack. Energizing the second circuit starts and accelerates the load without shock to the mechanical parts. Voltage disturbances, caused by the starting of synchronous motors from lines fed by small generators having voltage regulators, are minimized by using double winding starting. The drop caused by the first and larger increment is corrected by the regulator before the coming of the lesser actual starting and accelerating increment of load. The total voltage disturbance is much less than would be the case with single-winding full-voltage starting.

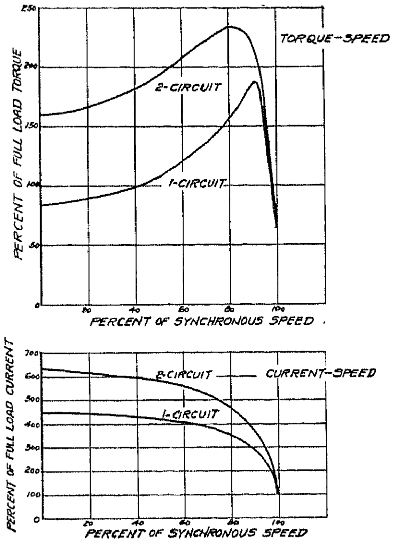

Fig. 3—Characteristics of 800-Hp. Unity Power-Factor, 1,200-R. P. M. Synchronous Motor with Normal Full-Voltage Starting and with Single-Circuit Starting

The use of double windings is somewhat restricted in small high-voltage motors because of the desirability of avoiding the use of very small conductors, necessitated by coils of many turns.

Increase of Starting Torques

The need for starting and pull-in torques greater than those of a normal motor occurs most frequently in applying low-speed motors, although the requirement of abnormally high pull-in torque is sometimes encountered also in applying high-speed motors. Considering the low-speed group, the following methods are available for increasing the subsynchronous torques:

1. Oversize high-torque motor, possibly with double squirrel-cage rotor windings.
2. Normal motor started on over voltage.
3. Delta-Y starting.
4. Normal motor started on tapped winding.
5. Oversize motor with phase-connected starting winding, and with salient or distributed pole-field windings.
6. The supersynchronous motor.

1. Oversize Motors. The oversize high-torque motor is similar to the normal motor but built to larger dimensions for a given rating to make room for a greater total magnetic flux. The wide pole pieces accommodate more bars without restricting the magnetic section. The bars may be arranged on a double squirrel cage using high- and low-reactance elements to improve the torque per ampere and to fit the speed-torque characteristic to the load requirement. The speed-torque curve shown in Fig. 4 is from a large low-speed rubber mill motor and it indicates the uniform torque obtainable from start to synchronism.

This type of motor is widely used on rubber mills, ball and tube mills, crushers, and on certain types of compressors. Simplicity of construction and in control, high efficiency, ample thermal capacity in the amortisseur winding and the design control of torque characteristics are the factors favoring this type of motor. Ample room makes it possible to reduce armature and field copper losses so that the efficiency exceeds that of normally designed motors. Iron losses tend to offset this gain only at light loads. Double-winding increment starting is available to meet objections to the sudden application of high torque, and to excessive voltage dips when high-torque motors have to start from a limited power supply.

Reliable intermittent rated capacitors of comparatively low cost are available for supplying part of the starting magnetizing current when connected in parallel with the motor; their use extends the high-torque motor field by modifying the starting current. While unity power factor starting is hardly feasible, the capacitor is justified by reduction in starting current and improvement in starting power factor particularly when the one capacitor can be used in starting several motors. By cutting out the capacitor in steps as the motor accelerates, the kva. demand may be regulated so as to prevent objectionable voltage variations.

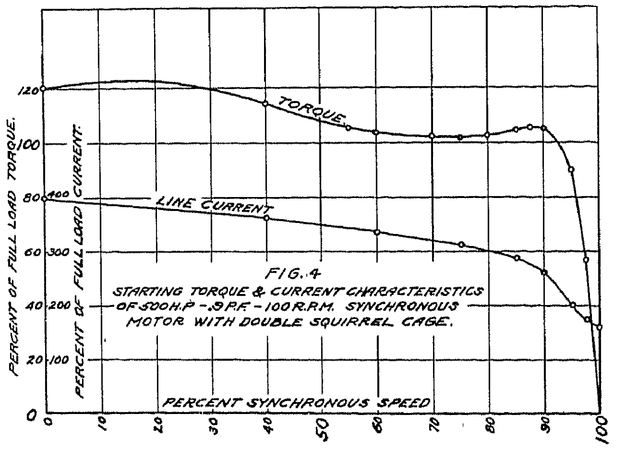

Fig. 4—Starting Torque and Current Characteristics of 500-Hp., 9-Power Factor, 100-Rev. per Min. Synchronous Motor with Double Squirrel Cage

2. Overvoltage Starting. Overvoltage starting can well be applied to a normal motor to obtain high torque in very slow speed ratings in which high-torque design is at a disadvantage because of the excessive sizes, weights and costs involved.

Undoubtedly there will be some initial objection to the application of overvoltage to motor windings as there was against full voltage starting of normal motors. However, to the manufacturer overvoltage starting of normal low-speed motors does not present any difficulties. With 150-170 per cent starting voltage, a low-speed, low-torque motor is not subjected to any higher per cent of current or torque than is a high-speed motor started on full voltage. The short stiff end turns of a low-speed motor's stator coils require little or no bracing to withstand starting stresses.

Overvoltage starting may be accomplished by stepping up the line voltage through an auto-transformer or by applying line voltage to only a part of the primary winding. The favored arrangement of an inverted auto-transformer with taps is shown in Fig. 5. Starting torque varies as the square of the applied voltage plus a slight increase which comes from the fact that saturation decreases the starting reactance. If high pull-in torque is also required, as for starting up a pulverizer with full charge, the motor can be synchronized before the voltage is reduced to normal. This is best done without interrupting power, so as not to let the motor drop out of step. The Korndorfer connection accomplishes this by opening the neutral and immediately closing the running contactor with only a slight reduction of voltage below normal.

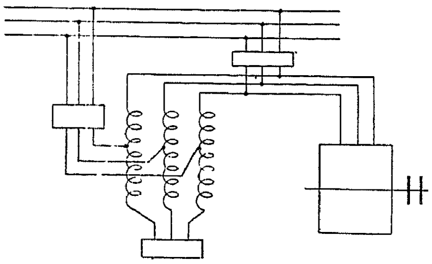

Fig. 5—Primary Connections for Overvoltage Starting of Synchronous Motor with Inverted Auto-Transformer

The inverted Korndorfer connection may be used to provide increment starting leading up to overvoltage, thus avoiding mechanical shock to machines in which there may be slack between motor and load and for which high starting torque is required. At start the Y point of the auto-transformer is left open and the motor terminal voltage is below normal because of the reactive drop in the auto-transformer which is carrying the impedance current of the motor. Excess voltage may now be applied without interrupting power by closing the neutral point of the auto-transformer.

3. Delta-Y Starting. Delta-Y starting has been successfully used on motors required to start a heavy load with moderate inertia characteristic but, since the circuit must be interrupted in changing to normal connections after synchronizing, it is doubtful if large high-voltage circuit breakers are fast enough to keep the motor in step. Moreover, no adjustment of starting characteristics is possible except in the motor design.

In a factory test made to simulate load conditions the changeover was made successfully in 4¾ cycles or 0.08 seconds by standard low-voltage a-c. contactors.

4. Tapped Winding Starting. Where it is found that a relatively small increase in starting or pull-in torque is required the service man may cut out a few coils of each phase to decrease reactance and increase input. This procedure lowers synchronous reactance, raises densities, and core loss, and calls for more excitation.

In a deliberate design it is possible to arrange the end connections so that a number of coils in each phase are cut out while starting on full voltage and picked up again after acceleration so that full synchronous reactance is restored. Tests indicate an effect similar to but weaker than overvoltage starting because a reduced number of coils with greater current does not set up a proportionately higher flux.

Tapped winding starting may be considered for instance for certain types of compressors requiring more than the conventional 40 per cent starting torque. The greater weight and dimensions of a high-torque motor may be objectionable and besides, its steep torque angle characteristic would call for a large flywheel. With tapped winding starting the flywheel requirement is normal and the control needs but a small addition.

5. Phase Wound Starting Windings. All of the high-torque arrangements so far discussed have approximately the "torque efficiency" (ratio of starting torque to starting inrush) of the normal motor, although somewhat better ratio is possible in the high torque, double squirrel-cage motor, and further improvement may be obtained by using starting capacitors.

The wound rotor induction motor with external rheostats in the secondary circuit has excellent starting torque and current characteristics, so many attempts have been made to combine these with the advantages of synchronous operation. By using such a motor with slightly larger air gap and fewer and larger rotor slots than normal for an induction machine, the synchronous induction motor, widely used abroad, is obtained. The slip rings are connected to rheostats in starting and to a direct circuit source at speed. The three fundamental difficulties with this type of motor are high cost, low efficiency, and high ratio of starting to rotor excitation voltage. The limited space for field copper in the rotor slots requires an oversize motor and a more expensive field winding than for a salient-pole motor of the same output and temperature rise. The same cause results in a higher excitation loss and a lower efficiency. The third difficulty arises from the fact that the $I\ R$ drop required to circulate full current in the field winding with a reasonable power loss is generally less than 2 per cent of the voltage induced in the field winding by normal flux at full frequency. The rotor starting voltage at full load torque is therefore some 50 times the excitation voltage, unless connection changes are made. These difficulties tend to limit the application of the synchronous induction motor to a few cases of high-speed motors, where the disadvantage of increased excitation loss is minimized.

Another proposal tried out both here and abroad has been to use a normal salient-pole synchronous motor with a phase winding in the pole faces. This has interesting possibilities, as it permits normal efficiency to be obtained, but it introduces a more complicated mechanical construction, and it does not permit as great a torque efficiency as the normal wound rotor induction motor construction. The phase winding must be carefully insulated and mechanically braced for high-speed motors, and this hampers accessibility of the field windings. To secure good starting characteristics and adequate heat capacity in the phase winding, it is desirable to use special pole construction, at added expense. To permit full flux and torque to exist at starting, the field winding must be open circuited, with resulting very high induced voltage or else special low-voltage excitation must be used. To avoid using very low-voltage excitation, it is customary to sectionalize the field winding while starting.

In general, it appears that the use of such special types of motors gives greater over-all cost for a given performance than when standard salient-pole motors with special control are used, though this conclusion is somewhat dependent on the manufacturer's facilities.

6. *The Supersynchronous Motor.* Controllable low-starting current combined with high torques is found in the low-speed supersynchronous motor. The external primary element is supported by bearings and is surrounded by a brake band. In starting, the primary is brought up to speed in the reverse rotation and synchronized while the field with its connected load stands still. Gradual application of the brake by hand or by pilot motor slows down the external element and starts and accelerates the field and load. The sum of the speeds of the two elements is synchronous speed and all the pull-out torque of the motor is available during starting and accelerating, and rated power factor prevails. When the primary comes to rest, say after 12 to 20 seconds, the unit operates as a normal synchronous motor. This motor is widely used in cement mills, on line shafts, mine fans, etc., not only for its low-starting current, but for the smooth control of the acceleration which is possible through regulation of the brake pressure. The design is not applicable to high-speed motors and unless its unique characteristics are important for a specific application, it is generally preferable to use a normal type motor with the proper control features.

Leading vs. Unity Power Factor for High-Torque Motors.

In normal low-torque designs, it is found that an 80 per cent leading power-factor motor develops more starting and pull-in torque than the unity power-factor motor of the same rating. Hence the question is often asked whether one should use a leading power-factor motor where high starting torques are the main objective.

The normal-torque, leading power-factor motor is larger than the unity power-factor machine because the excess magnetizing capacity must be capable of some adjustment without affecting the ability of the motor to carry its load. That is, the normal leading power-factor motor must have a higher pull-out torque than the unity power-factor motor, and will consequently be built in larger dimensions, with a more efficient amortisseur winding and higher starting torques.

In a leading power-factor motor designed oversize to obtain high starting torques, the advantage of the larger amortisseur cannot be fully realized; the magnetizing current of the motor itself must be kept low so that the excess field capacity may be used to supply magnetizing current to the line. Thus the magnetic densities must be lower than in a unity power-factor motor, and the leading power-factor motor will develop less starting torque than the unity power-factor motor, in which higher densities are used.

On the other hand, if high pull-out torque is the governing requirement, the leading power-factor motor may be built in smaller dimensions than the unity power-factor motor. Briefly, pull-out torque is the ratio of magnetizing field strength to demagnetiznig armature reaction strength. By over-exciting a motor of small gap, a high ratio is possible. But if unity power-factor is required, this high ratio is possible only by making the magnetizing current of the motor high and the armature reaction low. In extreme cases this results in a unity power-factor motor of larger size and larger air gap than are required for a leading power-factor motor. The extremely large gap penalizes torque efficiency, so that for applications requiring high pull-out as well as starting torque, the leading power-factor motor may be best adapted. However, it must be remembered that adjustment of power factor is sacrificed in order to maintain high pull-out torque.

Table II shows approximately how pull-out torque affects motor size at various power factors, and is based on 60-cycle motors at speeds up to 450 r. p. m.

TABLE II

Pull-out torque	Equivalent horsepower		
Per cent	1.0 power factor	0.9 power factor	0.8 power factor
150	1.0		
175	1.18		
200	1.37	1.37	
225	1.56	1.54	1.56
250	1.80	1.69	1.69
275	2.02	1.85	1.80
300	2.25	2.05	1.91
325	2.53	2.22	2.02
350	2.79	2.40	2.16

ACCELERATING AND SYNCHRONIZING HEAVY INERTIAS

The problem of synchronizing a salient-pole motor having heavy inertia load is discussed in other papers.[2]

In some applications of medium and high-speed synchronous motors, the pull-in torque requirement may be the limiting feature of design. Some of the most difficult cases arise in direct-connected drive of mine fans, oxide fans, and sintering fans. The air load on the fan may be as high as 100 per cent, since such fans are not always provided with shut-off gates. Typical values of inertia are such that the motor must reach 97.5-98 per cent speed before it can be synchronized. To develop 100 per cent torque at 98 per cent speed, the starting winding must be of very low resistance, with the maximum torque point occurring at approximately 95 per cent speed. Very few motors of standard design will meet such conditions. It is therefore, of interest to consider what can be done by special designs, both in the acceleration and the synchronizing of heavy inertias.

With the supersynchronous motor, the problem of pulling into step is entirely avoided, because the motor is in synchronism before the load is started. As regards heat storage and heat dissipation, the brake must be considered rather than the squirrel cage. Since the brake is outside the motor frame, it has a large radiating surface and considerable thermal capacity. It has been found practicable to apply this motor to heavy mine fans, line shafts (with large inertia in the various connected pulleys), and similar loads. However, this motor does not solve the problem on high-speed drive as it is not at present available for high speeds.

Both the synchronous induction motor and the phase wound damper motor are capable of accelerating a large inertia, since most of the loss in the secondary circuit is dissipated in the resistor. Normal type motors, however, can be built with amortisseur windings to accelerate heavy loads. Much of the heat generated in the bars during a gradual acceleration passes directly into the pole pieces, and the permissible temperature of the bars is not limited by contact with any insulating materials. A survey of a large number of designs of standard, medium, and high-speed motors indicates that for motors of 514-1,200 r. p. m., the inertia which can safely be accelerated is approximately as follows:

$$W R^2 = 25 \times H P \times \frac{(1000)^2}{(N)} \times \frac{1}{P F}$$

where

$H P$ = the rated horsepower of the motor
N = synchronous r. p. m.
$P F$ = rated power factor of motor

In any group of commercial designs in standard frames, some ratings will have more favorable characteristics than others, and in specific cases it is often possible to enlarge the amortisseur bars and increase the accelerating capacity well beyond the limits of this formula.

In low-speed motors the thermal capacity of the amortisseur seldom requires any modification of the normal design. Because of the relatively large diameter required for a low-speed motor, the amortisseur will normally contain enough material to give large thermal storage. While the $W R^2$ of many low-speed drives is very large, the stored energy at the low speeds is usually moderate, so that the size of the motor is generally fixed by other requirements.

At the other extreme, two-pole synchronous motors are found to have excellent accelerating capacity, in spite of the small motor diameter. These motors, built with round rotors and embedded field coils, are used principally for driving centrifugal compressors, many of which involve very high stored energy. By making the amortisseur in the form of very deep bars placed at the sides of rotor slots, a larger surface is provided for transferring heat to the rotor iron. In effect, the whole outer portion of the rotor is used to absorb the heat generated during acceleration.

In general, for synchronizing with a large inertia, either a phase winding or a squirrel cage can be designed with very low resistance to develop high torque at very low slip. It seems to be characteristic of most flywheel loads that the load at starting is not high, hence the moderate starting torque obtained with a low resistance squirrel cage of normal reactance is usually adequate for starting loads of this type.

SYNCHRONOUS CHARACTERISTICS

The use of synchronous motors on steady loads does not present any unusual problems involving the synchronous characteristics. But where irregular loads and high momentary peaks occur, it is necessary to modify the normal design to insure successful operation.

In any application which involves wide swings of load, the first question in selecting a motor involves the power supply. If the capacity is relatively large, and power swings corresponding in magnitude to the load swings are not objectionable, the first point is established in favor of the synchronous motor. If the supply is limited, and the load swings occur at very short intervals, the wound rotor induction motor with a flywheel and a slip regulator is still in demand. A number of large synchronous motor-generator sets has been installed in steel mills, to supply power to d-c. motors on reversing drives. Over a wide range of load, the losses of a high-torque synchronous motor are lower than those of an induction motor with slip regulating equipment, and the great concentration of power often permits taking advantage of this saving. But even in a steel mill the load swings of a reversing blooming mill motor are usually so great that they must be leveled out by a flywheel motor-generator set.

If it is found that the load swings can be assimilated by the power system, the characteristics of the synchronous motor itself must be considered. For any practical case involving peak loads of the order of one-half second duration or more, three alternatives are possible in applying a synchronous motor:

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[2] Edgerton, *Jour. I. E. E.*, Sept. 1930.
Edgerton & Fourmarier, see page 769.

1. Provide a motor with sufficiently high sustained pull-out torque to carry the load without losing synchronism.
2. Remove excitation and allow the motor to run on the amortisseur winding, resynchronizing after load has been reduced.
3. Automatically apply excess excitation to motor field during heavy overloads.

The first method, which amounts to building an oversize motor, is particularly applicable where the load swings occur frequently. Fig. 6 is taken from a wattmeter chart of a synchronous motor driving a band saw. Since load peaks up to 250 per cent may occur every few seconds, the synchronous motor must inherently be able to develop this torque with rated excitation current, hence motors for this service are designed for 250-300 per cent pull-out torque.

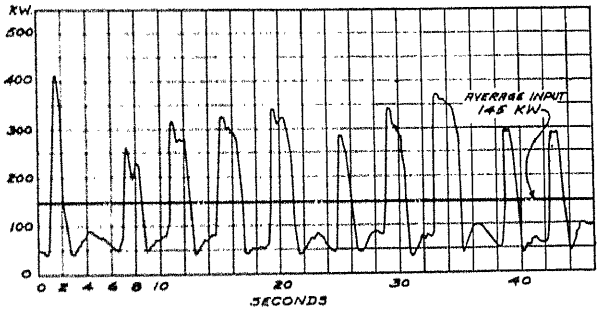


Fig. 6 Characteristic Input Curve of a Synchronous Motor Driving a Band Saw

Although unity power-factor motors can be designed to develop 300 per cent pull-out torque or more by resorting to a very large air gap and low armature reaction, it has been pointed out that such motors will in general be larger than leading power-factor motors which develop the same pull-out torque. Since the synchronous motor is often chosen primarily for power-factor improvement, motors of 80 per cent or even 70 per cent leading power factor are commonly selected.

Even with this torque available, motors are occasionally thrown out of step by extreme overloads. Then it must be determined whether the flywheel effect of the machines, together with the sustaining torque of the amortisseur winding, can carry through the peak load without too great a reduction in speed. It is found that medium and high-speed loads having severe fluctuations frequently include a very large WR^2. Loads of this character are found in lumber mills. They include band saws, fuel hogs and chippers, all traditionally driven by induction motors. These machines are built with large flywheel effect to take advantage of the slip of an induction motor in reducing the electrical input during momentary overloads. For synchronous motor drive, the speed reduction can be estimated closely if data are available regarding the inertia, the amortisseur torque, and the probable duration of the extreme load peaks. If the reduction is not large, the duty of accelerating back to synchronous speed will not endanger the amortisseur winding, since the heating of this element, for a given torque output, varies with the slip.

However, care must be taken in applying a synchronous motor where it must operate as an induction motor during extreme overloads. After a motor has been forced out of synchronism d-c. excitation must be temporarily removed, since the motor would otherwise generate a braking torque in opposition to the accelerating torque of the amortisseur winding. The load must be limited to a value which will enable the motor to resynchronize. In wood hog drive, this is done by automatically stopping the feed conveyor while the motor is out of synchronism. In sawing logs with band saws, where a heavy knot may overload the motor to the pull-out point, the loss of speed changes the whine of the saw to a lower pitch. The operator, accustomed to gaging the load by this sound with induction motor drive, can slow down the log carriage to let the motor resynchronize.

An element of judgment enters in estimating how often the motor will be pulled out of step. If the intervals are apt to be long, and the peaks short, the advantages of synchronous operation may be largely realized; but if the load is such as to cause pull-out frequently, the average power factor may be no better than that of a wound rotor induction motor, and the violent changes in power factor when synchronizing and when dropping out may seriously affect voltage regulation.

For meeting such conditions, the synchronous induction motor would be well suited, except for the inherently low pull-out torque of this type of motor. If designed for good starting characteristics, unity power-factor designs will have only 120-140 per cent pull-out torque. By designing for 0.8 leading power factor, 175-200 per cent pull-out torque can be obtained. If the overexcitation is further increased to obtain still higher synchronous torque, the loss in efficiency may be serious. Comparison of a synchronous induction motor with a salient-pole motor, both rated 125 hp. 0.8 power factor, 900 r. p. m., shows a differential of 2 per cent in efficiency at full load in favor of the salient-pole machine. In this case, the wound rotor machine is designed for a synchronous pull-out torque of 175 per cent at 0.8 power factor and can exert 300 per cent torque as an induction motor, without d-c. excitation. In designing this machine, the intention was to produce a motor capable of carrying moderate overloads in synchronism and to be used with a large flywheel to sustain the speed during high momentary loads. The salient-pole motor, also rated at 0.8 power factor, has

250 per cent pull-out torque, and can exert 200 per cent torque at 85 per cent speed without d-c. excitation. Obviously, the wound rotor machine can only be justified by very severe accelerating duty or starting current requirements.

Quick Response Excitation. The subject of high-speed excitation cannot be discussed within the limits of this paper, but a few of the possibilities may be mentioned. If a synchronous motor is subjected to occasional peak loads much greater than the average load, and the field excitation can be quickly increased to meet these overloads, a material reduction in the size and cost of the motor is sometimes possible. In addition, the motor may be designed for best efficiency at the average load condition, and the motor power factor will be held more nearly constant during the load changes. The problem is the same as that involved in overexcitation of generators to maintain synchronism during short circuits, except that most industrial motor installations cannot justify the refinement and expense of the equipment used with large generators.

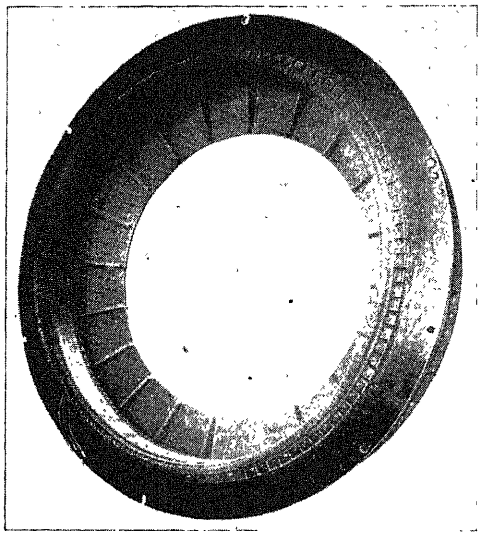


Fig. 7—Fabricated Frame of 500-Hp. 72-R. P. M. Synchronous Motor for Mounting in the Base of a Coal Pulverizer

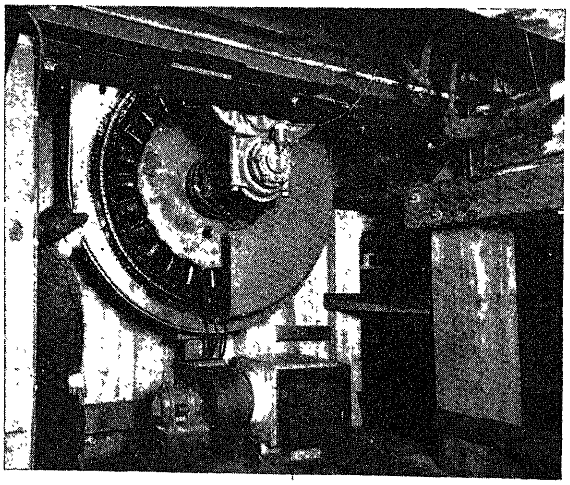


Fig. 8—Inverted Synchronous Motor Mounted on Lower Shaft of a Band Saw

When a heavy load is suddenly applied, a synchronous motor can momentarily exert a torque far greater than its sustained maximum torque. But unless excitation is quickly increased, the duration of this high transient torque will be only a fraction of a second, varying with the inductance of the field winding. Fortunately, this time interval is usually long enough so that the excitation of the motor may be controlled from the exciter shunt field, although in the larger sizes, modification of the exciter may be necessary to increase its rate of voltage rise. With small exciters, which inherently give quicker response, standard designs may suffice. The indication may be given by a quick acting current or power relay in the primary circuit of the motor.

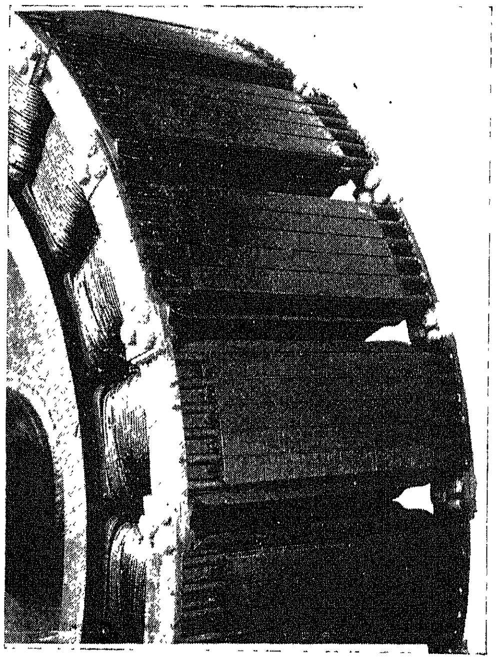


Fig. 9—Construction of Amortisseur for Heavy Starting Duty

For small motors such as 100 horsepower, it will usually be more economical to build a motor having inherently this higher torque, than to resort to overexcitation, with a larger exciter and additional control. But for a motor of say, 2,000 horsepower, the additional cost of field forcing equipment is relatively small. The gain in operating efficiency and the saving in first cost will often justify this equipment for large size motors on fluctuating loads.

Another method of overexcitation which is simpler but which involves some electrical loss has been used on a number of large synchronous motors on continuous rolling mills. With a normal excitation requirement of 125 volts the exciter is operated at 175-200 volts, the difference being dissipated in a fixed resistor in series with the motor field. A contactor is used to short circuit the resistor when the motor load exceeds a predetermined value. In spite of the additional loss in the motor field circuit, a study of several cases has shown that the over-all efficiency is higher than that of an 80 per cent power-factor motor for the same maximum torque conditions, and the overexcited motor has lower first cost.

Interesting possibilities are found in the use of electron tubes for overexciting a synchronous motor, although this development has not yet, to the authors' knowledge, been applied commercially. Circuits can be arranged, using the thyratron tube, whereby the excitation voltage will be automatically regulated as a function of the primary current in the motor windings, the response being practically instantaneous. Factory tests on a standard motor with controlled thyratron excitation have shown a great improvement in stability on sudden reductions of voltage, as well as increased synchronizing ability with a heavy load of high inertia.

In applying any field forcing scheme, the motor field must be of liberal design so that the intermittent high excitation will not overheat the field coils. With adequate margin provided, however, the field winding can still be considerably smaller than that of an equivalent high-torque motor which carries high-excitation current continuously.

MECHANICAL DESIGN

To adapt the synchronous motor to special applications without considerable development expense, there is required a greater flexibility in mechanical construction than is needed for normal applications for which lines of standard parts are available. The use of fabricated construction has materially assisted in promoting the "built-in" type of drive and has allowed the machinery manufacturers greater freedom in the design of the driven equipment. Fig. 7 shows the frame of a 500-hp. 100-pole 72-r. p. m. vertical motor for direct connection to a coal pulverizer. In this machine, it was necessary to keep the diameter to a minimum and space limitations did not permit an oversize high-torque motor. A normal-torque motor using overvoltage to obtain high starting and pull-in torques made it possible to keep within the required dimensions. A supporting ring built as part of the motor frame, serves as a foundation for the pulverizer. The result is a compact direct-drive unit in which all gears have been eliminated. The large air gap of the synchronous motor promotes mechanical reliability in this equipment in which considerable vibration may exist.

The flexibility of the welded steel construction often permits motorizing old compressors without rebuilding foundations, the motor frame being extended to meet specified dimensions. The forming of special stator frames to accommodate unusual mountings for other applications is illustrated in the case of the inverted motor, Fig. 8. For small, medium speed drives, overhung synchronous motors can sometimes be used thus eliminating the extended base, extra bearings and coupling. The rotors of these machines are often fabricated to permit short overhang.

Although the rapid progress of fabrication in recent years has received considerable attention, the art of casting in iron, steel, and alloys has also reached a high state of development. Each type of construction has certain advantages and disadvantages and the decision as to which type to use is always subject to full analysis from the standpoint of manufacturing facilities, economical production, and mechanical reliability.

The use of synchronous motors with loads requiring high-starting torques has made the amortisseur winding one of the most important parts in the machine. Fig. 9 illustrates the trend toward very sturdy construction to withstand heavy duty loads.

The progress in the improvement of insulations and of the magnetic and mechanical properties of iron and steel does not come within the scope of this paper other than to mention that these improvements have assisted in the extension of the synchronous motor to new fields of application.

Discussion

SYNCHRONOUS MOTOR WITH PHASE-CONNECTED DAMPER WINDING FOR HIGH-TORQUE LOADS

(HYDE)

SYNCHRONOUS MOTORS

(McLENEGAN AND FERRISS)

Fraser Jeffrey: Unless an oversized machine is used, it is questionable whether enough thermal capacity can be built into a salient-pole synchronous motor with squirrel-cage amortisseur winding to take care of the acceleration and pulling into step of an extremely heavy inertia load. This type of load is frequently to be found in cement plant tube mills where the starting conditions are often unusually severe especially after an extended period of shut down in cold weather. Under this condition of starting, the heat losses that have to be dissipated in the squirrel-cage amortisseur winding approximately equal the kinetic energy of the rotating load at full speed. Therefore it is desirable to be able to get rid of this heat loss external to the motor, as is done with a slip-ring induction motor, rather than to try and dissipate a large loss such as would occur in the rotor of a squirrel-cage induction motor under such conditions.

While the authors stress among other things, the high cost of the so-called synchronous induction motor and its use being restricted mostly to high-speed drives, nevertheless and contrary to this opinion, such machines are being successfully applied to low-speed high inertia drives at costs competitive to other modified types of salient-pole machines. While it is true that the synchronous induction motor has a somewhat lower efficiency than the salient-pole type synchronous motor, still for the acceleration of unusually heavy inertia loads, it has certain other features that make it desirable, if not necessary, for this kind of service.

The stator of the synchronous induction motor is like that of any ordinary induction motor, but the rotor is usually made with deep open type slots, the winding of which resembles that of a simple d-c. armature. This is fully distributed and the three phases are connected 120 elec. deg. apart, so that in starting, smooth and even acceleration equivalent to a regular slip-ring type induction motor, is obtained. By referring to Fig. 1, it will be noted that the three-phase leads are brought out to three slip rings between which are interposed two other slip rings to which the source of excitation is connected. Thus the maximum phase voltage in starting is separated by two ring spaces. The exciter armature, which is always in the circuit during both starting and running conditions, forms the neutral point of the rotor winding and thus has no excessive voltage across its terminals during the starting period. Starting is extremely simple and is usually accomplished by pressing a push button which closes the primary oil switch and then automatically starts cutting out sections of secondary resistance. The period of acceleration is governed by adjustable timing relays, and, as the motor has not over 1 per cent slip at full load as an induction motor, it easily pulls into step and operates as a synchronous machine, with very little line disturbance.

The additional complication of sectionalizing the fields to reduce initial starting voltages has not been found necessary because the highest open-circuit phase potential on say a 1,500 hp., 163½ r. p. m. synchronous induction motor has not exceeded 3,200 volts. This in turn required a direct current exciting potential of 65 volts. Neither of these conditions has been of any detriment to the successful operation of this type of motor. Such a machine is capable of exerting full load starting torque with 143 per cent line current (full load current being based on 100 per cent power factor) with 69.5 per cent power factor. The usual accelerating periods vary, depending largely on the local conditions, being anywhere from 30 to 60 seconds but rarely more than one minute under the most severe starting conditions. Once set, the time of acceleration usually remains unchanged for any given installation, but varies somewhat for different installations.

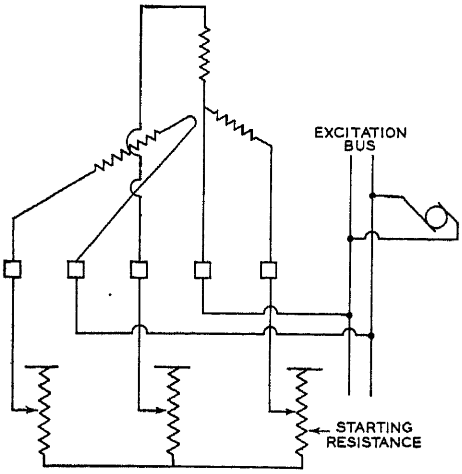


Fig. 1

There is a large number of low-speed synchronous induction motors in operation in this country today. They vary in size from 150 hp. to 1,500 hp., and are connected to high inertia loads. The fact that they are simple in mechanical and electrical construction as well as simple in operation and that they smoothly accelerate and pull into step these heavy loads and do it with minimum currents at high power factor, gives them an advantage over the other types of modified synchronous machines. Some of these motors have been in service for approximately five years, others somewhat less, and their operation over an extended period of time, has been uniformly successful.

F. K. Brainard: The authors are to be congratulated on their keen analysis and clear presentation of the problems connected with synchronous motor application. However, the subject is not complete without a brief discussion of the economic phases of the problem since one of the ultimate tests of any engineering project large or small is the total economic cost of the project under consideration compared with the cost of possible alternatives.

As stated by the authors the motive behind the change from induction to synchronous motors has been partly power-factor improvement and partly efficiency, both of which are primarily economic questions. That is, the losses in many synchronous motors are less than in the corresponding induction motors and the synchronous motors either draw no reactive magnetizing kva. from the system or furnish reactive kva. to be used by other apparatus on the system while the induction motors always draw reactive kva.

Normal unity power-factor 60-cycle synchronous motors with suitable belted exciters and manual control cost less and have higher efficiency than squirrel-cage induction motors with similar control at speeds of about 514 r. p. m. or less. Hence for these speeds and for applications for which they are suitable synchronous motors should be used. For speeds from about 600 r. p. m. to 3,600 r. p. m. however the induction motor is cheaper and the efficiencies nearly equal. Hence for this range in speed the synchronous motor must usually justify itself on the basis of the lack of reactive kva. drawn by the induction motor. This comparison is usually favorable to the synchronous motor up to about 1,200 r. p. m. although this depends largely upon the value of reactive kva. in the particular installation, but above that the induction motor generally has the advantage, *i. e.*, the additional cost of eliminating the relatively small amount of reactive kva. drawn by the high-speed induction motor is usually unjustifiable.

Similarly in the comparison between unity power-factor and 80 per cent power-factor synchronous motors the additional cost of the 80 per cent power-factor motor must be justified by the reactive kva. which it feeds into the system, after making due allowance for its lower efficiency. This is largely a question of speed as before, since the additional cost as well as the additional loss per reactive kva. increases rapidly as the speed decreases. The point at which the 80 per cent power-factor motor becomes uneconomical however depends to a very large extent upon the power-factor clause in the power company's contract in case the power is purchased. Years ago the power companies generally made no charge whatever for reactive kva. but now they frequently make a very considerable charge for it, so that this point must be carefully considered in any installation where purchased power is used.

Similar comparisons could be made between the various types of motors mentioned by the author but in any case the proposed motor must be justified economically if it is to survive.

D. W. McLenegan and A. G. Ferriss: As mentioned in the paper, the authors feel that there is a limited field for the synchronous induction motor, principally in cases where very severe accelerating duty is combined with the requirement of very low starting current. However, in the authors' experience the additional cost, complication, lower efficiency, and special exciter required by the synchronous induction motor are justified only in a relatively small percentage of the high-torque applications. Tube mills in cement plants, cited by Mr. Jeffrey, do not present severe accelerating duty to a normal high-torque motor. In many cases it has been found that the inertia of the tube mill was equivalent to less than the inertia of the motor rotor, when corrected for the difference in speed. Experience gained in the application of salient-pole motors to a wide variety of high inertia loads indicates that the majority of such applications can be successfully handled by salient-pole motors at lower cost, and with better efficiency than with synchronous induction motors.

With salient-pole motors having squirrel-cage starting windings, open field starting is seldom resorted to except where the very maximum torque efficiency is required. Hence the problem of excessive starting voltages in the fields of these motors seldom arises. Where open field starting is necessary, the induced voltage can be kept within reasonable limits by sectionalizing the field winding, or by using lower excitation voltage without sacrificing the advantages of the salient-pole design. It is interesting to note that the figure given by Mr. Jeffrey for rotor starting voltage of a synchronous-induction motor is almost exactly 50 times the excitation voltage, which is the ratio given in this paper.

With reference to Mr. Brainard's discussion, it is recognized that in the medium speed range the synchronous motor is seldom a competitor of the squirrel-cage induction motor except in cases where the slightly higher efficiency or the need for power-factor improvement justify the greater first cost of the synchronous motor. It appears, however, that at the higher voltages the differential in efficiency between synchronous motors and induction motors of standard design increases, since the synchronous motor is normally built with larger slots, and hence can better accommodate high-voltage insulation without causing excessive copper losses.

As pointed out in the paper, the choice between unity-power-factor and 80 per cent power-factor synchronous motors must be made by balancing the need for power-factor improvement against the increased cost and lower efficiency of the overexcited motor, since the torque requirements of most applications can be met with either design.

Lackawanna Suburban Electrification[1]

BY EDWARD L. MORELAND[2]
Fellow, A. I. E. E.

***Synopsis.**—The initial electrification on the Delaware, Lackawanna & Western Railroad includes approximately 68 route miles and 158 track miles of line out of Hoboken, New Jersey, and covers the more important parts of the suburban service and a small amount of freight transfer switching.*

The 3,000-volt d-c. system is used. Power is purchased in the form of three-phase, 60-cycle, high-tension, alternating current delivered by the power companies to five substations owned and operated by the railroad located along the right-of-way. Four of the substations are manually operated and one is fully automatic. Mercury-arc rectifiers are used exclusively for converting the alternating current to 3,000-volt direct current. The rectifiers have excited grids and are compounded to give substantially flat voltage up to 150 per cent load. All d-c. distribution is through the catenary system, auxiliary feeders not being necessary. All d-c. circuits are protected by high-speed magnetic blow-out circuit breakers.

The catenary system is of the tangent-chord double-contact wire type supported on steel structures in multi-track sections and on creosoted wood poles on the single track branch.

Multiple-unit trains are used exclusively in the passenger service, made up of two-car units consisting of a motor car and a trailer car, semi-permanently coupled together. Motors are insulated for 3,000 volts, are operated in series-parallel under electro-pneumatic control with automatic acceleration, and are self ventilated.

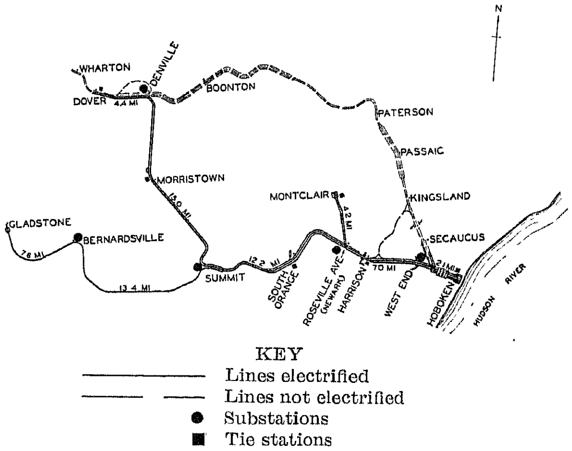
The freight transfer service is handled by two locomotives of the "three power" type, arranged for operation from the 3,000-volt d-c. contact system or on internal power provided by a diesel engine generator set supplemented by a storage battery.

GENERAL CONDITIONS

THE electrification program now in hand by the Delaware, Lackawanna & Western Railroad covers approximately 68 route miles and 158 track miles of line in the suburban district west of Hoboken, and includes the main passenger line from Hoboken to Dover (via Newark, the Oranges, Summit and Morristown) and the branch lines to Montclair and Gladstone. The steam service replaced by electrified trains aggregated approximately 23,000 revenue car-miles per day. The map shown in Fig. 1 indicates the lines now being electrified as well as other features, such as location of substations and tie stations, which will be referred to later.

The present electrification covers only local passenger service on the portion of the system now being electrified and electric transfer of freight trains between the freight classification yard at Secaucus and the terminal pier head yard in Jersey City, adjoining the Hoboken Terminal. Local passenger trains on the Boonton Line (via Paterson and Passaic) and all passenger trains to and from points beyond Dover will continue to be operated by steam power. Classified freight trains which do not need to be set out at the Secaucus yard for classification will also be taken to and from the Jersey City yard under steam power.

The principal reason for undertaking the electrification was a desire to give better service in the suburban territory by providing faster, cleaner, and more frequent trains possible only with electrification. One of the important factors in this connection was the discomfort to passengers caused by the comparatively long tunnels. These consist of two parallel tunnels just west of Hoboken, with two tracks each, approximately 4,300 ft. long, in which smoke conditions were bad under steam operation, particularly during rush hours. Under the present plan of operation one of the tunnels will be used exclusively by electric trains and all steam trains will be put through the other tunnel. During rush hours the one tunnel cannot handle all of the electric trains and it will be necessary to put some of them through the steam tunnel; the smoke conditions even in this tunnel will, however, be very much improved as compared with conditions under all steam operation.



Figures show distances between substations

FIG. 1—MAP OF LACKAWANNA SUBURBAN LINES EAST OF DOVER

Showing lines electrified and locations of substations and tie stations

Although the initial electrification is essentially confined to suburban passenger service, the choice of system used and all designs were worked out with the idea that the electrification must be suitable for both suburban service and for main line operation of all through passenger and freight service, which will presumably be electrified at some later time.

The 3,000-volt d-c. system is used, with power delivered to the trains by means of an overhead contact system fed from five substations owned and operated by the railroad located along the right-of-way. In the passenger service multiple unit cars are used exclusively.

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[1] This paper was written in October, 1930, at which time approximately one-half of the electrification was in operation, the program being scheduled for completion about the end of the year.

[2] Jackson & Moreland, Consulting Engineers, Boston, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

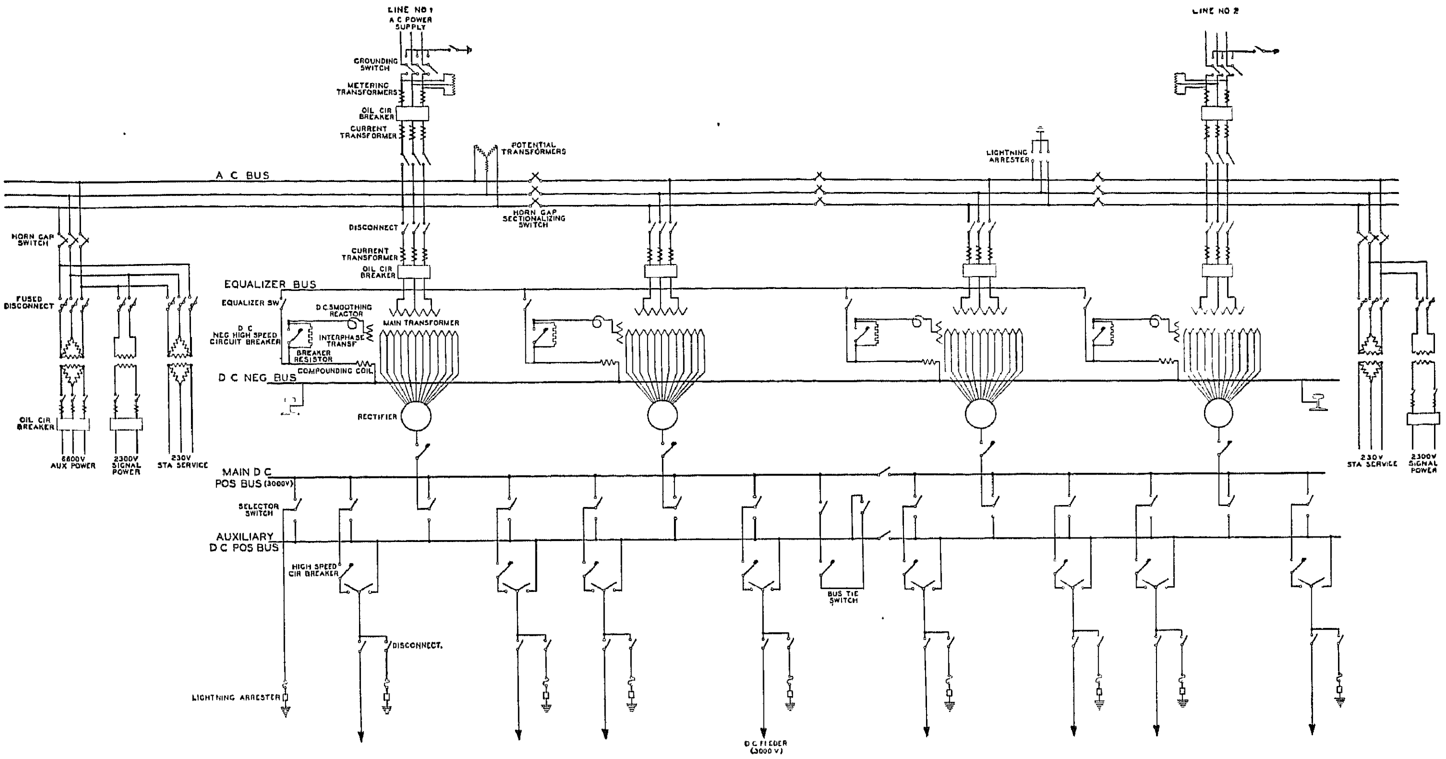

Fig. 2—Simplified Wiring Diagram of Typical Substation

In the freight transfer service "three-power" locomotives are employed.

Power Supply

Power for the electrification is purchased from the three power companies serving the territories in which the substations are located, and is delivered by the power companies to the five railroad substations in the form of three-phase, 60-cycle, alternating current, at the voltages available on the power companies' systems in the vicinity. There is no transmission of alternating current power by the railroad along the right-of-way for traction purposes.

The locations of the substations are shown on the map (Fig. 1). Power is supplied at the West End and Roseville substations by the Public Service Electric and Gas Company at 13,200 volts and 26,400 volts respectively, at the Summit substation by the Jersey Central Power and Light Company at 66,000 volts, and at the Bernardsville and Denville substations by the New Jersey Power and Light Company at 33,000 volts. The West End substation is supplied through three underground cables, any two of which are capable of transmitting the full requirements. Roseville substation is supplied by two underground cable circuits and each of the other three substations by two overhead circuits, in all cases one circuit being of sufficient capacity to supply the full requirements of the substation. In all cases, except Bernardsville, it is expected that all circuits will be kept in service during normal operation. At Bernardsville power will be taken normally from one circuit, with the second circuit open, but with provision for automatic throw-over to the second circuit in case of failure of voltage on the first circuit.

There are three separate power contracts with the three power companies. The contracts are all of the monthly-demand-charge plus energy-charge form, practically universal in railroad power contracts, with the demand determined as the average of the three highest clock-hour demands during the month, each month on its own basis (with certain minimum restrictions) and provision for elimination of unusual demands caused by unusual conditions and for compensation by the power companies for increased demands on the adjoining companies caused by failure of power supply. The fact that there are three separate contracts, each with its demand separately determined rather than one single contract for the entire power supply with a single demand, unquestionably adds materially to the total cost of power since it deprives the railroad of the benefit of diversity between the demands of the substations on different power systems. The three contracts were, however, necessary because the power companies would not offer a single combined proposition, and the cost of an alternating current transmission system along the right-of-way to distribute alternating current power to the substations, if all of the requirements could have been purchased from a single power company, would have far more than offset the added cost due to loss of diversity between substations.

Substations

Four of the substations, namely, West End, Roseville, Summit and Denville are at junction points and an operator will be kept in attendance at all times. They were, therefore, not equipped with complete automatic control, although many of the functions are automatic. The Bernardsville substation is fully automatic with supervisory control from the Summit substation 13.4 miles away.

Substation locations were determined by locations of junction points on the railroad and by other geographical conditions rather than by the economic spacing which would have been the criterion if it had not been for the function points. The spacings between the substations as shown on the map, Fig. 1, vary from 7.0 to 15.0 miles.

Mercury arc rectifiers are used exclusively for converting from 60-cycle alternating current to the 3,000-volt direct current required for traction purposes.

A-C. Supply Circuits and Control. Fig. 2 shows a schematic diagram of a typical substation, and Figs. 3 and 4 show respectively, a typical substation plan and a typical substation cross-section. In all substations the a-c. switching structure is of the outdoor type, and at all substations, except Bernardsville, the main a-c. buses are sectionalized by means of horn-gap air-break switches, and are at present being operated with the bus sectionalizing switches open with one supply circuit and one or two rectifiers on each bus section.

In all substations the connections from the a-c. bus to the rectifier transformers are through oil-circuit breakers with instantaneous overload protection. The main transformers have three-phase delta-connected

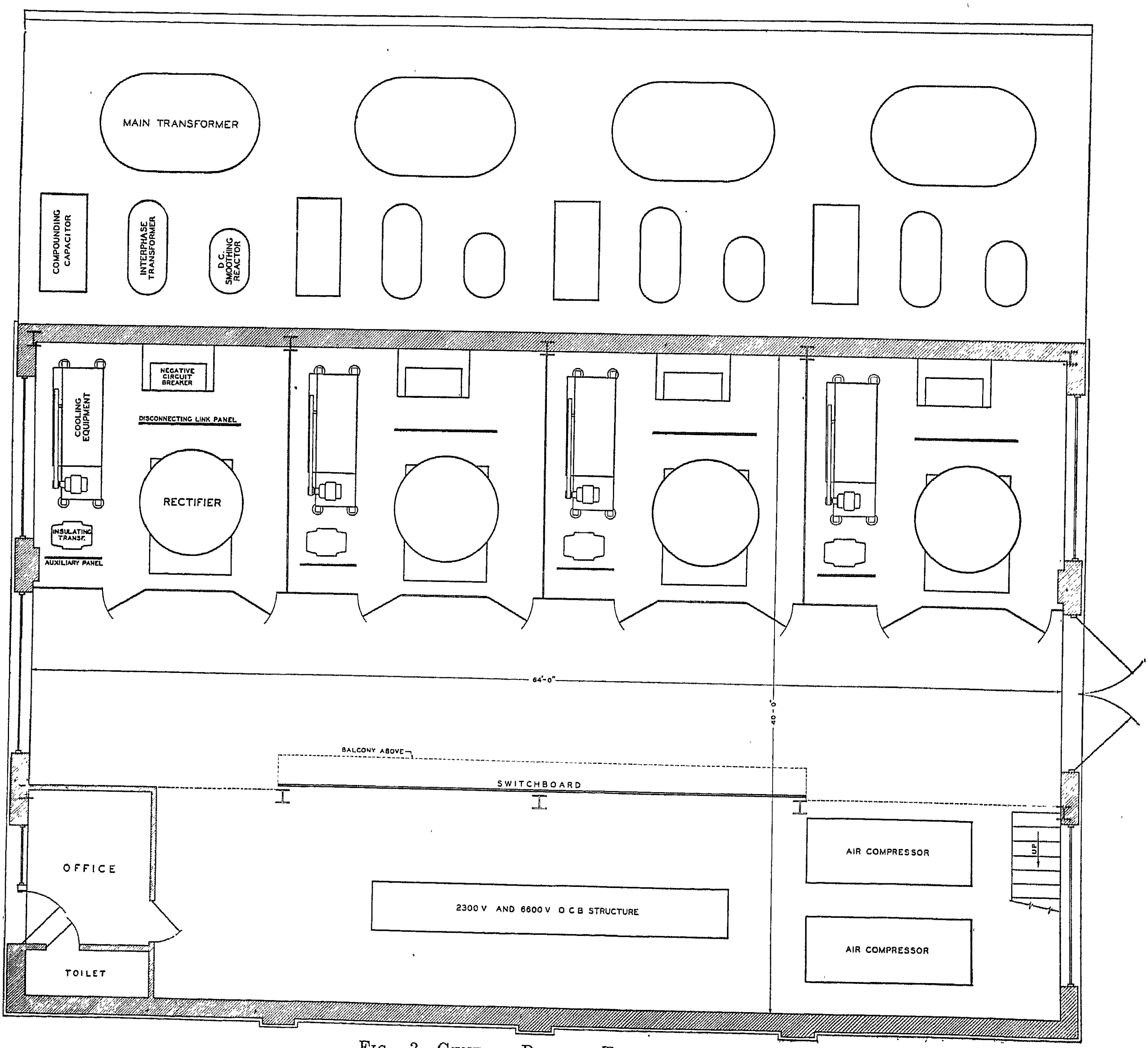

FIG. 3—GENERAL PLAN OF TYPICAL SUBSTATION

primaries, with ratio adjusters externally operated from the ground, and twelve-phase zig-zig secondaries, and are provided with thermal protection against overheating.

At West End and Roseville substations the incoming a-c. power feeders are connected to the bus through oil circuit breakers and disconnecting switches. Each of these substations is equipped with four twelve-phase transformers each supplying power to a twelve-phase, 3,000-kw. rectifier tank.

At Summit and Denville substations the incoming a-c. power feeders are connected to the bus through horn-gap air-break switches. In these cases the oil circuit breakers in the power companies' substations supplying the feeder circuits are relied upon to clear the associated circuit in the event of a fault on a section of the bus. Provisions are, however, made for later installation of oil circuit breakers instead of the horn-gap air-break switches if conditions on the power companies' systems should change so that the present method of operation would no longer be satisfactory. These two substations are each equipped with two twelve-phase transformers each supplying power to two six-phase, 1,500-kw. rectifier tanks. Normally, both rectifier tanks of each two-tank unit will be operated together, but in event of failure of one tank of a pair the faulty tank can be disconnected by opening disconnecting switches provided in the leads to the anodes, and the other tank operated alone.

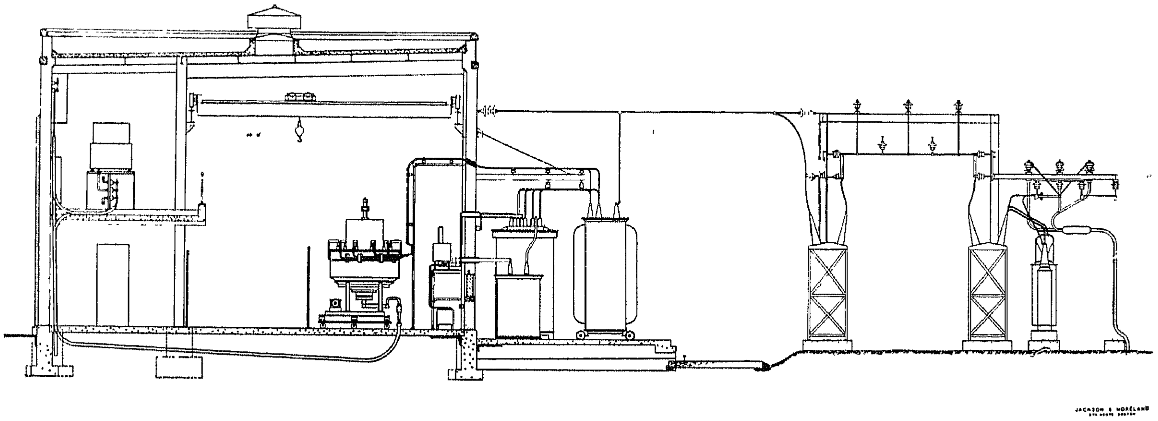

FIG. 4—CROSS-SECTION OF TYPICAL SUBSTATION

At Bernardsville substation the incoming a-c. power feeders are also connected to the bus through horn-gap air-break switches. As already pointed out, this station will normally be supplied from one circuit, the second circuit being used only in case of failure of power on the first circuit. At this substation there are two twelve-phase, 1,500-kva. transformers, each supplying power to a twelve-phase rectifier tank.

Fig. 5 shows a photograph of the outdoor structure at the Roseville substation, which is more or less typical of the kind of construction used throughout, except that at Summit, Denville, and Bernardsville the "strung bus" is used instead of the rigid type of bus employed at West End and Roseville. Lightning protection to the a-c. bus is provided by oxide film type lightning arresters.

D-C. Bus Arrangement and Circuit Control. On the d-c. side of each substation a main and auxiliary bus are provided, and each bus is sectionalized by a disconnecting switch, but is normally operated with the sectionalizing switch closed. The d-c. feeders connecting the substation bus to the catenary system are controlled by high-speed circuit breakers of the magnetic blow-out type, with holding coils energized from a storage battery. The feeders are staggered on the bus, as far as practicable; for example, if there are two feeders connecting to the catenary system west of a substation one feeder is connected to each section of the d-c. bus so that a single d-c. bus failure could not entirely cut off the supply of power to a section of the line. The arrangement of circuit breakers and disconnecting switches is such that any rectifier and any d-c. feeder may be connected to either the main or auxiliary bus with its associated high-speed circuit breaker in series. Any feeder circuit breaker can be by-passed and disconnected from the line for inspection or maintenance by connecting the feeder direct to the auxiliary bus, the feeder protection then being furnished by a high-speed bus tie breaker. This arrangement also permits the feeders supplying the contact system on both sides of any air gap to be tied together through disconnecting switches and the auxiliary bus so that live line maintenance or adjustment of air gaps can be carried out without danger of high voltage across the gap due to the opening of a circuit breaker. The d-c. bus, and the disconnecting switches on the d-c. feeder circuits and the rectifier positive connections, are mounted in an alberene stone cell structure, which in all substations except Bernardsville is in a balcony. At the Bernardsville substation the cell structure is on the main floor. The high-speed magnetic blow-out circuit breakers on the d-c. feeder circuits and on the rectifier positive circuits are mounted on top of the cell structure.

A lightning arrester of the electrolytic aluminum cell type with balancing resistor and shunted spark gap is provided on each d-c. feeder and on the d-c. main bus.

Fig. 6 shows a photograph of the switchboard in the Roseville substation, and indicates the uses of the various panels. Each rectifier unit has a separate panel, whereas three feeders (with space for a fourth) are controlled from each feeder panel.

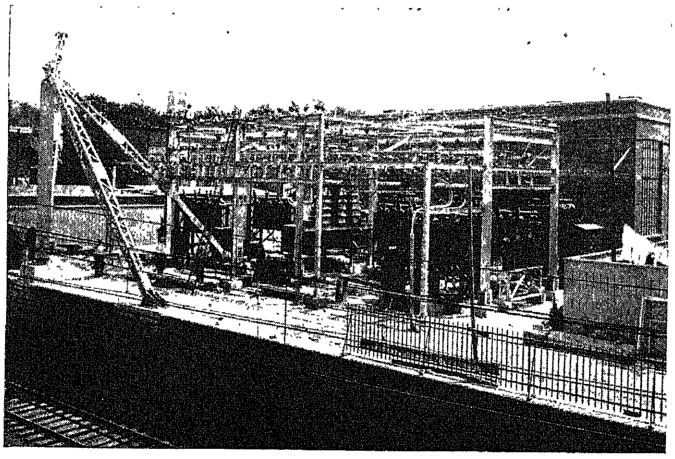

FIG. 5—OUTDOOR STRUCTURE AT ROSEVILLE SUBSTATION
Substation building in background
Derrick at left used to lift equipment from cars on the depressed tracks
Photograph taken during construction

A negative bus is provided at each substation to which the rail returns and rectifier negative leads are connected. The negative bus is mounted outdoors on the substation wall just behind the transformers, and is grounded to the station ground bus.

Two storage batteries are provided in each substation. One normally supplies energy for the holding circuit of the high-speed d-c. circuit breakers and the other normally supplies power for operating the high-speed d-c. circuit breakers, the a-c. oil circuit breakers, the pilot lights and the emergency station lighting. The circuits are, however, so arranged that any battery circuit can be switched to either battery. Two motor-generator battery charging sets are also provided. Normally one set is used to charge each of the batteries but either set can be used to charge both batteries.

Bernardsville Automatic Substation. As previously mentioned, four of the five substations are manually operated. The fifth, Bernardsville, is fully automatic with supervisory control from the Summit substation. The power supply at this substation will normally be taken over a single circuit, but provision is made for automatic throw-over to a second supply circuit in the event of failure of power on the first circuit. Once having changed over to the second circuit, it does not go back automatically to the first circuit on restoration of power on that circuit unless there is a failure of power on the second circuit.

The automatic features provide that either of the two rectifier units may be made the preferred unit and that the second unit will automatically come into service when the load on the first unit exceeds a pre-determined amount, and will automatically drop out after a pre-determined time delay when the total substation load drops below a pre-determined amount. The preferred rectifier may be kept on the line continuously or the control may be adjusted to bring it on only when there is a load demand on the substation as indicated by a drop in d-c. line voltage, and to take it off the line, after a definite time delay, when the load has decreased below a pre-determined amount as indicated by current flow. In the event of a fault on either rectifier or any of its auxiliary circuits that rectifier is automatically locked off the line. If the fault is on the preferred rectifier the second rectifier automatically takes its place.

Supervisory Control of Automatic Substation. The supervisory control is of the synchronous selector type, using four wires between the Bernardsville substation and the supervisory point at the Summit substation. Lights on the supervisory panel at the Summit substation automatically indicate:

1. The position (open or closed) of the switches on the two incoming high-tension a-c. supply circuits.

2. The position of the switches on the two rectifier circuits (the a-c. oil circuit breaker, the positive d-c. breaker, and the negative d-c. breaker for each rectifier unit, are all under common control and all open and close together).

3. The condition of either rectifier when not on the line, that is, whether or not the unit is available for service.

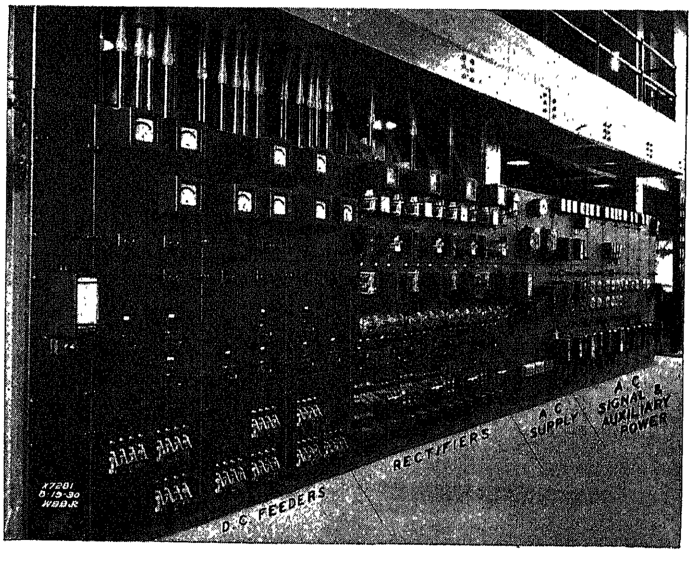

FIG. 6—SWITCHBOARD AT ROSEVILLE SUBSTATION

4. Lock-out of a unit due to some fault which requires manual release of the lock-out.

5. The position of the switches on the two signal supply circuits fed from the substation.

6. The position of the switch connecting the signal power bus to the transformer from which the signal power is normally supplied.

7. Whether or not the switch between the signal power bus and the signal circuit connecting between the Bernardsville and Summit substations, if in the open position, is set up for automatically closing in event of loss of voltage on the Bernardsville signal power bus.

8. Lock-out of the automatic reclosing breaker on the d-c. feeder to the stub-end catenary line to Gladstone or on the stub-end a-c. signal circuit to Gladstone, or lock-out of the oil circuit breaker connecting the signal power bus to the signal power transformer.

9. An alarm indication if either of the charging units for the two storage batteries shuts down or current ceases to flow to the battery; and

10. A warning indication if the voltage on the signal power bus is low.

The supervisory control permits the operator at the Summit substation; (1) to take control of both incoming a-c. supply lines together, and having taken control, to operate them individually, or restore both to automatic control at will; (2) to take control and operate, or restore to automatic control either rectifier individually; (3) to open or close either d-c. feeder breaker; (4) to open or close any one of the three signal power switches, and if the switch between the signal power bus and the signal power circuit between Bernardsville and Summit is open to set up this switch for automatic closing in event of loss of voltage on the signal power bus, or to take off the automatic set up at will; (5) to determine the amount of load being carried by either rectifier as indicated by the current in one phase of the a-c. supply circuit; and (6) to trip simultaneously, in case of emergency, all circuits which can feed power into the substation, both alternating and direct current.

Signal and Auxiliary Power Supply. In addition to the 3,000-volt direct current supplied from the substations for traction purposes, each of the substations also supplies 2,300-volt, single-phase, 60-cycle power for signal operation. The West End, Roseville and Denville substations also supply 6,600-volt, three-phase, 60-cycle power for railroad auxiliary power circuits for station lighting and miscellaneous power requirements along the right-of-way.

Rectifiers. Rectifiers were selected rather than motor-generators for the conversion of the 60-cycle a-c. power to 3,000-volt d-c. power because of the relatively low first cost of rectifier equipment, the high efficiency of such equipment, its ability to carry heavy overloads without injury, and the economy of substation design resulting from the small space occupied by the rectifiers and the absence of heavy foundations required for motor-generators. Fig. 7 shows the comparative efficiencies of 3,000-volt mercury-arc rectifier sets and motor-generator sets.

The substation buildings housing four 3,000-kw. rectifier tanks are only 64 feet long by 40 feet wide by an average height of 27 feet (all inside dimensions), which is small compared with the space which would be required for the same capacity in motor-generator sets. The Lackawanna rectifiers were bought under a specification requiring guarantee of successful operation in service under loads of 150 per cent of current rating (50 per cent overload) for two hours and 300 per cent of current rating (200 per cent overload) for five minutes. During the acceptance tests short-time overloads up to 600 per cent of current rating (500 per cent overload) were also applied and successfully carried for periods of 10 seconds.

The rectifier tanks are equipped with the usual arc-striking anodes, holding (or exciting) anodes, vacuum pumps, and gages. The tanks are water-cooled, using a closed cooling system with forced circulation under thermostat control. The circulating water for each rectifier tank is re-cooled in a cooler unit associated with that tank somewhat similar to an enlarged automobile radiator, using a circulating pump and motor-driven fan. Heating elements under thermostat control are also provided to prevent various parts of the rectifiers becoming too cool when shut down or lightly loaded.

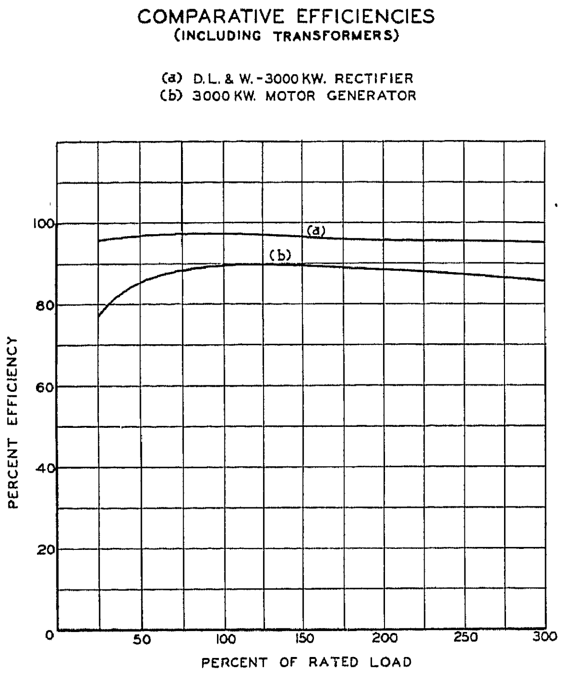

FIG. 7—COMPARATIVE EFFICIENCIES, 3,000-VOLT MERCURY ARC RECTIFIERS AND MOTOR-GENERATOR SETS (BOTH INCLUDING TRANSFORMERS)

Direct current smoothing reactors to reduce the ripple in the d-c. voltage, and resonant shunts to drain off high-frequency harmonics which would cause interference on communication circuits, are also provided. The resonant shunts are tuned for 360, 720, 1,080 and 1,440 cycles. Surge suppressors made up of capacitors with limiting resistors are provided to protect the transformer secondary windings and the interphase transformer windings against the high-voltage surges induced when the residual current flow following an arc-back or internal short circuit is interrupted by the arc going out.

The shell of the rectifier tank and some of the associated equipment is at substantially cathode potential (3,000 volts above ground) when the rectifier is in operation. This equipment is therefore insulated from ground and carefully protected from accidental contact while in operation. This is accomplished by enclosing each rectifier tank, with its associated cooling equipment, auxiliary switch panel, auxiliary circuit insulating transformers, resonant shunts, surge suppressors and the negative circuit breaker with its associated load limiting resistor in a grille enclosure, so arranged with interlocks on the locks of the enclosure doors that no high-voltage circuit within the enclosure can be alive when a door lock is in the open position.

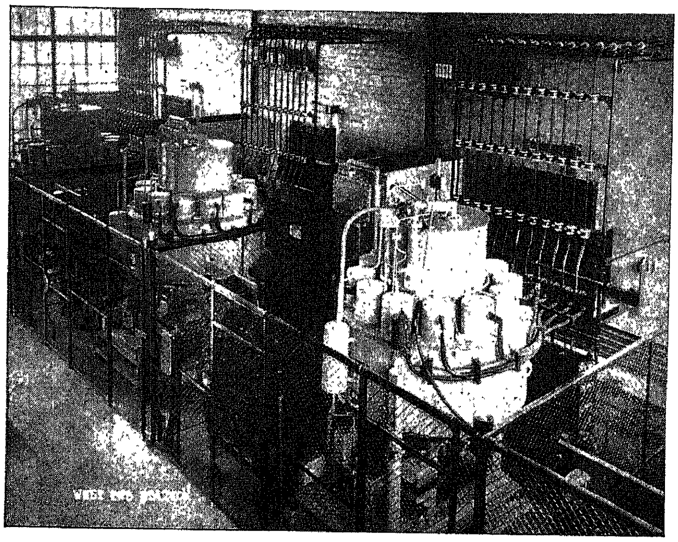

FIG. 8—RECTIFIERS AND GRILLE ENCLOSURES

Fig. 8 is an interior view of a substation and shows the arrangement of the rectifiers and the enclosing grilles.

In addition to the overload protection provided by the oil-circuit breaker on the a-c. side of the transformer, each rectifier unit is protected by a high-speed circuit breaker of the magnetic blow-out type on both the positive side (cathode connection) and on the negative side (connection to mid-point of the inter-phase transformer). The positive breaker is set to open on reverse current only and is given a low setting with zero rate-of-rise adjustment. The positive d-c. circuit breaker is interlocked with the a-c. oil circuit breaker in such a way that the opening of the a-c. oil circuit breaker of any rectifier unit trips the d-c. positive breaker associated with that unit. The negative d-c. circuit breaker, when closed, short circuits a series load limiting resistor of three ohms resistance, which is cut into circuit when the negative breaker opens. This breaker is set to open only on very heavy overloads or short circuits. An auxiliary timing relay connected across part of the negative load limiting resistor is so arranged that if the current through the resistor drops to a pre-determined amount within a pre-determined time the negative breaker recloses. If the current does not drop within the pre-determined time the relay functions to trip the a-c. oil circuit breaker, which in turn trips the positive d-c. circuit breaker. Closing the a-c. oil circuit breaker automatically closes the negative d-c. circuit breaker.

Rectifier Grid Excitation. The outstanding features of the Lackawanna rectifiers, aside from the voltage and capacity of the units, are the anode grid excitation and the automatic compounding. Fig. 9 shows a cross-section of the rectifier tank. One of the main anodes, which are made of graphite, is shown at the left. In addition to the usual shields and baffles provided to shield the anode from the mercury thrown up from the cathode mercury pool by the arc stream, each anode is also surrounded by an inner shield suspended from a porcelain insulator. This shield supports a mesh grid, made of special high-melting-point metal, mounted just below the face of the anode. The inner shield and grid, which are insulated both from the anode and the tank case, are connected by a separate bushing through the anode seal and through a resistor to an excitation circuit, the opposite pole of which is connected to the cathode. Voltage applied to the grid ionizes the mercury vapor near the face of the anode and thus assists in establishing the power arc. This permits much closer shielding of the anodes than would be possible without the grid excitation, and consequently reduces the tendency for the rectifier to "arc back" or short-circuit internally from anode to anode and also materially reduces the amount of power necessary in the holding anode circuit.

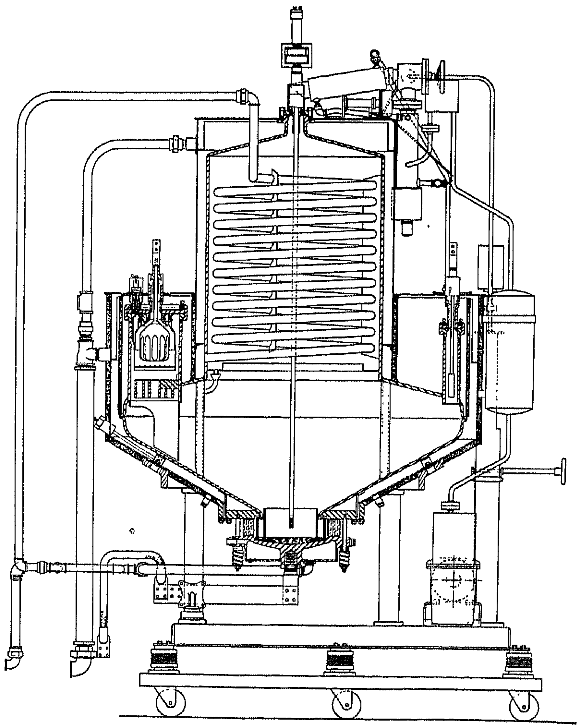

FIG. 9—CROSS-SECTION 3,000-VOLT, 3,000-KW. RECTIFIER TANK

Tests have been made both with d-c. grid excitation with a grid voltage of 96 volts (above cathode potential) and with twelve-phase a-c. excitation with the grid voltage leading the anode voltage by 12 to 15 degrees. When the rectifier units are run singly they operate satisfactorily with either d-c. or a-c. grid excitation. Difficulty has, however, been experienced in getting units to parallel satisfactorily with a-c. excitation, and consequently the units are now (October, 1930) being run with d-c. excitation supplied by tungar trickle charger battery sets pending further tests with a-c. excitation. The rectifiers parallel satisfactorily and divide load perfectly with d-c. excitation, but a-c. excitation would be simpler if equally effective. The shielding of the Lackawanna rectifier is so complete that it will not establish the power arc and pick up load until the grids are excited, or if the grid excitation is cut off while a rectifier is carrying load the rectifier immediately drops its load.

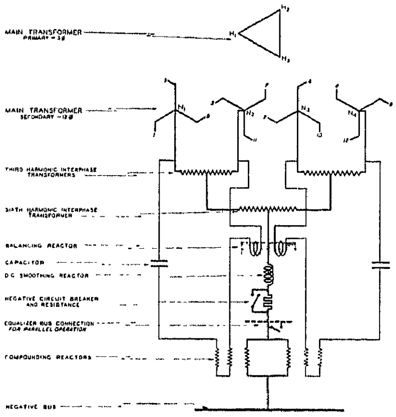
MAIN TRANSFORMER PRIMARY – 3ø
H1
H2
H3
MAIN TRANSFORMER SECONDARY – 12ø
N1
N2
N3
N4
THIRD HARMONIC INTERPHASE TRANSFORMERS
SIXTH HARMONIC INTERPHASE TRANSFORMER
BALANCING REACTOR
CAPACITOR
D C SMOOTHING REACTOR
NEGATIVE CIRCUIT BREAKER AND RESISTANCE
EQUALIZER BUS CONNECTION FOR PARALLEL OPERATION
COMPOUNDING REACTORS
NEGATIVE BUS

FIG. 10—COMPOUNDING CIRCUIT, SIMPLIFIED DIAGRAM

Rectifier Compounding. The rectifier compounding circuit is shown diagrammatically in Fig. 10. The main power circuits are indicated by the heavy lines and the auxiliary compounding circuits by the lighter lines. As shown on the diagram the twelve-phase secondary of the main transformer is made up of four three-phase secondaries with their neutrals connected together through interphase transformer windings. From the standpoint of the external circuit the four three-phase windings make up a true twelve-phase secondary, each phase displaced 30 degrees from the next. Due to the action of the interphase transformers, however, when the rectifier is running non-compounded, each of the three-phase windings functions as a separate three-phase unit, and the voltage on the d-c. side of the rectifier is the voltage corresponding to a three-phase rectifier.

The shifting of the arc from anode to anode in the three-phase units results in triple frequency or third harmonic voltages, which are impressed on the third harmonic interphase transformers. If neutral N^1 were connected directly to neutral N^2 and neutral N^3 were connected directly to neutral N^4 instead of through the interphase transformer windings, the rectifier would function as two six-phase units and the d-c. voltage would be the voltage corresponding to a six-phase rectifier, which is approximately 15 per cent higher than the voltage of a three-phase rectifier with the same a-c. secondary coil voltage.

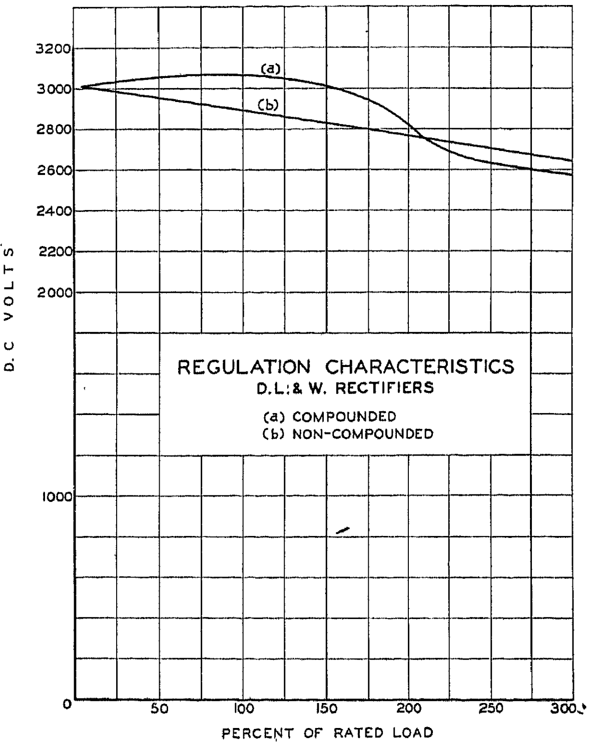

FIG. 11—RECTIFIER CHARACTERISTIC CURVES

The compounding is accomplished by shunting the third harmonic interphase transformers (which connect neutral N^1 with N^2 and N^3 with N^4) by means of two compounding circuits each containing a capacitor and a winding of a compounding reactor. The d-c. windings of the compounding reactors are connected in series with the main power circuit, and consequently the saturation of the cores of the compounding reactors is varied with the output of the rectifier. Obviously, as the saturation of the core varies the reactance of the compounding circuit and its admittance to the third harmonic voltage across the third-harmonic interphase transformer winding also vary. The compounding circuits may, therefore, be tuned so that the capacity and reactance balance at the core saturation corresponding to the d-c. load at which the maximum compounding effect is desired. At that point the reactance of the compounding circuit to the third harmonic voltage is substantially zero and the third harmonic interphase winding is therefore substantially short-circuited so far as the third harmonic current is concerned. The two three-phase windings then function as a six-phase winding, and the rectifier d-c. voltage corresponds to the voltage of a six-phase rectifier. In other words, at very light loads the d-c. voltage corresponds substantially to the voltage of a three-phase rectifier and as load comes on, due to the shunting of the third harmonic current from the interphase transformer winding through the compounding circuit, the condition approaches more and more nearly that of the six-phase rectifier, and the voltage is varied accordingly. The two compounding circuits are linked together by means of the balancing reactor to insure balancing the load between the two groups of anodes. The compounding circuit can be adjusted to hold the d-c. voltage substantially constant up to as high as 225 per cent of rated load with relatively little increase in voltage at intermediate loads. The compounding of the Lackawanna rectifier is adjusted to give approximately 3,000 volts up to substantially 150 per cent load, using a total of 416 kva. of capacitors in the compounding circuits of each 3,000-kw. rectifier unit. The characteristic curve of one of the Lackawanna rectifiers, as determined by test, when running compounded, is shown in Fig. 11. For comparison the characteristic when running non-compounded is also shown.

The power factor of the compounded rectifier unit, including the transformer, is notably high, showing approximately 96½ per cent lagging at 25 per cent load and increasing with load to 99 per cent lagging at 50 per cent load, slightly leading at 150 per cent load and back to 99 per cent lagging at 300 per cent load. The effect of the compounding capacitors on the power factor is greatly increased due to the fact that they are located in circuits carrying third harmonic frequency.

When compounded rectifiers are run in parallel it is necessary to provide an equalizer bus connected above the compounding reactor, as indicated in Figs. 2 and 10, in order to assure equal compounding effect and equal division of the load between parallel units.

MULTIPLE-UNIT CAR EQUIPMENT

The multiple-unit car equipment in the initial electrification consists of 141 two-car units, each unit consisting of one motor car and one trailer car semi-permanently coupled together, with a control position and motorman's cab at each end of each two-car unit. Trains of two to twelve cars are made up by coupling units together, the train control circuits connecting from unit to unit (and from car to car within the unit) by means of jumpers with plug connectors. The units are operated with the trailer car west of the motor car in order to secure the benefit of cleaning and drying the rails by the passage of the trailer car ahead of the motor car on west-bound movements against which the grades are heaviest.

All motor cars are new steel cars, 70 ft., 3¼ in. long over bumpers, seating 84 passengers, are equipped with vestibules with hinged doors and trap doors, and without toilets. All motor cars except five are equipped with roller-bearing trucks. The trucks on five-motor cars were equipped with standard journal bearings for comparative purposes. The motor cars, completely equipped, weigh approximately 147,000 lb. without passenger load.

All trailer cars are remodeled steel cars formerly used in steam service. They have been equipped with vestibules, seat 78 or 82 passengers, depending upon the type (except combination baggage and passenger cars and club cars, which seat less), are 70 ft., ¾ in. to 70 ft. 6 in. over bumpers, and weigh fully equipped from 105,000 to 110,000 lb., exclusive of passenger load. Each trailer car is provided with a toilet. All trailer cars, except five, have standard journal bearings on the trucks. The trucks of five have been equipped with roller bearings for comparative purposes.

Fig. 12 shows a view of an eight-car train, made up of four of the two-car units.

Fig. 13 shows a simplified diagram of the car wiring. All of the main equipment, except the pantographs, main fuse, master controller, heaters and heater bus jumper (connecting the trail car with the motor car), is mounted underneath the motor car.

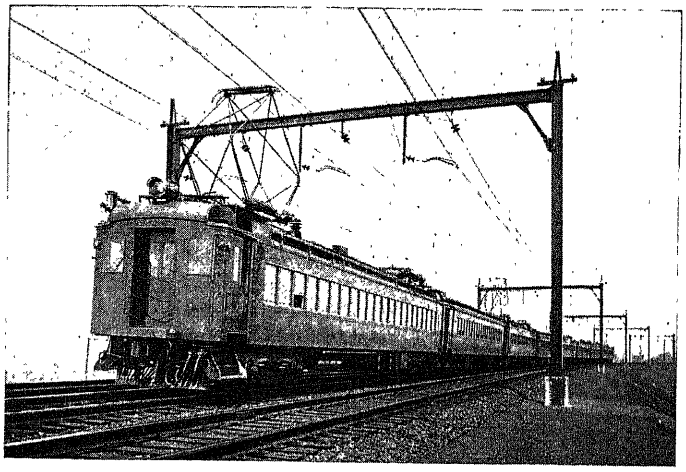

FIG. 12—MULTIPLE-UNIT TRAIN WITH FOUR TWO-CAR UNITS
Showing typical overhead construction on curved track

Motor Equipment. Each motor car is equipped with four motors of 235 horsepower (one-hour rating) each, the capacity of the motors being fixed by the requirements of local service with frequent accelerations. On express runs the motors would be good for an additional trailer car on each unit, but all units were made alike in order to permit maximum flexibility in use. Advantage is being taken of this fact to permit hauling an extra express car (not electrified) on certain trains during non-rush hours. Fig. 14 shows the motor characteristic curves. The motors are operated with series-parallel connections, starting with four motors in series and cutting over to two parallel groups of two in series as the train comes up to speed. All motors are, however, insulated for the full 3,000 volts.

The motors are self-ventilated, using a series pull-push system. The cooling air is taken in through louvers at the top of the car, passes through settling chambers over the vestibules, then through ducts built into the sides of the car and under the car, and through flexible "pants-leg" ducts to the motors. The air enters the motor at the commutator end through an end housing connection admitting the air to the armature inside the commutator bell. It is then sucked through the armature by the fan on the pinion end and forced back through the field windings to a discharge port at the commutator end of the housing. The air taken in at the roof of the car is relatively clean and the motor housing being under pressure there is no tendency to draw dirt into the motor. Results indicate that the design is effective in keeping the motor free from dirt or snow without the necessity for filtering the air.

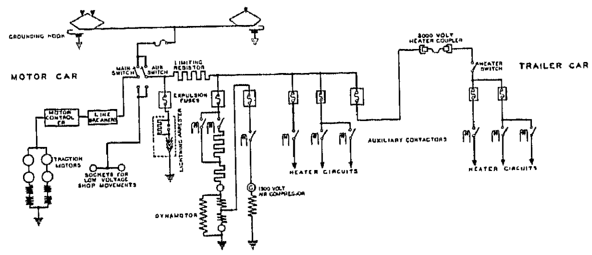

FIG. 13—CAR WIRING, SIMPLIFIED DIAGRAM

There are four brush studs per motor, each brush stud holding two brushes. The armature shaft and axle bearings are of the constant oil level type. The gear ratio is 59 to 22 and the gears are equipped with special gear silencers. The gear cases are mostly malleable iron but some motors are equipped with pressed steel cases.

Control and Auxiliary Equipment. Each motor car is equipped with two pantographs, only one of which will normally be used at a time. The pantographs have four self-alining roller bearings, are spring raised and air lowered, and have a working range from 15 ft., 6 in. to 25 ft., 3 in. above top of rail. The standard pantograph has two contact shoes each equipped with two wearing strips with a grease groove between. For comparative purposes, however, fifty pantographs have been provided with single contact shoes, each with two wearing strips and a grease groove. These pantographs are idential with the standard pantographs in all other particulars and the parts are interchangeable. The normal pressures against the contact wire are 28 lb. for the standard pantograph (with two contact shoes) and 18 lb. for the single shoe pantograph. The maximum variation between upward and downward pressure throughout the working range of the pantograph is approximately 4 lb.

The control system operates at 32 volts, supplied from a storage battery carried by the motor car. A master controller is located in each end of each two-car unit. The main controller (which is under electro-pneumatic control by the master controller), the contactors, and auxiliary equipment are mounted beneath the motor car. The control equipment provides automatic acceleration adjusted for an acceleration of 1.5 mi. per hr. per sec. The operator can, however, take control at any point and hold up the automatic acceleration. With motors in the series parallel connection, and 50 per cent of the field shunted, the balancing speed of a six-car train is approximately 65 mi. per hr.

A dynamotor (with two windings in series), mounted under each motor car, provides 1,500-volt power for operation of a 1,500-volt air compressor and also drives, by direct shaft connection, a 4-kw. battery charging generator for charging the control battery, which also supplies the car lighting for the motor car and the associated trailer car. The dynamotor is self-ventilating, taking its ventilating air directly from beneath the car through a centrifugal air separator.

The air compressor has a displacement of 36 cu. ft. per min., and is adequate to supply the requirements of a four-car train with an ample margin of reserve in the event of failure of one compressor of a two-unit train.

In the event of failure of one unit in a train the control of that unit can be cut out and the faulty unit dragged as a trailer by the other units in the train. In the event of failure of one motor, or pair of motors, on a single-unit train that pair of motors can be cut out by a motor cut-out switch provided for the purpose underneath the car, and the unit operated on the other pair of motors.

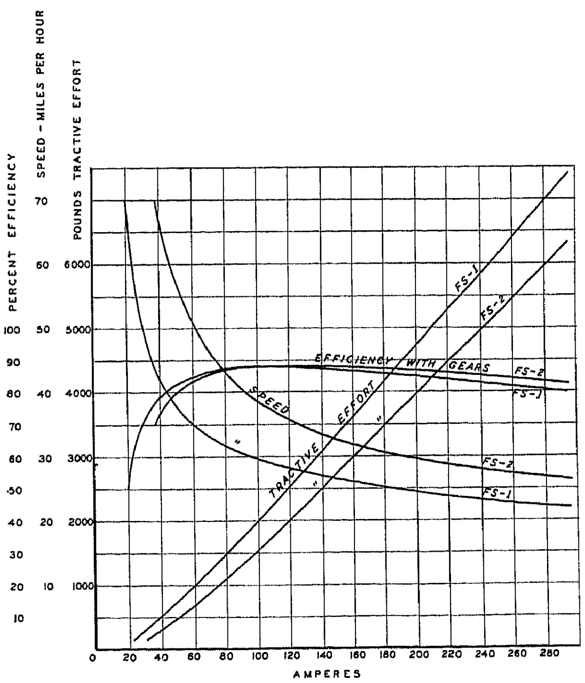

FIG. 14—CHARACTERISTIC CURVES FOR MULTIPLE-UNIT TRACTION MOTOR

Push-button control switches mounted in the operator's cabs provide for controlling the pantographs, the control circuit, the heater circuit, the main line breaker, the dynamotor, the number and gage lights and the head lights.

Heaters. Heating on all cars is supplied by totally enclosed resistance type heaters operated in series on 3,000-volt circuits. On the standard passenger cars the heaters are mounted under the seats with 40 heating elements in series. The heating elements, which are themselves totally enclosed and insulated, are supported on porcelain block insulators mounted in steel cases, so designed as to guard fully against the possibility of coming in contact with the heating element. The heater cases and the conduits leading to them are thoroughly grounded. There are two separate heater circuits in each car with two heating elements in each heater case, one on each heater circuit. The two heater circuits are separately fused but are both under control of the same thermostat. Either circuit may, however, be cut out by the trainman, leaving the other circuit under thermostat control. In the baggage compartments of combination passenger and baggage cars and in the toilet rooms wall type heaters are used, and in the club cars the heaters are mounted along the sides of the cars just above the floor line in the position formerly occupied by the steam pipes. The total heater capacity in each passenger car is approximately 28 kilowatts. In addition, heaters are also provided in the motorman's compartments at the two ends of the unit. These are under the control of the operator (without thermostat control) and can be turned on only when the door of the cabinet enclosing the master controller and other control equipment is open. The heater circuit in the trailer car is supplied from its associated motor car by a heater bus jumper on the roof of the cars. There is no 3,000-volt connection between separate units, each unit being entirely independent of the other units in the train, except for the control circuits connected through by jumpers underneath the cars and for the air lines which are also connected through.

Metering Equipment. Ten of the motor cars (five of those with roller bearings and the five with standard journal bearings) are equipped with watthour meters with shunts so arranged that the power used by the whole car or by any circuit can be metered. With these ten cars and the five trailer cars equipped with roller bearings (all other trailer cars having standard bearings) one ten-car train (five units) can be made up with all cars having roller bearings and another with all cars having standard journal bearings. Thus means will be available for accurately determining the effect of roller bearings on the power consumption under all conditions of service.

Car Circuit Protection. The general scheme of the protection to the various circuits on the cars is shown by the simplified car wiring diagram, Fig. 13. The motor circuit is protected by a main line breaker consisting of four magnetic blow-out contactors in series and an overload relay. All auxiliary circuits are fed as a group through a limiting resistor of 1.5 ohms (which would limit a short-circuit current to a maximum of 2,000 amperes). Each auxiliary circuit is protected individually by an expulsion fuse, and the dynamo circuit has an additional permanent resistance in series of 13.5 ohms.

There is also a main fuse, of a new combination expulsion—compression type, mounted on the roof of the car through which all car circuits are supplied. In the event of a heavy short circuit on the car or a motor flashover not cleared by the main line breaker, the high-speed circuit breakers at the substations and tie stations through which power is supplied to the overhead contact system, will normally operate faster than the main fuse on the roof of the car. If, however, the high-speed circuit breakers should fail to function properly, the fuse will clear the fault. Only one case of this kind has as yet occurred.

The main fuse has a continuous rating of 300 amperes, or 650 amperes for 30 seconds. The dynamotor circuit and the heater jumper circuit to the trail car are each fused for 20 amperes, and the heater circuits are each fused for 8 amperes.

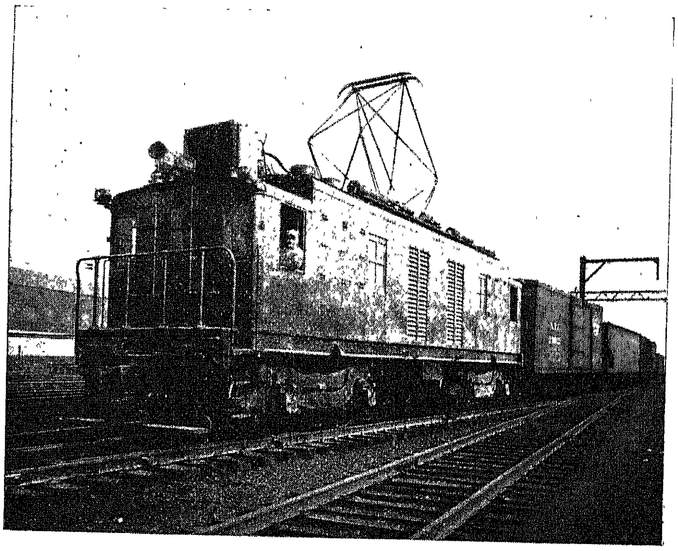

FIG. 15—THREE POWER LOCOMOTIVE

Lightning protection is provided on each motor car by an electrolytic aluminum cell lightning arrester with balancing resistor and shunted spark gap.

LOCOMOTIVES

Two locomotives of the three-power type shown in Fig. 15 have been provided for handling the freight transfer service between the Secaucus and the Jersey City yards, a distance of approximately four miles. When on electrified tracks, these locomotives operate from the 3,000-volt contact system, and when on non-electrified tracks on internal power provided by a 300 hp. diesel engine direct connected to a 200-kw., 750-volt generator supplemented by a storage battery of 360 cells with a rating of 242 kw-hr. (340 ampere hours). Each locomotive weighs 248,000 lb. with all the weight on four driving axles, is equipped with four motors of 400 horsepower each (one hour rating) and has a tractive effort of 60,000 lb. Each locomotive is capable of hauling a train of 2,500 tons, trailing load, at a balancing speed of approximately 28 mi. per hr. when operating on external power from the 3,000-volt contact system, or at approximately 12½ mi. per hr. when operating on internal power only. When running light on internal power the locomotive has a balancing speed of approximately 30 mi. per hr.

DISTRIBUTION SYSTEM

With two minor exceptions the 3,000-volt power for operation of the trains is distributed through the contact system only, without the use of auxiliary positive feeders. The two exceptions are at points where there is a change from three electrified main tracks to two electrified tracks, and the third circuit is carried through the two-track zones in order to make the three circuits independent throughout.

Type of Catenary. The catenary system is of the tangent chord type. On main line tracks it consists of a composite main messenger, an auxiliary messenger and two contact wires. The main messenger is made up of a core of seven strands of high strength bronze surrounded by thirteen strands of hard-drawn copper, the total having a breaking strength of 31,000 lb. and an equivalent conductivity of 350,000 cm. hard-drawn copper. The auxiliary messenger is 2/0 hard-drawn copper and the contact wires are two 4/0, grooved bronze wires of 55 per cent conductivity. A relatively small amount of special 80 per cent conductivity bronze contact wire has also been installed for comparative purposes. The total equivalent conductivity of the standard main line catenary system is 720,000 cm. hard-drawn copper per track. On yard tracks and cross-overs a lighter system is used with a 7/16-in. bronze messenger and a single 4/0, 55 per cent conductivity bronze contact wire.

Catenary Supports. Steel catenary supporting structures are used throughout except on the branch line from Summit to Gladstone, which is a single track line and on which wood poles with steel brackets are used except at special locations where steel structures are required. The standard steel supporting structure has H-columns and either an H-beam or a lattice truss cross member, the H-beam being used up to spans of 53 ft. and lattice truss for longer spans. Dead end structures have A-frame columns. Copper-bearing steel was used in all small members. The H-columns on standard structures are set with the web at right angles to the track and are bolted to the foundations with two bolts. On tangent track the standard spacing between catenary supports is 300 ft.; on curved track, or where there are overhead bridges or other obstructions, the spacing is reduced, or pull-off poles are located between supporting structures. In general the signals and catenary system are supported on the same structures. Signal bridges installed during recent years had been designed to take future catenary load, and the older signal bridges were replaced by new structures. In a relatively few cases it was impracticable to locate the catenary structure at the desired signal location, and existing signal bridges (raised if necessary) were left between catenary supporting structures.

Fig. 12, already referred to, shows typical catenary construction on three-track curve section. Fig. 16 shows typical construction on three-track tangent section and also illustrates the method of mounting signals on catenary supporting structures.

Tie Stations and Catenary Sectionalizing. Except on the single track branch line from Summit to Gladstone, tie stations (or circuit breaker sectionalizing stations) are located approximately midway between substations and also at or near the ends of stub end lines. The locations of the tie stations and the distances between substations are shown on the map, Fig. 1, already referred to. At the tie stations the catenary system is sectionalized and all circuits are brought to a common bus through high-speed circuit breakers, the holding coils of which are energized from the line. With this arrangement advantage is taken of multiple feed with reduced copper losses between substations and tie stations without sacrificing selective protection of the line. In the event of a line fault the faulty section is isolated by the substation and tie station breakers connected with that particular section of the line without disturbing the rest of the line. All tie stations are largely automatic in function but are also under remote or supervisory control from the nearest railroad signal tower. Lights on the control panel in the signal tower indicate the position of each feeder circuit breaker and a voltmeter shows the voltage of the control battery. The towerman, under direction of the load dispatcher, can operate any switch in the tie station or restore it to automatic control, at will. The high-speed circuit breakers connected with circuits which are also fed at the other end from a substation are arranged so that after opening under a line short circuit or overload they reclose automatically as soon as voltage is restored on the circuit (*i. e.*, when the circuit has been energized

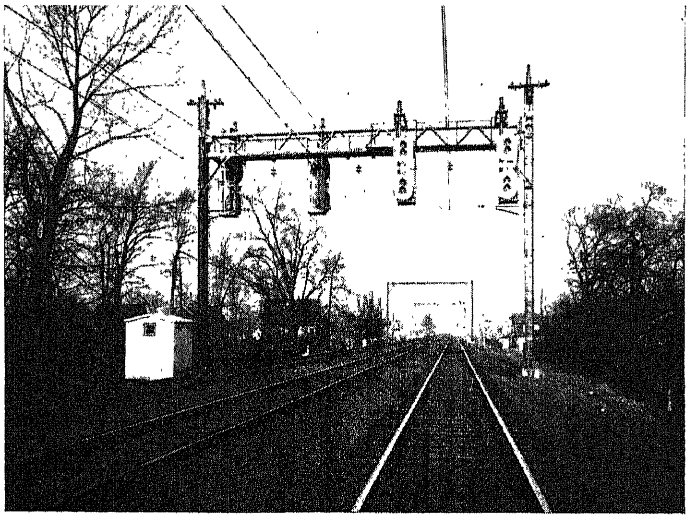

FIG. 16—TYPICAL CONTACT SYSTEM ON THREE-TRACK TANGENT SECTION

Method of mounting signals on catenary supporting structure shown in foreground from the substation end by the operator at the substation, reclosing the feeder breaker). On stub end lines, which receive power at one end only, the breakers, after opening under short circuit or overload, automatically reclose three times at definite intervals and then lock out if the fault has not been cleared. The remote control permits the towerman to close any breaker which has remained open as a result of automatic operation.

As previously pointed out, the holding coils of the high-speed circuit breakers on the feeder circuits and on rectifier circuits at the substations are energized from a storage battery, whereas the holding coils of the feeder breakers at tie stations are energized from the line. This arrangement aids in getting selectivity in the event of line fault since the holding power of the breakers at the tie station is reduced as the line voltage falls under short circuit.

The trip points of the high-speed magnetic blow-out circuit breakers are adjustable as to current and also as to sensitivity to the rate-of-rise of the current. New features of these breakers permit increased flexibility in the rate-of-rise adjustment. Means are also provided for calibrating the breakers with low-voltage battery current (maximum, 20 amperes at 12 volts).

With the breakers properly adjusted with respect to each other a short circuit on any part of the catenary system will normally open only the breaker or breakers directly connected with the faulty section of line. The time from application of the short circuit until the current starts to decrease following the opening of the breaker is as short as 0.012 seconds under maximum short-circuit conditions. Longer times may be required for lesser short circuits.

In addition to the sectionalizing provided at tie stations and substations, other intermediate line sectionalizing switches are provided on the catenary system to permit cutting out smaller sections of line in event of fault or for inspection and maintenance work. These sectionalizing switches are of the horn-gap air-break high contact pressure type and are mounted on the catenary supporting structures, and equipped with operating mechanisms permitting operation from the ground. Those in most important locations are equipped with motor-operated mechanisms under control from the nearest signal tower or other point where an attendant is always available who can operate the switches under instructions from the power dispatcher.

Ground Wires. In order to provide lightning protection both to the catenary system and to auxiliary power circuits which are carried by the catenary structures, two 1/0 hard-drawn copper ground wires are carried on bayonets on the catenary structure except on the Gladstone branch, which is referred to later. These ground wires are connected directly to the steel of the catenary structures and are also connected to the mid-point of approximately every alternate impedance bond. They also serve to give a low resistance return path in the event of a faulty insulator on the catenary system and thereby insure proper breaker operation. In addition they serve, to a minor degree, as auxiliary negative return circuits in parallel with the rails.

On the single track line from Summit to Gladstone a supplementary negative return cable is provided consisting of a 4/0 hard-drawn copper cable carried on the catenary supporting brackets close to the poles, and connected to the brackets. This negative return is bonded to the mid-point of impedance bonds at suitable intervals. Its most important function is to guard against the possibility of track workers being exposed to high voltages across open track circuits in event of broken rails or removal of rails during track repairs. It also serves to insure proper circuit breaker operation and prevent burning of the wood poles in event of a faulty catenary insulator, provides considerable lightning protection to the catenary system, and decreases the resistance of the return circuit by supplementing the rail return.

TRACK BONDING

The standard track bonding consists of a single 4/0, 8½-in., gas-welded, stranded-copper, steel-terminal, U-bond at each joint on each rail, except where only one rail is bonded through interlocking plants, at which locations two of the same size and type of bonds are installed on each joint of the one rail. Impedance bonds are provided at all signal locations, the distance between the impedance bonds averaging about 2,500 ft. Two sizes of impedance bonds are used, 500 amperes and 1,000 amperes, depending upon the location. Cross bonds are provided between mid-points of impedance bonds at suitable intervals ranging from 5,000 to 8,000 ft.

SHOPS

Periodic light inspection of the cars and locomotives is taken care of in a light inspection shop in the Hoboken yard. This shop is 480 ft. long and has three inspection pits, each sufficient to accommodate a six-car train, and two shorter truck tracks. It is also provided with galleries for facilitating inspection and adjustment of pantographs and with a crane at one end of the shop of sufficient capacity to lift one end of a car at a time to permit change of trucks. This shop is equipped for making light repairs only. The 3,000-volt contact system is not carried into the shop. The cars are moved in and out of the shop by 250-volt power supplied to the cars (from motor-generator sets) by means of jumper cables plugged into receptacles provided for the purpose on the motor cars, as indicated on the elementary car wiring diagram, Fig. 13.

Heavy repairs, general overhaul and painting will be carried out at the railroad's car shops at Kingsland approximately seven miles from Hoboken on the Boonton line. Two sections of the Kingsland shop have been modified to make them thoroughly suitable for this work and facilities for dipping and baking armatures and other motor repairs have been provided. As the Boonton line is not now being electrified beyond Secaucus the cars will be hauled to the shop by steam switching locomotives or by the three-power electric locomotives running on internal power.

SIGNAL SYSTEM

In connection with the installation of the electrification program it was necessary for the railroad to make a complete change in its signal system. The signals and track circuits were formerly battery-operated and the signals throughout most of the territory electrified were of the old semaphore type. All signals in the electrified zone have been changed to color light signals with a-c. track circuits. They are now operated at 60 cycles, alternating current, but are so arranged that the frequency can be changed to 100 cycles if this should later be necessary in connection with automatic train control. Power for operation of the signals is supplied from signal power circuits carried on the catenary supporting structures. In most cases these circuits are in duplicate with provision for automatic throw-over from one circuit to the other in the event of failure of power on the first circuit.

PROTECTION AGAINST ELECTROLYSIS, INDUCTIVE INTERFERENCE AND CONTACT WITH OTHER CIRCUITS

Extensive surveys and tests were made both before and after electric operation of trains was begun to determine the points at which there might be danger of electrolytic damage to underground cable sheaths and other metallic structures, and drainage cables have been installed where they were found desirable.

Where necessary to provide adequate physical clearance, communication circuits located along the right-of-way, consisting of the railroad's communication lines and some other telephone and telegraph lines, have been moved or put into cables. Inductive interference from the d-c. power current flowing through the catenary system has been practically eliminated by the use of twelve-phase rectifier transformers, d-c. smoothing reactors and resonant shunts, installed at the substations. Some slight difficulty on railroad communication circuits with long parallel exposures was experienced at first but was substantially eliminated by more careful adjustment of the resonant shunts.

The mercury arc rectifiers also cause a distortion of the wave form on the a-c. supply circuits, and where the a-c. power is supplied by overhead transmission lines inductive interference in neighboring telephone circuits may be experienced unless steps are taken to guard against it. This effect can, however, be largely eliminated by proper coordination and transposing of the power circuits and the neighboring telephone circuits. This has been taken care of by the cooperative effort of the power companies and the telephone companies.

Communication circuits and low-voltage power circuits crossing the right-of-way, except those on overhead highway bridges, have been removed.

No radio interference has been reported.

METHODS OF HANDLING CONSTRUCTION WORK

The electrical equipment on new motor cars was installed by the car builders with the exception of the motors which were installed by the railroad forces after delivery of the cars. The electrical equipment on the trailer cars, consisting of control and heater equipment, was installed by the car builders adding the vestibules to these cars. The substation and tie station buildings were built under contract by building contractors, except for two tie stations where existing buildings were used, and all equipment in substations and tie stations was installed by electrical contractors. All work along the right-of-way, including catenary foundations, catenary supporting structures, erection of the catenary system, and auxiliary power and signal circuits, relocation and modification of signals and track bonding was carried out by railroad forces.

Discussion

H. M. Trueblood: This electrification has been of considerable interest from the inductive coordination standpoint, not only to the operating telephone and telegraph companies, but also to those of us who are studying this question from a more general point of view. For one thing, this is the first electrification in the United States depending solely on rectifiers as the source of propulsion power. For another, the contact-wire voltage is 3,000 volts as against 1,500 volts for the Illinois Central terminal electrification in Chicago, and 600 volts for the New York Central and Pennsylvania terminal electrifications in New York. This matter of trolley voltage is of interest because, other conditions being the same, it is a fair approximation to say that the voltage induced in exposed communication circuits, when trolley short circuits occur, is proportional to the trolley voltage. Again, in the territory traversed by the Lackawanna Electrification, there is a considerable concentration of important communication facilities. In a rather unusual degree, telephone central offices are located in close proximity to the railroad right-of-way and there was some concern at first because of the possibility of ground potentials sufficient to cause disturbances during periods of train acceleration, when fairly heavy currents are drawn for several seconds.

The telephone company has therefore followed the work of getting this electrification into operation with a great deal of interest. Through excellent cooperative arrangements, the voltages produced under short-circuit conditions, and also ground potentials under conditions of heavy current flow, have been quite fully studied in a series of field tests. While a few measurements are still to be made, the results of these tests indicate that little or no difficulty is to be expected in Bell System plant either from induction or from ground potentials. Measures mentioned by Mr. Moreland have been effective in the avoidance of noise from the trolley-circuit currents and voltages. Aside from such arrangements to avoid electrolysis as are mentioned in the paper, we therefore do not expect that specific measures of coordination will be needed, so far as concerns effects in Bell System plant arising from the d-c. propulsion system.

A number of tests on the effects of wave-shape distortion on the a-c. side of the rectifiers has been carried out, with the cooperation of the power companies supplying energy to the electrification. Here measures to which Mr. Moreland has referred will, we believe, take care of the situation so far as exposures to the rectifier feeders themselves are concerned. The problem, however, is not confined to such exposures, since the harmonics originating in the rectifiers tend to spread in the connected a-c.

system, and in fact, appear on the 2,300-volt buses used for general distribution to certain localities. The solution to this part of the problem is now being worked out. In certain places, it appears that specific measures of some sort will have to be taken to avoid difficulty.

W. B. Potter: The 3,000-volt multiple unit equipment principally because of its application to suburban service is of particular interest. The scheme of ventilation for the traction motors seems better than anything that has been done heretofore. Putting the air intake at the top of the car away from the dirt and brakeshoe dust, together with using the empty space at the end of the car roof for a settling chamber, is obviously a good way to supply the motors with clean air. It is surprising that this plan has not been more generally used.

The use of 3,000-volt rectifiers to the exclusion of any rotary conversion apparatus is probably the most novel feature of this electrification. Reference has been made to arcovers. Something that is akin to commutator flashing but different in that it is not destructive to the rectifier. The momentary even though infrequent interruption of power is, however, an annoyance. It is gratifying to know that progress is being made toward the elimination of this arcing phenomena.

C. H. Lydall: I have been very interested in Mr. Moreland's paper as I spent some years in South Africa on the Maine Line Electrification of the Natal Railway.

One hundred and seventy route miles, mostly single line, were electrified, primarily to increase the volume of traffic over the hilly portion of Central Natal. The railroad in the center of the section electrified climbs up to a height of 5,000 ft. above sea level and then drops down to 1,000 ft. above sea level at the southern end. This will give some idea of the gradients which the electric locomotives had to negotiate. Locomotives working on 3,000-volt d-c. were used throughout on the electrification and three 1,000-hp. units were used on all fully loaded freight trains.

Very little trouble was experienced in the construction and starting to work of the scheme with one exception, namely, trouble from lightning. In parts of South Africa lightning is, I suppose, as severe as in any part of the world. It was not realized how severe this was until we got to the preliminary running stage of the electrification. During that period it was found that in bad storms lightning struck the trolley lines and arced over, generally at the point where the pull-off arms were attached to the masts. The insulators at this point were smashed, the pull-off arms hung down, suspended from the trolley wire and the next locomotive that came along caused a short circuit. Maintenance gangs had to get to the point of breakdown and repair the line before traffic could be resumed. In South Africa, where the roads are few and very bad, the greatest time was taken in the gangs getting to the breakdown.

As soon as it was realized how severe the lightning was in Natal and the serious effect it would have on the running of the service, high-speed circuit breakers were installed on the feeders to the track. The breakers operated so satisfactorily that interruptions caused by lightning ceased entirely. When the lightning hit the trolley wire and the insulators flashed over the breakers opened so quickly that the flashover was extinguished before the insulators were damaged. Locomotive drivers were given two minutes in which to switch off their controllers, the line breakers were closed again and service continued in normal operation.

Transverse Fissure Detector Car

BY HARCOURT C. DRAKE[1]

Associate, A. I. E. E.

INTRODUCTION

TRANSVERSE fissures in rails are fatigue failures of internal origin, whose rate of growth depends largely upon the density of traffic. The typical air-tight fissure, Fig. 1, has a characteristically bright, silvery appearance produced by the peening action of the fissure faces when the rails are subjected to the impact of heavy wheel loads.

This type of failure was brought to the attention of railway engineers about 19 years ago when a passenger train was derailed by a rail which broke into 17 pieces, disclosing several transverse fissures. In the years which followed, rail failures due to transverse fissures were reported in ever increasing numbers. The presence of large transverse fissures in relatively new rail under the best conditions of track maintenance, together with the utter impossibility of seeing or locating such fissures, placed this type of failure in the class with a cyclone, a lightning stroke, or other "act of God."

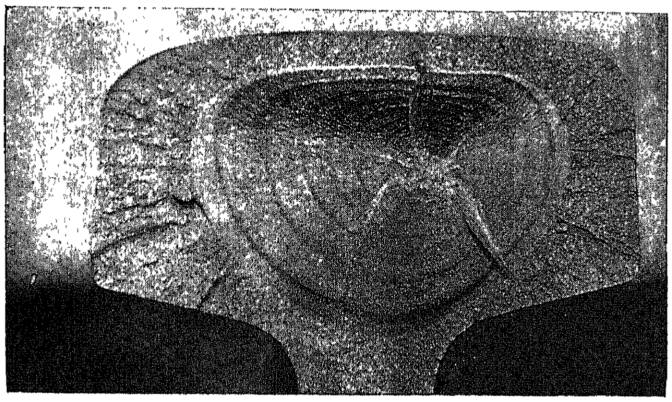

FIG. 1—A TYPICAL INTERNAL TRANSVERSE FISSURE

In the early part of 1927, internal fissures were successfully located in rails at the laboratory by the electric method and after several months of experimental work a fissure detector car (Fig. 2) was constructed for the American Railway Engineering Association. The car passed its acceptance test and was placed in testing service on the railroads of the country in November, 1928.

THEORY OF OPERATION

The fissure detector depends for its operation upon the fact that electric current flowing through a rail is compelled to pass around any fracture, inclusion, or separation in the metal. When a rail is sound the axis of current is substantially straight, but if a fissure is encountered, the current axis at that point is displaced. An inductive pick-up device is mounted directly over the rail head, where it cuts the magnetic lines of force surrounding the rail. In the case of a sound rail carrying current, the magnetic flux surrounding the rail is uniform and no e. m. f. is generated as the inductive pick-up passes along, but when the axis of current is deflected, as at a fissure, the flux density around the rail varies, resulting in a generated potential at the terminals of the pick-up. This output from the pick-up is amplified and led through relays connected to recorder pens, thus giving a record of the received impulse (see Fig. 3).

FACTORS GOVERNING DESIGN

In applying the electrical method of fissure detection to inspection of rails in track, the following factors materially affect the final design:

1. Many spots are encountered where locomotive drivers have slipped in starting and gouged out the rail.

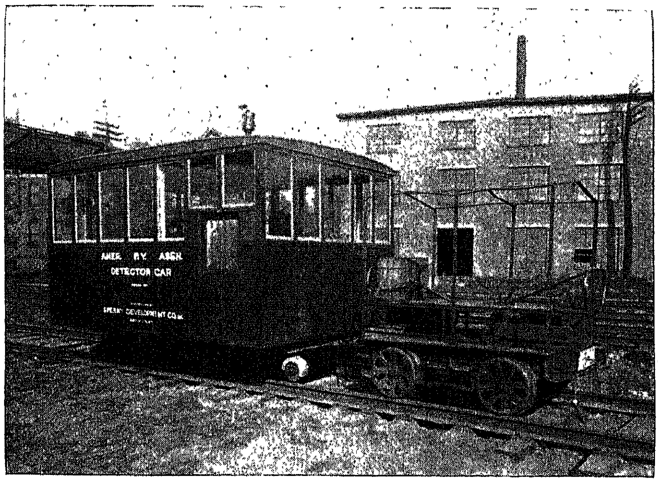

FIG. 2—THE FIRST FISSURE DETECTOR CAR, AS BUILT FOR THE AMERICAN RAILWAY ASSOCIATION

2. Worn or flat spots occur frequently, especially opposite the rail joints.
3. In certain stretches of track the rail surface has acquired a pronounced ripple, termed corrugated or washboard rail.
4. The rail is frequently covered with a hard insulating film, the resistance of which may run up to several thousand ohms.
5. The gage of the track varies between certain limits.
6. The rail surface may be level or canted.
7. The size of the rails varies from 70 to 130 lb. per yd.

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[1] Research Engineer, Sperry Products, Inc., Brooklyn, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

DESCRIPTION OF THE DETECTOR CAR

The single unit type of detector car (Fig. 4) is 45 ft. long, self-propelled, equipped with extra heavy frame and axles, and capable of a speed of 45 mi. per hr. when running to and from work. Just back of the driver's compartment are located the cook's gallery and comfortable living quarters for the crew. Back of the living quarters is the compartment containing the power plant of the detector equipment. This power plant consists of a specially designed double commutator, 6,000-ampere 2-volt d-c. generator, direct connected to a 50-hp. four-cylinder gasoline engine. The exciting generator and air compressor are also driven by this engine. Current from the 2-volt generator is carried by heavy buses to the brush units located on either side of the car. The brush unit assembly (Fig. 5) consists of two sets of eight solid phosphor-bronze brushes mounted about 3 ft. apart and rigidly connected by a cast frame. These brushes are supplied with individual springs which insure a pressure of 50 lb. per brush. Flanged wheels located at the ends of the brush holder engage the rail when the brushes are down and, due to their angular mounting, cling to the gage side regardless of varying track conditions. The brush units are applied to the rail by air-operated pistons electrically controlled from the operator's switchboard. A safety switch is connected to the gear shift of the motor-car drive so that the brushes will lift if the driver attempts to back up while they are on the rail. It was found that brush contact resistance was greatly reduced by the use of water on the rail. Provision is therefore made to carry a sufficient supply of water for this purpose and the brush control switch applies the water to the rail automatically when the brushes are lowered. Mounted between the brushes is a separate carriage upon which is supported the pick-up unit. This carriage with its eight wheels and swivel mounting holds the pick-up unit at a fixed distance above the rail and rides over burns and flat spots without interruption. Initially driver burns and flat spots gave indications which tended to slow up the testing. This source of trouble was materially reduced by the eight-wheel mounting. Recently a new pick-up device of radically different design has been on test on five of the cars. It eliminates the indications due to flat spots and all but the largest of the burns.

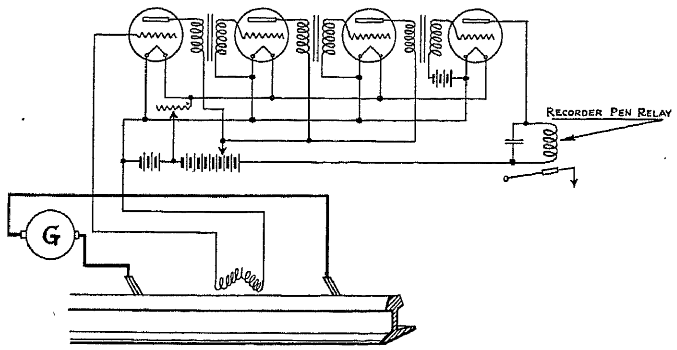

FIG. 3—SCHEMATIC DIAGRAM OF THE FISSURE DETECTOR CIRCUIT

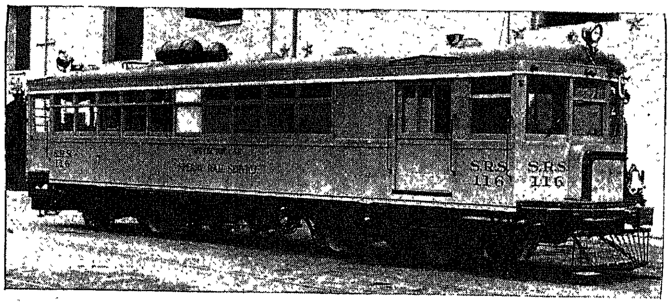

FIG. 4—THE SELF-PROPELLED TYPE OF DETECTOR CAR

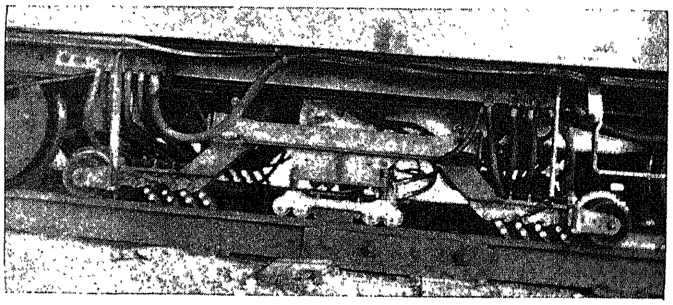

FIG. 5—MAIN CURRENT BRUSH UNIT

The operating room is located at the back end of the car and contains the recording table and control board (Fig. 6) as well as amplifying equipment and batteries. The output from the pick-up unit is brought up to a four-tube amplifier which is properly shielded and protected from shock. Three recorder pen relays are connected to the output of the amplifier and adjusted to respond to different values of plate current, thus giving an indication of the size of the defect detected. Other relays are provided to operate paint guns so that the fissure will be automatically marked by a spot of paint. The record (Fig. 7) is made up of nine lines; the three

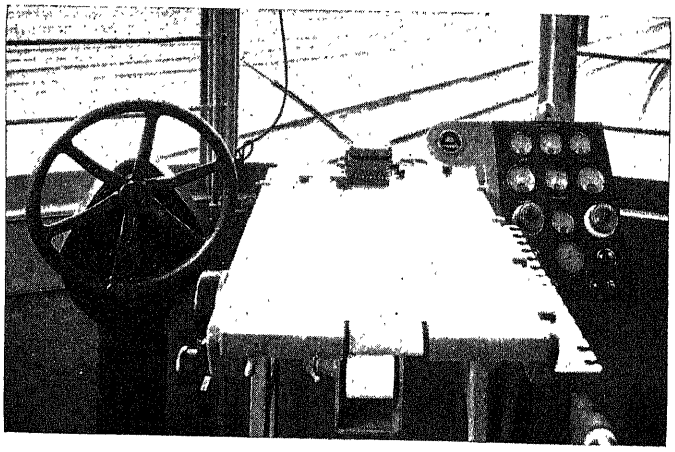

FIG. 6—RECORDING TABLE AND CONTROL BOARD

pens on either side (numbered 1, 2, 3) give the relative size of the fissure while the lines in the center record the angle bars. Joint cut-outs mounted on the brush holders engage the angle bars and cut out the fissure pens and paint gun at the joint, at the same time operating the joint pens. The line to the extreme right is a land-mark pen operated from the motor-car driver's switch-board. He presses the button whenever a mile post, bridge, signal tower or other land-mark is passed and the operator at the recording table stamps in the proper designation.

Operation of Car

In operation, the car passes along the track at approximately 6 mi. per hr. with from 2,000 to 3,000 amperes flowing through each rail, the current value depending upon the weight of rail being tested. The operator, seated at the recording table in the rear of the car, has the record before him and a clear view of the tested track. When the car passes over a fissure an indication appears on the record and the defect is automatically marked with a spot of paint. The operator notes the record, sees the paint mark on the rail, and stopping the car, backs it up to the paint mark for a hand test of the suspected spot. To make the hand test, 1,500 amperes is introduced into the rail by means of air-operated contactors and the unit I. R. drop is measured with a galvanometer. By means of this test the size of a transverse fissure can be determined within a few per cent. The detector car not only locates transverse fissures, but split heads (Fig. 8), horizontal fissures (Fig. 9), compound fissures, pipes, cracked webs, broken bases, and other defects. When a defect is located by the car, the first question asked by the railroad representative is, will it be safe to leave the rail in track until there is a lull in the day's traffic, or must the next train be tied up while the rail is replaced? The answer to this question lies in the result of the hand test. When the hand test shows no transverse crack, the rail is marked for removal at the next opportune time, but if a large transverse fissure is indicated the railroad takes no chance, removing the rail at once.

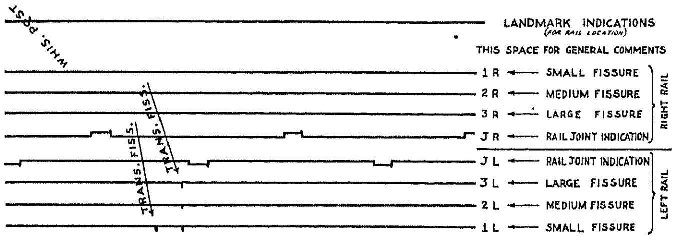

Fig. 7—Sample of Record Made by Detector Car

Although the testing speed of the car is 6 mi. per hr., there are many elements which enter in to reduce the total mileage tested per day.

1. Traffic delays due to clearing for trains frequently consume considerable time, but are unavoidable.

2. When a fissure is located, the railroad representative always makes a record of the manufacturer, the year rolled, ingot letter, and heat number of the rail. If the rail has been in track for several years or has been oiled by the track oiler, it is often difficult to locate and decipher the heat numbers and it is not at all uncommon on some roads to have from five to fifteen minutes wasted in obtaining the rail data.

3. A repeat run over a suspected rail takes 30 seconds and from two to three minutes are necessary for a hand test.

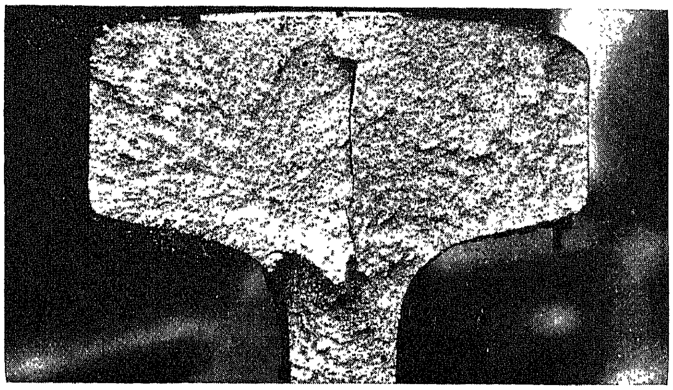

Fig. 8—A Split Head

Testing Procedure

The testing procedure on the ten cars now in service is practically identical, but the railroads differ somewhat in their method of conducting the test. A survey of the various methods employed indicates that the following procedure results in a minimum of time lost.

The railroad has one man on the car and one on a hand car which follows the detector car. In addition there are, of course, the conductor and flagman. When an indication is recorded the car is stopped, a check run made over the suspected rail, and the car backed up for a hand test. The operator examines the rail and if this visual inspection discloses exterior evidence of a split head, horizontal fissure, or other defect he gives the word to the railroad man, who makes a red paint mark

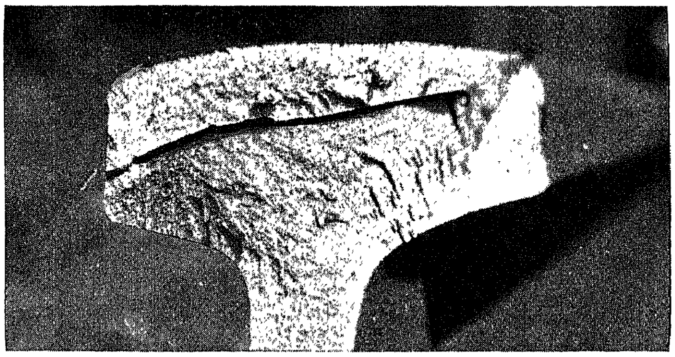

Fig. 9—A Horizontal Fissure

on the tie at the defective spot and chalks the identification number on the rail. If no visible defect is noted, the hand test is made and the rail is marked as before. The car then proceeds with the testing while the man on the hand car stops to take down all of the data together with the location of the rail, and then continues on after the test car. In this way the maximum time taken is the two or three minutes required for hand testing.

CONCLUSION

Since the first detector car was placed in service two years ago, many interesting fissure facts have been accumulated.

1. Rails have been found containing many defects as, for example, one which had 24 transverse fissures.

2. Fissures have been found in rails 28 years old; and also in rails 5 months old.

3. Many miles of track have been found free of fissures; other short stretches have had 20 or 30 fissures per mile.

While these isolated cases are of interest, the most important point is that the transverse fissures are now being located and removed from track before they can cause any damage, whereas prior to two years ago their presence was not known until they had grown to the surface or broken the rail.

Inclined Catenary Calculations

BY B. M. PICKENS[1]

Associate, A. I. E. E.

Synopsis.—*This paper gives a simple method of determining the general U and S curves for a catenary system. They are both obtained from the same equation and give the general shape the contact wire must take in order to lie in a horizontal plane.*

The specific shape the contact wire will take on any curve is found by adding together multiples of the U and S curves. Methods of obtaining the multipliers for track curves of varying radii and for constant radius curves are given. The method used postulates a smooth continuous curve for the contact wire from span to span lying in a horizontal plane.

The contact wire does not necessarily have the same shape as the track curve. For a given catenary system the shape taken is determined only by the relative location of the points of messenger support. The condition of design necessary to give a contact wire curve which has practically the same shape as the track curve, is shown.

THE use of "inclined catenary" on curves in railway catenary construction has become quite general in the United States on all systems having a single contact wire whether simple or compound catenary. The calculations involved in determining the dimensions of the inclined catenary are not as simple as for tangent catenary and are not generally known. For the inclined catenary it is necessary to determine the dimensions of a space figure while tangent catenary can be considered as lying wholly in a plane. Also there are more variables affecting the shape of the inclined catenary. A fairly simple method of solution can be followed if the projections of the inclined catenary on horizontal and vertical planes are considered.

The contact wire will lie wholly in a horizontal plane and will be supported from the messenger by hangers inclined to this plane. If the hangers are equally spaced each one will carry the same portion of the contact wire weight since there can be no transfer of weight from one to another without a vertical dimension to the contact wire. Since each hanger carries an equal load to the messenger the vertical load on the messenger is the same as on tangent catenary, neglecting the variation in hanger weights, and the vertical projection of the messenger will be the same as for a tangent catenary messenger of the same system in a span of the same length.

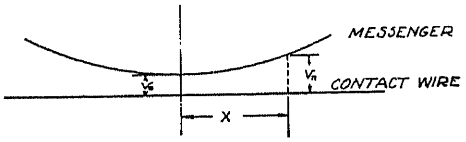

FIG. 1—VERTICAL PROJECTION OF MESSENGER AND CONTACT WIRE

The shape of the vertical projection can be determined with sufficient accuracy if it is considered to be a parabola. The curve actually lies somewhere between a catenary and a parabola. If all the weight of the system were concentrated in the messenger it would take a catenary shape, if in the contact wire, a parabolic shape. The parabola has the simplest formula and is generally used for catenary calculations of this type. If a system of axes is taken as shown in Fig. 1 with the origin located on the vertical projection of the contact wire directly below the low point of the messenger, then the vertical projection v_n of any hanger x feet from the origin will be

$$v_n = v_o + W x^2/2 M \qquad (1)$$

where

v_o = vertical projection of shortest hanger
W = weight of system per ft. lb.
M = messenger tension lb.

Equation (1) gives either the tangent hanger length or the vertical projection of any curve hanger.

If the horizontal projections of the contact wire and messenger are now considered it can be shown[2] that their shapes are mutually reflected at some scale depending on their tensions. An axis can be drawn between the projections so located that the perpendicular distance from any point on the axis to the projection of the contact wire is to the perpendicular distance from the same point, to the messenger projection, as the messenger tension is to the contact wire tension. This is shown in Fig. 2 where

$$t/m = M/T$$

$$T = \text{contact wire tension}$$

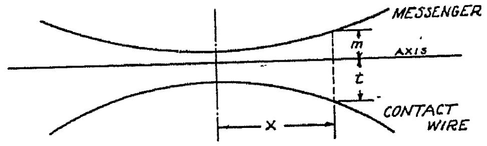

FIG. 2—HORIZONTAL PROJECTION OF MESSENGER AND CONTACT WIRE

If the horizontal projection of any hanger is h

$$t + m = h$$

and

$$h = t\,(1 + T/M) \qquad (2)$$

Since the relationship between the horizontal pro-

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

[2] *Catenary Design for Overhead Contact Systems*, H. F. Brown, A. I. E. E. TRANS. 1927, p. 1082.

jections of messenger and contact wires is known, it is only necessary to determine the shape of one of the wires in order to find the hanger lengths. It is convenient to work with the contact wire projection and determine the values of t for various hangers. The curve hanger length will be the square root of the sum of the squares of the vertical and horizontal hanger projections.

Curve hanger length

$$g = \sqrt{h^2 + v^2} \qquad (3)$$

In Fig. 3 the relationship between the values of t at three adjacent hanger locations at any place in the span is

$$t_{n+1} = t_n + (t_n - t_{n-1}) + \Delta \qquad (4)$$

The horizontal load f_n applied to the nth hanger by the change in direction of the contact wire is directly proportional to change Δ and the contact wire tension T and inversely proportional to the hanger spacing s.

$$f_n = \Delta\, T/s \qquad (5)$$

The slope of the hangers is determined by the horizontal and vertical loads applied at the lower end. The horizontal load on the nth hanger is f_n and the vertical load w is the weight of s feet of the contact wire or wires. Usually half the weight of the hanger is also considered as part of the vertical load.

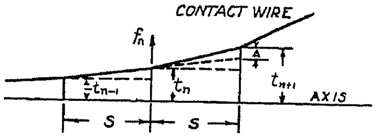

FIG. 3—HORIZONTAL PROJECTION OF THREE ADJACENT HANGER LOCATIONS AT ANY PLACE IN THE SPAN

From Fig. 4 it will be seen that

$$f_n = w\, h_n/v_n \qquad (6)$$

Combining equations (2), (5), and (6)

$$\Delta = s\, w\, t_n\, (1 + T/M)/v_n\, T \qquad (7)$$

and substituting (7) in (4)

$$t_{n+1} = t_n\, [2 + s\, w(1 + T/M)/v_n\, T] - t_{n-1} \qquad (8)$$

if we change equation (1) to

$$v_n = v_o\, (1 + x^2\, W/2\, v_o\, M)$$

and substitute in (8)

$$t_{n+1} = t_n\, [2 + c/(1 + x^2/c_1)] - t_{n-1} \qquad (9)$$

where

$$c = s\, w\, (M + T)/v_o\, T\, M$$

and

$$c_1 = 2\, v_o\, M/W$$

also

$$x = \text{value of } x \text{ for } n\text{th hanger.}$$

The values of c and c_1 are constant for any given system.

Equation (9) is a general equation which gives the relationship between the values of t for any three adjacent hangers. The equation holds for any type of span, uniform curve, reverse curve or changing curve as no assumptions were made concerning the nature of the curve. It gives the general shape the contact wire must take in order to lie in a horizontal plane. If the relationship between any two adjacent values of t in a span is known, the value for the adjoining t can be determined and all other values of t in the span can be found by applying (9) progressively. From equation (9) two general shape curves can be plotted for any given system. Assume that the values of t adjacent to and equally spaced from the origin, are equal, then a curve can be calculated which will have a U shape. Also, if it is assumed that the values of t are equal but of opposite sign another curve develops which has an S shape. It is convenient to select a catenary system in order to plot and show the use of these curves.

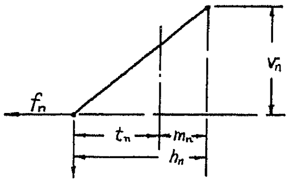

FIG. 4—CROSS SECTION OF CATENARY AT NTH HANGER

	Wgt. per ft.	Tension
Messenger bronze	0.744	3820
Hangers	0.080	
Auxiliary wire 2/0 copper	0.403	1200
Contact wire 4/0 bronze	0.641	2500
Weight of system per ft	1.868	

Hanger spacing s = 15 ft.
Length v_o = 0.5 ft.
Weight w = 15 (0.641 + 0.403 + 0.040) = 16.26
Weight W = 1.868

From these data the values of the constants c and c_1 can be calculated.

$$c = \frac{15 \times 16.26\ (3820 + 3700)}{0.5 \times 3820 \times 3700} = 0.260$$

$$c_1 = \frac{2 \times 0.5 \times 3820}{1.868} = 2048$$

For the system selected equation (9) can be written

$$t_{n+1} = t_n\, [2 + 0.260/(1 + x^2/2048)] - t_{n-1}$$

the value of the quantity in the brackets depends only on x and can be solved for the value of x for each hanger location. If we assign a value of + 1 to t_1 and + 1 to t_{-1} then we can calculate all other values of t in the span.

$$t_2 = 1 \times 2.2530 - 1 = 1.253$$

$$t_3 = 1.253 \times 2.2081 - 1 = 1.770$$

$$t_4 = 1.770 \times 2.1540 - 1.253 = 2.56$$

$$t_5 = 2.56 \times 2.111 - 1.770 = 3.63$$

and the other values for $t\,(U)$ can be obtained in a similar manner.

The values for $F(x)$ for each hanger location for a 330-ft. span are given in Table I, together with the values of t for the U and S curves. The S curve is obtained by assigning to t_1 a value of $+1$ and to t_{-1} a value of -1.

$$t_2 = 1 \times 2.253 + 1 = 3.253$$
$$t_3 = 3.253 \times 2.2081 - 1 = 6.18$$
$$t_4 = 6.18 \times 2.154 - 3.253 = 10.15$$
$$t_5 = 10.15 \times 2.111 - 6.18 = 15.24$$

and the other values of $t\,(S)$ can be obtained in a similar manner and are given in Table I.

TABLE I

Hanger location	Ft. from low point	$F(x)$	$t\,(U)$	$t\,(S)$	$0.25\,t\,(S)$
1	7.5	2.253	1.00	1.00	0.25
2	22.5	2.208	1.25	3.25	.81
3	37.5	2.154	1.77	6.18	1.55
4	52.5	2.111	2.56	10.15	2.54
5	67.5	2.081	3.63	15.24	3.81
6	82.5	2.060	4.99	21.65	5.41
7	97.5	2.046	6.64	29.36	7.34
8	112.5	2.036	8.59	38.40	9.60
9	127.5	2.029	10.84	48.74	12.18
10	142.5	2.024	13.41	60.50	15.13
11	157.5	2.020	16.36	73.90	18.48
			19.60	88.50	22.13

In order to have curves of about the same scale it is convenient to plot 0.25 of the $t\,(S)$ values. The values given in Table I are for one half of each curve. It is not necessary to plot both halves as the U curve is symmetrical about the t axes and the S curve to the left of the origin has the same values as to the right but they are negative. Fig. 5 shows the U and S curves plotted from the values given in Table I.

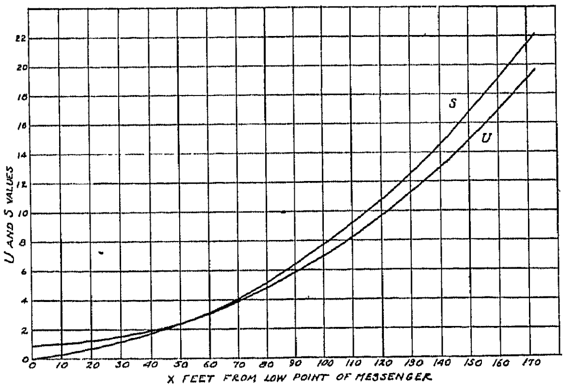

FIG. 5—U AND S CURVES FOR CATENARY SYSTEM

Where two spans of inclined catenary join, the contact wire must have a common tangent since the wire is continuous from span to span and there is no attachment to the contact wire to cause a change in direction where pull-offs are not used. This is one of the conditions for which the hangers must be designed. In the solution it is, therefore, necessary to know the slope of the general curves U and S. Since equation (**9**) is not in a form that can be differentiated the simplest method of obtaining the slope is to do it graphically. dU/dx and dS/dx curves are given in Fig. 6. These curves were obtained graphically from the U and S curves of Fig. 5.

The values of t given for either the U or S curves satisfy equation (**9**). Also any multiple or any portion of either of the curves satisfies the equation or the two curves may be added together to give a new curve which satisfies the equation. It follows that a general equation for t may be written

$$t = a\,U + b\,S \tag{10}$$

where a and b are constants to be determined and the values of U and S vary with the distance x from the origin and are obtained from curves which can be drawn for any given catenary system. The slope of this curve at any point will be

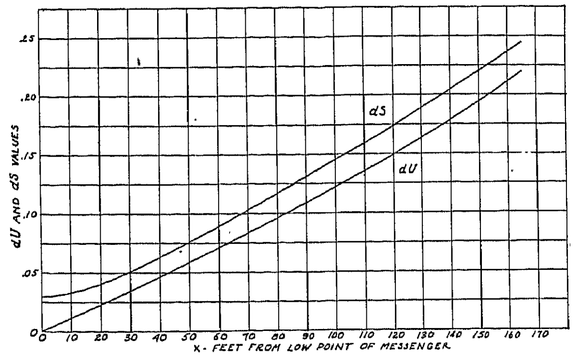

FIG. 6—dU AND dS CURVES FOR CATENARY SYSTEM

$$d\,t = a\,d\,U + b\,d\,S \tag{11}$$

the values of dU and dS are obtained from the dU and dS curves plotted for the system.

In a catenary system where a constant length of shortest hanger is used on tangent and the vertical projection of the shortest curve hanger is made equal to this length, the vertical height of any messenger support will be determined by the span length and will be the same on both tangent and curve. Where pull-offs are not used the only connection to the catenary is at the point of messenger support; since the vertical height of this support is the same on both tangent and curve it follows that the shape of the catenary is influenced only by the relative location of the points of support in a horizontal plane and the degree of the track curve influences the catenary only in as much as it causes greater or lesser angles between the lines connecting the points of support. The catenary is designed to follow the path of the center of the pantograph as closely as possible and the only way its shape can be varied is to change the support locations. The support locations must be such that they will bring the contact wire very close to the path of the center of the pantograph. In Fig. 7, A, B, and C are points on the path of travel of the center of a pantograph shoe, located at supports 1, 2 and 3. The perpendicular distance e from B to the line $A\,C$ can be obtained from field measurements or track curve characteristics. Super-elevation and height of contact wire should be taken into account in determining e.

The catenary should be designed to pass through points A, B, and C. Then if the curve taken by the contact wire is similar to the track curve the contact wire will fall on the path of the center of the pantograph. The angle between lines $A\,B$ and $B\,C$ is α. This is a measure of the change in direction of the track in the spans. The change in direction of the *axes* of the catenary in the two spans may be something different. It is shown in Fig. 7 as β. In the same figure

$$\alpha = e/L_1 + e/L_2 \qquad \textbf{(12)}$$

since e is small compared to L.

The angle between the axes of spans 1-2 and 2-3, β, is equal to $\alpha + \theta_1 + \theta_2$. It is also equal to the sum of the slopes δ_1 and δ_2 that a tangent to the contact wire curve at support 2 makes with the axes. In Fig. 7

$$\theta_1 = (t_2 - t_1)/L_1$$
$$\theta_2 = (t_2 - t_3)/L_2$$

approximately,

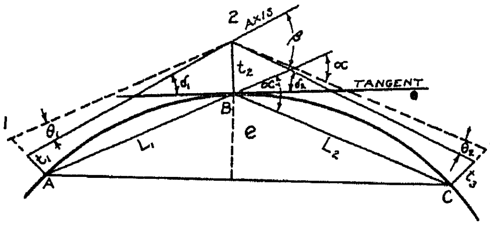

FIG. 7—ANGULAR RELATION BETWEEN CATENARY SUPPORTS ON CURVES

and

$$\beta = \alpha + \theta_1 + \theta_2 = \delta_1 + \delta_2$$

which can be changed to

$$\delta_1 + \delta_2 = (e + t_2 - t_1)/L_1 + (e + t_2 - t_3)/L_2 \qquad \textbf{(13)}$$

the slope of the t curve for span 1-2 at support 2 is

$$\delta_1 = d\,t_1 = a\,d\,U_2 + b\,d\,S_2 \qquad \textbf{(14)}$$

and for span 2-3 at support 2 is

$$\delta_2 = d\,t_2 = a'\,d\,U_2 - b'\,d\,S_2 \qquad \textbf{(15)}$$

where a and b are constants for span 1-2 and a' and b' are the constants for span 2-3.

Other equations which may be written for these two spans are

$$t_1 = a\,U_1 - b\,S_1 \qquad \textbf{(16)}$$
$$t_2 = a\,U_2 + b\,S_2 \qquad \textbf{(17)}$$
$$t_2 = a'\,U_2 - b'\,S_2 \qquad \textbf{(18)}$$
$$t_3 = a'\,U_3 + b'\,S_3 \qquad \textbf{(19)}$$

where the sub numbers are used to indicate the support. In equations (**13**) to (**19**) inclusive there are nine unknowns and seven equations. However, t_1 can usually be determined or made equal to 0 by starting from a tangent section of track and t_3 can be roughly calculated from the value e at support 3. A small error in the value of t_3 does not affect the values of a, b, and t_2 very much. So it is possible to begin at one end of a curve and work through it one span at a time using the values of a, b, and t_2 from each set of calculations and discarding the a', b' and assumed t_3 values. In this way the t_3 of one set of calculations is the t_2 of the next set. Equations (**13**) to (**19**) may be solved to determine the values of a, b, a', b' and t_2.

$$b' = \frac{K + K_1 - Z_1 - Z_2 - K_2}{Y_2\,(Y_1 - Z) + Y_3} \qquad \textbf{(20)}$$

$$b = U_2\,(k_2 - k_1)/Y - b'\,Y_2/Y \qquad \textbf{(21)}$$
$$a' = k_2 - b'\,r_2 \qquad \textbf{(22)}$$
$$a = k_1 + b\,r_1 \qquad \textbf{(23)}$$
$$t_2 = U_2\,k_2 - b'\,Y_2 \qquad \textbf{(24)}$$

where

$$K = Y_1\,U_2\,(k_2 - k_1)$$
$$K_1 = d\,U_2\,(k_2 + k_1)$$
$$Z_2 = (e - t_3)/L_2$$
$$Z_1 = (e - t_1)/L_1$$
$$K_2 = U_2\,k_2\,Z$$
$$Z = (L_1 + L_2)/L_1\,L_2$$
$$Y = r_1\,U_2 + S_2$$
$$Y_1 = (r_1\,d\,U_2 + d\,S_2)/Y$$
$$Y_3 = r_2\,d\,U_2 + d\,S_2$$
$$Y_2 = r_2\,U_2 + S_2$$
$$S_1/U_1 = r_1$$
$$S_3/U_3 = r_2$$
$$t_1/U_1 = k_1$$
$$t_3/U_3 = k_2$$

UNIFORM CURVES

Where the spans have the same length and the curve has a uniform radius $L_1 = L_2 = L$ and $t_1 = t_2 = t_3$. It can be seen from equations (**16**) and (**17**) that if $t_1 = t_2$, $U_1 = U_2$ and $S_1 = S_2$, b must equal 0, also $b' = 0$.

$$t_1 = t_2 = t_3 = a\,U_1 = a\,U_2 = a\,U_3$$

and

$$t = a\,U \qquad \textbf{(25)}$$

for a uniform curve

$$\delta_1 = \delta_2 = \beta/2 = \alpha/2$$
$$\alpha = 2\,e/L = 2\,a\,d\,U_2$$
$$a = e/L\,d\,U_2$$

for a uniform curve

$$e = L^2/2\,R$$

therefore

$$a = L/2\,R\,d\,U_2 \qquad \textbf{(26)}$$

The values of L, R and $d\,U_2$ are all known so that a can be determined.

$$t = L\,U/2\,R\,d\,U_2 \qquad \textbf{(27)}$$

where $d\,U_2$ is the slope of the contact wire curve at the support.

SHAPE OF CURVE

The path of the center of a pantograph on a curve of constant radius and super-elevation will be a circle. If the contact wire follows this path it would exert an equal force on all hangers in the span

$$f_1 = f_2 = f_3 = f\,n$$

since the vertical load on all the hangers must be equal for a horizontal contact wire it follows that all the hangers must have the same slope if the contact wire is to have a circular shape. The horizontal curve pull exerted by the contact wire on each hanger is

$$f = s\,T/R \qquad (28)$$

where R is the radius of the curve and the vertical load on each hanger is W.

In Fig. 8 all the hangers are drawn with the same slope. The horizontal deflection of the messenger depends on the curve pull of the contact wire and will be

$$d = f\,x^2/2\,M \qquad (29)$$

and the vertical sag of the messenger will be

$$i = W\,x^2/2\,M \qquad (30)$$

the horizontal deflection of the contact wire will be

$$u = x^2/2\,R \qquad (31)$$

if the section of contact wire in a span is considered to be a parabola. (The difference between a circle and a parabola in the range of span lengths and radii encountered is very small.) In Fig. 8

$$\begin{aligned} w/f &= v_o/h_o = v_n/h_n = (v_n - v_o)/(h_n - h_o) \\ i &= v_n - v_o \\ d + j &= h_n - h_o \\ w/f &= i/(d + j) \qquad (32) \end{aligned}$$

If equations (28), (29), (30) and (31) are substituted in (32) and for W its equivalent $w/s + w_m/s$ where w_m is the weight of s ft. of messenger plus ½ the weight of one hanger, the result is that

$$M/T = w_m/w \qquad (33)$$

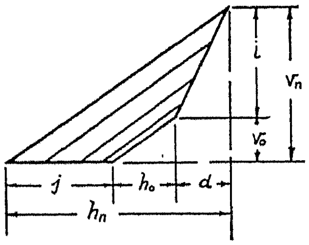

FIG. 8—CROSS SECTION OF INCLINED CATENARY TAKEN AT CENTER OF SPAN

which indicates that with a given set of wires the ratio of messenger to contact wire tension must equal the ratio of their weights if the contact wire is to follow a parabolic curve which closely approximates the circular track curve. For any other ratio of tensions the contact wire will follow some other curve which deviates from the track curve as the difference in the ratios increases. However, for most catenary systems this deviation is small. It can easily be checked by comparing the difference in the values of t at the support and center of the span with the curve middle ordinate for the same span.

If the condition of equation (33) is fulfilled in designing a catenary system the slope of any hanger on a regular curve will depend on the radius of the curve and the contact wire tension. A chart can be drawn up giving the vertical projections of the hangers and the angles of inclination for various curves, and from this chart regular curve hanger lengths can be taken directly without the use of U and S curves. However, for transition curves or curves where the radius is not known or varies, the hanger lengths must be determined by some other method and that outlined previously may be used.

It also follows that where the contact wire curve is a parabola the U curve is a parabola. Its equation is

$$U = 1 + x^2/p \qquad (34)$$

where

$$p = 2\,v_o\,M/W$$

and the slope curve $d\,U$ is a straight line

$$d\,U = 2\,x/p \qquad (35)$$

Equations (34) and (35) may be used for plotting the U and $d\,U$ curves where the messenger and contact wire tensions have the same ratio as their weights but for the S and $d\,S$ curves the method previously outlined should be used.

Discussion

For discussion of this paper see page 646.

Design of Catenary System for Cleveland Union Terminal

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Associate, A. I. E. E.

and

E. E. RICHARDS[2]
Non-member

Synopsis.—The article presents a brief description of the new electrification at Cleveland, Ohio, and the methods used in designing the overhead catenary system.

The tangent chord compound catenary is used, the system being quite heavy. The main line construction weighs 3.855 and the yard construction 2.321 lb. per ft. dead load.

The main line system is composed of a main messenger, an auxiliary messenger, and two contact wires hanging side by side. The yard system has a main messenger, and two contact wires, one above the other.

The parabola formulas were used for the calculations, and where feasible, were made up in the form of charts, illustrations of which appear in the text.

* * * * *

I—GENERAL DESCRIPTION

THE electrification of the Cleveland Union Terminal at Cleveland, Ohio, extends about eleven miles east of the Public Square, where the station is located, and about six miles west of the same point.

At both Collinwood on the east and Linndale on the west, there were large yards and roundhouses, which made these two locations strategic points for changing from steam to electric operation.

An inspection shed for electric locomotives is located at each of these points, together with the necessary power change and engine standing tracks. At Collinwood, in addition, there is located a repair shop fully equipped to handle all repairs necessary on the electric locomotives.

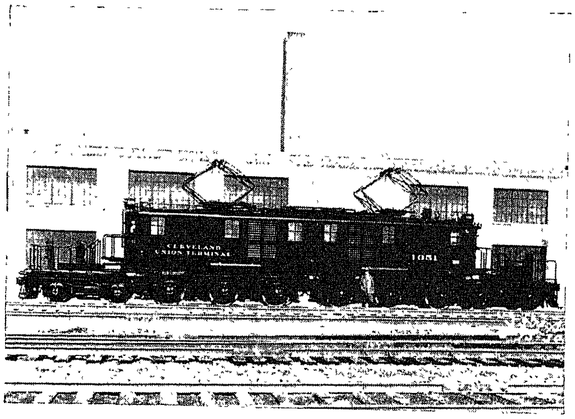

FIG. 1—PASSENGER LOCOMOTIVE

Two main line tracks are electrified from Linndale to Collinwood, and with the crossovers, and yard tracks, the total electrified mileage is about 59 track miles.

The 3,000-volt d-c. supply is furnished by two substations, each approximately midway between an end of the electrification and the Cleveland station. There are also six circuit breaker houses which act as tie points where the overhead systems are joined through circuit breakers to a common bus.

The electric locomotives weigh about 200 tons. They are capable of pulling a 15-car pullman train up the maximum grade. About 20 of these will be in active service, with two more as spares. The type of locomotive is shown in Fig. 1.

Passenger stations are located at the Linndale terminal, and at the Cleveland Public Square. In addition, on the east side, there is a station at Superior Avenue, about half way between Cleveland and Collinwood.

II—DESCRIPTION OF CATENARY SYSTEM

The tangent chord type of construction is used, as the weighty system necessary for the carrying of heavy currents does not readily lend itself to the inclined type of construction.

The main line construction consists of a 7/8-in. primary or main messenger, a 4/0, 19-wire hard drawn copper auxiliary messenger suspended from the primary messenger by hangers, and two 4/0 grooved bronze-contact wires clipped to the auxiliary messenger.

The primary messenger has a 19-wire core of high strength bronze, and a 16-wire outer layer of copper, the total conductivity being equal to 350,000 cir. mils of copper. The minimum breaking strength is 39,300 lb.

The hangers, of No. 1 hard drawn copper wire, are spaced, normally, every 20 ft. and the contact wires are clipped to the auxiliary messenger in 20-ft. spans. The two contact wire spans overlap, which results in a clip at each quarter point between hangers.

This main line construction is used over the two main line tracks from one end of the line to the other, except through the Cleveland station, where messenger construction is not used.

For dead ending the main messenger, a yoke set is used, with two strings of three insulators each, in parallel. The 10-in., 18,000-lb. insulators give, with this arrangement, a strength of 36,000 lb., and a wet flash-over voltage of 115 kv. It will be noticed that this dead end develops the full strength of the cable attached to it. This principle has been adhered to, as far as possible, in the whole design.

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[1] Formerly with The Cleveland Union Terminals Company.

[2] Catenary Designer, The Cleveland Union Terminals Company, Cleveland, Ohio.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

The auxiliary messenger and the two contact wires are dead ended with a separate assembly similar to the above. While the mechanical strength developed by this dead end is greater than that required for these three wires, the electrical strength is the same as that of the primary messenger dead end, and the whole arrangement has a more symmetrical appearance than if different types of dead end assemblies were used. Fig. 2 shows some main line dead end assemblies.

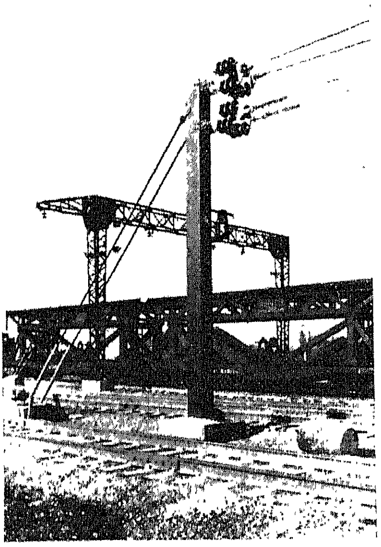

FIG. 2—MAIN LINE DEAD-END CONSTRUCTION

For yards and crossovers, the auxiliary messenger is dispensed with. The two 4/0, grooved bronze contact wires are placed one above the other, the upper one supporting the lower one by clips, it in turn being suspended by means of hangers from a ⅝-in. bronze messenger. This yard messenger has a conductivity of 31,700 cir. mils copper equivalent and a minimum breaking strength of 28,300 lb.

The dead-end assembly for the yard messenger consists of two 12-in. insulators, in series, the assembly having a rating of about 24,000 lb. tensile strength. The same arrangement is used for the two contact wires. Turnbuckles are used on both ends of yard messenger and yard contact wire systems. Fig. 3 shows yard dead-end construction.

In the station area, tandem contact wires only, are used. Two 4/0, grooved bronze-contact wires, one above the other, as in yard construction, are suspended in approximately 70-ft. spans. This construction was necessary in order to keep the line down to around 16½ ft. above the rail as most of the station area is covered by buildings, with an overhead clearance of 17 ft.

Each main line catenary is divided into five sections electrically, by four air gaps, or isolated sections. Standard main line dead ends are inserted in the main line, using, however, four insulators instead of six. Another short section of main line construction is run parallel with the main catenary, 14 in. away from it. The pantograph rides on this section and off the main catenary for a distance of about 150 ft. Separate feeders are run to the isolated sections. Two sections are on the east and two west of Cleveland station.

At the station area where the tandem construction is used, the main messenger and auxiliary are dead-ended on both sides of the station, and one 1,500,000-cir. mil cable for each main line track is carried through a duct line from one dead end to the other, to supply the necessary conductivity.

Several different types of catenary structures are used. Where more than two tracks are to be spanned, as in the yards and on the Cuyahoga Viaduct, bridges, made up of latticed columns and truss are used. Center poles, formed of an *H* beam column and a cross arm of two channels are used on most of the double track section. For single yard tracks and sidings, bracket poles, of the same type of construction as the center poles, are used. There are also certain other special types of structures, made necessary by unusual physical conditions along the right-of-way. The pull-off poles and anchor poles are *H* beams.

III—DESIGN OF CATENARY SYSTEM

A. *Location and Height of Structures.*

The maximum span on tangent track was fixed at 300 ft. This gives a sag of approximately 5 ft. at 60 deg. fahr. with the assumed weights. The suspension assembly, of two 10-in. suspension insulators with clevis, eyebolt and messenger clamp, is about 21 in. long. The center hanger was chosen at 6 in., and the distance between the auxiliary messenger and the contact wires was assumed to be 3 in. The sum of these distances amounts to about 7 ft. 6 in., which must be added to the normal contact wire height, to give the height of the structures. Due to the great number of low clearance overhead street bridges, it was decided to use a rather low normal contact wire height, 18 ft. 6 in.

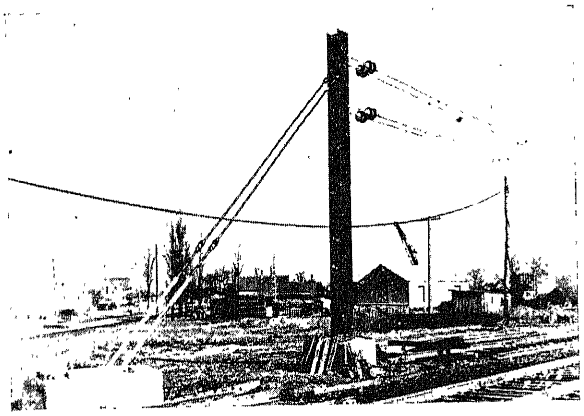

FIG. 3—YARD LINE DEAD-END CONSTRUCTION

at 60 deg. fahr. This, then, gives the height of structure, for a 300 ft. span, of 26 ft.

On curves, the contact wire was not allowed to be more than 12 in. from the inclined center line of the track. Assuming that the wire would be 12 in. outside of the center line at the ends of the span, and 12 in. inside at the center of the span, the maximum allowable middle ordinate is 2 ft. Two charts, for spacing structures on curves, were made up. These are shown in Figs. 4 and 5. These are similar to the charts used by the designers of the Illinois Central catenary system.

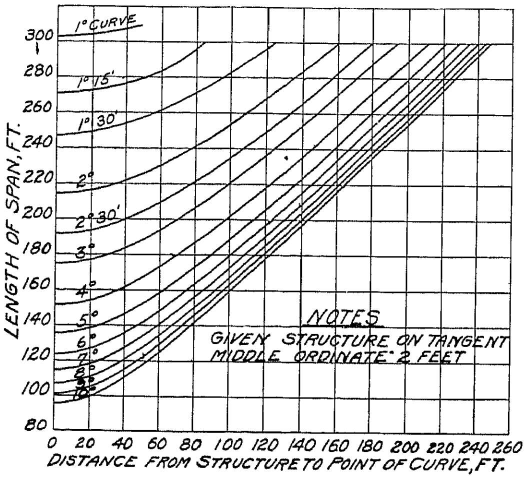

FIG. 4—SPACING CHART FOR LOCATING STRUCTURES

At crossovers and turnouts, the structures were located, as far as other conditions would permit, to act as the pull-off for the crossover or turnout catenary thus eliminating a separate pull-off pole.

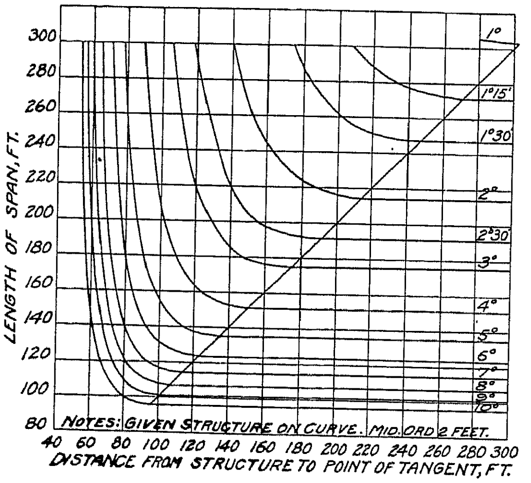

FIG. 5—SPACING CHART FOR LOCATING STRUCTURES

At the same time that the structures were located on the track plans, a profile was drawn on cross section paper. The track profile was put on first, with vertical lines representing the structures spaced according to the plans. The contact wire was then drawn in parallel to the track rail at 18 ft. 6 in. elevation, except where low clearance necessitated lower elevations. A set of celluloid curves was made, representing the messenger curve at 0 deg., 60 deg. and 120 deg. fahr. These were used to draw the messenger profile, thus obtaining the height of all structures.

When approaching low clearance overhead street bridges, the line was gradually lowered so that the messenger took its natural curve under the bridge, avoiding upthrust on the suspension insulators attached to the bridge. The natural curve used in these cases was the 0 deg. curve.

The structure heights obtained from the profile were assembled into groups varying by 6 in. The foundation depths were also varied by 6-in. steps so that the structures could be designed in groups varying by one ft.

B. *Sag and Tension Calculations.*

The parabolic formulas were used in the sag and tension calculations, as the parabola of the simple curves, gives probably the closest approximation to the true curve of the messenger.

The A. E. R. E. A. sag and tension chart was drawn for a 300-ft. span. This chart was modified slightly from the A. E. R. E. A. chart in that the load lines were made curved instead of straight, thus allowing the use of a uniform scale for sag.

On account of the decision to use a non-corrosive messenger, it was found that a large vertical movement could not be avoided. The messenger finally selected has a 3 ft. 10½-in. sag at 0 deg., 4 ft. 10½-in. at 60 deg., and 6 ft 1-in. at 120 deg. fahr. in a 300-ft. span.

The equivalent span was found for the main and yard lines, using Idail's formula:

$$L_{eq} = L_{av} + 2/3\ (L_{max} - L_{av})$$

where

L_{eq} = equivalent span length
L_{av} = average span length
L_{max} = maximum span length

The main line equivalent length was 260 ft., while the yard lines had values ranging from 120 to 280 ft. As it was impracticable to string every yard line at a different tension basis, the yard line values were divided into two sets, those having equivalent span lengths up to 200 ft. using 160 ft., and from 200 ft. up using 240 ft.

The A. E. R. E. A. charts were then drawn up using the equivalent spans as a basis. The equivalent span for the auxiliary and contact wires was 20 ft. From these charts, curves of tension against temperature were plotted. These stringing charts were made up for all the wires; main messenger, yard messengers, auxiliary messenger, and contact wire. The main messenger chart also had curves of sag against temperature, so that the messenger could be strung by either the sag stick method or the dynamometer method. Figs. 6 and 7 show respectively, the sag and tension chart and the stringing chart, for the main messenger.

C. Design and Spacing of Hangers.

After consideration of the various methods of spacing hangers, a system was worked out which is simple, goodlooking, and easy to lay out in the field. The end hanger is always 10 ft. from the suspension clamp. Then, working toward the center, there are as many 20-ft. spaces each side of the center as will go evenly. The space left in the center of the span, if any, is divided up into from one to three spaces, all equal, varying from 10 ft. 6 in. to 19 ft. 6 in. in length. The chief engineer of the construction company designed an instrument which rides along on the messenger and marks the spaces, so that no tape lines were necessary. The marking was done from both ends toward the center, so that the variations in the actual length of span from the length shown on the plans were taken in the center spaces.

A large parabola, representing the messenger at 60 deg. fahr. was drawn. Over the tracing was laid a piece of cross section cloth, with the vertical and horizontal scales to which the parabola was drawn, marked on it. By moving this cross section cloth so that the horizontal scale was always in the same relation to the parabola, as the auxiliary messenger was to the primary messenger, (as indicated by the profile), the hanger lengths could be read off directly to the nearest half inch.

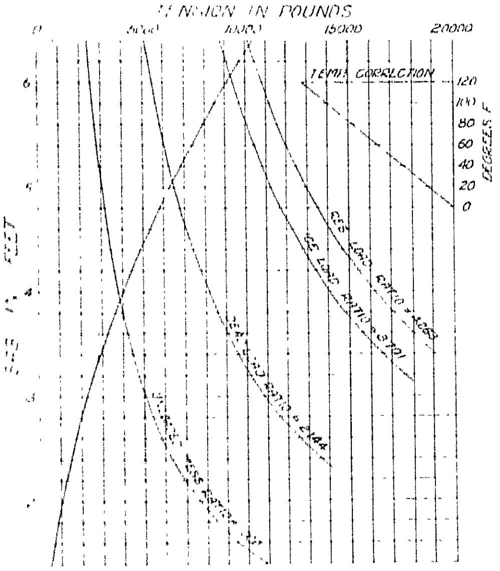

FIG. 6—SAG AND TENSION CHART FOR MAIN MESSENGER

The hanger data were made up in diagram form as shown in Fig. 8. These sheets were bound up in books, covering different sections of the line.

D. Steadies and Pull-Offs.

It was decided to use a steady arm (Fig. 9) at all structures on tangent track. At those structures which were followed immediately by a curve or a spiral, a pull-off attachment was added to the steady arm. This is merely a clamp which grips the steady arm pipe, and has two rods with clips on one end to hold the contact wires. These were necessary because of the low auxiliary messenger tension. If the line has a slight change of alinement, as on the beginning of a spiral, and the contact wires are not held, they pull toward the inside of the curve enough so that the auxiliary messenger gathers slack from both sides of the steady arm, and a large horizontal sag occurs in it at the point where it is clamped by the steady arm.

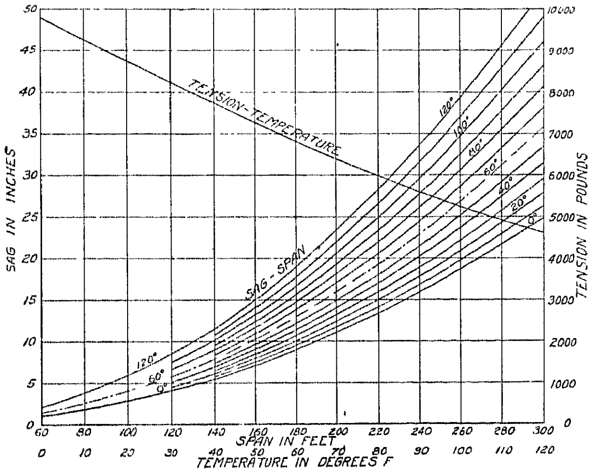

FIG. 7—STRINGING CHART FOR MAIN MESSENGER

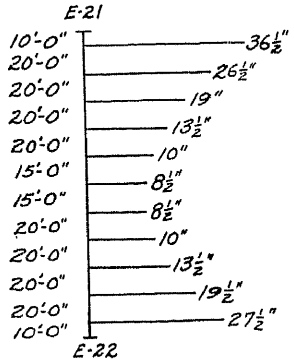

FIG. 8—HANGER DATA SHEET

Hanger lengths
1-210-ft. span
Between structures Nos. E-21 and E-22

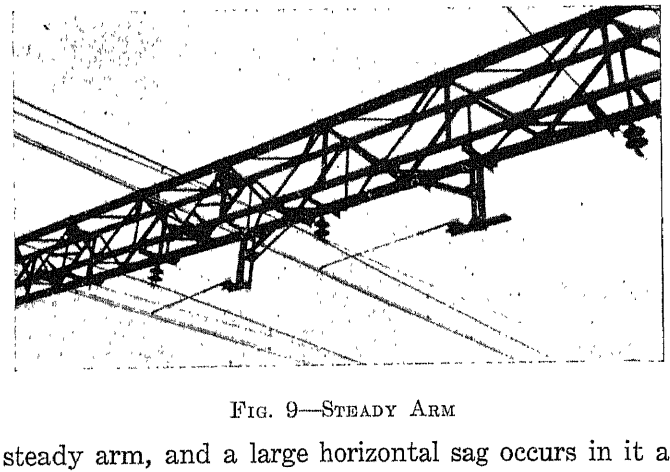

FIG. 9—STEADY ARM

Two types of pull-off arms are used. One clamps the auxiliary and two contact wires and is for main line catenary. The other takes two contact wires, one above the other, as in yard construction. This yard pull-off has the clamp detachable from the arm, so that it can be used alone where a short pull-off is needed. The clamp has an additional fitting for gripping the top contact wire or the auxiliary messenger of an adjacent line when pulling off a turnout.

Much time was saved in erection, and much more will be saved in maintenance, by using chain instead of cable and Crosby clips to hold the pull-off. This idea was originated by M. J. Griffith, of the Terminals Company. The chain link is dropped down into a slot in the vertical leg of an angle, which in turn is bolted to the column or bracket as illustrated in Figs. 10 and 11.

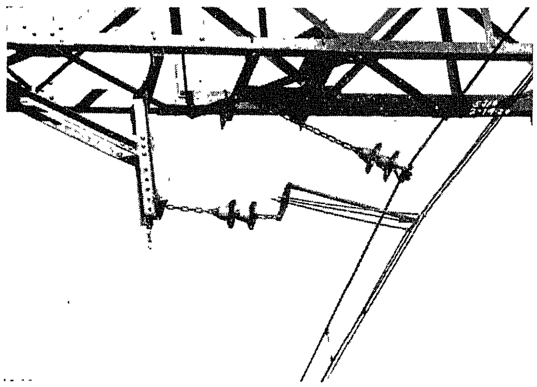

FIG. 10—MAIN LINE PULL-OFF

The steady arms were attached to the structures so as to be approximately horizontal at 0 deg. fahr., and the pull-offs so that their slope at 0 deg. would be parallel to a line across the top of the track rails.

E. Attachments to Structures.

Nearly all brackets, angles, etc., for supporting the catenary and holding it in the correct horizontal relation to the track are fastened to the structures with bolts. This facilitates shifting them to conform with changes in track alinement. The steady arm and pull-off attachments are designed to permit a vertical adjustment of one foot.

A convenient chart was prepared to quickly determine the angle of a suspension string supporting a turnout messenger. From this angle is calculated the offset from the center line of the main track at which the supporting bracket is to be placed.

IV—DATA FOR ERECTION

The data supplied the contractor consisted, first, of drawings showing a side elevation of each catenary structure. On these elevations was shown the location and type of each catenary attachment. The contractor was next supplied a set of tables giving the distance down from the bottom of the truss or cross arm at which the steady arm or pull-off angle was to be bolted. This list also showed the number of links of ½-in. chain at the different suspension locations.

The next information furnished the contractor was the wiring diagrams and stringing charts. The wiring diagrams were simply track plans on which single lines representing the catenary systems were drawn in colored inks. They also showed the location of dead ends, pull-offs, etc.

Following the stringing of the messengers came the hangers. Hanger books, as previously described, were made up, giving the spacing and length of the hangers. These were given to the contractor along with a list showing the length of steady arms and pull-offs.

In many cases special conditions required entirely special equipment or arrangement of fittings. Letter size drawings were made of each special case and furnished in book form to the contractor's engineers.

The contractor's storekeepers and material men were furnished with books containing letter size drawings of every assembly used on the system, whether standard or special.

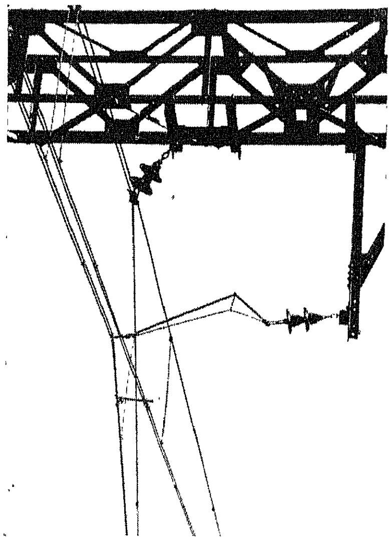

FIG. 11—YARD LINE PULL-OFF

After the erection was completed a final alinement of the overhead system was made, using the pantograph of a locomotive as a guide. The pantograph was marked with white lines one foot on each side of the center line, and the catenary was adjusted so that the contact wires were within these lines at all points.

Discussion

INCLINED CATENARY CALCULATIONS

(PICKENS)

DESIGN OF CATENARY SYSTEM

(CLEMENT AND RICHARDS)

W. B. Potter: Mr. Pickens has presented an excellent paper on a method of calculating the mechanical details of inclined catenary. This type of construction is undoubtedly neat in appearance. It is my opinion, however, that it does not provide for the same flexibility on curves as the chord system. The essential feature of an overhead contact line is to provide for the collection of current without arcing at the collector. Arcing more than mechanical wear reduces the life of the contact line and the collector surfaces. The contact between the line and collector is best assured by such flexibility of the line and freedom from hard spots so as not to cause any jumping of the collector.

H. F. Brown: Mr. Pickens in his paper has not only presented a valuable addition to the meager literature existing on inclined catenary design, but has simplified somewhat the calculations required not only in the development of the shape factors, but in their application to the track alinement.

It is of interest to note that equation (**33**) of Mr. Pickens' paper is the same as the fundamental relationship proposed by Mr. Jorstad,* for a simplified design for inclined catenary. This relationship, however, is difficult to maintain, taking into account the various weights and tensions of the contact and messenger members as determined by the conductivity requirements, and the height limitations of the supporting structures, so that very often deviations from the simple parabolic shape must occur.

It may be said that for short, symmetrical spans (up to say 150 to 175 ft.) of fairly light catenary, the error using a parabolic shape factor alone is not very serious; but for longer spans, transitional spans, asymmetrical spans, and with heavier messenger and contact members, the deviation becomes more noticeable and more accurate calculations should be employed. Mr. Pickens' paper outlines a somewhat simplified means for such calculations.

One factor, which is not brought out by either Mr. Pickens, or my paper[2], and which may cause a large error in determining the true shape, is that in the method of design it is assumed the hanger rods are all parallel, whereas they are really installed nearly radial to the contact wire. The error involved on long rods, may become a considerable amount, necessitating a slight trimming of the rods, or a slight change in their location along the contact wire to fit the actual distance between the messenger and the contact, to make a smooth curve.

It is of interest to note that in general the catenary system which lends itself most readily to the inclined type on curves, is the lighter type used on the high-voltage railroad electrifications. There are very good examples of this design on the recently installed Reading electrification in Philadelphia, on the Pennsylvania Railroad electrification between Wilmington and Trenton and on the older Broad Street-Paoli installation; on the Great Northern (where the spans are fairly short); on the Norfolk & Western, on the Virginian and on the New Haven, where this construction originated.

The railroads which have recently electrified using the lower voltages, such as the Illinois Central, the Cleveland Union Terminal, and the Delaware, Lackawanna and Western, have not used this type of construction with the heavier type of catenary which was required.

The paper on *Design of Catenary System for Cleveland Union Terminals* just presented is valuable for the graphical methods outlined and for the description of the methods used during construction.

One thought occurs in connection with the design of the heavier types of catenary, such as the one described, especially those systems using a bronze messenger for conductivity (or for resisting corrosion). On spans of 250 to 300 feet, the temperature movements of the center of the span are much larger than in the case where steel messengers are used. If the catenary is designed for level contact wire at 60 degrees, this means that at the lowest temperatures there will be an appreciable raising of the contact wire in the center of the span, which, with the increased tension also due to temperature, in the contact wire or wires, will exert considerable compression in the long hangers adjacent to the support. When the hangers are of small diameter, as on the system just described, such hangers are liable to become bowed or even permanently bent or "dog legged," to the extent that they will not resume their former straight line appearance when the temperature rises, thus detracting somewhat from the general good appearance of the rest of the design.

The suggestion is offered for consideration in such cases, that a larger diameter rod, having greater resistance to bending under compression be used, or that possibly, if light rods are desirable, the catenary design be based on a normal tension of say 30 deg. fahr. with level wire at that temperature instead of at 60 deg., thus allowing a greater deviation from the normal under maximum temperature conditions (when the tensions are always lower in all members) but keeping the deviation under minimum temperature conditions, to values which will not over stress any of the lighter members.

On the New Haven Railroad the smallest hanger rods are not less than 3/8 in. in diameter if of bronze, and 1/2 in. if of steel, and the spacing is somewhat closer than on the more recent installations. I believe, however, that both the larger size and frequent spacing has been of extreme value under abnormal temperature as well as broken wire conditions, as affording a better equalization of both tension and compression stresses between the contact wire and the messenger.

The use of chains is somewhat novel for suspensions and pull-off members, and it is admitted that it gives great flexibility of adjustment. Twenty-five years of operation of the overhead system on the New Haven have shown that parts in moving contact with each other and exposed to the weather and corrosion, may wear very thin in a few years of such exposure, coupled with vibration, and that such parts should be reduced to a minimum. It will be of interest to see whether the chains now in use on the Cleveland overhead are still there in 1955, or have been replaced by single members of the proper length.

N. Litchfield: It is of interest to note that the use of the so-called "curved" or "inclined" catenary was originated by the engineers of the New Haven Railroad in 1909. The methods of calculation employed were described in a paper presented before the Franklin Institute in 1914. These methods enabled them to calculate the length of the trolley hangers, and have them manufactured and delivered ready for use without further adjustment.

Mr. Brown stated in his paper,[2]—

"On this type of compound catenary spans were fairly short, being but 150 ft. for the maximum condition. The calculation involving the inclined rods on curves was therefore comparatively simple, as the errors made in assuming the hangers had the same inclination throughout the span, were very small. These calculations were based on the theory that the contact wire had a parabolic shape, with the horizontal 'sag' equal in value to the middle ordinate of the circular curve of the track for that span. The horizontal force or 'curve pull' was assumed to be uniform for each hanger rod. Both of these assumptions are fallacious, but do not introduce appreciable errors until it is attempted to apply this method of design to single catenary having much longer spans, and on sharper curves."

After the New Haven electrification, when other electrifications were encountered having long spans and sharp curves, it became necessary to modify the original method of calculation. This modification, applying particularly to transition curves, was worked out by Messrs. H. S. Richmond and Philip Goldstone in 1913-14 for the consulting engineers for the Norfolk & Western Railway on the electrification of the Elkhorn Grade. It was later used by them on other electrifications and has become practically the standard method in general use. Inasmuch as Messrs. Richmond and Goldstone never presented their theory in any paper before a technical society, it seems proper at this time to record their connection with the development of this art.

H. S. Richmond: Through the ability to calculate the horizontal offsets of the messenger suspensions and the lengths of hangers on irregular curves and transitions as well as on so-called regular curves, a great economy was effected; as previously a great deal of the calculation was done in a physical manner

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[1] *Railway Inclined Catenary Standardized Design*, O. M. Jorstad, A. I. E. E. Trans., 1927, p. 1125.

[2] *Catenary Design for Overhead Contact Systems*, H. F. Brown, A. I. E. E. Trans., 1927, p. 1082.

and on top of the work train, an expensive method of calculation. It has indeed been found that calculations may be made with such refinement that positions of insulators may be determined and steel supports fabricated with attachments provided in advance to receive the suspensions, provided careful track information is obtained. This, however, must be specially obtained for the purpose as, generally, theoretical map location is not sufficiently accurate.

In the opinion of the writer, equation (33) is not of significance as the U-curve is practically quite a close approximation to the circle and most track curves are not actually very uniform. We have found it convenient, by abbreviated methods, to rapidly perform the calculations without going to the trouble of investigating the track curvature for regularity.

The suggested relation expressed in formula (33) $\dfrac{M\,W}{T\,W_m} = 1$ imposes an arbitrary limitation on design which is not desirable. In the example offered $\dfrac{M\,W}{T\,W_m} = 1.4$. Assuming that the trolley tension chosen is a desirable one, to lower the messenger tension to bring about the proposed relationship would result in an excessive sag and an uneconomical spacing of supporting structures. This would work directly toward excessive cost since the cost of the individual structure is very nearly a constant, irrespective of the spacing.

The theorems presented will warrant the careful study of the designer who decides to adopt this type of construction.

J. B. Cooley: Mr. Pickens' paper furnishes a valuable text for anyone desiring to design an overhead contact system employing inclined catenary. I believe that it is the only paper of its kind which not only furnishes a method of computing the general shape curves U and S, but also supplies us with a definite means for using them in determining the characteristics of a catenary system.

I have been designing inclined catenary for a number of years and would like to offer the following amendment to Mr. Pickens' solution. The method I am about to describe is, I believe, much shorter than that of the paper, and although not quite as exact, it is within the bounds of practicability.

The U and S curves are combined to form a T curve, a transition curve. This is a curve having value of the horizontal distance between messenger and contact (called the offset), zero at some point X distance from the low point of the messenger; real value at any other point. With point X established, an infinite number of curves may be drawn, all having the general equation $t = C\,T$, where the multiplier C depends on the value of t at any point, such as the offset at a support. Therefore, in any span of catenary, two transitions may be computed, the sum of which will give the complete catenary for the span. If t_1 and t_2 are the offsets at supports 1 and 2, then one transition will have zero value at support 1, value t_2 at support 2; the other will have zero value at support 2, value t_1 at support 1.

Since the distance X from low point to support where value equals zero varies as the span lengths vary; it would seem necessary to compute a T curve for every half span. This is not, however, the case, as it has been found that if distance X is so chosen that it represents the mean of the span lengths encountered in any design, and a T curve computed, this curve may be used for any other distance X without appreciable error. The equation for any span of catenary may then be written

$$t = C\,T + C'\,T$$

and

$$C = \frac{t_2}{T_2} \qquad C' = \frac{t_1}{T_1}$$

Where T_2 signifies value of base curve at support 2, T_1 value at support 1.

The several equations listed by Mr. Pickens to find the values of t for a catenary system having no pull-offs, may be condensed to one equation.

$$t_2 = \frac{e\left(\dfrac{1}{L_1} + \dfrac{1}{L_2}\right) - t_1\left(\dfrac{1}{L_1} - \dfrac{p}{T_1}\right) - t_3\left(\dfrac{1}{L_2} - \dfrac{p}{T_3}\right)}{\dfrac{2}{T_2}\left(\dfrac{d\,T}{d\,x}\right)_2 - \left(\dfrac{1}{L_1} + \dfrac{1}{L_2}\right)}$$

Where e, L_1, L_2, t_1, t_2, t_3 refer to the same variables as indicated in the paper, T_1, T_2, T_3 are values of the transition curve at support 1, 2, 3; $\left(\dfrac{d\,T}{d\,X}\right)_2$ is the slope of the transition curve at support 2; and p is the slope of the transition curve at point X where T equals zero. To solve start at the beginning of the curved track and assign values to t_1, t_2, t_3, t_4, ... t_n. These are found by assuming a regular curve as described in the paper. Insert assigned values of t_1 and t_3 in above equation, and solve for t_2. Proceed to next structure, and use solved value of t_2, assigned value of t_4, and solve for t_3, etc. If any solved value of t differs from assigned value by more than 10 per cent, the equation for the preceding structure must be recomputed by inserting the corrected value and solving again.

I would like to say in reference to equation (33) of the paper that a catenary system bearing those relations will only simplify computations if the railroad curvature is of a regular degree. It has been my experience that most railroad curves are compounded with a spiral at each end, and the exact radius at any point is difficult to determine. Thus the method of base curves for determining the characteristics of any catenary system must be employed, and any advantage or reduction of labor in computation, by using equation (33) is lost.

B. M. Pickens: In connection with equation (33,) it should be noted that where materials of the same density are used in the messenger and contact wires, the equation requires that the same average unit stress be used in each. This gives a picture of the limitations imposed by the equation.

In those systems where it is desired to make use of high-strength messengers and an auxiliary wire at low tension as part of the contact system, it is impractical to use this relationship. If the conditions of equation (33) were fulfilled, advantage could not be taken of the high-strength messengers available and the sag would be excessive. However, in the case of a simple catenary using a composite messenger and bronze contact wire the unit stresses can be made the same without imposing any undesirable limitations on the design. If these conditions are fulfilled the design of the curve catenary on uniform curves is very simple. This is, of course, only of advantage where there are uniform curves.

N. F. Clement: There has been doubt expressed from many sources as to the advisability of using chain to connect the pull-off arms to the structures.

As far as I know there is no installation using chain in this manner, so we had no experience or precedent to guide us in its use. However, there is a number of points which seemed to us to make it worth while to take the step.

In the first place the cost of installation is small. Then the cost of replacement and adjustment is very much less than that of the clip and cable system. This latter is important in view of the fact that there are apt to be many changes in track layout the first few years of operation over a new track system.

Of course it was known there would be wear due to vibration, but to just what extent it was impossible to determine without extensive tests. The chain system is very easily inspected and excessive wear or corrosion can be recognized at once.

The most wear will come between the link held in the slot of the angle on the structure, and the next one out from it. In case the rest of the chain is sound it will be possible to pull the pull-off up one link thus making it practically as good as new with no expenditure for material and very little for labor.

I am not inclined to agree with Mr. Brown on the matter of stiff end hangers. We knew at the start there would be some bowing of end hangers and we let them bow.

Due to our large vertical movement, about 2 ft. 2 in., and consequent tendency for the contact wires to rise at the structures, a stiff hanger would probably cause a hard spot. Furthermore, it would push the auxiliary down toward the contact wires with danger of having the pantograph hit the lower hanger clamp.

With loop hangers conductivity is lost as the loop lifts off the primary messenger. There is also a liability of arcing at this point. Flexible connections are expensive and as a rule not lasting.

I will admit there is a question about the economy of using chain on the pull-offs. There is of course no doubt about the saving in material and installation cost at the beginning. We can only estimate the life and replacement costs, and time only will show how correct that estimate was.

Some Experiments with Arcs Between Metal Electrodes*

BY G. M. SHRUM† and H. G. WIEST, Jr.‡
Non-member Non-member

Synopsis.—The paper describes experiments on the arc between metal electrodes in argon and other gases, in which the current was controlled by means of a kenotron. By varying the temperature of the cathode in the kenotron, it was found possible to obtain volt-ampere characteristics of the discharge for the transition from glow to arc or the reverse. It was also observed that in the arc stage, for at least smaller currents, the spectrum of the discharge is characteristic of the cathode material.

INTRODUCTION

THE phenomena associated with the arc discharge between cold metallic electrodes are unusually complex, and, in general, are not well understood. The growing use of the arc commercially, and especially in industrial welding operations, warrants a further study of the fundamental process underlying this particular type of discharge. In this report we have considered chiefly arcs between iron and copper electrodes separated by only a few millimeters of air, or some inert gas. The gas pressures varied from one mm. of Hg. to one atmosphere.

One group of experiments included arcs controlled by means of external series resistance, in which currents ranging from one to twenty amperes were used. Later it was decided to extend the investigation to the discharges which precede the arc as the current is gradually increased from zero. In this region the negative characteristic is usually very steep and it was necessary to work with much higher line voltages and limit the current through the discharge by means of a kenotron. A galvanometer and bridge circuit, such as used in this laboratory, was arranged whereby the volt-ampere characteristics could be obtained for currents ranging from 1×10^{-6} to 100×10^{-3} amperes. It was necessary to modify this method in order to obtain the volt-ampere characteristic of the transition region between the glow and the arc discharge. The galvanometer and bridge circuit were replaced by an oscillograph, by means of which a record of the voltage and current was obtained simultaneously. It was thus possible to determine volt-ampere characteristics for discharges where the current varied continuously from 50×10^{-3} to 2 amperes.

A combination of the above methods thus enabled us to study the characteristics of discharges for currents varying from 1×10^{-6} to 20 amperes. In this range of currents, the discharge passes by continuous gradations from the stage of normal cathode glow through that of the abnormal cathode glow into the low-voltage, high-current type which constitutes an arc. It was found possible, as shown in this paper, to obtain the continuous transition not only from the glow to the arc, but also in the reverse direction.

ARCS WITH CURRENT LIMITED BY EXTERNAL RESISTANCE

Apparatus. For this work a 250-500-volt d-c. generator was used, the current through the arc being limited by means of an external series resistance. The early experiments indicated that reproducible results could be obtained only if the shape, size, and condition of the surface of the electrodes were kept constant. Some preliminary experiments indicated that electrodes 1/8 in.—3/16 in. diameter and 2 in. long were very satisfactory as they could be mounted in a holder and when necessary, replaced very easily. This effected a considerable saving in electrode material, since, owing to the heating and consequent melting, new electrodes were required for each experiment. They were prepared by cutting from longer rods of the material and thus required no further shaping.

Characteristics of Arcs in Various Gases It was found that the values obtained for the arc voltage for a given current, electrode material, and arc length, were in good agreement with values given by Seeliger, Nottingham and others. Some typical data are given in Table I. The arc voltage given depends, of course, upon the arc

TABLE I
ARC CHARACTERISTICS FOR CLEAN ELECTRODES IN VARIOUS GASES AT ATMOSPHERIC PRESSURE

Electrode material	Gas	Arc voltage	Current in amperes	Arc length mm.	Remarks
Fe	Air	32	2	1	Very steady.
Fe	Argon	24	2	1	Very unsteady. Cathode spot wanders.
Fe	N_2	30	2	1	Unsteady.
Fe	CO_2	29	2	1	Very steady.
Fe	H_2	—	2	1	Would not strike.
W	Air	40	2	1	Voltage fluctuates.
W	Argon	—	2	1	Arc would strike but could not be maintained.
W	N_2	22	2	1	Fairly steady.
W	CO_2	37	2	1	Fairly steady.
Cu	Air	25	2	1	Steady.
Cu	N_2	38	2	1	Fairly steady.
Cu	CO_2	30	2	1	Steady.
Cu	Argon	22	2	1	Fairly steady.
Cu	N_2	—	2	1	Would not strike

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*The experiments were conducted at the Research Laboratory of the General Electric Co., Schenectady, N. Y.

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length which was not determined very accurately in this part of the work.

It was not always possible to duplicate the readings for air, owing, possibly, to variations produced by the chemical action at the molten surfaces of the electrodes. The results given in Table I are for gases at atmospheric pressure and for electrodes which had been thoroughly cleaned before striking the arc. It will be noted that in all cases the arc was steady in air and CO_2, while it would either not strike or was unsteady in H_2, Ar and N_2. This phenomenon was particularly true for iron electrodes. At pressures lower than one atmosphere, the arc was less stable for all gases. Later it was found that by pretreatment of the electrodes, stable arcs could be maintained in all these gases, even at pressures of the order of 2 cm. of Hg.

Experiments with Oxidized Electrodes. It was found that electrodes which had been used for arcs in air and later used in argon gave very steady arcs. This phenomenon was investigated and it was found to be due to the oxidation of the electrodes when used in air. The same degree of stability was obtained by oxidizing the ends of the electrodes by other methods. Thus when the tip of the electrode was melted in an oxidizing flame, or oxidized by electrolysis, it served equally well. It was found too that it was necessary to treat the cathode only, in order to produce a stable arc. Thus a clean iron anode and an oxidized cathode gave a steady arc in argon and N_2, and even in H_2 for a short time, until the oxide was reduced. The combination of an oxidized anode and a clean cathode, however, gave an unstable arc. It appears that in air both electrodes must become oxidized since either the anode or the cathode serves later when used as a cathode in argon to give a steady arc. It is true, however, that an electrode which has been used as an anode in air serves to make a better cathode than one which has been used as the negative electrode. It was found too that Fe electrodes coated with either the red or black oxide of iron make good cathodes, while other oxide coatings on the cathode, including barium, calcium, and thorium, were found to be very effective, although for some purposes they would not be satisfactory as some of them tend to lower the potential drop across the arc.

With the treated cathodes it was observed that as the arc was struck, the cathode spot was established at the point where the electrodes had been in contact. As the arc was established, there seemed to be no wandering of the cathode spot, whereas with the clean electrodes, it wandered so rapidly over the surface of the cathode that the arc soon broke. A possible explanation is that the oxide prevents the wandering of the cathode spot, and in order to test this point, a flat plate was fastened on the end of a rod and used as cathode. One part of this plate was oxidized, while the remainder was thoroughly cleaned. When the arc was struck on the oxidized surface, it remained steady, but when struck on the clean surface, it wandered very rapidly, but never crossed over on to the oxidized portion, thus supporting the explanation that the oxide prevents the wandering of the cathode spot.

It is known that in air the cathode spot wanders less on carbon than on other materials, and this has been ascribed to the roughness of the carbon surface. This, however, cannot be the explanation of the phenomena observed here, because clean roughened surfaces showed little improvement over similar surfaces polished. Then again iron anodes from arcs in air are often very smooth, and it has been shown that these make excellent cathodes.

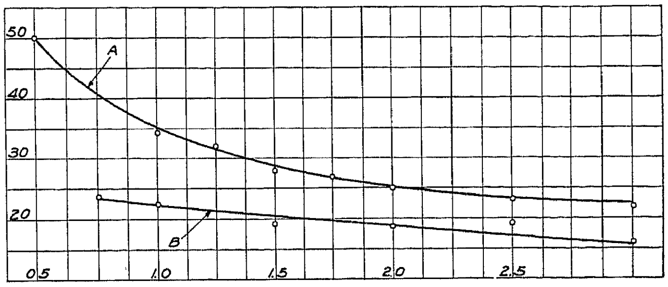

FIG. 1A—ORDINATE—VOLTS ABSCISSA—AMPERES

Effect of moist argon (curve *A*) as compared with dry argon (curve *B*) on volt-ampere characteristic of arc between iron electrodes.

When the cathode spot is prevented from wandering, the temperature rises locally on the cathode surface and a steady arc results. An explanation of the process whereby the oxide prevents the wandering of the cathode spot is probably to be found in the observations that oxygen on the surface of metals, such as tungsten, molybdenum, and tantalum, decreases both thermionic emission and secondary electron emission due to positive ion bombardment. Probably the effect of oxygen on iron is similar to that observed for the other metals mentioned.

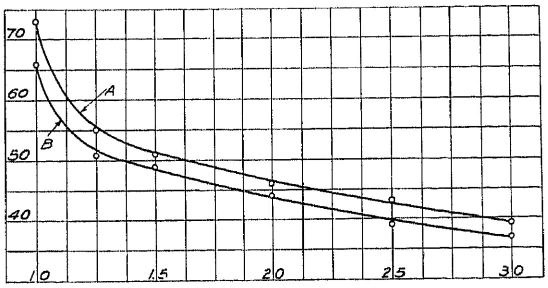

FIG. 1B—ORDINATE—VOLTS ABSCISSA—AMPERES

Effect of moist air (curve *A*) as compared with dry air (curve *B*) on volt-ampere characteristic of arc between iron electrodes

Experiments with Water Vapor and CO. The volt-ampere characteristic of an iron arc was taken in commercial argon, dried by passing it over P_2O_5, and also when the same gas had been bubbled through a flask of water, thus saturating it with water vapor before entering the arc chamber. It will be noted from Fig. 1A that the voltage drop across the arc for the moist argon was from 15 to 20 per cent greater than for dry argon.

Fig. 1B represents the corresponding results for dry and moist air. The difference is not quite as great as it is in argon, but otherwise the effect is about the same. At low current values the arc in moist argon was much more stable than in dry argon, and thus curve A (moist) for Fig. 1A was extended to lower currents than those given for curve B (dry). This was probably due to the oxidation which took place at the electrodes in the argon containing water vapor.

If this difference could be extrapolated to welding currents, then it is fairly obvious that any coating which would supply water vapor to the arc would tend to raise the arc drop and thus, for the same current, increase the rate of melting.

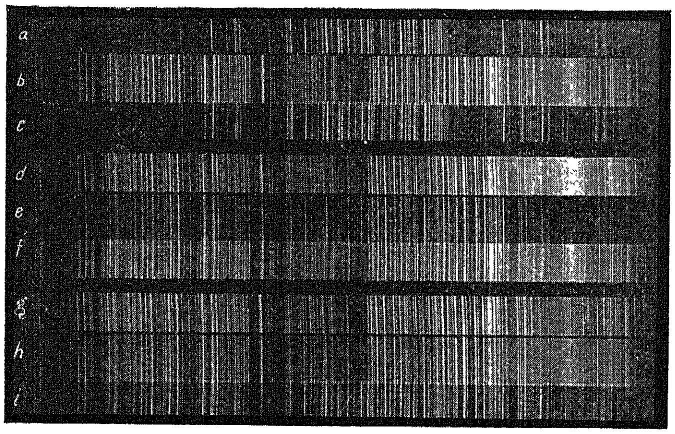

Fig. 2—Spectra of Arcs with Various Cathode Materials

In the case of carbon monoxide, the volt-ampere characteristic was found to be approximately the same as for the arc in air.

Spectroscopic Observations

Some experiments were carried out with arcs in air to determine which electrode contributed the greater amount of vapor to the arc stream. Currents of 1.5 amperes were passed between Fe and Ni, Fe and Cu, and Cu and Ni electrodes. The corresponding spectra taken with a Hilger constant deviation glass spectrograph are shown by Fig. 2. On this plate, *a* and *c* represent the spectrum of the nickel, while *b*, *d*, and *g* are for pure iron. An iron anode with nickel cathode is represented by *e*, while *f* is the spectrum for a nickel anode and iron cathode. An iron cathode and nickel anode are represened by *h*, while *i* gives the spectrum for a nickel cathode and iron anode. Thus it will be seen that while there are a few faint lines due to the anode, *the spectra in each case are typical of the cathode material.*

At the comparatively low currents at which these spectra were obtained, little or no fusion of the cathode occurred, but evidently enough vapor was emitted from the cathode spot to produce the characteristic spectrum of the metal.

Spectrograms were taken of the *normal and abnormal cathode glow* and also of the anode glow for an iron arc in commercial argon (14 per cent N_2). The negative glow for both the normal and abnormal arc showed the argon lines and the negative bands of N_2, while in the anode glow it was the positive bands of N_2 which predominated. Similar experiments in H_2 showed the Balmer lines very *strongly in the negative glow, while the secondary or molecular spectrum predominated in the anode glow.*

These results indicate that for a short gap such as used in these experiments, the degree of excitation at the cathode is much more intense than that at the anode. The difference in the types of spectra observed for the glow discharge and the arc is in agreement with the observation that for the normal and abnormal glow the cathode drop is a hundred volts and higher, while for the arc the cathode drop is 30 volts or lower.

The Transition between the Glow and the Arc Discharge by Galvanometer-Bridge Method

Apparatus. A diagram of the apparatus used is shown in Fig. 3. K was an 8-in. Pyrex bulb which served as an arc chamber. The electrodes were supported by molybdenum rods R_1 and R_2 sealed in ground glass stoppers S_1 and S_2. In order to study the region of the abnormal cathode fall, it was necessary to limit the area of the cathode exposed to the discharge. Thus the cathode C was made up of two quartz protecting tubes C_2 and C_3. The exposed surface of the cathode, C, had an area of 0.22 cm. and was slightly convex on the side towards the anode. The threaded stem of the cathode fitted closely into the quartz tube C_2 which in turn was ground to fit the opening in the larger piece of quartz C_3. The large copper piece C_4 was arranged with disks for cooling purposes. The length of the gap was determined by means of a traveling microscope, M, in conjunction with the knife edge, R, attached to the rod of the soft iron plunger, P. Glass beads, B, in the bottom of the flash served to prevent drops of molten metal from cracking the glass. F was

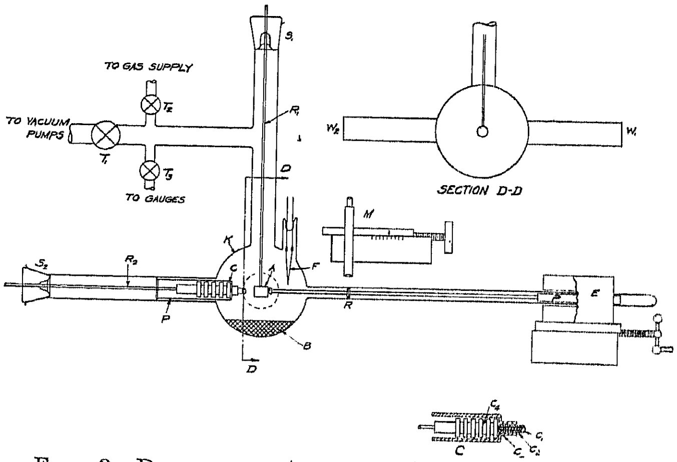

Fig. 3—Diagram of Apparatus Used for Experiments on Arcs Between Metal Electrodes in Gases at Varying Pressures

a heavy tungsten filament which was used for cleaning up traces of oxygen in the argon.

The electrical circuit is shown diagrammatically in Fig. 4. For these experiments the switches, S_1 and S_2, were closed. When the kenotron, U, was required, the switch, S_3, was in position 1, while with external resistance for limiting the current, S_3 was at position 2. B represents an air circuit breaker, R_1 a potentiometer resistance, R_4 a megohm resistance for protecting the galvanometer G, and R_3 a current limiting resistance. A was a multiple range ammeter, while V was a high resistance voltmeter,* in some cases an electrostatic voltmeter. For currents from 50×10^{-3} to 5 amperes, a UV-214 kenotron was used, while the lower currents were obtained by using a smaller three element tube with grid and plate tied together. This gave a low saturation voltage, and consequently a lower line voltage was required, which in turn helped to keep the leakage currents very small. The 2-μf. condenser, C, across the terminals of the generator served to eliminate disturbances in the voltage wave. The resistance, R_2, and the oscillograph elements E_1 and E_2 were not used in this part of the work.

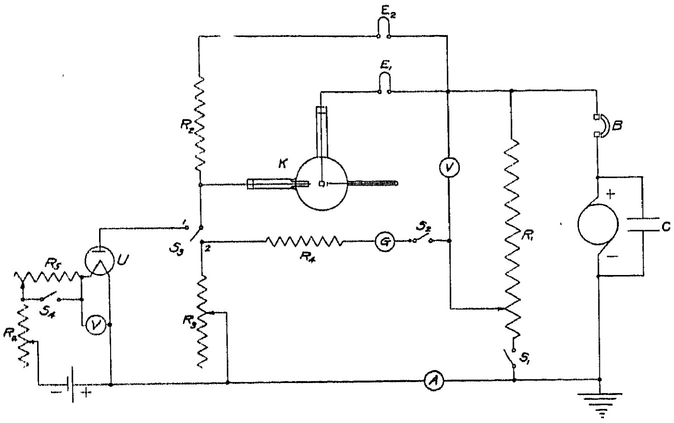

FIG. 4—ELECTRICAL ARRANGEMENT FOR EXPERIMENTS ON ARCS

Observations. Voltage readings were taken for various types of electrodes for currents varying from 1×10^{-6} amperes to 2 amperes. (With the apparatus as arranged, the leakage currents were such as to make it impossible to extend the readings to lower current values.) By this method it was possible to trace the development of the discharge from the corona or brush discharge through the normal glow and into the region of the abnormal cathode fall. Fig. 5 shows a plot on semi-log paper of the results obtained for copper electrodes with a separation of 2 mm., in commercial argon at 2-cm. pressure. It will be noted that the volt-ampere characteristic is negative for the brush and normal glow discharge, while it is positive for the abnormal glow discharge. The maximum voltage reached in the abnormal glow varied somewhat, but was of the order of 500 volts, while the corresponding current was approximately 100 milliamperes. As the current was increased further, the voltage across the gap decreased, and this continued until a drop of 30-50 volts was reached when the discharge had all the characteristics of an arc. It was almost impossible to obtain reproducible results in this transition region between the abnormal glow and the arc, due, mainly, to the excessive heating of the cathode and of the protecting refractory material. At high temperatures the discharge characteristics were modified by the nature of the refractory material. A number of high temperature refractory materials was tried, including oxides of magnesium, aluminum, and thorium, but generally it was found that clear fused quartz was the most satisfactory.

Very often the arc would strike at the junction between the quartz and the metal, and when this happened, the quartz fused and decomposed very rapidly.

The real difficulty is the problem of dissipating from such a small cathode surface the energy corresponding to voltages up to 500 volts and to currents of the order of 100 milliamperes. If larger cathodes are used, then correspondingly greater currents have to be used in order to bring the current density up to the required value for the abnormal glow.

This phase of the problem was partially solved, however, by using an oscillograph in conjunction with a large kenotron for limiting the current.

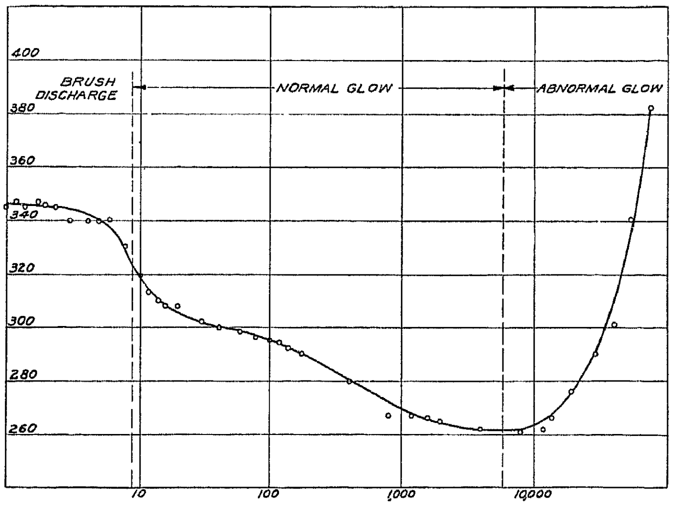

FIG. 5—ABSCISSA—LOG (MILLIAMPERES)
ORDINATE-VOLTS

The Study of the Transition between the Glow and the Arc Discharge by Means of an Oscillograph

Apparatus. The arrangement of apparatus and electrical connections are shown in Figs. 3 and 4. The current was supplied by a 4,000-volt d-c. generator. As a kenotron, a high-voltage, water-cooled radiotron UV-214 was used. This tube was rated for a maximum peak plate current of 7.5 amperes. The volt-ampere characteristic for plate currents less than 2 amperes was fairly flat for plate voltages beyond 1,500 volts.

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*Figure shows two voltmeters, V. The text refers, of course, to the instrument across the arc.

In Fig. 4 the switches S_1 and S_2 were open, and S_3 at position 1. E_1 represents the current element of the oscillograph and E_2 a very high sensitivity voltage element. R_2 was used as a multiplier for the voltage element. The temperature of the kenotron filament was adjusted so as to give a plate current of approximately two amperes when the switch S_4 was closed. The procedure was to strike the arc, then start the oscillograph motor, and immediately afterwards open the switch S_4. As the temperature of the kenotron filament dropped, the current through the arc gradually decreased, the discharge passing through the various low current stages. The resistances R_5 and R_6 served to control the initial and final currents through the kenotron filament, and thereby the rate at which the arc current changed.

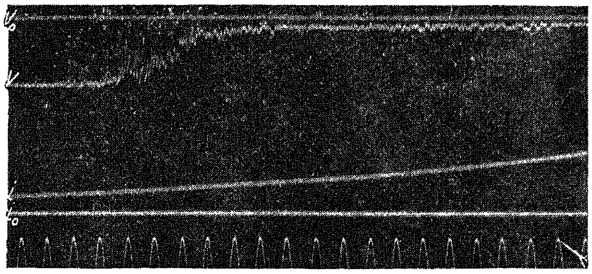

Fig. 6—Volt-Ampere Characteristic of Discharge Between Copper Electrodes in Argon at 2-Cm. Pressure

Oscillogram for transition from arc to glow for copper electrodes in argon at 25-cm. pressure

(i_o and V_o represent zero lines for current and volts respectively

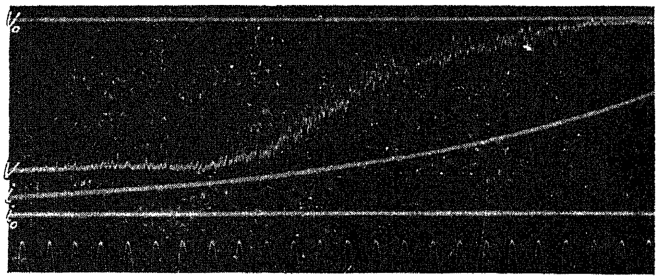

Fig. 7—Oscillogram for Transition from Arc to Glow for Iron Electrodes in Argon at 2-Cm. Pressure

Transitions from the glow to the arc were obtained by starting with the very low current discharge and with S_4 open. As the oscillograph motor was started S_4 was closed, and the filament temperature increased rapidly, causing a corresponding increase in the arc current. As the current in the discharge increased, the latter passed through the glow stages and finally into an arc.

Results. At pressures of 25 cm. of argon, it was found that the transition from the arc to the glow was very abrupt. It was noted also that the transition from the glow to the arc was practically identical with that from the arc to the glow, taking place at practically the same values for the voltage and current. Since the time taken for the filament to arrive at the temperature corresponding to a *particular value of the discharge current must be different in the two cases,* it would appear that *the transition is governed more by the current density at the cathode than by the temperature.* Fig. 6 is an oscillogram record of the transition from the arc to the glow for copper electrodes, 1 mm. separation, in argon at 25 cm. pressure. The zero line for current is represented by i_0. while i is the discharge current. The zero for the voltage is marked V_0, while V is the voltage across the discharge gap. The duration of the experiment is indicated by the 60-cycle timing wave. Fig. 8 has been plotted from the oscillogram record. Similar results were obtained for the transition from the glow to the arc.

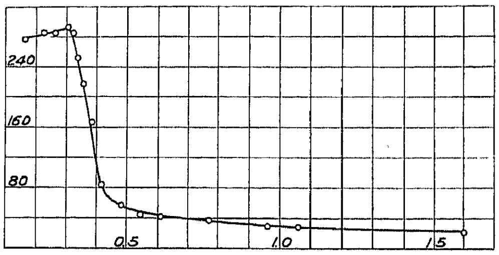

Fig. 8—Volt-Ampere Characteristic for Case of Oscillogram in Fig. 6

At low pressures the transition is much more gradual. Fig. 7 represents a typical oscillograph record of the transition from the arc to the glow for iron electrodes in argon at 2-cm. pressure for a gap length of 2 mm. Fig. 9 represents the corresponding volt-ampere characteristic as plotted from the oscillogram. It will be noted that the transition is quite gradual at these pressures, and especially at the short gap lengths. From observations on gaps of varying lengths, it appears that the arc persists down to lower current values as the length of the gap is increased, while the

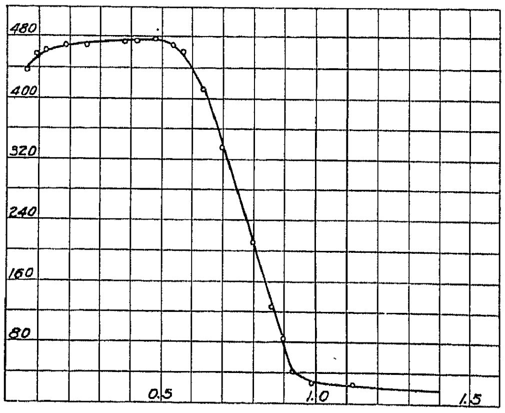

Fig. 9—Volt-Ampere Characteristic for Case of Oscillogram in Fig. 7

voltage peak for the abnormal glow remains at about the same current density, causing a much more abrupt transition. Thus either high pressures or longer gap lengths tend to produce sudden transitions. It should be noted that for the shorter gap length the abnormal voltage rises to higher values than for the longer gaps. This is in agreement with the low value observed for the maximum abnormal voltage at high pressures. At pressures lower than 2 cm. the oscillogram records were very erratic, probably due to the much higher abnormal cathode voltages, which caused excessive heating and bombardment of the refractory material.

The voltage oscillations in the transition region are probably caused by the steep negative volt-ampere characteristic of the discharge at this point. It is possible that a kenotron with a much flatter space charge characteristic would tend to control such fluctuations. Also it was found that these voltage fluctuations were greatly enhanced when the argon was replaced by hydrogen.

CONCLUSION

The chief interest in this work lies in the characteristics which have been obtained for the transition stage between the two types of discharges. If, as Seeliger[1] notes, the arc discharge always develops from the glow or Townsend discharge, then this transition stage precedes every arc. Many workers, however, have doubted the existence of such a transition region, and it has usually been assumed that the transition from arc to glow is so abrupt that the different stages cannot be obtained. Stark[2] studied this transition and came to the conclusion that as the glow spreads over the cathode surface, the latter gradually gets hot enough at some point to start an arc. Seeliger[3] investigated experimentally the development of the arc from the glow discharge. He used a special discharge tube with the cathode a mercury surface at the end of a small quartz tube, the discharge tube being filled with a rare gas. Wehrli[4] also has investigated the transition from the glow to the arc in the case of arcs between tungsten electrodes in pure nitrogen.

Our results obtained with the oscillograph indicate that at low pressures the abnormal cathode fall rises to very high values and then as the current is increased, the drop across the electrodes decreases and this continues until the discharge eventually has all the characteristics of an arc. The volt-ampere characteristic for the transition stage is very steep. Our results do not support the assumption that it is a temperature effect at the cathode and the resulting thermionic emission which causes the glow discharge to go over into an arc. The results obtained indicate rather that it is really a matter of current density at the cathode. Some experiments were tried in which the exposed area of the cathode was increased, and it was found that much higher currents were required before the arc could be developed. This was true in spite of the fact that the electrode temperature was occasionally high enough to cause fusion of the metal.

It was found to be very difficult to prevent the cathode spot from forming on the insulating material which was used to limit the area of the cathode spot. The explanation for this is not clear, although it may be due to a contamination of the adjacent metal surface of the cathode by material sputtered from the insulator under the positive ion bombardment of the abnormal cathode drop. In this connection it may be interesting to note that Oliphant[5] found for helium positive ions that the secondary electron emission was much greater for contaminated surfaces than for clean surfaces. He also found that it was less for hot surfaces than for coldones.

In a very interesting paper, Güntherschulze[6] has discussed the distribution of energy between the anode and cathode in the glow discharge. He found for the abnormal glow discharge that the number of positive ions required to liberate one electron from the cathode rapidly decreased with increasing cathode fall and for a given cathode fall was greater for iron than any of the other metals investigated.

It may be pointed out that an extension of this study of the transition from the glow to the arc and from the arc to the glow under various conditions seems to be a line of investigation which should lead to interesting results. An arrangement permitting the control and accurate measurement of the temperature of the electrodes, especially that of the cathode, together with oscillograph records of the transition stages, should lead to a better understanding of the actual mechanism by which electrons are emitted at the cathode.

In conclusion, the authors wish to express their appreciation of the interest shown in this investigation by Dr. S. Dushman and Mr. C. G. Found.

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2. Stark, *Ann. der Phys.*, (1903), Vol. 12, p. 692.
3. Seeliger, *Physik Zeitsch*, (1926), Vol. 27, p. 730.
4. Wehrli: *Zeitsch f. Physik*(1927), Vol. 44, p. 301; *Helvetica Physica Acta*, (1928), Vol. 1, pp. 247, 323, 449.
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6. Güntherschulze: *Zeitschf. Physik* (1926), Vol. 37, 12, p. 868.

Discussion

S. Dushman: One of the most important results which follows from this investigation is the conclusion that the transition from the glow to the arc, or vice versa, is governed more by the current density at the cathode than by the temperature of the latter. It is of interest to discuss this conclusion from the point of view of prevalent theories on the phenomena occurring in the transition from glow to arc and on the origin of electrons at the cathode of an arc discharge.

As is shown in Figs. 8 and 9, the transition from glow to arc is accompanied by a decrease in voltage drop between the electrodes. This is accounted for by the change in cathode drop. For the glow, this drop is of the order of 100 volts or higher, and there is built up a positive ion space charge at the boundary between the glow and the cathode dark space. At the immediate surface of the cathode itself, the space charge is negligible and the electron emission is limited by the rate at which positive ions strike the cathode. However, when the arc forms, a redistribution occurs of potential in the space between the electrodes so that the cathode drop decreases to the order of the ionizing potential of the metal vapor.

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[1] For references see Bibliography.

Owing to the fact that the cathode is now emitting all the electrons that are required for the arc current, an electron space charge is built up at the surface of the cathode, while the positive ion space charge region is formed at a distance from the cathode which is of the order of a mean free path.

In consequence of this there exists a gradient at the cathode of an arc which is of the order of 10^5 to 10^6 volts per cm. It is this gradient which is held responsible, according to Dr. Langmuir's theory, for the emission of electrons from the cathode; for as pointed out by Dr. Compton the theory of pure temperature emission will not account for current in such arcs as those between electrodes of copper, iron or mercury.

However, it would appear from the observations on cold cathode emission and as a consequence of the theoretical considerations of Fowler that the gradients of the order of 10^6 volts per cm. are not quite high enough to pull electrons out of cold metals and therefore it is necessary to look for still another process by which electrons may be emitted. Mr. Found's theory that a great part of the emission is due to metastable atoms certainly appears very plausible, especially in view of the results which he has obtained in a study of arcs in neon, mercury, and other gases. Also the results obtained in the present investigation on the part played by current density in governing the transition from glow to arc would not be in contradiction with such a theory, although it is rather difficult at present to work out the quantitative relations.

C. G. Found: The paper by Shrum and Wiest is a valuable contribution to the study not only of arcs but also of general discharges in gases, particularly the transition stages from the normal to the abnormal glow and from the abnormal glow to the arc. The transition from the normal to the abnormal glow is accompanied by an increase in cathode drop, which causes the positive ions to strike the cathode with increasing velocity and thereby the secondary electron emission from the cathode is increased. The transition from the abnormal glow to the arc is most interesting, and, I believe, indicative of the processes responsible for this transition. As pointed out by the authors, a thermionic emission does not account for the experimental results, and Dr. Dushman has mentioned the unsatisfactory results of assuming the cathode current to be due to the electrostatic field.

In order to account for the development of the arc on a basis of secondary electron emission by positive ions, it is necessary, from space charge consideration, to assume a high efficiency of secondary electron emission. Moreover, comparing Figs. 8 and 9 it is noted that at the higher pressures although the abnormal cathode drop is lower, the arc develops more rapidly at a lower current than at the lower pressure where the abnormal cathode drop is larger.

Since the efficiency of secondary emission by positive ions increases with voltage, it would appear the opposite would be true and the arc would strike at a lower current when the cathode drop was greater. I would like to suggest a mechanism of electron production at the cathode which is not directly dependent on the voltage distribution, namely, the secondary electron emission by metastable gas atoms. Results of investigations of discharges in neon show that the efficiency of secondary emission by metastable atoms is very high and that this process is responsible for the low arc-back voltage in neon.

The number of metastable atoms reaching the cathode is independent of the voltage distribution and determined only by their rate of production and their rate of diffusion. As the current in the abnormal glow is increased more and more metastable atoms are produced and gradually the secondary emission due to them exceeds that due to positive ions.

The formation of metastable atoms is produced mainly by collision of the second kind between excited and normal atoms. Since the number of such collisions increases with pressure, the concentration of metastable atoms is higher at the higher pressures and thus the secondary electron emission from this source is greater and the arc starts at a lower current at the higher pressure in agreement with the observations given in Figs. 8 and 9.

J. Slepian: A study of the transition between the glow and arc discharge such as the authors have made is particularly interesting and important because it is able to subject the various theories of the cathode of the arc to quite searching tests. It was a study of this transition in connection with the early development of the autovalve arrester which first made me doubt the universal validity of the thermionic emission theory of the cathode at a time when this theory was generally accepted. The authors also find that the transition is independent of the temperature of the cathode and thus add another nail to the coffin of the thermionic theory.

The results obtained by the authors would seem to be highly significant also in relation to the theory of the cathode of an arc which is generally accepted today, namely, the high field theory proposed by Langmuir. According to this theory, in the electric arc, electrons are drawn from the cold cathode by the very intense electric field produced by positive space charge very close to the cathode. To produce the large space charge necessary for the maintenance of so intense a field, very high current densities are necessary, of the order of thousands of amperes per square centimeter. Such current densities are observed usually at the cathodes of arcs and thus the theory seems plausible. However, if arcs should be found with current densities at the cathode of much smaller magnitude, then the high field theory becomes impossible, for these arcs at least.

Hence the finding of a continuous transition between glow and arc with cold cathodes as observed by the authors is very important, because in these transition states the voltage is much less than the normal glow voltage, and at the same time the current density is much less than the thousands of amperes per cm.^{2} usually found in an arc. Since the voltage is below that of the glow, some ionizing agent must be active other than those which are normally effective in the glow, namely ionization by collision and freeing of electrons from the cathode by positive ion bombardment and impact of excited molecules. This ionizing agent cannot be an electric field of sufficient intensity to draw electrons out from the cathode, because the current density is not great enough to support a space charge corresponding to so large a field.

Some time ago, I suggested that this new ionizing agent might reside in the gas next to the cathode, where there is a very great energy input density, and might be akin to thermal ionization of the gas. I say akin, because thermal ionization refers to the ionization existing in a gas in thermodynamic equilibrium, whereas the gas next to the cathode is in a state in which it probably cannot be described as having a definite temperature at all. If the gas next to the cathode was sufficiently ionized, the current to the cathode could be carried by positive ions entirely without requiring the liberation of electrons from the cathode. Hence, the need of an intense electric field at the cathode would be obviated, and the requirements of current density at the cathode would be less drastic.

Because of the importance of these transition states between glow and arc with cold cathodes, it is necessary to subject the demonstration of their existence given by the authors to the closest scrutiny to make sure that no valid objections can be raised. While ready, myself, to accept their existence, I do see one possible objection which I bring up hoping that the authors may be able to continue their experiments and set it aside. It is this. May not the transition states observed by the authors be only apparently equilibrium transition states? May they not consist of very rapid alternations between the glow state, and the high current density arc state? These alternations might take place with such high frequency as to escape detection on the oscillograph used by the authors.

An Improved Arc Welding Generator

BY J. H. BLANKENBUEHLER[1]

Non-member

Synopsis.—*The welding operator may encounter trouble in starting the arc or in having the arc extinguish itself too readily. Those causes of trouble which are attributable to the welding generator are analyzed and explained in terms of generator design. Development of means for eliminating the generator characteristics causing these troubles, is outlined and the results obtained are shown by oscillographic records. Other benefits accruing from this improvement are mentioned.*

GENERALLY speaking, an electric welding operator has two kinds of trouble when using an inferior welding generator, *i. e.*, the electrode may adhere firmly to the parent metal when touched to it and during welding the arc may be extinguished by the slightest disturbance.

The sticking of the electrode to the parent metal is caused by a fusing together of the molten end of the electrode and the molten parent metal. When an arc is first struck with a new electrode on a cold plate, neither should be melted until an arc is drawn. They can be melted, however, at the point of contact, if the electrical resistance of the contact and the short-circuit current of the welding generator create enough heat. This fusing together of the contacted metals immediately forms a lower resistance contact, which decreases the heating at that point, permits the molten metal to freeze and prevents the electrode from melting loose. This analysis of the process of electrode sticking indicates one requirement of a good arc welding generator, that is, to have a low ratio of short-circuit current to welding current.

However, from a study and tests of a number of different types of single-operator inherently-regulating arc welding generators it was observed that different generators with an identical ratio of short-circuit current to welding current showed entirely different arc striking characteristics. Oscillograph films of the current from these generators at short circuit revealed the difference. The generator with which it was most difficult to keep the electrode from sticking showed a large surge of current at short circuit. All of the generators tested, showed relatively large surges, and one produced surges more than ten times the final stable short-circuit current. Fig. 1, curve *A*, shows a surge from a generator which was reported to be a reasonably good welder. This figure is plotted to scale from an oscillograph picture taken during the short circuit of the welding generator. Curve *B* of Fig. 1, shows the short-circuit current characteristics of an improved arc welding generator, that is, one without any surge at short circuit.

The operator's other annoyance, instability of the arc, is explained by the fact that there is probably a minimum value below which the current in any particular arc cannot drop without extinguishing the arc. This minimum current is dependent on the length of the arc, the size and material of the welding electrode and the material being welded, the magnetic conditions about the arc, the presence of flux or stabilizing gases, the velocity of the air blowing on the arc and a number of other factors. Assuming this minimum current to exist, it can then be said that any disturbance at the arc, which reacts on the generator so as to allow the current to drop below the minimum stable arc value, will extinguish the arc.

It has been found that one of two different generators with identical volt-ampere characteristics may produce a very unstable arc even though the other be very stable. The explanation is that the inferior welding generator allowed the arc current to drop below the minimum stable value during arc disturbances. Oscillograph films of the current from these generators showing the change of current caused by an instantaneous increase in load resistance revealed the difference. The generator, with which it was difficult to maintain the arc, allowed the current to droop during the change considerably below the final value which the current assumed after the change. Fig. 2, curve *A*, shows this droop. If the current i on this curve is less than the minimum current for the arc, generator *A* would permit the arc to break. The stability of the arc supplied by generator *B*, which does not allow the current to droop below the final value I_2 is entirely dependent upon the slope of the volt-ampere curve of the generator. That is, if the volt-ampere curve of generator *B* as shown in Fig. 3 causes I_2 to be so much less than I_1 as to be below the minimum stable current, the arc will break even though there be no transient droop below I_2. At normal arc voltages the volt-ampere curves of practically all commercial welding

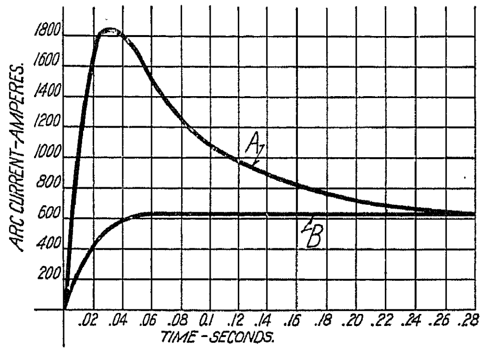

FIG. 1—WELDING CURRENT AT SHORT CIRCUIT

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generators give too low a change of current per volt change in arc length to render this condition important. Therefore a generator which has a short-circuit characteristic as shown in Fig. 1, curve *B*, and an arc lengthening characteristic as shown on Fig. 2, curve *B* should and does produce a very flexible and easily managed arc.

All single-operator inherently-regulating arc welding generators are commercially used with considerable reactance in their welding circuits, which reactance tends to prevent changes in the welding current. The reactance required to eliminate current surge at short circuit and to eliminate current droop when the arc is lengthened, with a generator of this type and of a size comparable to an ordinary generator of equivalent output, is undesirably large and gives off a large amount of power as heat. This power lost in the reactor must come from the supply line through the motor and generator, thus increasing the input to the set for a given output which raises the cost of welding. Since the motor and generator must also transmit this extra power, both must be larger than necessary. In some commercial sets this reactor loss at rated output is 40 per cent of the output of the set and still it does not possess the ideal characteristics outlined above. Elimination of current surge at short circuit and current droop when the arc is lengthened requires that the reactance delay the change of current long enough to allow the generator to change its voltage to suit the new conditions. Due to various retarding forces in the generator, this may be a considerable time. If the change of current is delayed too long however, the conditions at the arc may have undergone a second change before the welding generator has adjusted itself to the first change.

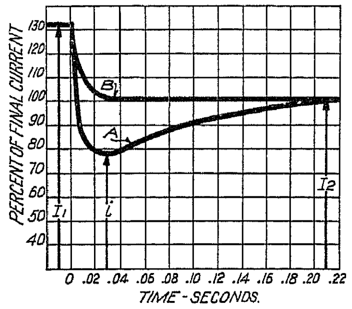

FIG. 2—CHANGE OF CURRENT FROM SINGLE-OPERATOR WELDING GENERATOR WITH AN INCREASE IN VOLTAGE FROM 15 TO 25 VOLTS

The approximate constant-current volt-ampere characteristic desired for a single-operator welding generator can be most efficiently obtained with a separately excited differentially compound wound generator. The voltage of this type of generator follows the arc voltage with any particular separately excited field current because of the variation of the current in the series field. Thus, for a certain separately excited field current, there is a definite no-load voltage induced in the armature by the flux produced by the field magnetomotive force. Any current flowing in the armature circuit produces a proportionate amount of differential series ampere turns, thus decreasing the total field ampere turns and, therefore, the flux and the voltage. This type of generator gives a series of volt-ampere curves as shown in Fig. 3, one curve for each field current.

When the arc is struck, the generator's voltage must change from the no-load voltage to zero. This change is produced by the current in the series field rising enough to reduce the total field ampere turns to that value which will only induce enough voltage to force the short circuit current through the resistance of the circuit. As the series ampere turns increase, the flux should decrease from the no-load value to the short-circuit value. It is difficult, however, to make this flux change rapidly since all the coils linking this flux have induced in them a voltage which produces a current causing a magnetomotive force opposing the change of flux. This action delays the change of the voltage from no load to zero and the current flowing through the short circuit assumes a value higher than the ultimate value until the flux change is completed. This produces the surge shown in curve *A*, Fig. 1. The elimination of the surge requires that the flux shall have changed to its final value by the time the current builds up to its final value. Reactance in the welding circuit works toward this end by delaying the rise of current. Enough reactance would delay the rise of current long enough to

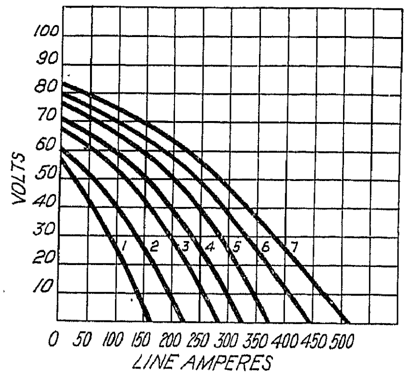

FIG. 3—VOLTAGE AMPERE CURVES

7½-kw., 25-volt, 1,750-r. p. m., differentially compound-wound separately-excited generator

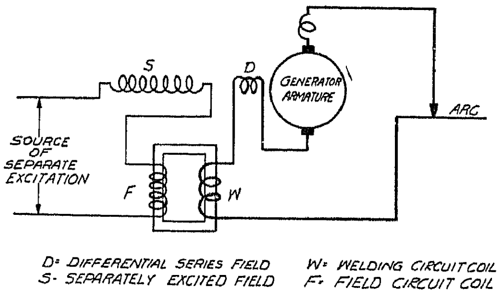
S
D
GENERATOR ARMATURE
ARC
SOURCE OF SEPARATE EXCITATION
F
W
D= DIFFERENTIAL SERIES FIELD
W= WELDING CIRCUIT COIL
S- SEPARATELY EXCITED FIELD
F= FIELD CIRCUIT COIL

FIG. 4—WIRING DIAGRAM

allow the flux change to be completed, but this reactor would cause too much power loss and would be undesirably large.

Instantaneous lengthening of the arc during welding requires a change in the generator almost the reciprocal in all respects to the change at short circuit. Lengthening the arc requires an increase in generator voltage which requires that the flux raise from one value to a higher value. This change is also delayed by all the coils linking the flux, and since the current at any instant is equal to the voltage at that instant divided by the circuit resistance, the current will droop below its ultimate value until the voltage reaches its final value. This gives the effect shown in curve *A*, Fig. 2. As in the case of elimination of surge at short circuit, elimination of this droop in current with a rise in arc voltage requires that the flux shall have changed to its final value by the time the current has dropped to its final value. As before, reactance in the welding circuit works toward this end by delaying the drop of current. Enough reactance to give the droopless curve *B*, Fig. 2, would be undesirably large and would cause a prohibitive power loss.

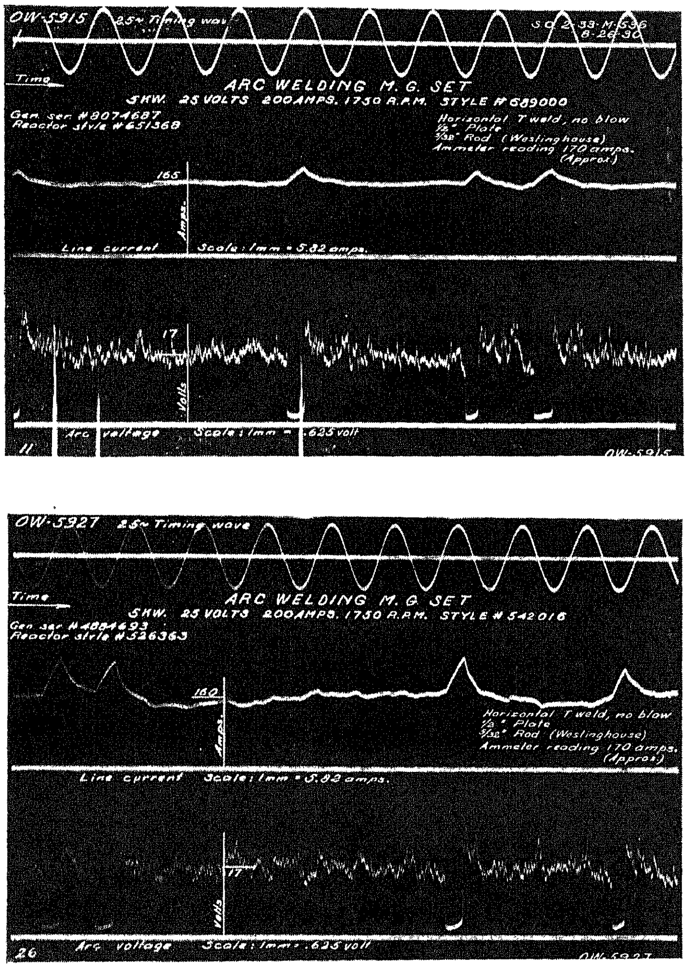

FIG. 5—MANUAL WELDING, HORIZONTAL T WELD, NO. MAG. BLOW, 1/2-IN. PLATE, 5/32-IN. ROD, 170 AMPERES (APPROX.)

Top, with transformer
Bottom, with reactor

Both difficulties, evidently require the same cure, that is, to increase the speed of change of flux by removing or neutralizing the forces acting to retard its change. These forces are magnetomotive forces produced by currents induced in any conductor linking the changing flux. Investigation of the relative effects of the various sources of delay in all types of generators, showed the field coils to be the worst offenders, since the voltage induced in these coils increased or decreased the field current in surprising amounts with the resultant effect of greatly retarding the flux change. Elimination of this effect required the prevention of the development of this voltage in the field coils or the neutralization of its effect by introducing an equal and opposite voltage in the field circuit. This neutralizing voltage must, of course, be approximately equal to the voltage induced in the field coils at every instant so that the field current cannot change and retard the flux change.

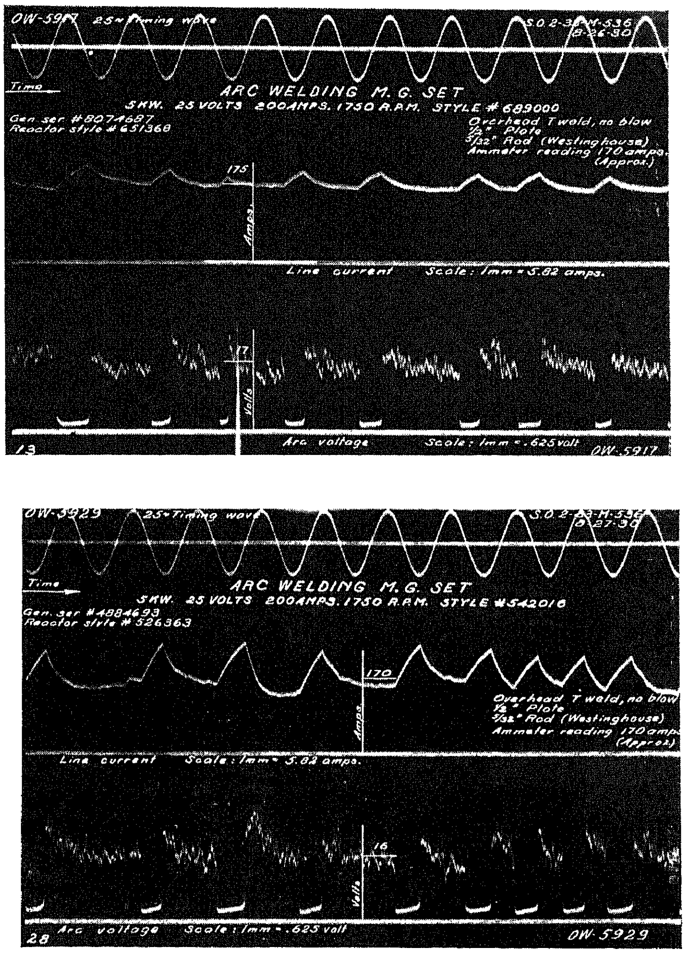

FIG. 6—MANUAL WELDING, OVERHEAD T WELD, NO MAG. BLOW, 5/32-IN. ROD, 170 AMPERES (APPROX.), 1/2-IN. PLATE

Top, with transformer
Bottom, with reactor

The solid iron parts of the generator's magnetic circuit and the rivets in the laminated field poles would have induced in them currents opposing any flux change. Tests on a generator with complete and perfectly laminated magnetic circuits showed that the retarding effect of these factors is negligible except when the iron parts are worked at very high flux densities which are seldom reached in industrial generators and never obtained at any possible arc voltages in welding generators.

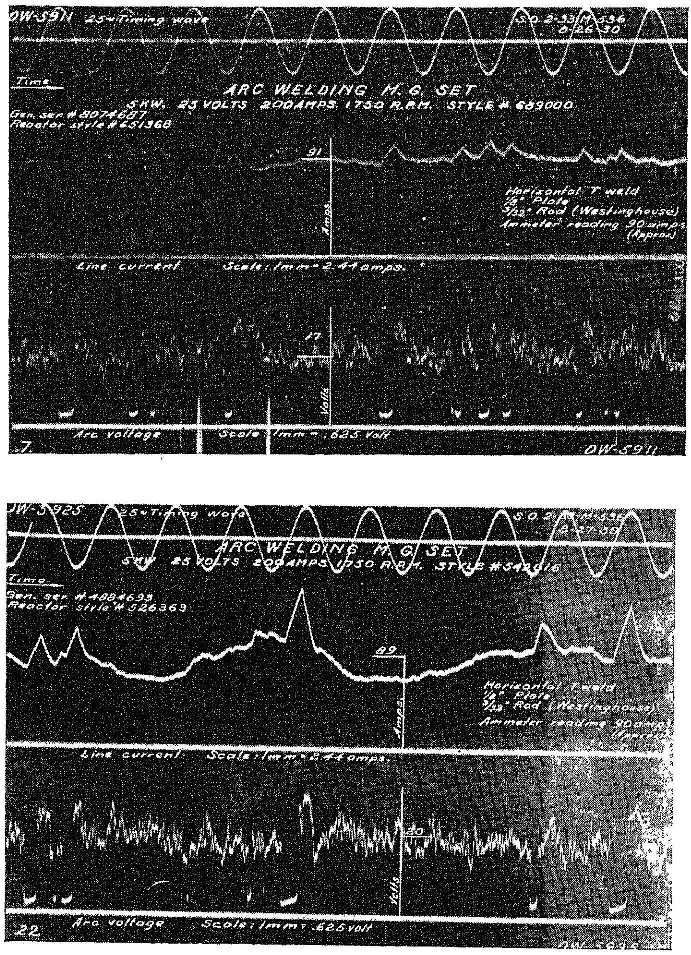

Fig. 7—Manual Welding, Horizontal T Weld, 1/8-In. Plate, 3/32-In. Rod, 90 Amperes (Approx.)

Top, with transformer
Bottom, with reactor

The logical method of producing this neutralizing voltage is to use a transformer with the primary in the welding circuit and the secondary in the field circuit, as shown by Fig. 4. It is quite obvious that the transformer can be proportioned so as to cause the voltage induced in the secondary by a current change in the primary to be the same as the voltage induced in the field coils by the same current change in the differential series coils. Since the series field and the primary of the transformer are in series, and carry the same current, the change in both occur at the same time, and the voltages in the secondary of the transformer, and the field coils will neutralize at every instant.

Another feature of this system lies in the fact that by making the proportions between the primary and secondary of the transformer such that the voltage induced in the secondary is greater than the voltage induced in the field coils by a change in welding current, then the field current can be changed to introduce a magnetomotive force promoting the flux change. This force can be used to offset any retarding forces other than in the field coils.

While the description of the functioning of this device has only considered its results when used with a separately-excited differentially-compoundwound generator, nevertheless the scheme can be used with any type of welding generator, whether separately excited or self excited, to neutralize the retarding effect of the generator field coils.

As previously stated the voltage induced in the secondary is about equal to that induced in the field coils. Its actual value is dependent on the number of turns in the field coils and the rate of change of the flux linking these coils. This voltage will reach a maximum value several times that of the excitation voltage. However, this is nothing unusual in electrical machine design, since much higher voltages are encountered every time the field circuits of d-c. motors or generators are opened. Considered from a safety standpoint,

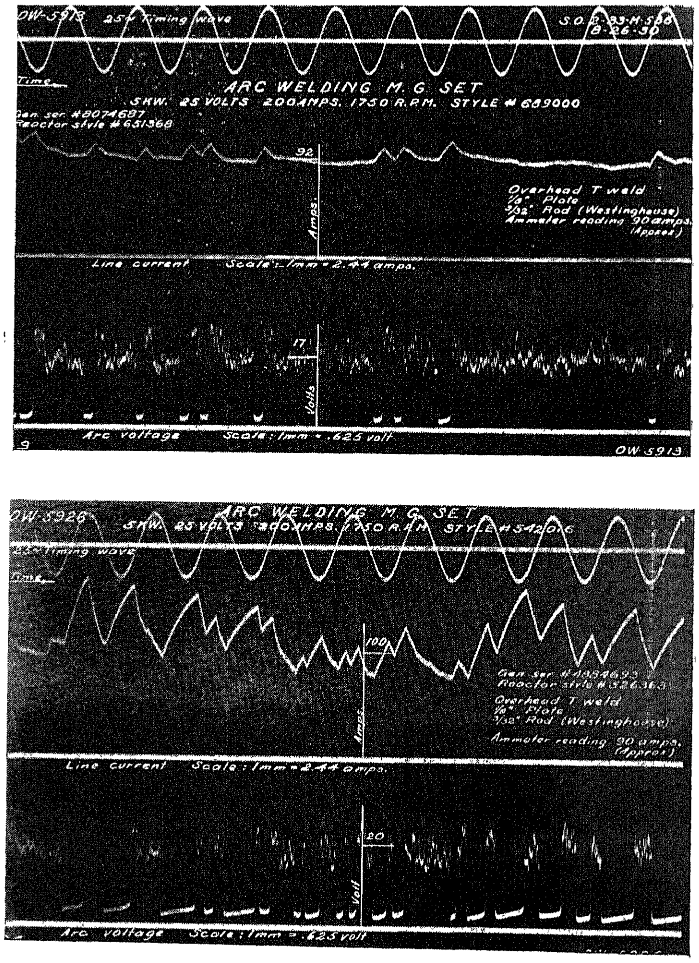

Fig. 8—Manual Welding, Overhead T Weld, 1/8-In. Plate, 3/32-In. Rod, 90 Amperes (Approx.)

Top, with transformer
Bottom, with reactor

the welding operator is in no danger of being shocked by the secondary voltage, because the welding circuit is electrically distinct and completely insulated from the secondary and the excitation circuit.

Curves *B* of Figs. 1 and 2 are plotted to scale from oscillograms taken on generators with this device, and curves *A* of the same figure show the action of the same generators with the reactor previously used.

Figs. 5, 6, 7 and 8 show oscillograms of the arc voltage and current during manual welding under various conditions with a 200-ampere welder. Each figure shows welding under identical conditions with the same welding generator both with the above device and with the reactor previously used. The reduction of current surge when the arc is short-circuited by a globule of metal is obvious, since in one case it is limited to the value shown by the volt-ampere curve of the generator, and in the other case it greatly exceeds this value.

A secondary benefit obtained from this device, in addition to the ease of striking and maintaining the arc, is a decrease in the porosity of the weld. Tests show that sudden increase of the current flowing in molten iron such as occurs in the molten globule of weld metal when it short circuits the arc, will cause sudden vaporization of parts of this metal. These vaporized parts expand tremendously and subsequent cooling of the globule leaves it full of holes where this vaporization has occurred and the gases have not been released. This sudden vaporization will also give the effect of explosion of the globule and parts of it will be blown away from the arc, thus decreasing the efficiency of deposit. Minimizing the increase of the arc current when the molten globule short circuits the arc, which occurs from 10 to 25 times per second, will, therefore, give a more dense deposit and allow a larger percentage of the electrode to be deposited as useful weld metal.

SUMMARY

The use of a d-c. transformer in the welding and excitation circuits of any single-operator inherently-regulating arc welding generator will (if properly designed with relation to the field circuit constants) produce ideal arc characteristics.

1. The arc obtained with such a combination of equipment is very easy to strike because all tendency for the electrode to freeze to the work is eliminated. This is due to the elimination of the usual short-circuit current surge of several times the normal welding current value.

2. The arc will not fail without apparent reason when disturbances take place in the arc circuit such as momentary slight increases in arc length which follow after each globule of molten metal leaves the electrode and is deposited on the work. This is due to the elimination of the usual current dip or drop when the arc is momentarily lengthened.

3. The deposited metal is surprisingly free from porosity. This is due to the elimination of current surges above the desired value which over-heat the welding materials and produce excessive amounts of occluded gas.

As a result of the first two features a novice can readily strike and maintain an arc on his first trial, whereas with other types of single-operator machines he will require considerable practise and coaching before he acquires the skill and technique of striking the arc, to say nothing of learning to weld satisfactorily.

Discussion

For discussion of this paper see page 688.

The Neutralized Welder—A Means of Controlling Transients

BY F. CREEDY*
Member, A. I. E. E.

THE characteristics of d-c. welders are of several different types. In the first place there are machines with straight line characteristics. Such machines usually have both a shunt and separately-excited coil, and by varying the currents in these two coils by means of the rheostat, the slope of the characteristic curve can be altered and the curves made to cross. In this way the welder may be adjusted so as to work at various currents. All these machines, however, have serious disadvantages, because the short-circuit current must be materially larger than the welding current and hence the ill-effect of shortening the arc or accidentally touching the work with the electrode are very much accentuated. Another type is the plain shunt machine working in the vertical portion of the characteristic. In this machine the short-circuit current is if anything less than the welder current, which, while better than having it larger than the welding current, is still not precisely what is required. In order to prevent the machine demagnetising when it is short-circuited, it is usual in plain shunt machines to use a certain amount of resistance in series with the arc circuit. While this has the advantage as pointed out below of steadying the operation of the set, it gives rise to a great reduction of efficiency, which should be avoided if possible. A combination of these two types may be made in which the machine regulates for almost exactly constant current over the range required during welding. This may take several forms and one of these has finally seemed most suitable for practical purposes. This is shown in Fig. 1. In this C represents the armature of the welding dynamo, which is supplied with a neutralizing coil N, for reasons which will be described below. On the commutator of this armature rest brushes B B and the neutralizing coil N is connected to a series coil S in such a manner that its ampere turns are opposite to those of the shunt coil T. The shunt coil T is connected across the brushes in series with the exciter E also shown as a neutralized machine, though this is less essential, the field of this exciter being regulated by means of the resistance R. Finally connected across the brushes is the arc P and the steadying reactance Q. The operation of such a circuit may readily be understood as follows: If we call E_1 the voltage across the main commutator and E_3 that across the exciter, then the current in the shunt coil T will be

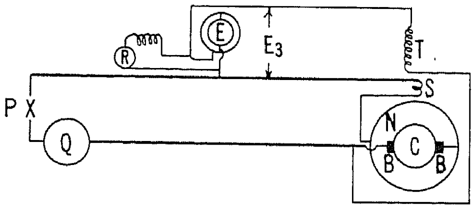

FIG. 1

$$\frac{E_1 + E_3}{R_3} = \frac{E_1}{R_3} + \frac{E_3}{R_3}$$

where R_3 is the resistance of T.

Thus such a circuit is equivalent to the two circuits shown in Fig. 6 in which T_1 is connected directly across the main commutator, the current in it being $\frac{E_1}{R_3}$, and V is connected across the exciter, the current in it being E_3/R_3. The ampere turns in the single coil T are clearly the same as those in the two separate coils T_1 and V, and hence the single coil is adopted as being the simpler. It is not difficult to prove that such an arrangement may have a constant current characteristic.

Let

K = armature volts per ampere turn (assuming no saturation during welding)
R = external resistance taking the place of the arc
r_1 = armature resistance
n_3 = shunt turns
n_1 = series turns
r_3 = shunt resistance
e_1 = commutator volts
e_3 = exciter volts
i_1 = line current
i_3 = shunt current

$$i_1 = \frac{K\left(\frac{n_3}{r_3}(e_1 + e_3) - n_1 i_1\right)}{r_1 + R}$$

Terminal voltage $e_1 = R\, i_1$.

$$i_1 = \frac{K\left(\frac{n_3}{r_3}(R\, i_1 + e_3) - n_1 i_1\right)}{r_1 + R}$$

Solving for i_1

$$i_1 \left\{ (r_1 + R) - K\left(\frac{n_3}{r_3} R - n_1\right) \right\} = K \cdot \frac{n_3}{r_3} \cdot e_3$$

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

$$i_1 = \frac{K \,.\, \dfrac{n_3}{r_3} \,.\, e_3}{r_1 + K\, n_1 + R\left(1 - \dfrac{K \,.\, n_3}{r_3}\right)}$$

If $\left(1 - \dfrac{K\, n_3}{r_3}\right) = 0$ or $K_{n3} = r_3$

$$\text{giving } i_1 = \frac{e_3}{r_1 + K\, n_1} \qquad (1)$$

variations in the external resistance or load R make no difference to the current i_1, or in other words the machine has a constant current characteristic.

Examining equation (1) above, we see that the line current, although it is independent of the external resistance or arc current, is directly proportional to e_3, that is, the voltage across the exciter. By varying this voltage, for instance by means of the field rheostat R in the exciter circuit, we change from one constant current characteristic to another. For each particular voltage across the exciter the machine regulates automatically for a particular current, which will be adapted to some particular size of electrode, and this regulation is effected solely by manipulating the single rheostat R.

We may generalize the above investigation a little.

We may write:

$$i_1\,(r + K\, n_1) + e_1\left(1 - \frac{K\, n_3}{r_3}\right) = K \,.\, \frac{n_3}{r_3} \,.\, e_3$$

remembering that $e_1 = R\, i_1$.

Consider how this may be plotted on a sheet of coordinate paper with e_1 as ordinate and i_1 as abscissa. For different values of e_3 the exciter voltage, it represents a family of parallel straight lines whose slope depends on the value of the coefficients of i_1 and e_1. For $r_3 = K\, n_3$ these lines become parallel to e_1, that is, we have a constant current characteristic as shown above. For other values of r_3 we have other slopes for the curves. In the ordinary operation of the machine, r_3 would not be varied, the operator would choose the slope of curve he preferred and stick to it for all sizes of electrode, still using only one rheostat for adjustment.

A study of the curves will make it clear how the constant current machine works with only one rheostat. We only obtain the constant current characteristic for a particular value of r_3 the resistance of the shunt circuit. Thus any variation of this means a departure from the constant current characteristic. Hence, the circuit in addition to its improved welding characteristic, has the further advantage of simplicity, since it requires only one rheostat. This is an immense point from the practical welder's point of view. For very low currents, below about 70 or 80 amperes, the striking voltage becomes rather low if the same number of turns is used on the series coil as with the higher currents.

In order to maintain a sufficiently high striking voltage for the arc, an extra tap in the series winding may be provided for use with currents below about this value, there being in all three welding terminals, *A*, *B*, and *C*. *A* and *B* are used for currents from say 80 to 300 amperes and *A* and *C* from 0 to 80 amperes. A single pole two-way switch can, of course, be used to change from *B* to *C*.

The first step in the investigation, therefore, consists of the simplification of the methods of excitation and elimination of unnecessary circuits. The circuit chosen has the practical advantage that the number of turns on the shunt field winding is approximately halved, and the size of the wire doubled, which conduces very strongly to reliability and long life.

STABILITY

Arc welders differ from one another in a quality which is vaguely called stability or steadiness. With some types of welder the arc apparently has much less tendency to splutter or fail than with others. These variations in characteristics are known to be due to the transient effects which occur during the change of circuit conditions and whose magnitude varies in different types of apparatus. Is it possible to define clearly what we mean by stability, and to reduce it to calculation?

What Occurs During "Spluttering." In order to throw light on the question of stability it will be worth while to discuss in considerable detail what happens in the welding dynamo during a "splutter" on the arc. This paper is naturally confined entirely to the electromagnetic occurrences, which are best studied from the mathematical point of view. A good beginning has been made in this direction by Mr. K. L. Hansen. (A. I. E. E. TRANS., Vol. 39, p. 1357.)

Suppose that welding is proceeding with a constant current, $i_1\, s$ say, and that some change in the circuit conditions occurs which we may represent by a change in the total resistance. For instance, the operator may momentarily shorten the arc or cause it to touch the work. If, for instance, the operator brought the electrode into permanent contact with the work, the current would change from the steady value of $i_1\, s$ to another steady value of $i_1\, p$. It is possible, as mentioned above, so to regulate the welding dynamo that it operates on a practically constant current, that is, so that the short-circuit current is practically the same as the welding current. In this case $i_1\, s = i_1\, p$, and as this is the simplest case for purposes of explanation, this is discussed first. The question to be investigated is; what happens during the change from $i_1\, s$ to $i_2\, p$? As is well known, owing to the presence of certain transient voltages in the circuit, the instantaneous currents during welding do not coincide with the steady currents, as shown by the characteristic curve, but on the other hand a sudden short circuit involves a momentary kick of current, which may be considerably greater than the steady short-circuit current. Can the value of this momentary kick of current be calculated to find out what factors influence it, so as to be able to reduce it to a minimum? It is clear that only by reducing this kick to a minimum can we get the steadiest possible operation. A study of the differential equations of the circuit shows that any change in circuit conditions sets up a transient wave of very low-frequency alternating current, superposed on the continuous current and rapidly dying away in accordance with a logarithmic decrement. The nature of this change is shown in Fig. 2 where on the left is shown the constant welding current $i_1\, s$ and on the right the constant short-circuit current $i_1\, p$. If, at the point C the arc is shortened or momentarily short-circuited, a kick is produced as shown, the current increasing to a value perhaps twice as great as the permanent short-circuit value, then decreasing to a value less than the permanent short-circuit value before it becomes steady. Theoretically several such waves might exist. Practically, the depression which follows the first kick has often been observed on the oscillograph, but owing to the slight unsteadiness which always exists in practise, preventing the current from showing us a perfect straight line on the oscillograph, the remaining waves of very small amplitude are difficult to detect. Such a wave may be represented by the formula $B_2\, (e^{-pt} \sin q\, t)$. Thus the wave is enveloped by the curves of logarithmic decrement $B_2\, (\pm\, e^{-pt})$ and touches it at the various points where $\sin q\, t = \pm 1$. Clearly the value of the current kick depends on the value of B_2, p and q.

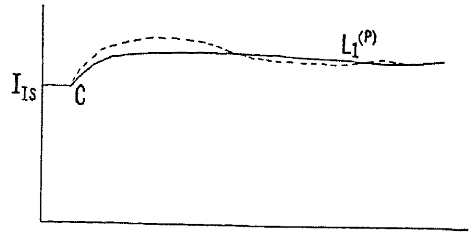

FIG. 2

These may be worked out and p and q represented by the following formulas (which neglect armature resistance).

$$p = \frac{r_1\, n_1 \left(L\, o + \dfrac{n_1^2}{\rho} \right)}{2\, L\, o\, \dfrac{n_1^2}{\rho}}$$

$$\left\{ -1 \pm \sqrt{1 - \frac{K}{r_1} \cdot \frac{4\, L\, o\, \dfrac{n_1^2}{\rho}}{\left(L\, o + \dfrac{n_1^2}{\rho} \right)^2}} \right\}$$

(See Appendix for definitions of symbols.)

For certain values of the constants the expression under the radical can clearly be negative, giving an oscillatory solution. Put $\dfrac{n_1^2}{\rho} = L_{11}$ and examine the expression

$$\frac{4\, L_{11}\, L\, o}{(L_{11} + L o)^2}$$

It is clearly zero if $L\, o = 0$ or if $L\, o = \infty$ and must therefore, have a maximum for some intermediate value of $L\, o$.

Differentiating and equating to zero we get

$$L\, o^2 - L_{11}^2 = 0$$

$$L_{11} = \pm\, L0$$

If we put $L_{11} = L\, o$ the maximum value of the expression is unity.

That this expression should be a maximum, that is, that the external self induction should be equal to that due to the main field, is clearly the worst condition for producing oscillations. If the external self induction is much *less* (an unpractical condition, since it does not permit of the current being kept steady) or much *more* (as it should be) then oscillations are impossible. Assuming this worst condition, then, we shall obtain oscillations if $\dfrac{K}{r_1} > 1$.

But this is the condition studied in the appendix in the case where no armature self induction exists, and it is there found to mean that the $I^2\, R$ loss in the field is *greater* than the power liberated by the decrease of the flux, so that some must be supplied by the rotation volts.

Oscillations can therefore always be prevented by increasing the resistance of the field.

But the condition $\dfrac{K}{r_1} = 1$ is that which gives rise to a constant current characteristic in a machine with shunt, separately excited, and reverse series coil. Hence we should endeavour to keep this quantity about unity, neither decisively greater nor decisively less, and rely for the prevention of oscillations on making the external self induction decisively greater than the internal. This may be done in two ways either, by making the external self induction large, or, the internal self induction *small* the latter being by far the better plan. Or of course, both may be adopted.

Summarising the investigation given in full in the Appendix, the main conclusions are as follows:

In the welding generator, two distinct problems exist.

1. To keep the current constant during rapid changes of conditions. This is solved by the use of reactance.
2. To design the machine so that the volts generated may vary rapidly to meet changes of conditions without much change of current.

It is essential to recognize that there are two distinct problems, and not one, and that it is only the second of these which requires the use of a special generator. Every change in voltage means a change in field strength.

1. Rapid changes require that field variations can take place with only a small inflow or outflow of magnetic energy to the field, that is, that *the magnetic energy stored in the field shall be as small as possible.*

2. For this purpose the generator should be designed as follows:

(a) With low saturation. This will be secured in any case, owing to the need for a high voltage on open circuit.

(b) The shortest possible air gap involving semi-closed rotor slots and laminated pole tips.

3. Low magnetic energy means very rapid increment or decrement of flux to meet changed conditions, as proved in the appendix.

4. It also means absence of oscillations if used in conjunction with a large external self-induction.

5. It is proved below that oscillations are due primarily to the fact that the magnetic energy stored in the field is about equal to that in the reactance. This condition should be avoided, and the energy stored in the reactance should be much greater than that stored in the field.

6. To store a large amount of energy, the reactance should have many turns, and a fairly long air gap, (it may of course be made of cast iron, or cast steel).

7. It is a common practise to use the reactance of the armature circuit due to the cross flux, for this purpose. This is bad practise for the following reasons:

(a) The necessity for a short air gap has just been pointed out in the interests of low stored field energy.

(b) If there are many armature turns with a short air gap, the cross flux will produce high saturation, tending to increase the stored field energy.

(c) If the pole arc is wide this cross flux will create sparking.

(d) If we reduce the pole arc to avoid this, we reduce the reactance formed by the cross flux and increase the reluctance of the field.

(e) A concentrated interpole winding diminishes the effectiveness of the machine as a reactance.

8. It is better therefore, to use a neutralizing winding or distributed interpole, completely abolishing the cross flux, and to have the reactance in separate form. The endeavor to combine the conditions for maximum reactance, and the opposite conditions for minimum field energy, means that neither set of conditions can be met satisfactorily. Only separate pieces of apparatus can satisfy both in the best possible manner.

9. This is true, even with a purely separately excited machine. If in addition there is a shunt coil, placing the reactance external to the tapping points of the shunt circuit, has further advantages. It shields the shunt winding from the effect of the rapid oscillations of the arc.

10. Owing to lag, caused by the self-induction of the main field, connecting the shunt winding directly across the arc, does not enable it to follow rapid variations, though it can follow fairly slow ones. It is therefore better to rely on the reactance to deal with rapid variations, and shield the shunt winding from them.

11. The lower the field energy, and the higher the field resistance the more rapid are the variations which can be followed.

12. The higher field resistance also tends to check oscillations but, it means more power required for excitation, and it is desirable in a machine with a shunt circuit, that the field resistance should be such as to give the constant current condition. It is therefore better to rely on other means, namely—

Low field energy, and high external reactance to avoid oscillations, and *not* to connect the shunt circuit directly across the arc. This last is perhaps a point of secondary importance, as to which some difference of opinion might be allowed.

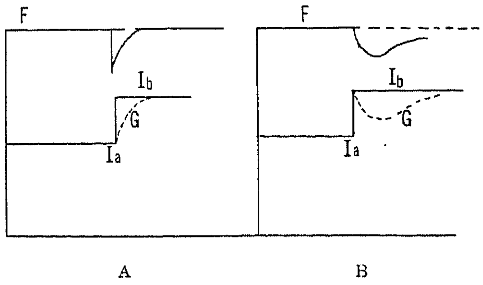

Fig. 3

13. Leakage self-induction in the shunt or separately excited circuit, prevents rapid response by permitting the shunt current to vary at a different rate from the armature current. Such leakage is the *sole* cause of these variations.

14. If there is no leakage between the series coil and the shunt or separately excited coil, all currents must vary at exactly the same rate so "sluggishness" is impossible.

The Effect of Field Leakage in the Welder

It may be of interest to discuss further the effect of field leakage in the welder. It has been previously known that currents in the separately excited coil produce a damping effect, and since this prevents rapid variation leakage has been thought advantageous. Their action may be most easily understood by assuming that the steady currents flow at all times and transient currents are superposed on them whenever the flux alters.

Let Fig. 3 refer to a separately excited machine in which a constant voltage is applied to the separately excited coil and flux change is due entirely to changes in the current in the series coil. If the series current suddenly changes from $I\,a$ to $I\,b$, then for the case of no leakage, the change in ampere turns is instantly masked by transient currents in the separately excited coil whose current is shown at F so that at the first instant there is no resultant change of flux. These transient currents die away at a rate which may be made very great by sufficiently reducing the energy stored in the main field. The resultant curve of the ampere turns is shown at G in Fig. 3A. Using leakage the damping currents will not be so great *and will die away more slowly* as shown in the curve F of Fig. 3B the resultant ampere turns being shown by the dotted curve G of Fig. 3B thus giving rise to an oscillation of a detrimental nature. The physical meaning of this damping current is that even if the series ampere turns made a sudden change, the flux cannot change until *time* has been allowed for the necessary energy to flow in and the less the energy the less the time.

The desirability of rapid increment or decrement is of course, the foundation for the well known and perfectly sound rule that the time constant of the field circuit should be made large in a welder.

It cannot be decided which is best of the two types of welder possible until it is clear as to what characteristics a welder should have. These types are:

1. That just discussed which may be called the "series type."
2. That in which there is a shunt coil having a variable instead of a constant e. m. f. impressed on it, the variations of the flux being due to the variations in the current in this shunt coil rather than to variations in the main current which in this type of welder might be maintained absolutely constant without impairing its operation.

In the writers view the conditions to be met with by a welder are as follows:

Current:

1. Welding current must not rise too much even on short circuit or it will cause "spluttering."
2. Current must not fall too rapidly with increasing arc length or bad welding will result as well as excessive voltage required to maintain the arc.

Voltage:

3. Voltage must as rapidly as possible assume a fixed minimum value as soon as the electrodes are separated and never fall below it or the arc will go out. This minimum voltage is about 38.5 (according to Mrs. Ayrton) for the carbon arc and probably 15 to 20 for the iron arc.
4. Voltage must be capable of *rapid* increase above this value to take care of variations of arc length. On open circuit it must be capable of being doubled or more.

Frequency. There are two types of variations high frequency and relatively low frequency.

In the writers opinion the reactance takes care of the high frequency variations and the machine of the relatively low frequency variations that no practical reactance could deal with.

The conditions connected with the current mean that the welding current should be kept nearly constant a slight reduction only being allowed to permit the reactance to produce a voltage tending to keep the arc going. Hence a small change in current should produce a comparatively large change in voltage. The voltage generated by the machine should follow that required by the arc as rapidly as possible.

In order to obtain a large change of voltage with a small change of current, and particularly if there are independent changes of voltage and current it is better to have some means of changing the voltage (such as a shunt coil) independently of the current.

Without a shunt coil there is only one machine voltage corresponding to every current. Every time the current assumes a particular value the voltage assumes its corresponding value. Thus a welder with shunt coil, separately excited, and reverse series coil such as that discussed above is likely to be more sensitive and responsive than one without shunt coil if there are no countervailing disadvantages.

It seems in fact, inconsistent to take elaborate precautions to reduce current variations by using a reactance and then rely on them to produce voltage variations.

In spite of the above *prima facie* advantages the use of the shunt coil has fallen into disrepute. A common opinion is that "it is notoriously sluggish" and one cannot help agreeing to this for machines constructed as usual. But this is due to the fact that in machines of the usual construction the shunt coil has a heavy magnetic leakage, combined with high stored magnetic energy, and if this magnetic leakage and high stored energy can be eliminated the use of such a coil can be rehabilitated. This will make a better welder than one without a shunt coil.

Leakage in the separately excited coil as pointed out has not the same effect as leakage in the shunt coil, and in fact may even be beneficial by reducing damping currents. (However, resistance seems preferable for this purpose, as is the common practise.)

Returning therefore to the machine with shunt coil instead of or as well as the separately excited coil, it is clear that variations of the current in it will tend to produce damping currents in the series winding. But the series winding already has a very large self-induction connected in series with it, so that any tendency it may have to damp out variations of flux due to variations of shunt current must be small. If, however, there is a distinct separately-excited winding it will tend to damp out variations due to the shunt winding as already discussed. It is to this cause the difference between the two oscillograms of Figs. 4 and 5 is attributed. The oscillogram in which there was a distinct separately excited winding (Fig. 5) shows larger current rise on short circuit than Fig. 4 the one in which the arrangement of Fig. 1 is used, which obviates these damping currents altogether.

It is true also that damping currents will be induced in such a shunt coil by variations of the series ampere turns just as if the coil were separately excited. But such damping currents may be reduced to a minimum by keeping the welding current nearly constant which can be better effected with this type of machine than with the series type. As stated above if the field energy is small they die away so rapidly that they cause little harm. This can be shown from the differential equations.

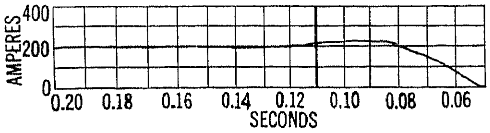

FIG. 4

If the shunt coil however is to produce an armature voltage proportional to the e. m. f. applied to its terminals the *shunt current must be proportional to this e. m. f.* that is, the self induction of the shunt circuit (due to the main and the leakage flux) should be reduced to a minimum while the resistance remains high. The self-induction due to the main flux as already shown should be minimized by reducing the field energy and it is also important to eliminate the leakage flux.

The use of series resistance to eliminate high frequency oscillations is the common practise, and it is clear that it is effective by increasing the rate of increment or decrement of the currents and fluxes. But this is simply because adequate precautions are not taken in the welder of ordinary design to reduce the field energy and when it is reduced to a minimum the decrement or increment of the currents and fluxes is quite sufficiently rapid without the use of series resistance. The welder described above has no series resistance, and gives an over-all efficiency between the watts input on the a-c. side as registered on the a-c. wattmeter and the actual energy going into the arc of 60 per cent. This includes all the a-c. losses, all the d-c. losses, exciter losses, and losses in the reactance and cables. It seems therefore, that series resistance is wholly needless if the welder is properly designed.

STABILITY RATIO

It is now possible to answer the question asked above, as to how it is possible to define clearly what is meant by stability and to reduce it to calculation and measurement.

It is proposed that the stability of all welding apparatus should be defined in terms of what may be called the *stability ratio*, and that the stability ratio should be defined and measured by the following standard test.

1. Load the welder up to its maximum load, say 30 volts, 200 amperes on a resistance.
2. Arrange a switch so that the resistance may be instantly short-circuited, thereby instantly short-circuiting the terminals of the machine through the steadying reactance.
3. Measure the maximum current kick produced, (say 600 amperes by way of example) when the short-circuiting switch is closed. The *stability ratio* is the ratio of this maximum kick to the normal full load current. In the example given, where the kick is 600 amperes and the full load current 200 amperes, the stability ratio is three. When this stability ratio has a small value the welder will operate steadily.

There is no serious difficulty in measuring this kick by well-known methods, with the instruments ordinarily found in a test room. In order to do this, remove the glass front from an ammeter, preferably of the moving coil or dynamometer type and whose moving system has a low moment of inertia. Hold the needle by means of a pencil point or otherwise at approximately the value which the current is expected to have, say at about 600 amperes. Short-circuit the machine and if the current is less than the value indicated by the pointer, it will continue to rest against the pencil point which prevents it from moving back to zero. If the current is greater, the needle will momentarily flutter away from it to a greater or less extent. If there is no flutter it indicates that the test should be repeated with the needle held opposite a lower reading on the scale, say 500 amperes, and if the flutter is vigorous it indicates that the needle should be held opposite a higher reading on the scale, say 700 amperes, the test being repeated until a point is found where the needle only just shows a faint indication of movement. By this means, using only standard d-c. instruments, the instantaneous short-circuit current may be measured with very fair accuracy.

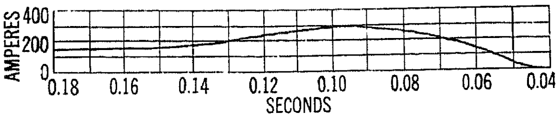

FIG. 5

In Figs. 4 and 5 oscillograms are shown of the neutralized welder switched on from open circuit to full load (a severer test than that stated above). Fig. 4 shows the machine arranged with shunt and separate excitation combined as in Fig. 1 while Fig. 5 shows it with shunt circuit and separately-excited circuit independent. The reason for the difference between these two oscillograms is suggested in the paper.

It is desirable to describe in detail the methods of construction adopted to take practical advantage of the theoretical results just obtained. In order to reduce the magnetic leakage between the shunt and series circuit, it is necessary to reduce the self induction of the armature circuit to a minimum. The most effective means of doing this is by making use of a neutralizing winding connected in series with the armature circuit, which may be constructed in one of several manners, as for instance those shown in Figs. 6 and 7. Fig. 7 shows a welding dynamo with a uniform airgap all round the circumference. The armature A is fitted with a commutator C on which rest brushes B B, and is surrounded by a stator having in the figure shown, 24 slots D. These slots contain a neutralizing winding N, connected in series with the armature, the connections not being shown, and such that the ampere conductors in its slots are nearly equal and opposite to those of the armature, being made in most cases slightly greater in order to produce the effect of an interpole. In addition to the neutralizing winding, certain of the slots contain a series winding S S, also connected in series with the armature, and a shunt winding T connected across the terminals of the armature through a suitable regulating resistance R. In addition to this there is frequently a separately exciting winding V, which is excited from some external source and controlled by a regulating resistance R_1. As explained in detail later, it is to be desired that the series coil and the shunt coils lie in the same slots, so that the mutual induction between them may be as high as possible, though since both of these excite the same magnetic circuit, it must in any case be high. The series winding is so connected as to oppose the action of the shunt winding and the separately-excited winding, where this exists. The armature circuit is then completed through the steadying reactance Q and the arc P.

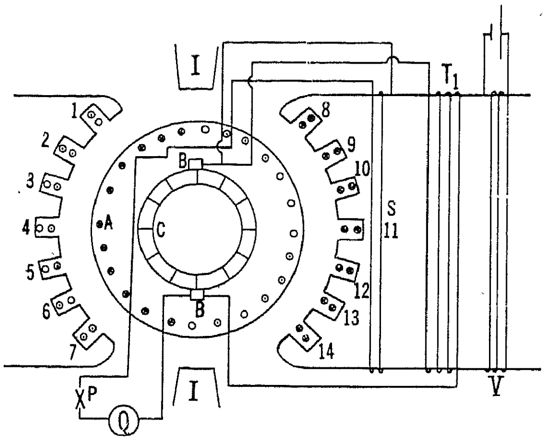

Fig. 6

Another construction of machine is that shown in Fig. 6, in which the machine is of the ordinary salient-pole type, fitted with interpoles and having slots made in the pole face. In this machine A, B B and C represent the armature, brushes, and commutator as before, while D represents the slots, which again contain a number of ampere conductors almost equal and opposite to those in the armature, but slightly greater as a rule. The interpole in such a machine need not be wound, as the ampere conductors, due to the winding in the slots will be sufficient to excite it. Alternatively, the ampere conductors in the slots may be as exactly as possible equal and opposite to those in the armature, while a small number of ampere turns are placed on the interpole, in order to produce the commutating flux. The use of a neutralizing winding permits the airgap to be reduced to a value similar to that used in induction motors, thus minimizing the field energy. The various field windings, S, T, and V are the same as before, similarly connected, but they are now, of course, wound on the salient poles in the form of a spool in the usual manner.

It has been pointed out above that in order to eliminate transient e. m. f.'s the leakage between the shunt and armature circuits must be reduced to a minimum. By far the greatest source of this leakage in machines of ordinary construction, is the armature flux, which is eliminated by the use of a neutralizing winding, and when this has been eliminated the remaining leakages are of much less importance. However, in order still further to reduce the magnetic leakage between the series and shunt coil, the series and shunt coils may be wound in alternate layers, making up a combined coil containing both windings. Since the number of turns on the shunt coil will usually be large, and that on the series coil small, it will be desirable, to wind the series coil with a number of conductors in parallel. Suppose, for example, that the number of turns on the shunt coil is 2,400 and that on the series coil six, the shunt coil might for instance conveniently be arranged with 80 conductors per layer and 30 layers.

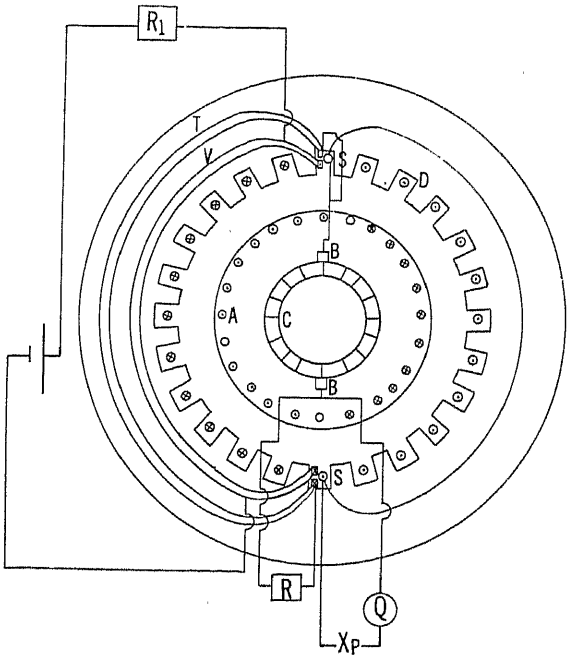

Fig. 7

It is not necessary that there should be a layer connected to the series winding adjacent to each layer of the shunt winding. It will be sufficient, for instance, if a layer of the series winding be interleaved with, say, every 10 layers of the shunt winding. In order to bring this about, the series winding may be wound in two parallels, each parallel containing six turns and being disposed in a single layer. This is shown in Fig. 8, in which *S S* show the two layers of the series winding which are in parallel and *T T T* show three sections of the shunt winding, each containing 10 layers and 80 conductors per layer, between which the layers of the series winding are placed. Where there is a separately exciting winding it may take the place of half the shunt windings in the manner shown dotted at *V*. Where the shunt and series windings are contained in slots as in Fig. 7, it is desirable though not essential that the series and shunt windings should be contained in the same slots as shown in Fig. 7, where again *S S* represent the series winding and *T* the shunt winding, a portion of which may be replaced by a separately exciting winding shown at *V*. It is further desirable that the whole of the shunt windings should be carried in a single slot per pole, in which case the winding *T*, Fig. 7 should be regarded as being the whole of the shunt winding, corresponding to a particular pole. In a machine with slotted stator the number of shunt turns may be very much less than in a machine of the ordinary type, and consequently, the number of layers of the shunt winding measured circumferentially will be small. Hence it is not difficult to bring the series and shunt windings into very close proximity.

Of the various arrangements possible, that shown in Fig. 7 in which the series windings occupy one side and the shunt windings the other, is somewhat preferable to one in which the series windings occupy the top and the shunt winding the bottom, in cases where the circumferential width of the slot is less than its depth.

The usual plan of operation on a-c. circuits is, of course, to couple the welding dynamo to a standard a-c. motor forming a motor-generator set, and this is the source of much of the first cost of the set. To eliminate this, the next step is to combine the welding dynamo and a-c. motor into one structure in which the magnetic circuit carries two distinct fluxes of different pole number. The a-c. motor may be either of the synchronous or induction type, though the latter is usually adopted in the sizes here described. To explain the principle of the machine clearly it will be convenient to consider the synchronous type first. Consider a machine carrying four windings—two on the stator and two on the rotor, which we may call *a*, *b*, *c*, *d*. Suppose, for instance that of the rotor windings *c* and *d*, *c* is arranged for six-pole and *d* is a two-pole winding, and of the two stator windings *a* and *b*, *b* is arranged for two poles and *a* is a *six-pole* field winding. Suppose direct current to circulate in the two-pole field winding *d* on the rotor and suppose the machine to be running at the synchronous speed of this two pole winding, namely 36,00 r. p. m. on 60 cycles. Clearly the two-pole winding can induce no voltage in the rotor winding *c* since it revolves with it and hence does not cut it. Similarly, if direct current circulates in the field winding *a* on the stator, clearly this direct current can induce no e. m. f. in the other winding *b* on the same core, since both windings are fixed. This winding *a* induces e. m. f.'s only in winding *c* and winding *d* induces e. m. f.'s only in winding *b*. The flux due to winding *d*—stationary with respect to the rotor—will revolve in respect to the stator, while that due to winding *a* cuts the rotor and is stationary with respect to the stator.

If winding *b* is supplied with 60-cycle single-phase or polyphase current the machine will work as a synchronous motor, and if winding *c* is joined to a commutator a continuous e. m. f. will be produced across that commutator.

Winding *d* cannot induce any e. m. f.'s in winding *a* since the two are wound for dissimilar numbers of poles, and for the same reason winding *a* cannot induce any e. m. f.'s in winding *d*.

In practise the use of two windings on the rotor of such a machine is inconvenient. This is rendered unnecessary if the single rotor winding attached to the commutator is given a pitch which is equally adapted to operate on six or two poles, such, for instance, as the full two-pole pitch which is also equally well suited to six poles, for in both cases north and south poles are diametrically opposite. If this winding is tapped off at points differing by two pole pitches of the six-pole field, that is, at three points 120 deg. apart, and these points brought out to sliprings, no e. m. f.'s will be induced by the six-pole flux between any two of these sliprings. If, therefore, a pair of them is short-circuited and direct current inserted between the short-circuited pair and the remaining ring, a two-pole excitation of the rotor may be brought about without requiring the use of a separate winding, and in this way the six-pole and two-pole windings may be combined. Means exist for combining the two separate stator windings also, but it is thought sufficient to consider that separate d-c. and a-c. windings are employed on the stator, as is actually the case.

The alternating side of the machine may also be caused to operate as an induction motor if the points which before were connected to sliprings are short-circuited together. Under these circumstances currents of slip frequency will flow through the short-circuited connections and the sliprings may be eliminated. In this case, of course, the short-circuited points need not be limited to three so long as care is taken that only points two pole pitches apart of the six-pole field are connected. This type of machine, owing to the absence of sliprings and of the necessity of direct current excitation of the rotor, is the most suitable for small sizes.

In the induction type of machine it is no longer true to say that one flux is stationary with respect to the rotor and the other with respect to the stator. The flux which is now generated by the alternating current winding on the stator is in slow rotation with respect to the rotor also. Of course, the machine is not limited to two and six poles, but can be built for many other ratios of poles, though for welding purposes this ratio is convenient.

It will next be of interest to describe the practical construction of a machine arranged for two and six poles, and converting from three-phase to direct current.

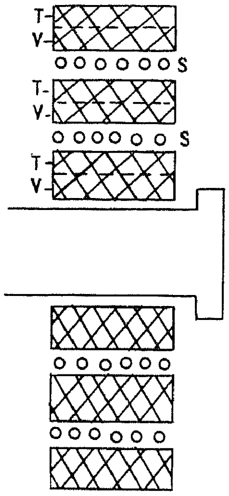

FIG. 8

A machine constructed as in Fig. 7 may readily be modified so that it acts as a converter from alternating current to direct current as well as a welding dynamo pure and simple. This modification is shown in Fig. 9 where the windings *S*, *T*, and *V* of Fig. 7 occupy slots 1, 5, 9, 13, 17 and 21, the welding dynamo being a six-pole machine. The neutralizing winding *N* occupies the rear portions of the slots, being arranged as a two-circuit drum winding, while those portions of the slot next to the airgap contain the alternating current windings *L* arranged for two poles, which may, for instance, be of the standard three-phase kind. The armature *A*, the commutator *C* and the brushes *B B B B B B* are shown in the figure (though not lettered) there being equalizers *F* joining points on the commutator one-third of the circumference apart in order to form circuits for the current induced by the a-c. windings, while at the same time acting as equalizers for the welding dynamo. For the sake of clearness the rotor winding is shown as a ring winding. If the brushes are, connected, for instance, in series with the tappings Y_1 Y_2 on the interpole winding, it will act purely as a neutralizing winding, but if they be connected as shown to certain other tappings, Y_1' Y_2' the m. m. f. of the neutralizing winding will have a certain component along the axis of the field circuit, and hence this winding will act both as a neutralizing winding and a series coil combined. The shunt winding *T* is connected in series with an exciter *E* (of the same type as shown in Fig. 1) across the main commutator. The coil *T* is thus excited by the voltage of the main machine and the exciter in series. The circuit is then completed through the arc *P* and the steadying reactance *Q*.

It is sometimes convenient in order to improve the starting characteristics to connect a part of the field winding *T* into the a-c. winding. For this purpose the winding *T* should be made up of three coils *a*, *b*, and *c*, each spanning one-half of the circumference, the coil *a* lying, for instance, in slots 1 and 13, the coil *b* in *c* and 21, and the coil *c* in 17 and 5. These are all connected in series to form the shunt coil *T*. To employ them for starting purposes, they may be disconnected from the direct current side of the machine and the ends short-circuited. If taps are brought out from between the individual coils, they will form a mesh as shown in Fig. 10 and this may be connected into the star point of the a-c. winding, where it will be equivalent to an added number of turns and will improve the starting characteristics. Instead of the whole of the shunt coil being so treated, only a portion may be used for this purpose, the remainder being open-circuited during the process of starting. In many cases the short circuit between the coils *A* and *C* may be omitted, the shunt coil being connected to the d-c. circuit in the normal manner and the ends of the three-phase winding simply tapped on to connections intermediate between the coils, *A*, *B*, and *C*, whether these form the whole or only part of the shunt winding.

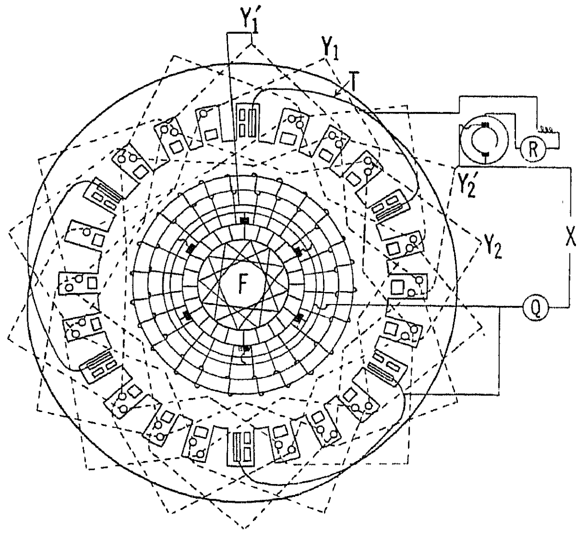

FIG. 9

Such a machine is usually arranged to operate at 3,400 r. p. m. on 60 cycles since this is the most economical speed. In Fig. 11 which shows a cross section of this type of machine are shown what are known as "fan risers." These consist of a number of blades attached, say, to every tenth segment, and at a high speed of rotation this produces a very powerful current of air which is extremely effective in cooling the commutator, so much so in fact, that in spite of its carrying a current of 300 amperes, the temperature rise is seldom more than 20 deg. fahr. to 40 deg. fahr.

The starting gear is in many cases that appropriate to a squirrel-cage induction motor. For sizes suitable for most welding operations three-phase star-delta starters are employed, somewhat modified where the connections of Fig. 10 are employed, while in sizes exceeding this one may employ an auto-transformer starter. The apparatus completed and ready for operation is seen in Fig. 12. Owing to the short distance between bearings the machine may be readily designed with vertical shaft. In this set as will be seen the ammeter and voltmeter are at the top coming next the rheostats (two in this case). Below this comes the star-delta starter and then comes a single-pole two-way switch arranged to change the tap on the series winding, as desired to weld, from 0-80 amperes or from 80-200 amperes. Below this are the two welding terminals and then the large cast-iron reactance. The small exciter is seen mounted on top of the generator.

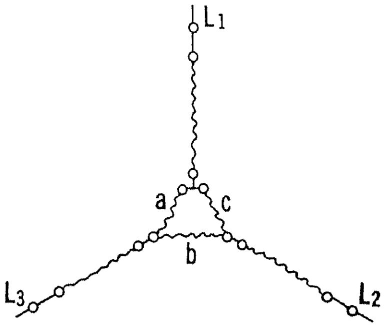

Fig. 10

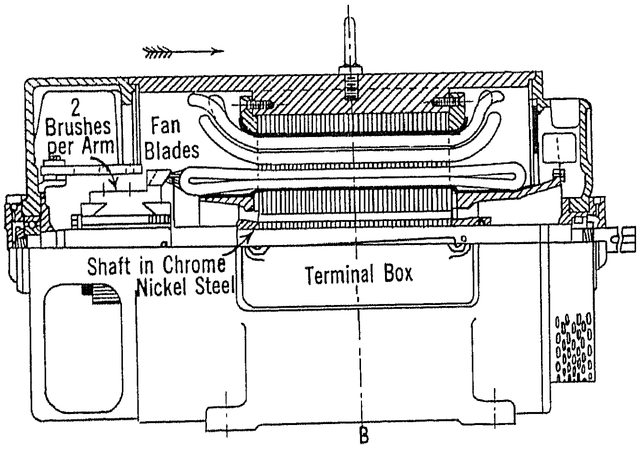

Fig. 11

Appendix I

A mathematical study of a series of simple cases of the welding generator is given in this appendix, each one elucidating some particular aspect of the general problem and by means of such studies we endeavor to obtain a clear physical grasp of the whole of the phenomena.

In the first place, Hansen treats the arc as if it were a resistance R in his paper referred to above.

The short period oscillations in voltage across the arc are due in the main to varying length l, if the mean current is nearly constant. It is well known that the voltage across an alternating (iron) arc is nearly constant during a half period, while the current rises from zero to a maximum and falls to zero again and then abruptly reverses when the current changes sign. This should be sufficient proof that the voltage across the arc is nearly independent of the current.

Thus instead of representing the arc by resistance R it will be represented either by a constant e. m. f. E or when we wish to study its variations by an e. m. f. $E_0 + E_1 \sin p\,t$. Clearly the sine wave could be replaced by any Fourier's series, but for the sake of simplicity this has not been used. The type of circuit to be studied will be that previously described in which the shunt is connected across the main commutator and exciter in series. By choosing this circuit we obtain differential equations of the second order only, instead of those of the third order, obtained when there are three independent circuits, as Hansen points out, and this is a very great simplification.

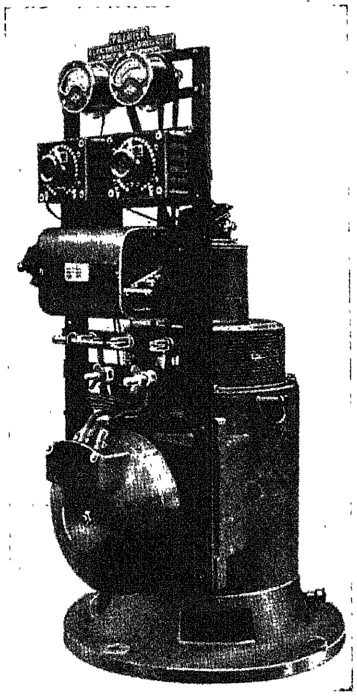

Fig. 12

It is generally admitted that the resistance of the armature circuit (unlike that of the field circuit) plays no essential part in the phenomena. Accordingly this is assumed zero throughout for the sake of simplicity.

If

$$K = \frac{\text{Total armature condrs.} \times \text{speed (r.p.s.)}}{\rho \times 10^8} = \frac{Z\,S}{\rho \times 10^8}$$

Let ρ be the reluctance of the main magnetic circuit (assumed constant).

i_3 the shunt current

i_1 the armature current

n_3 the number of shunt turns

n_1 the number of series field turns

$K\,(n_3\,i_3 - n_1\,i_1)$ the e. m. f. generated in the armature.

r_0 the shunt field resistance per turn (so that $r_0\,n_3$ is the resistance of the shunt)

$a_3 = n_3\,i_3$

$a_1 = n_1\,i_1$

L_{33} coefficient of self induction of the field circuit due to the main flux

L_{31} coefficient of self induction of the field circuit due to the field leakage flux

L_{11} coefficient of self induction of series field circuit due to the main flux

Lo coefficient of self induction of the series field circuit due to the field leakage flux

M coefficient of mutual induction between field and armature circuit

e_1 exciter volts (assumed constant).

Then the main flux will be

$$\phi = \frac{n_3\,i_3}{\rho} - \frac{n_1\,i_1}{\rho} = \phi_3 - \phi_1 \text{ say} \qquad (1)$$

The e. m. f. induced in the shunt circuit.

$$n_3 \frac{d\,\phi}{d\,t} = n_3 \frac{d}{d\,t}\left(\frac{n_3\,i_3}{\rho} - \frac{n_1\,i_1}{\rho}\right) = L_{33}\frac{d\,i_3}{d\,t} - M\frac{d\,i_1}{d\,t} \qquad (2)$$

Thus

$$L_{33} = \frac{n_3^2}{\rho} \text{ and } M = \frac{n_3\,n_1}{\rho}$$

The e. m. f. induced in the series field circuit will similarly be

$$n_1 \frac{d\,\phi}{d\,t} = n_1 \frac{d}{d\,t}\left(\frac{n_3\,i_3}{\rho} - \frac{n_1\,i_1}{\rho}\right) \qquad (3)$$

$$= M\frac{d\,i_3}{d\,t} - L_{11}\frac{d\,i_1}{d\,t} \qquad (4)$$

Thus

$$L_{11} = \frac{n_1^2}{\rho}$$

Let us assume first of all that the magnetic leakage is zero then the differential equation of the main circuit is—

$$\frac{n_1}{\rho}\;\frac{d\,(a_3 - a_1)}{d\,t} + K\,(a_3 - a_1) = E \qquad (5)$$

and that of the field circuit

$$\frac{n_3}{\rho}\;\frac{d\,(a_3 - a_1)}{d\,t} + r_0\,a_3 = e_1 + E\,. \qquad (6)$$

In symbolic form this may be written

$$\left(\frac{n_1}{\rho}\,D + K\right)a_3 - \left(\frac{n_1}{\rho}\,D + K\right)a_1 = E \qquad (7)$$

$$\left(\frac{n_3}{\rho}\,D + r_0\right)a_3 - \frac{n_3}{\rho}\,D\,a_1 = e_1 + E \qquad (8)$$

Eliminating a_1

$$\frac{n_3}{\rho}D\left(\frac{n_1}{\rho}\,D+K\right)a_3 - \left(\frac{n_3}{\rho}\,D+r_0\right)\left(\frac{n_1}{\rho}\,D+K\right)a_3$$

$$= -\,K\,(e_1 + E) \qquad (9)$$

or on cancelation

$$\left(\frac{n_1}{\rho}\,D + K\right)a_3 = \frac{K\,(e_1 + E)}{r_0} \qquad (10)$$

the solution of which is

$$a_3 = C_3\,\epsilon^{-\frac{K\,\rho\,t}{n_1}} + \frac{e_1 + E}{r_0} \qquad (11)$$

Eliminating a_3

$$-\left(\frac{n_3}{\rho}\,D+r_0\right)\left(\frac{n_1}{\rho}\,D+K\right) + \frac{n_3}{\rho}\,D\left(\frac{n_1}{\rho}\,D+K\right)$$

$$= r_0\,E - K\,(e_1 + E) \qquad (12)$$

or on cancelation

$$\left(\frac{n_1}{\rho}\,D + K\right)a_1 = \frac{K\,(e_1 + E)}{r_0} - E. \qquad (13)$$

The solution of which is

$$a_1 = C_1\,\epsilon^{-\frac{K\,\rho t}{n_1}} - \frac{E}{K} + \frac{e_1 + E}{r_0} \qquad (14)$$

Clearly the two constants C_3 and C_1 are not independent. To determine them the most convenient way is to assume

$$a_3 - a_1 = 0 \text{ when } t = 0$$

This gives

$$C_3 - C_1 = -\,\frac{E}{K} \text{ and } \frac{d\,(a_3 - a_1)}{d\,t} = -\,\frac{K\,\rho}{n_1}\,(C_3 - C_1)$$

$$= \frac{E\,\rho}{n_1}$$

Substituting in equation (6)

$$\frac{n_3}{n_1}\,E + r_0\,C_3 + e_1 + E = e_1 + E$$

$$C_3 = -\,\frac{n_3}{n_1}\;\frac{E}{r_0} \qquad (15)$$

$$C_1 = \frac{E}{K} - \frac{n_3}{n_1}\frac{E}{r_0} \qquad (16)$$

Thus finally

$$a_3 = \frac{e_1}{r_0} + \frac{E}{r_0}\left(1 - \frac{n_3}{n_1}\epsilon^{-\frac{K\rho t}{n_1}}\right) \qquad (17)$$

$$a_1 = \frac{e_1}{r_0} + \frac{E}{r_0}\left(1 - \frac{n_3}{n_1}\epsilon^{-\frac{Kpt}{n_1}}\right) = \frac{E}{K}\left(\epsilon^{-\frac{Kpt}{n_1}} - 1\right). \qquad (18)$$

$$a_3 - a_1 = \frac{E}{K}\left(1 - \epsilon^{-\frac{Kpt}{n_1}}\right). \qquad (19)$$

The steady condition being as one would expect that

$$K(a_3 - a_1) = E \qquad (20)$$

$$a_3 = \frac{e_1 + E}{r_0} \qquad (21)$$

It will be worth while to devote some considerable attention to working out a clear physical interpretation of these equations.

In the first place all terms of second order cancel the equations reducing to the first order. Consequently all the currents (or ampere turns) a_3, a_1, and $a_3 - a_1$ vary together *at exactly the same rate* which is measured by

$$\epsilon^{-\frac{Kpt}{n_1}}$$

This corresponds physically to the fact that there is only one flux, the main flux of the machine which can clearly only vary at one rate. Thus so long as the magnetic leakage is zero, the "sluggishness" of one coil (which means essentially that the current in it rises more slowly than in the others) is impossible.

Substituting the above values in the equations themselves we get

$$\frac{n_1}{\rho}\frac{d(a_3 - a_1)}{dt} + K(a_3 - a_1) = E \qquad (22)$$

$$E\,\epsilon^{-\frac{Kpt}{n_1}} + E\left(1 - \epsilon^{-\frac{Kpt}{n_1}}\right) = E \qquad (23)$$

This shows that starting from $t = 0$ when the flux is zero. *The rate of increase of the flux is such that that portion of the terminal E M F which cannot be supplied by the rotation voltage is supplied by the transformer voltage induced by the flux variation.*

$$\frac{n_3}{\rho}\frac{d(a_3 - a_1)}{dt} + r_0 a_3 = e_1 + E \qquad (24)$$

$$\frac{n_3}{n_1}E\,\epsilon^{-\frac{Kpt}{n_1}} + e_1 + E - \frac{n_3}{n_1}E\,\epsilon^{-\frac{Kpt}{n_1}} = e_1 + E. \qquad (25)$$

The field current may be considered as made up of two components, the steady component $\frac{e_1 + E}{r_0}$ *regarded as always flowing and whose resistance drop balances the applied E M F and a transient component flowing in the opposite direction, whose resistance drop balances the transformer voltage induced by the variation of the main field.*

The series field ampere turns (a_1) *and therefore the armature current takes up such a value as to balance the field ampere turns* (a_3) *except for such portion as is needed to produce the rotation voltage necessary to balance the resultant of the terminal voltage and the induced or transformer volts.*

The short-circuit conditions ($E = 0$) are worth a brief examination. We then have

$$a_3 = a_1 = \frac{e_1}{r_0},\ a_3 - a_1 = 0 \text{ at every instant.}$$

That is to say the flux is infinitesimally small being just enough to urge a sufficient current round the (zero) resistance of the short-circuited circuit to produce a number of ampere turns in the series field coil exactly equal and opposite to the shunt field ampere turns. Since there is no field at any instant, there is no stored magnetic energy and no reason why the currents should not instantaneously assume their final values.

If at the moment of short circuit the flux has a particular value $\phi = \frac{A}{\rho}$ and the ampere turns therefore a particular value A, then since at the instant of short circuit the only e. m. fs. in the armature circuit are—

1. The rotation voltage.
2. The transformer voltage due to the decrement of the flux. At the moment of short circuit the flux still retains the value $\phi = \frac{A}{p}$ and hence its rate of change must instantly assume a value such that the transformer voltage can balance the rotation voltage according to the equation.

$$\frac{n_1}{\rho}\frac{d(a_3 - a_1)}{dt} + K(a_3 - a_1) = 0$$

or

$$\frac{d(a_3 - a_1)}{dt} = -\frac{K\rho}{n_1}(a_3 - a_1)$$

of which the only solution is

$$a_3 - a_1 = (C_3 - C_1)\,\epsilon^{-\frac{Kpt}{n_1}}$$

This is the physical explanation of the value of the exponent of ϵ and why it does not contain the shunt resistance. The actual energy of the field is absorbed by the shunt resistance at the rate determined as above.

$$\frac{K\rho}{n_1} = \frac{Z\,s}{n_1} \times 10^{-8}$$

This becomes greater and the decrement therefore faster, the fewer the turns n, which for a given flux and therefore a given $Z.s$ means the less the reluctance. Hence a low field reluctance means a machine capable of rapid adjustment. The power represented by the product of the rotation volts and armature current goes to maintain the flux by transformer action and is ultimately dissipated in resistance or pumped into the exciter.

SELF INDUCTION IN THE FIELD CIRCUIT

The Cause of Sluggishness. Suppose there is a leakage self induction L_{31} in the field circuit $\left(\text{so that } \frac{L_{31}}{n_3}\frac{d\,a_3}{d\,t} = L_{31}\frac{d\,i_3}{d\,t}\right)$.

The equations will now be

$$\frac{n_1}{p}\frac{d\,(a_3 - a_1)}{d\,t} + K\,(a_3 - a_1) = E \qquad (28)$$

$$\frac{n_3}{p}\frac{d\,(a_3 - a_1)}{d\,t} + r_0\,a_3 + \frac{L_{31}}{n_3}\frac{d\,a_3}{d\,t} = e_1 + E. \qquad (29)$$

As before the variables are separable and equation (**5**) may be directly solved for $(a_3 - a_1)$ giving

$$a_3 - a_1 = (C_3 - C_1)\,\epsilon^{-\frac{K\rho t}{n_1}} + \frac{E}{K} \qquad (30)$$

and

$$C_3 - C_1 = -\frac{E}{K} \text{ if } a_3 - a_1 = 0 \text{ when } t = 0$$

$$\frac{L_{31}}{n_3}\frac{d\,a_3}{d\,t} + r_0\,a_3 = e_1 + E - \frac{n_3}{n_1}\,E\,\epsilon^{-\frac{K\rho t}{n_1}} \qquad (31)$$

General solution

$$a_3 = C_3\,\epsilon^{-\frac{r_0 n_3 t}{L_{31}}} \qquad (32)$$

Particular integral

$$a_3 = \frac{e_1 + E}{r_0} - \frac{\frac{n_3}{n_1}E}{r_0 - \frac{L_{31}}{n_3}\,\frac{K\rho}{n_1}}\,\epsilon^{-\frac{K\rho t}{n_1}} \qquad (33)$$

Finally

$$a_3 = C_3\,\epsilon^{-\frac{r_0 n_3 t}{L_{31}}} + \frac{e_1 + E}{r_0} - \frac{\frac{n_3}{n_1}E}{r_0 - \frac{i_{31}}{n_3}\,\frac{K\rho}{n_1}}\,\epsilon^{-\frac{K\rho t}{n_1}} \qquad (34)$$

$$a_3 - a_1 = \frac{E}{K}\left(1 - \epsilon^{-\frac{K\rho t}{n_1}}\right) \text{ or } a_1 = a_3 - \frac{E}{K}\left(1 - e^{-\frac{K\rho t}{n_1}}\right)$$

C_3 may be independently determined by assuming that for instance when $t = 0$; $a_3 = \frac{e_1 + E}{r_0}$ when we have in general

$$a_3 = \frac{e_1 + E}{r_0} - \frac{\frac{n_3}{n_1}E}{r_0 - \frac{L_{31}}{n_3\cdot}\,\frac{K\rho}{n_1}}\left(\epsilon^{-\frac{K\rho t}{n_1}} - \epsilon^{-\frac{r_0 n_3 t}{L_{31}}}\right)$$

Let us restate the equations (**5**) and (**6**) above in terms of the currents. First substitute $i_{33} = \frac{n_3}{n_1}\,i_3$.

We obtain

$$L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + K\,n_1\,(i_{33} - i_1) = E \qquad (36)$$

$$\frac{n_3}{n_1}\,L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + r_0\,n_1\,(i_{33}) = e_1 + E$$

$$L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + r_1\,n_1\,i_{33} = \frac{n_1}{n_3}\,(e_1 + E) \text{ if } r_1 = \frac{n_1}{n_3}\,r_0. \qquad (37)$$

By the general theory of linear differential equations the general solution of these equations is the general solution of the equations in which the terms independent of i_1 and i_{33} are put equal to zero as below

$$L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + K\,n_1\,(i_{33} - i_1) = 0 \qquad (38)$$

$$L_{11}\frac{d\,(i_{33} - i_1)}{d\,t} + r_1\,n_1\,i_{33} = 0 \qquad (39)$$

together with a particular integral usually giving the steady state of the original equations.

From the physical point of view, this means that the steady state may be regarded as always in existence, the transient effects being superadded at the moment of switching on or off and dying to zero after a few seconds.

If therefore these transient effects are studied by *themselves* it is possible to obtain a better physical grasp of them than if the study is complicated by a superposed steady state. Equations (**38**), (**39**) represent the case in which *both* armature and shunt field are short-circuited. We suppose that at $t = 0$ a certain initial value of $n_1\,(i_{33} - i_1) = A$ say exists and study in what manners this dies way to zero. Clearly if the shunt coil is short-circuited it cannot matter how many turns it contains, and hence the substitution $i_{33} = \frac{n_3}{n_1}\,i_3$ has been introduced equivalent to assuming that it contains the same number of turns, n_1 as the series coil.

Solving the above equations we have from (**38**)

$$i_{33} - i_1 = A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

$$\frac{d\,(i_{33} - i_1)}{d\,t} = -\frac{K\,n_1}{L_{11}}\,A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

From (**39**)

$$r_1\,n_1\,i_{33} = L_{11}\,\frac{K\,n_1}{L_{11}}\,A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

$$i_{33} = \frac{K}{r_1}\,A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

$$i_1 = \left(\frac{K}{r_1} - 1\right)A\,\epsilon^{-\frac{Kn_1}{L_{11}}} \qquad (\mathbf{40})$$

It has before been pointed out that with a short-circuited armature the rate of decrement of the flux must be such as to induce an e. m. f. in the series coil equal and opposite to the rotation volts.

If however the armature circuit be opened, the field circuit has a natural rate of decrement of its own, determined by solving equation (**39**) for the case when $i_1 = 0$.

This gives us

$$i_{33} = C\,\epsilon^{-\frac{r_1\,n_1\,t}{L_{11}}} \qquad (\mathbf{41})$$

Now if the natural rate of decrement of the field is equal to that determined by short-circuiting the armature then

$$\frac{r_1\,n_1}{L_{11}} = \frac{K\,n_1}{L_{11}} \text{ or } \frac{K}{r_1} = 1 \qquad (\mathbf{42})$$

Under these circumstances, as the above equation (**40**) shows $i_1 = 0$ even though the armature be short-circuited. In other words the natural rate of decrement of the field produces an e. m. f. in the series coil, which just balances the rotation volts without the need for any armature current to forcibly modify the rate of decrement.

From the point of view of power and energy the natural rate of decrement of the flux is such that the energy released by the falling of the flux exactly supplies that converted into heat by the field resistance.

If the natural rate of decrement of the field is *less* than that determined by the armature we have—

$$\frac{K}{r_1} \pm 1$$

Total energy stored in flux

$$w = 1/2\,L_{11}\,(i_{33} - i_1)^2$$

Power liberated by its decrease

$$\frac{d\,w}{d\,t} = (i_{33} - i_1)\,L_{11}\,\frac{d\,(i_{33} - i_1)}{d\,t} = -K\,n_1 A^2\,\epsilon^{-\frac{2Kn_1t}{L_{11}}}$$

$I^2\,R$ loss in field

$$i_{33}^2\,r_1\,n_1 = \frac{K^2\,n_1}{r_1}\,A^2\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

If $\frac{K}{r_1} > 1$ clearly the $I^2\,r$ loss in the field is greater than the power liberated by the flux. Some therefore must be supplied by the rotation volts. This is confirmed by the fact that the armature current

$$i_1 = \left(\frac{K}{r_1} - 1\right)A\,\epsilon^{-\frac{Kn_1t}{L_{11}}}$$

and the rotation volts

$$K\,n_1\,(i_{33} - i_1) = K\,n_1\,A\,\epsilon^{-\frac{dKn_1t}{L_{11}}} \qquad (\mathbf{43})$$

are in the same direction, or the rotation volts generates power. If the $I^2\,R$ loss in the field is *less* than the power liberated by the flux, some of this power must be devoted to motoring the machine. In this case $\frac{K}{r_1} < 1$ and i_1 reverses its sign flowing *against* the rotation volts. Thus the machine is a motor.

Returning to equations (**5**) and (**6**)

The steady state solutions are—

$$a_3 - a_1 = \frac{E}{K} \text{ or } n_1\,(i_{33} - i_1) = \frac{E}{K} \qquad (\mathbf{44})$$

$$a_3 = \frac{e_1 + E}{r_0} \text{ or } n_1\,i_{33} = \frac{n_1}{n_3}\,\frac{e_1 + E}{r_0} \qquad (\mathbf{45})$$

$$a_1 = \frac{e_1 + E}{r_0} - \frac{E}{K} \text{ or } n_1\,i_1 = \frac{n_1}{n_3}\,\frac{e_1 + E}{r_0} - \frac{E}{K}\,. \qquad (\mathbf{46})$$

these being arrived at on the assumption that when $t = 0$

$$a_3 - a_1 = 0$$

Now

$$a_3 - a_1 = n_1\,(i_{33} - i_1)$$

If therefore we put

$$n_1\,A = -\frac{E}{K}$$

we may suppose that the steady state solutions exist at all instants (including $t = 0$) but that superposed on these there is a transient solution whose value is equal and opposite to that of the steady solution when $t = 0$. In the resultant therefore, we have $a_3 - a_1 = 0$ at $t = 0$ gradually approximating to the steady state as the transients die away.

The value of A given above gives us

$$a_3 = -\frac{E}{r_1}\,\epsilon^{-\frac{Kn_1t}{L_{11}}} \qquad (\mathbf{47})$$

on which we superpose a "steady" solution $\frac{e_1 + E}{r_0}$ to get

$$a_3 = \frac{e_1}{r_0} + \frac{E}{r_0}\left(1 - \frac{n_3}{n_1}\,\epsilon^{-\frac{K n_1 t}{L_{11}}}\right) \text{ remembering}$$

$$\text{that } r_1 = \frac{n_1}{n_3}\,r_0 \qquad (48)$$

and

$$a_1 = -\left(\frac{K}{r_1} - 1\right)\frac{E}{K}\,\epsilon^{-\frac{K n_1 t}{L_{11}}}$$

on which we superpose

$$\frac{e_1 + E}{r_0} + \frac{E}{K}$$

$$a_1 = \frac{e_1}{r_0} + \frac{E}{r_0}\left(1 - \frac{n_3}{n_1}\,\epsilon^{-\frac{K n_1 t}{L_{11}}}\right) + \frac{E}{K}\left(\epsilon^{-\frac{K n_1 t}{L_{11}}} - 1\right)$$

$$L_{11} = \frac{n_1^2}{\rho} \text{ so that } \frac{K\,n_1\,t}{L_{11}} = \frac{K\,\rho\,t}{n_1}$$

thus making the identity of the two formulas obvious.

From the point now reached it is not difficult to understand why the presence of leakage self induction in the shunt coil, should permit the current in it to have a different rate of decrement from that of the main flux.

Clearly it is only the main flux of the machine which is determined by the condition that the transformer e. m. f. in the series coil due to its decrement shall balance the rotation voltage on short circuit. If there is an additional flux which produces no rotation voltage —namely, the flux of leakage self induction this condition does not apply to it, and there is no reason why it should not be determined by the further condition (usual in inductive circuits) that the energy released by the decrement of the whole flux (main flux and leakage flux) shall be equal to that transformed into heat by the resistance of the circuit.

SELF-INDUCTION IN THE ARMATURE CIRCUIT

The Cause of Oscillation. The equations now become (ignoring the steady state as before)

$$\frac{n_1^2}{\rho}\,\frac{d\,(i_{33} - i_1)}{d\,t} + K\,n_1\,(i_{33} - i_1) - L_0\,\frac{d\,i_1}{d\,t} = 0 \qquad (51)$$

$$\frac{n_1^2}{\rho}\,\frac{d\,(i_{33} - i_1)}{d\,t} + r_1\,n_1\,i_{33} = 0 \qquad (52)$$

or in symbolic form

$$\left(\frac{n_1^2}{\rho}\,D + K\,n_1\right) i_{33} - \left\{\left(\frac{n_1^2}{\rho} + L_0\right) D + K\,n_1\right\} \quad L_1 = 0$$

$$\left(\frac{n_1^2}{\rho}\,D + r_1\,n_1\right) i_{33} - \frac{n_1^2}{\rho}\,D\,i_1 \qquad = 0$$

$$\left\{L_0\,\frac{n_1^2}{\rho}\,D^2 + r_1\,n_1\left(L_0 + \frac{n_1^2}{\rho}\right) D + r_1\,n_1\,K\,n_1\right\} i_{33} = 0$$

The solution of this is

$$i_{33} = C_1\,\epsilon^{p_1 t} + C_2\,\epsilon^{p_2 t} \text{ with}$$

p =

$$\frac{-r_1\,n_1\left(L_0 + \frac{n_1^2}{\rho}\right) \pm \sqrt{n_1^2\,r_1^2\left(L_0 + \frac{n_1^2}{\rho}\right)^2 - 4 r_1 K n_1^2 L_0\,\frac{n_1^2}{\rho}}}{2\,L_0\,\frac{n_1^2}{\rho}}$$

or

$$p = \frac{-\,r_1\,n_1\left(L_0 + \frac{n_1^2}{\rho}\right)}{2\,L_0\,\frac{n_1^2}{\rho}} \times$$

$$\left\{-1 \pm \sqrt{1 - \frac{K}{r_1}\,\frac{4\,L_0\,\frac{n_1^2}{\rho}}{\left(L_0 + \frac{n_1^2}{\rho}\right)^2}}\right\}$$

This equation is fully discussed above.

Returning to the differential equation

$$\frac{n_1^2}{\rho}\,\frac{d\,(i_{33} - i_1)}{d\,t} + K\,n_1\,(i_{33} - i_1) - L_0\,\frac{d\,i_1}{d\,t} = 0$$

Consider three successive instants assuming that the solution is oscillatory.

$$i_{33} - i_1 \text{ a maximum } \frac{d\,(i_{33} - i_1)}{d\,t} = 0. \qquad (1)$$

At this instant the transformer voltage due to the main flux is zero, but the flux is not. Hence the rotation voltage exists and in fact is a maximum. The current i_1 is not zero, and hence the rotation voltage generates power which stores energy in the external self induction.

$$i_1 \text{ a maximum } \frac{d\,i_1}{d\,t} = 0. \qquad (2)$$

At this instant the rotation voltage is balanced by the transformer voltage due to the variation of the main field, and since by hypothesis $\frac{K}{r_1} > 1$ it is still generating power to increase the field energy.

$$i_{33} - i_1 = 0 \text{ rotation voltage zero.} \qquad (3)$$

At this instant an exchange of energy takes place between the reactance and the main field that in the reactance say, falling off, and that in the main field increasing towards a maximum, when the cycle repeats.

Briefly the rotation voltage supplies energy to the field and the external reactance which is partly stored and partly consumed by the field resistance and the energy oscillates between these two. If so large a proportion of the energy of the circuit is contained in the external reactance that any variations in the field energy produce but a slight effect on it oscillations cannot occur.

The reason why in a shunt machine the self induction should be outside the shunt tappings rather than inside, as when it is formed by the cross flux of the armature, can be explained without differential equations.

If the shunt coil be (1) connected directly across the arc, all variations in the latter directly affect the shunt coil.

Whereas (2) if a self induction is interposed between them the shunt is shielded from these oscillations.

Suppose for instance, the voltage across the arc be represented by $E_0 + E_1 \sin p\,t$ then in case (1) a partly alternating e. m. f. is impressed on the shunt coil giving rise to an alternating component of current therein, *which may lag approximately 90 deg. behind* $E_1 \sin p\,t$. and produce a correspondingly lagging component of rotation voltage which will by no means balance $E_1 \sin p\,t$.

Thus unless the variations are so slow and the resistance of the shunt circuit so high that this lag does not exist, connecting the shunt circuit across the arc does *not* permit it to follow rapid variations of the latter. It is better therefore, to rely on the self induction to take up very rapid variations and to shield the shunt coil from them.

But the less magnetic energy stored in the field the easier is it for the machine to follow rapid variations, and it should be remembered that the variations in the arc are probably not as rapid as those of an ordinary alternating current.

Discussion

For discussion of this paper see page 688.

A New System for D-C. Arc Welding

BY S. R. BERGMAN*

Associate, A. I E. E.

Synopsis.—*A short description is first given of a d-c. generator having characteristics suitable for arc welding. Curves showing performance under steady conditions of load are shown. It is pointed out that for quick changes of load transients appear detrimental to arc welding. In order to eliminate these transients, an external inductive device called a transforming reactor is introduced in the arc welding circuit. This transforming reactor has a dual purpose. First, it depresses the transients as is shown by oscillograms, resulting in quick recovery of the voltage and a small short-circuit current. Thus it is shown that under arc welding load the generator performs in accordance with the curves corresponding to a steady load. Second, the transforming reactor introduces reactance in the arc circuit which, experience has shown, is desirable in order to make the arc stable and therefore easy to handle.*

The system for d-c. arc welding which is described consists, therefore, of two units,—a generator and a transforming reactor, forming a combination capable of producing uniform welds of high strength.

Arc welding, particularly when applied to ferrous metals, has, during the last decade, developed into an art of great importance in industry. As this art has developed, the requirements for a proper power supply have been studied with the view of obtaining uniform and great strength of the welds as well as securing ease in welding. To meet this demand, a motor-generator set has been developed and, in addition, an auxiliary device of inductive character to aid the generator in producing ideal welding conditions.

The requirements for the generator may be briefly stated as follows:

1. The open-circuit voltage was chosen to range from 70 to 90 volts. Experiments have shown that it is desirable to have this voltage above 70. With less voltage available, the arc has a tendency to break as it is elongated, due to unsteady welding, or the blowing from influences of external magnetic fields. This voltage is also required in order to break through scale on the iron. Tests have also shown that there is no advantage derived from a voltage higher than 80 volts, although a margin of 10 volts above this is advantageous to compensate for the voltage drop in extra long leads.

2. The arc voltage runs from 14 to 25 volts depending upon the strength of the welding current. It is thus required that the voltage of the generator should inherently drop from an open-circuit voltage of 70 to 80 volts to an arc voltage of 14 to 25 volts. The regulation of the generator is shown in Fig. 1, giving approximately a straight-line relation between voltage and current.

The arrangement of the generator is shown in Fig. 2. The principle involved is that of a dual magnetic circuit. The armature is wound for two poles and the field contains four poles. In general, the armature should be wound for half the number of poles contained in the field. In d-c. machines of standard design, adjacent poles have opposite polarity, but in this machine, the poles are paired in groups of the same polarity. Thus, there is a group of two north poles followed by a group of two south poles, etc.

In order to establish a working theory, assume that the flux distribution is such as shown in Fig. 2. There exist two fluxes ϕ_m and ϕ_c displaced 90 electrical deg. from each other. The flux ϕ_m will be designated the main flux and the flux ϕ_c the cross-flux.

The load current of the armature is taken from the two brushes A and B, placed in neutrals located between poles of opposite polarity. In addition, there is an exciting brush, E. This machine is self-excited, the excitation being taken from brushes A and E, as shown in Fig. 2. The main magnetic circuit is so designed that magnetic saturation exists, whereas the cross-magnetic circuit is designed without any saturation. As soon as the armature is loaded, there is set up an armature reaction, R, which may be resolved in two

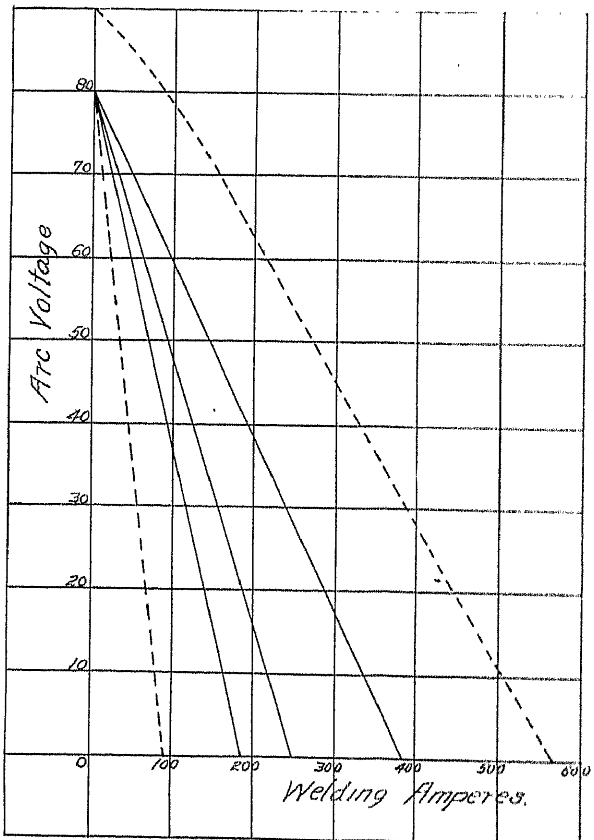

FIG. 1—CHARACTERISTICS OF GENERATOR

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

components; one, $R\,M$, in the direction of the main flux and the other, $R\,C$, in opposition to the cross-flux. Since the main magnetic circuit is saturated, the component, $R\,M$, can not force any more flux through this circuit and the main flux remains practically constant.

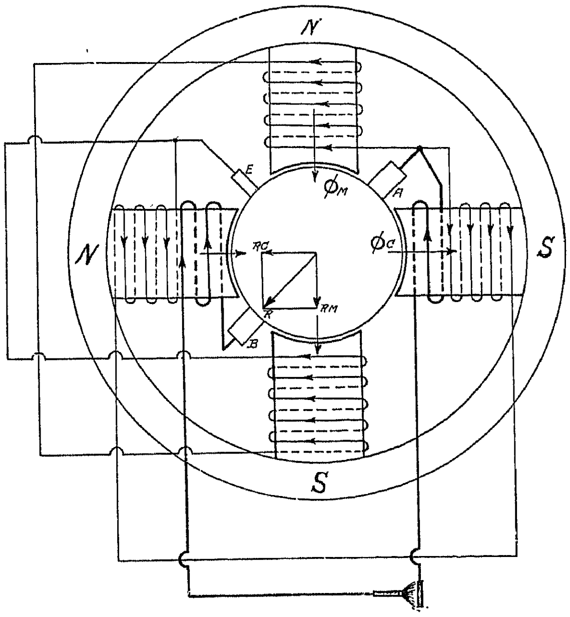

Fig. 2—Principle of Generator

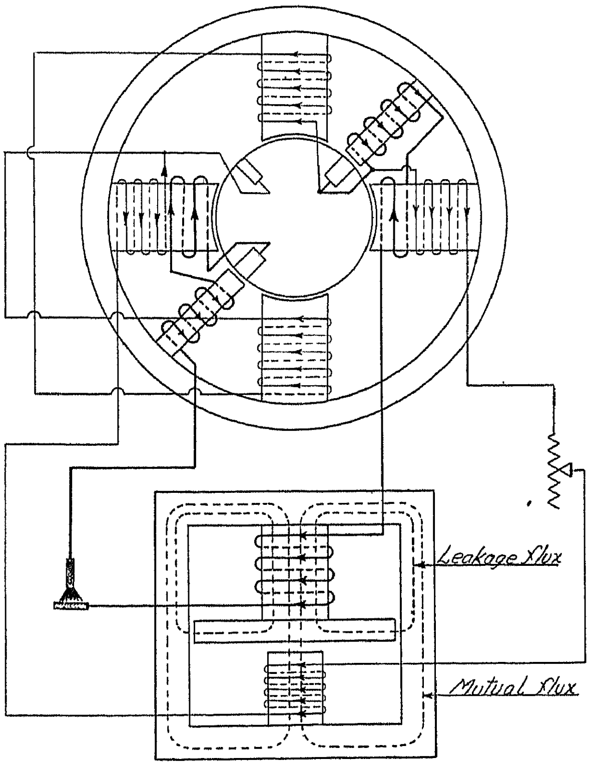

Fig. 3—Generator and Transforming Reactor

The component, $R\,C$, however, blows out the cross-flux which thus decreases as the load increases. The machine voltage $A\,B$ is the algebraic sum of $A\,E$ and $E\,B$. Thus, $A\,B = A\,E + E\,B$. The voltage $A\,E$ is induced by the main flux and remains practically constant since the main flux is constant, and the speed is constant. The voltage $E\,B$, however, decreases with the load since the cross-flux decreases as described As a matter of fact, the cross-flux passes through zero, and is finally reversed.

If it is desired to weld with smaller currents, a series winding is placed on the cross-poles (Fig. 2) supporting the armature reaction. Since it is desirable to be able to obtain a large number of different welding currents, the series winding is provided with taps. To secure a close adjustment of the welding current, a field rheostat is inserted in the cross circuit. To obtain good commutation, commutating poles are added.

A set of welding currents may now be obtained as shown in Fig. 1, which particular curves apply to a machine rated at 300 amperes. The rated current range of this machine is from 90 to 375 amperes although a minimum current of 75 amperes, and a maximum current of 450 amperes is obtainable.

In actual welding the current is never constant, because the circuit is frequently short-circuited by small globules of metal and because the electrode is not fed at exactly the proper rate, particularly in hand welding.

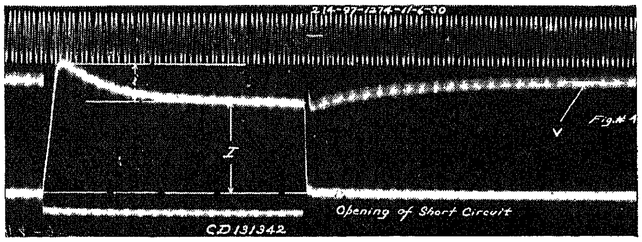

Fig. 4

Under such conditions, transient currents and potentials appear in the generator and the regulation deviates from the curves shown in Fig. 1, which correspond to static conditions. In striking the arc, the current overshoots its steady value, and in breaking the arc, a certain time interval is required for the voltage to return to its normal open-circuit value. It is desirable that the short-circuited current should not be excessive, and it is also desirable to have the voltage build up rapidly.

In order to cope with these transient conditions, a special inductive device which may be called a transforming reactor was developed as shown in Fig. 3. This device contains two windings, a primary winding connected in series with the arc, and a secondary winding connected in series with the shunt field of the cross-magnetic circuit. These two windings are connected in such a way that the direct current flowing through them will magnetize in the same direction.

The function of the transforming reactor is to provide a magnetic coupling external to the generator between the arc circuit and the field circuit, which will be opposite in effect to that between the two circuits within the generator. Suppose that the arc is suddenly broken; then the ampere turns of the arc circuit, both in the generator and in the transforming reactor, suddenly disappear and since the secondary winding attempts to maintain the mutual flux, the secondary current has a tendency to increase, counteracting the tendency of the generator magnetic circuit to make it decrease. Thus, the transforming reactor tends to keep the cross-shunt field current constant, regardless of changes in the arc-current. The result of this is to make the voltage build up more rapidly to its normal value when the arc is broken and to limit the peak value of current when a short circuit is made.

To illustrate the principle of this generator in conjunction with the transforming reactor, an oscillogram is shown in Fig. 4, showing what happens when the arc is short-circuited and when the short circuit immediately thereupon is opened. The steady value of the short-circuited current is I, and the transient value, i, is quite small. When the short circuit is removed, it will be noted that the voltage of the generator nearly instantaneously builds up to the open-circuit voltage, V.

This oscillogram proves that the transforming reactor fulfills its purpose in practically eliminating the transients, making the generator, during sudden changes of arc current, follow the regulation curves obtained under steady conditions.

This device should, in order to function ideally, be designed in such a manner that the mutual induction between its two windings is the same as the mutual induction existing between the cross-magnetic shunt winding and the armature winding with its series winding.

Let N_1 = total effective number of turns on the cross axis of the generator carrying the welding current.

N_2 = total number of cross-shunt field turns.

n_1 = number of turns on the transforming reactor carrying the welding current.

n_2 = number of turns on the transforming reactor in series with the cross-shunt field.

F = cross-flux per ampere turn of the generator.

f = mutual flux per ampere turn of the reactor.

The design of the transforming-reactor should then satisfy:

$$f \times n_1 \times n_2 = - K \times F \times N_1 \times N_2$$

The minus sign is used, since the two windings on the transforming reactor support each other, whereas in the generator corresponding windings oppose each other. In order to eliminate all transients, K should be equal to 1. However, fractional values of K give very good transient performance.

Experience has shown that in order to stabilize the arc, a certain amount of series reactance is necessary in the arc circuit. The generator itself has a certain amount of reactance, but in order to further stabilize the arc, additional reactance is desirable. This additional reactance is supplied by shaping the magnetic circuit of the inductive device so as to enlarge its primary leakage, as shown in Fig. 3. Thus, the transforming reactor has two fluxes, namely, the mutual flux which links both the primary and the secondary windings, and the primary leakage flux interlinked with the primary alone. These fluxes are indicated in Fig. 3.

This generator is self-excited, requiring no exciter. It has inherent regulation suitable for arc welding. The transforming reactor serves the dual purpose of eliminating transients in the generator and supplying reactance to the welding circuit. Experiments have shown that this combination welds with great ease both for hand and automatic welding. It lays down the metal with ease and accuracy under all conditions met in welding, producing welds of uniformity and strength.

Discussion

For discussion of this paper see page 688.

Resistance Welding of Motor Frames

BY MALCOLM THOMSON*

Associate, A. I. E. E.

Synopsis.—Cost reduction calls for a continual revision of methods. The making of motor frames using rolled plate in place of castings formerly used, the object being to reduce cost is one such revision. The method of manufacture is briefly outlined including the rolling, welding, sizing, and other operations. The electric resistance welding method is emphasized and results of electrical tests on equipment are outlined. The oscillograph records give the character of a resistance flash weld in some detail.

The conclusion is reached that not only is there a saving in cost of frames, but also a reduction in machining costs. The result is a better product at less cost.

* * * * *

THE continual demands of modern industry have caused and are causing a continual revision of methods employed. Perhaps no more interesting application of electricity to shop manufacture occurs than in its use in various forms of electric welding. The methods in most common use are the arc methods, the carbon arc and the metallic arc, and the resistance method. The carbon arc process was invented by DeMeritens in France in 1881 and the metallic arc process by Slavianoff in Russia in 1890. The resistance welding process was invented by Dr. Elihu Thomson in 1886. It is this latter process which is particularly adapted for use where there is mass production.

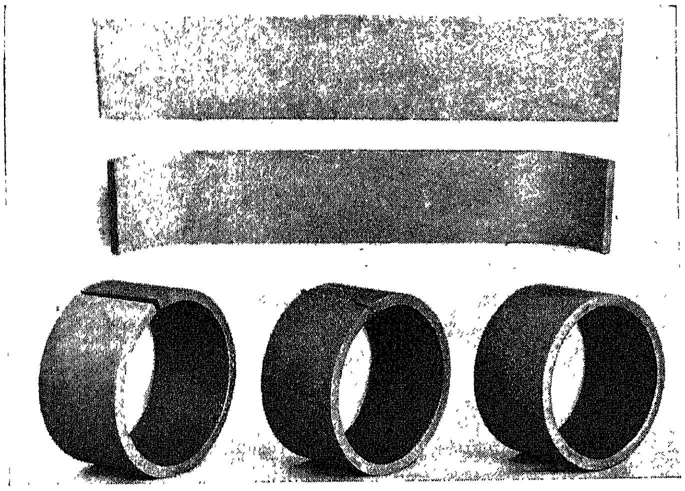

FIG. 1—ROLLED-PLATE AUTOMOTIVE-MOTOR FRAME. VIEWS SHOWING SUCCESSIVE STAGES IN FORMING

In the manufacture of motor frames, competition brings in the question of reduced costs. It was, therefore, natural that the General Electric Company took steps in 1928 to equip a modern shop for fabrication of motor frames, as well as other products which could be fabricated from structural steel. A section of a building formerly used as a steel foundry was set aside for this purpose. The manufacture of rings for frames of d-c. motors is briefly outlined as follows:

The steel for the rings is ordered from the mills in carload lots of the proper width and length, the length being figured so as to include the necessary amount for welding and sizing operations. The steel which has been found most suitable for magnet frames is a basic open hearth process manufacture of low carbon content. The slabs of steel are unloaded at one end of the department, and are of various sizes in accordance with different sizes of magnet frames. The slabs are passed to a 1,200-ton hydraulic press which crimps or curls the ends to avoid flats and waste of stock at portions which cannot be rolled.

A view showing the steel as purchased and also illustrating the various steps in fabrication is given in Fig. 1. A typical ring in bending rolls is shown in Fig. 2.

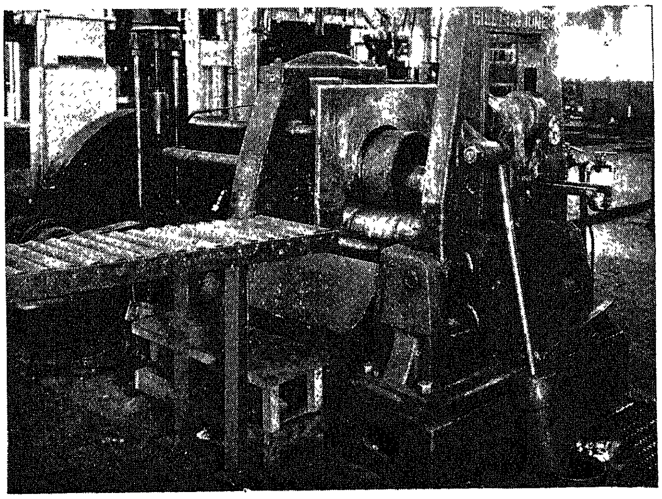

FIG. 2—ROLLING MACHINE FOR FABRICATING AUTOMOTIVE-MOTOR MAGNET FRAMES

Attention is called to the vertical plate which insures accurate alinement of the ends of the ring during rolling process. The rolls have electric motor drive for raising and lowering rolls, as well as motor drive for rolls. The end section is pivoted and controlled by an air cylinder which is shown in the figure. This feature is for quick removal of rings.

From the rolls the rings pass by a conveyor which has

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considerable storage capacity to a specially designed resistance flash welder of the Thomson-Gibb make. Fig. 3 shows this welder in action. The rings are held with powerful clamps actuated by hydraulic cylinders against special aluminum bronze die blocks. The operator separates the ends of the rings slightly, then starts slowly feeding the jaws of the machine together with a hand valve control of a hydraulic push up cylinder.

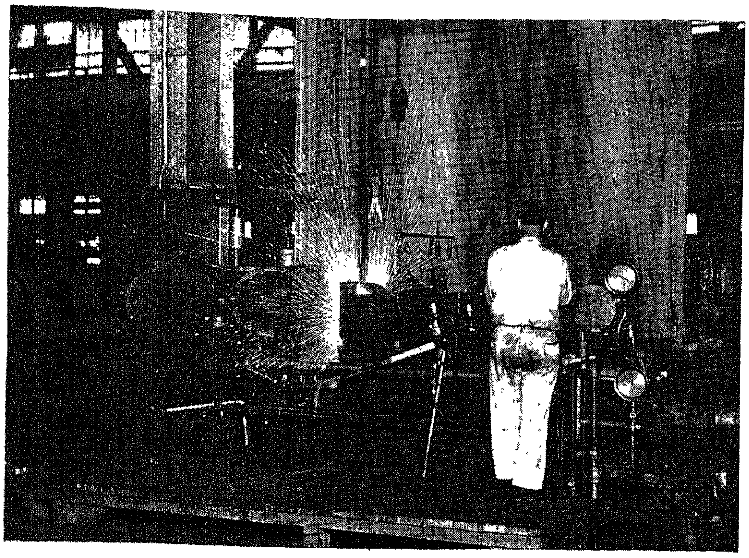

FIG. 3—SPECIALLY DESIGNED THOMSON-GIBB RESISTANCE FLASH WELDER IN OPERATION

The current is turned on by a push button working a small contactor which in turn controls a large contactor, the latter closing the connection of the primary of the transformer to the power lines. As the ends of the ring meet, at first feeble then violent flashing occurs. A predetermined amount is flashed off, averaging slightly less than an inch total. The flashing heats the abutting faces to the melting point of steel and the area immediately behind to a welding heat mainly by conduction, but partly by resistance. At the end of the flashing period, the full pressure of the hydraulic ram or about 100,000 lb. is applied, suddenly forcing the molten faces together. The result is a perfect and homogeneous weld, all burnt or oxidized metal being forced out. This welder while rated at 12 sq. in. maximum, cross section is used on sections up to 18 sq. in. The welder has a 750-kva. water-cooled transformer. The open-circuit voltage at welding jaws is 12 volts.

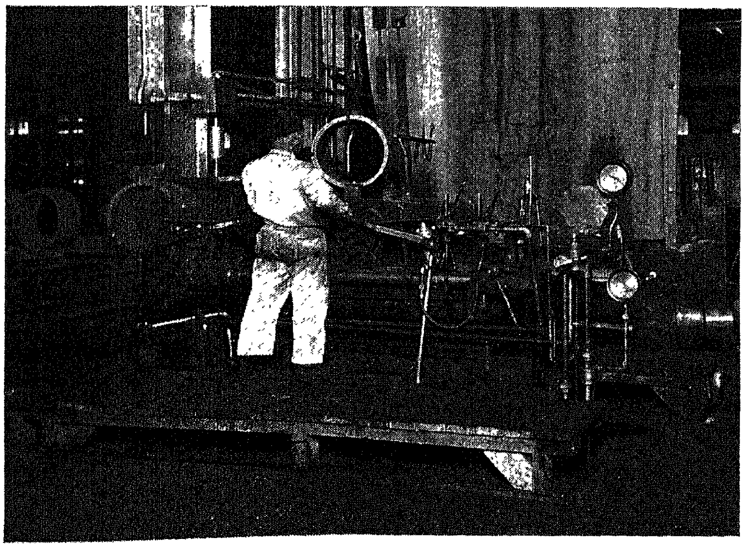

FIG. 4—ELECTRIC HOIST REMOVING RING FROM WELDER

The demands on welding transformers call for different features of design other than those used with the usual types. The service is intermittent but large amounts of current are necessary, particularly at the final or upset period of the welding cycle. Due to the severe nature of the service, accessibility is an important feature, hence the coils are usually assembled on a straight core section, then a "U" shaped section is placed in position and fastened to the straight section with bolts and straps. This allows for more easy repair in case of trouble. On 60-cycle supply, the core is worked at a flux density of 60,000 to 80,000 lines per sq. in.

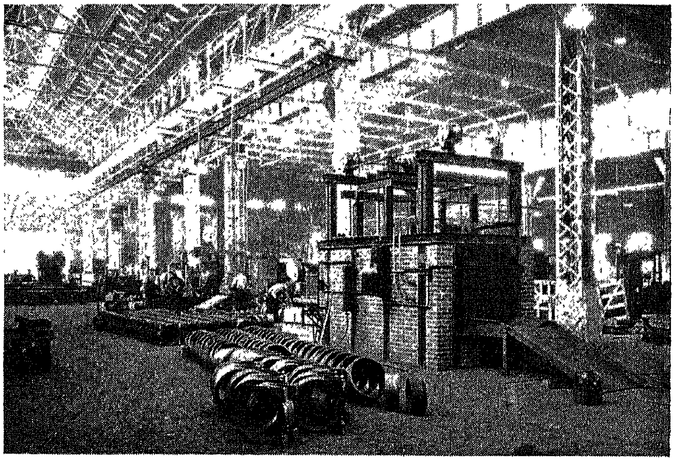

FIG. 5—ARC WELDING, BUILDING 30, RIVER WORKS

The secondary is usually of cast copper with cooling tubes for water cast in and of single turn type. In larger transformers usually several cast copper sections are connected in parallel. The ends of the copper secondary are machined, one end being fastened solidly to the fixed platen. The other end is bolted to a flexible lead made up of many thin sections of copper strip, this lead being connected to movable platen. The welding dies are located on top of the platens and are of hard-rolled copper or special wear-resisting material, depending on the nature of service. The work is very solidly clamped to the die blocks because of the push-up pressure required. The cast copper is worked at a current density of 1,800 to 2,200 amperes per sq. in., the flexible leads at 2,000 to 2,400 amperes and the joints at 400 to 600 amperes per sq. in. The dies are worked at about 3,000 amperes per sq. in. current density. Wherever possible dies and platen are water cooled internally. The two sides of the secondary are kept as close together as possible and the length of this circuit kept at a minimum to avoid any unnecessary secondary voltage drop.

Primary coils of the larger type transformers are usually built up of copper strip insulated between layers by a sleeve of cloth or layer of asbestos paper. The coils are impregnated and baked; then wound with tape and finally painted.

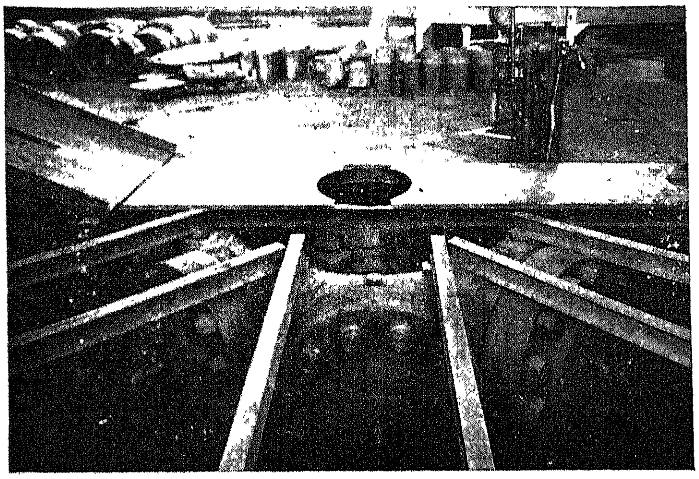

Fig. 6 Hydraulic Press for Fabricated Automotive-Motor Magnet Frames

Voltage regulation on the primary is obtained either by use of a separate auto-transformer or by bringing out taps, a regulating switch being employed. Usually there are from five to ten points of voltage regulation, the lowest being about 50 per cent of the highest. Since on production work the breaking of the current may occur several times a minute, contactors used to switch on the power supply must be able to break full load of welder without undue arcing.

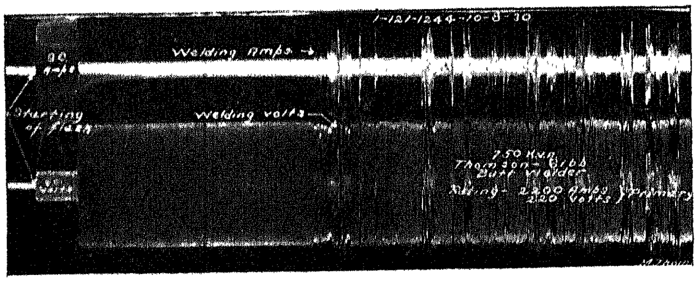

Fig. 7

The transformers are usually designed for one particular job; that is, a transformer designed for flash welding of steel would not be suitable for fast upsetting, and one designed for steel not suitable for welding copper or aluminum.

The transformer in the Thomson-Gibb machine described was designed especially for welding of rings up to 12 sq. in. cross section, but as previously stated, is used on somewhat heavier sections also.

Readings taken while welding section of 18 sq. in. show that there is a primary current of about 900 amperes taken during the flashing period with a peak load of 2,250 amperes at the final or completion period of weld when the ring is forced together. The primary voltage is 236. Current is at 60 cycle frequency. At high point of weld, readings taken show 461,250 volt-amperes and 252,000 watts which gives a power factor of about 55 per cent. The average during flashing was 126,000 watts; with the secondary open, 1,400 watts.

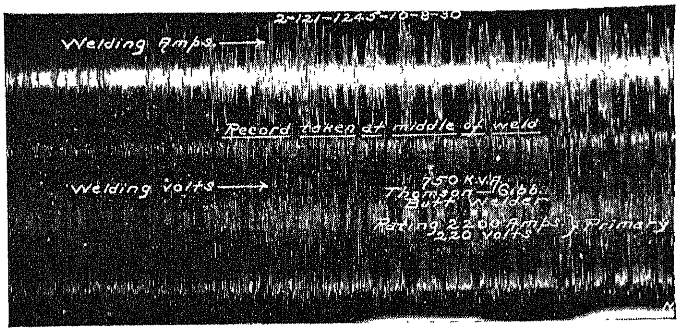

Fig. 8

Oscillograph records have been made on this welder which are reproduced in Figs. 7 to 11 inclusive. Fig. 7 shows conditions at start of flash, first a brief interval of short circuit, then open circuit followed by beginning of contact of pieces. This represents a period of feeble flashing. As the flashing becomes more violent, conditions are as in Fig. 8. Toward the end of the flashing period, as shown in Fig. 9, when flashing appears continuous, the wave form still shows no continuous flow of current, but rather closely compact intervals of short circuit and open circuit. This picture also shows the upset or final push-up section of the weld. Figs. 10 and 11 show views of wave analysis at early stages and near final stage of flashing period.

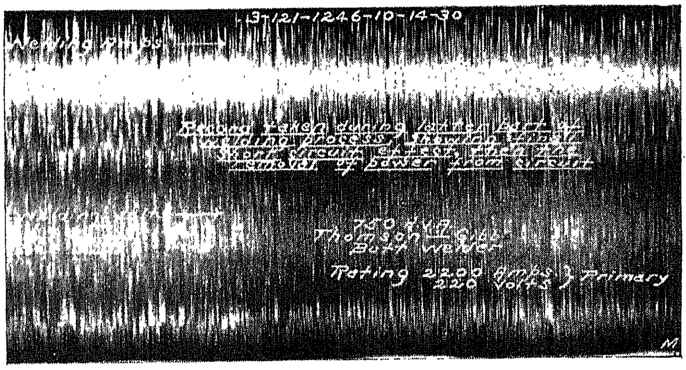

Fig. 9

After the welding operation is completed, the ring is removed by an electric hoist shown in Fig. 4. The upset and flash resulting from the welding operation show clearly in this view. This is removed, the operation being done in a vertical motor-driven broach which trims outside and inside while the metal is still red hot. The rings then pass by a conveyor similar to that previously shown, to an electric furnace. The electric furnace, (shown in Fig. 5) while primarily used for heating rings for sizing, also is advantageous in that it gives a strain anneal. The rings on the floor are complete rings ready for shipment. Those in the front rows have feet welded on.

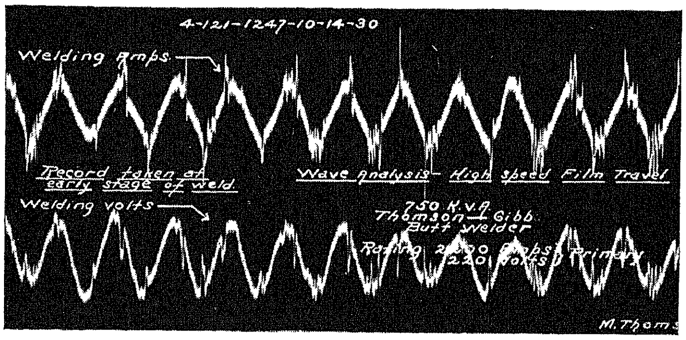

FIG. 10

The rings are heated to about 1,400 deg. fahr. in the furnace and then pass down a slide to an eight-cylinder hydraulic press located in the floor as shown in Fig. 6. Part of the flooring has been removed to show construction of the press. The press is so constructed that each cylinder has a removable head portion so that various sizes of rings may be accommodated. The rings are lowered horizontally into the press by means of an air-operated table and then squeezed to vary within 1/16 in. to 3/32 in. of correct size, depending on size of ring.

The ring is now complete with the exception of feet. The feet are made from steel bent into a U form in the hydraulic press, then gas cut apart. Each U makes one pair of feet and by inverting the U's, the right and left are made. It is the practise to cut all four feet at once using a double torch machine for this purpose.

The sized rings pass by conveyor to arc welding stations where the feet are tack welded, then finished welded into place. The completed frames are then sand-blasted, after which they are shipped to the motor building for machining and assembly. Some of the rings are for automotive equipment and do not have feet. These latter rings have dropped forged arms and bearing retainers welded in place by metallic arc process.

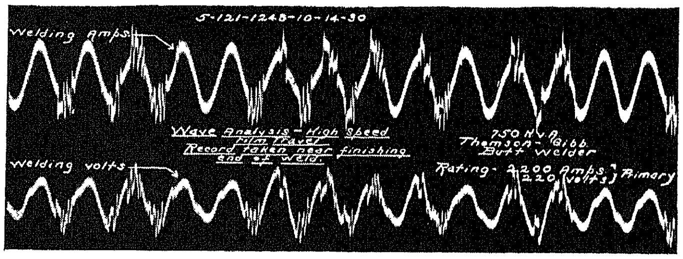

FIG. 11

By the use of electric welding of two kinds, it has been possible to produce frames 25 to 50 per cent cheaper than corresponding steel castings with added advantage of much greater uniformity in dimensions and structure. This, in turn, reflects in lowering machining costs and better motors.

Discussion

For discussion of this paper see page 688.

Electric Resistance Brazing

BY SAMUEL MARTIN, JR.*
Non-member

Synopsis.—*This paper describes in general the equipment for and process of making brazed joints. The lead soldered joint has been in use for a long time and has done its work well but because of certain inherent weaknesses, it is slowly being replaced by the brazed joint. The electric brazing equipment is listed and a short description of it given. The preparation of various sizes and shapes of work is taken up in detail and a short discussion of the different types of joints is given. Three kinds of brazing mediums are discussed and applications of each are mentioned. A description of the actual brazing process follows and the paper is concluded with a list of advantages of the brazed joint over the lead-soldered joint, including: increased factor of safety, saving in time, reduced cost, reduced size, and increased accessibility of joints for inspection.*

* * * * *

ALTHOUGH the lead solder joint will probably not be entirely superseded for a long time; it is, due to its inherent weaknesses, slowly being replaced by a new type of brazed joint. The lead solder joint is not all that could be desired as far as mechanical strength is concerned; when heated, it breaks apart at relatively low temperatures; and when a joint is made, its resistance is not always a known quantity. Such is the problem confronting the electrical industry if it desires to continue to set the pace for the present industrial age. A joint with high mechanical strength, high conductivity and high heat resisting qualities is needed.

Electric resistance brazing has been developed to meet these needs. This process is simple to accomplish, utilizing the high resistance of carbon blocks to the flow of electricity. The pieces to be joined are clamped between a pair of carbon blocks and current is passed through them and the joints. The carbon electrodes heat up quickly and the heat passes from them into the parts being joined, raising their temperature to the melting point of the brazing alloy. The alloy, in the form of a thin strip for convenience, is then applied to the contact surfaces of the parts being brazed. It melts and flows through the joints, uniting the parts into a solid mass.

The equipment required for making this type of joint is simple, consisting of an air-cooled transformer, primary and secondary leads, the latter being connected to the tongs for holding the carbon blocks, and a foot switch. The two most useful sizes of transformers are:

1. Five-kva., weighing approximately 45 lb. and capable of brazing conductors, cables, etc., up to 300,000 cm. in cross-section.
2. Ten kva., weighing approximately 95 lb. and capable of brazing conductors, bars, cables, etc., between 300,000 and 2,000,000 cm. in cross-section.

The five-kva. transformer is wound to give a single secondary voltage of 8 volts, while the 10-kva. transformer is wound to give 8, 10, and 12 volts on the secondary. Fig. 1 shows a complete 10-kva. equipment ready for connection to a 220-volt, single-phase, 60-cycle power line. The primary taps to give the different secondary voltages can be clearly seen. The momentary overload of 400 per cent can be carried by either of the transformers.

There are many variables which affect the operating characteristics of the transformer. Consider first that the transformers have a turn ratio of 220 to 12, 10 or 8. For the 12-volt secondary, the turn ratio is 18.4; for the 10-volt secondary, it is 22; and for the 8-volt secondary, it is 27.5. For simplicity, consider one tap only, the 12-volt one. For every ampere in the primary, there are 18.4 amperes in the secondary. Then, for 100 amperes in the primary, there are 1,840 amperes in the secondary. With comparatively high current and low voltage, the reactance of the leads to the tongs has quite an effect on the power factor. The load, which is the resistance of the carbon blocks plus that of the piece being brazed, also affects the power factor. For any given length of lead, 7-ft. ones are recommended for use with the equipment, the kind of carbon block, the kind of joint and the amount of contact between the carbon blocks and the joint affect the power factor, high resistance tending to raise it, and low resistance tending to lower it.

For comparison, the following test data were obtained for (A) no joint between the carbon blocks, and, (B) a joint between the carbon blocks:

(A) Short circuit through 1⅜-in. soft carbon blocks:

Volts	Amperes	Watts	Volt-amperes	Power factor (%)
222	0	0		
212	118	15,600	25,000	62.4
210	120	16,000	25,200	63.5

(B) Brazing terminal using 1½-in. hard carbon blocks:

Volts	Amperes	Watts	Volt-amperes	Power factor (%)
237	0	0		
236	40	6,800	9,440	72.2
235	58	10,000	13,600	73.5

As the weights of the equipment indicate, the outfits are portable, thereby making it possible to apply the tongs to joints in any position. It is sometimes convenient to mount the 10-kva. transformer on a small truck supported by casters.

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For joints larger than 2,000,000 cir. mils. cross-section, stationary machines or presses have been adopted, using much larger transformers.

For different types of joints, various styles of tongs have been developed. Fig. 2 shows one of these tongs.

The preparation of the work for the brazing process is important. The parts to be brazed should be as flat as possible so that the carbon blocks will come in intimate contact with the full surface of the material. This will prevent arcing and hot spots occurring between the carbon blocks and the metal parts. Round bars, rods, solid wire, etc., should be flattened with a hammer or machining process to insure good contact. If the round bar or irregular shaped piece cannot be flattened as mentioned above, the carbons may be cut to fit the piece. Flat plates replace tubular terminals as shown in Fig. 3. If the flat plate terminal or bus bar is more than 1 in. wide, it is slotted to prevent excessive heat radiation. The brazing operation is then performed on each individual section as a separate operation. Stranded cable is prepared in a slightly different manner. It is either wrapped with thin copper strip or inserted in a thin copper tube and then flattened before clamping in the tongs. If it is desired to make a very flat joint, the thin copper strip should be wound around the strands rather loosely. The wrapping or tubing should extend on the cable ½ to 1 in. beyond the portion being brazed.

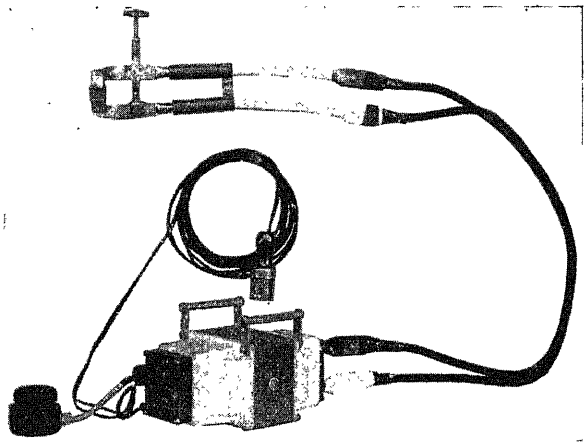

Fig. 1—10-Kva., 8—10—12-Volt Brazing Transformer, Complete with Leads, Tongs, Foot-Switch and Plug

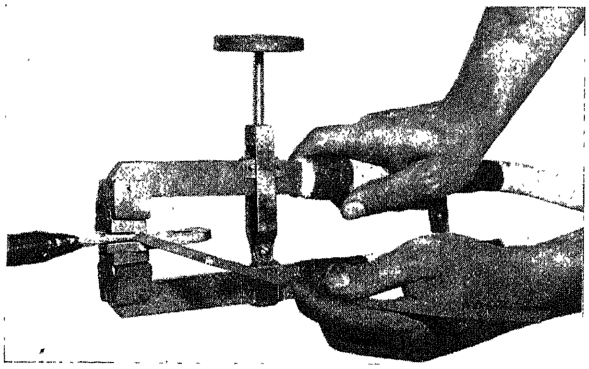

Fig. 2—Application of Tongs to Brazing Cable to Flat-Plate Terminal

Copper braid is prepared in a somewhat similar manner as shown in Fig. 3. Here a copper clip about 0.02 in. thick and slightly wider than the braid is used. The copper clip should extend on the braid about ½ in. beyond the contact surface. When several flat strips of braid are used in parallel, a thin piece of brazing alloy about ½ in. square is usually placed in the end of alternate braids throughout the group and one clip is put over the end of the group before clamping in the tongs.

Depending on the size of the cable, the insulation on it should be removed for a distance of ½ in. to 2 in. from the joint. The bare cable should be wrapped with damp asbestos at the end of the insulation to protect it from over-heating during the brazing process.

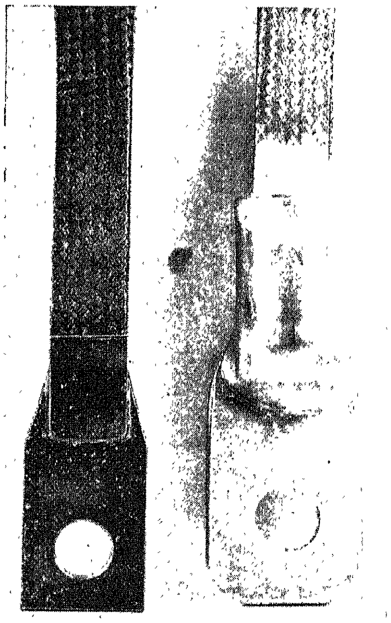

Fig. 3—Comparison of a Brazed Flat-Plate Terminal and Soldered-Tubular Terminal

Lap joints on cables, conductors, etc., are made by lapping pieces to be brazed about 1½ times the thickness of the conductor. If it is desired to make a butt or scarf joint on rectangular conductors, a clamp can be made for holding the two ends together, applying the carbon blocks to the edges of the conductor and feeding the alloy in from the flat side.

In soft soldering, it is very essential that the surfaces be clean and free from oxides and other impurities It is not so essential to clean the surfaces for brazing, but certain oils and varnishes, however, are not attacked by the flux used in brazing and should be removed. The flux most generally used in brazing is fused borax. In some cases, it is more satisfactory to use potassium acid fluoride in paste form. The fused borax lends itself very readily in a dry powder form to being shifted from a small shaker or applied by a small power bellow. It can also be mixed in water or mixed with vaseline and applied to the joints. Any of the above methods of applying the flux is more economical than throwing it on the joint with the hand.

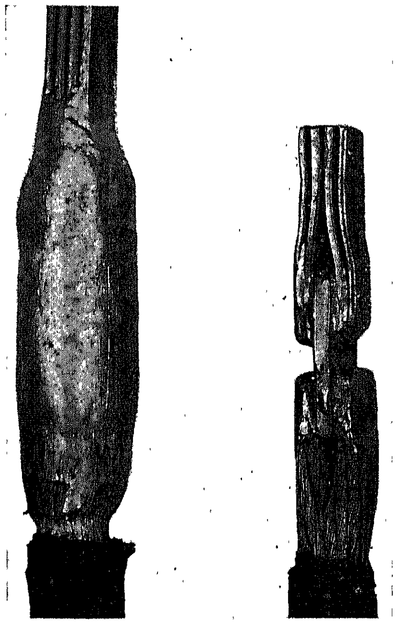

FIG. 4—COMPARISON OF SOLDERED AND BRAZED JOINTS SHOWING POSSIBILITY OF INSPECTION OF JOINT

Depending upon the joint to be made, there are three different silver alloys used as brazing medium. A comparison of these alloys can be obtained from the following table:

Per cent silver	Melting point deg. cent.	Melting point deg. fahr.
10%	830	1526
30%	735	1355
50%	725	1337

The three different silver solders have their particular applications, depending on the permissible temperature determined by the physical conditions of the joints. For example, to prevent over-heating of very small copper wire, it is essential to keep the brazing temperature to a minimum and the 50 per cent alloy is used. On larger cables, terminals, etc., the 30 per cent alloy is used, because it is less expensive, while on the heavy bus bar plates, etc., where there is no insulation, the 10 per cent silver solder is satisfactory.

When the copper bars have been prepared as described, the tongs are tightly clamped on the joints by turning the handwheel. The flux is then applied to the joints and the current is turned on by pushing the foot-operated switch applying power to the primary of the transformer. The carbon blocks heat quickly and if the parts to be brazed are small, the metal will reach a brazing temperature in a few seconds, but if the parts to be brazed are large, more time will be required and the current should be applied intermittently to permit the heat to distribute itself throughout the mass, thus insuring a well-brazed joint.

The borax will melt when the brazing temperature has been reached, and a strip of silver solder is applied at the line of contact between the parts being joined. As the alloy melts, it will follow the borax through the joints. When the borax rises to the surface of the molten alloy, it is an indication that the heat is sufficient and the current can be turned off. Care should be taken not to remove the tongs or to release the pressure on them until the joint has cooled to a dull red.

The economics of brazing are sound. Many cost studies have been made on various size cables, plates, bars, etc. For comparison, consider only three cases, (A) terminal on 180,000-cir. mils cable, (B) terminal on 1,500,000-cir. mils. cable, and (C) twelve ⅛-in. by ¼-in. conductors fastened to a 770,000-cir. mils cable.

Case	Preparation time	Brazing or soldering time	Finishing time	Cost of alloy or solder	Cost of labor at 60c./hr.	Total cost
A-B	3.41	3.17	1.2	0.027	0.077	0.104
A-S	3.41	5.17		0.025	0.085	0.110
B-B	10.90	7.28	4.0	0.225	0.220	0.445
B-S	10.90	17.97		0.247	0.288	0.535
C-B	5.35	15.57		0.225	0.209	0.434
C-S	5.35	24.80		0.46	0.302	0.762

In the column headed "Case," the B is for brazing and the S for soldering. The preparation time includes skinning, binding, and forming of the cable. The finishing time is that taken for cleaning. The above costs are figured for the most favorable conditions for soldering. If labor were figured at 90c. per hr., the saving on the brazed joint would be much greater than that indicated.

Copper bus bars, conductors, cables, etc., are of particular interest to the electrical industry. When the silver solder is applied to a joint formed of these parts, it dissolves a thin film of the metal surface and forms a new alloy having a melting point higher than that of the original solder. This alloy is very rich in copper which makes it an excellent conductor. Since the melting point of the new alloy formed is higher than that of the silver solder, a higher temperature is always required to open such a joint than to make it.

There are several distinct advantages of joints made with silver solder or other high melting point alloys over the ordinary soft solder joints. If the lap joints are designed to have a contact area of at least 1½ times the cross section of the conductor, the factor of safety of such a joint is very high for it will carry as much overload as the conductor itself. Time is saved in making a joint by the resistance brazed method, some joints being made in from ½ to ⅓ the time required to make a lead-soldered joint on the same size conductor. As a result of the saving in time and the small amount of alloy used in making the brazed joint, it is invariably less expensive to make. As illustrated by Fig. 4, brazed joints are considerably smaller and are easily accessible to inspection, since all of the work is open. When a tubular terminal is soldered to a cable, it is impossible to inspect the joint thoroughly, but a cable brazed to a flat terminal can easily be examined to see that there are no defects.

The mechanical strength, durability and high electrical conductivity of brazed joints marks a step forward in the electrical industry.

Discussion

AN IMPROVED ARC WELDING GENERATOR

(BLANKENBUEHLER)

THE NEUTRALIZED WELDER—A MEANS OF CONTROLLING TRANSIENTS

(CREEDY)

A NEW SYSTEM FOR D-C. ARC WELDING

(BERGMAN)

RESISTANCE WELDING OF MOTOR FRAMES

(THOMSON)

ELECTRIC RESISTANCE BRAZING

(MARTIN)

A. Churchward: My study and interpretation of these papers lead me to believe that the constant potential type of welding machine is the one which we would all be putting out as single arc units if its design and construction were not so expensive and the inherent losses on short leads great. It therefore seems logical to assume that we are all trying our utmost to obtain the same results, as nearly as possible, that the constant potential 60-volt generator gives.

These inherent losses disappear entirely when long leads are used, such as specified in the latest Navy Specifications, but I still do not see why the single operator is being used at so much higher voltage than the constant potential, namely 60 volts. Of course this may be due to the inherent characteristics of the bucking series type of machine. I would state that I used the transformer reactor early in 1924 on a self-excited drooping characteristic welding generator, and my tests confirm Mr. Blankenbuehler's results.

D. W. Ver Planck: It is interesting to consider a method of analyzing the current and voltage transients which occur in self-excited arc-welding generators of the type described by Mr. Bergman. The results which can be obtained are of value in showing how the transients vary with the design of the machine and in predetermining approximately the transient characteristics when the design has been decided upon.

To bring the problem within the range of simple analysis it is necessary to make certain assumptions regarding magnetic saturation which fortunately are fulfilled closely in actual machines. They are:

1. The cross magnetic circuit of the generator and the magnetic circuit of the transforming-reactor have no saturation.
2. The main magnetic circuit of the generator is so highly saturated that the main flux is independent of changes in excitation on the main axis.

These assumptions make it possible to replace the generator for purposes of analysis by an equivalent linear electrical network. In this network the armature generated voltage is separated into two parts: a constant positive part equal to the open-circuit voltage of the machine; and a variable negative part which is directly proportional to the change from no load in the cross axis net excitation.

The behavior under various transient conditions of the generator equivalent circuit together with the transforming reactor can then be quite readily determined by the methods of operational circuit analysis.

Applying this method of analysis to finding the voltage transient at the welding terminals following the sudden opening of a steady short circuit shows that the transient consists of the following parts, which may be seen in Fig. 4 of Mr. Bergman's paper:

1. A voltage impulse due to the sudden decay of the flux which links the welding circuit alone, largely in the leakage paths of the transforming-reactor.
2. A sudden rise of the voltage, exclusive of the impulse, which is sometimes called the "instantaneous recovery voltage." This instantaneous recovery voltage is shown by the analysis to consist of the following three parts:
   a. The voltage generated in the armature by rotation through short-circuit flux, which voltage is numerically equal to the total resistance drop in the welding circuit caused by short-circuit current.
   b. An additional voltage generated in the armature by rotation through flux which suddenly appears in the generator as a result of the redistribution of field flux linkages between the transforming-reactor and the generator.
   c. A voltage induced in the welding circuit by transformer action as the field flux begins to change toward its final open-circuit value. This voltage is normally small compared to parts a and b.
3. From the instantaneous recovery voltage the terminal voltage approaches its final open-circuit value exponentially with time. The analysis shows that the time constant of this part of the transient is equal to the total self inductance of the cross field circuit divided by the total resistance of this circuit; that is, the cross field circuit time-constant.

Thus the analysis shows to what particular quality of the machine each part of the transient is due and hence it suggests to the designer the way in which to emphasize or suppress any part of the transient.

The current transient following the making of a short circuit can be handled in a similar fashion, though with somewhat less ease.

One of the most important features of Professor Creedy's paper is the demonstration that the steady-state and some of the transient characteristics of welding generators can be determined analytically. This should go a long way toward dispelling the mystery which seems to shroud welding transients.

However, all of the transient problems have not been cleared up yet by any means. So far all that has been shown is what happens after a single change in terminal conditions such as the sudden change from one arc voltage to another, or the sudden opening of the circuit. While this information is very valuable, it must be remembered that during actual welding the terminal conditions are always fluctuating so that the generator operates continually under a series of transient conditions. The treatment of this problem would of course involve the variable resistance of the arc, making the solution very difficult.

While Professor Creedy's analysis leads to the same results as the equivalent circuit method, I believe that the latter is considerably easier to handle. Also the equivalent circuit method should lead to the definition of certain machine constants, the knowledge of which would completely specify the steady-state and transient characteristics of the generator.

Such machine constants for welding generators would be analogous to the various steady-state and transient reactances used in synchronous machine theory. Their use would make it no longer necessary to solve the differential equations for every welding transient problem. And more important than this, the general use of such constants should do much toward making it easier for the welding engineers to tell the generator engineers what kind of machines to build.

C. J. Holslag: I was the first to discover in 1915 this transformer action during the making and breaking of the arc in a welding generator. This effect is very simple, despite the elaborate papers concerning it. When you touch the electrode to the work, the armature reaction increases the field current to double, triple, or four times its normal value and the effect is cumulative because the armature current increases, etc., until they all come back to rest at the steady value.

If an attempt is made to strike the arc during this period, all is well, but by waiting a small fraction of a second, the opposite effect occurs, *i. e.*, field current is cut in two or even reversed by the transformer action of the laminated field pole with series and shunt winding, or even the armature current alone. This action is very detrimental because a good fraction of a second is taken by the field to recover. During this period, the arc generally goes out and the operator has to "pick" again and again until he gets the right combination to hold it. The whole problem is the above analysis of the interaction of field and armature current. There are many ways to overcome this, once the problem is made clear. The simple way to do it is to use a compound exciter of approximately twice the voltage induced in the field circuit. This prevents the flux dropping below the knee with accompanying recovery.

J. F. Lincoln: The systems employing a transformer in the circuit of a welding set, or the so-called transformer reactor described in the papers by Messrs. J. H. Blankenbuehler and S. R. Bergman are old. They were first described in 1926, being later described by this company, in 1928, and used by another manufacturer for about a year in so far as the transformer and the wiring scheme are concerned.

There is no doubt that this transforming reactor will overcome some of the difficulties which are found in the usual set if no reactor were used. It is extremely doubtful that it is as good either from the point of view of cost, or operation, as the usual well-known reactance coil which has been used for a number of years. The company with whom the writer is connected came to that conclusion, after very complete tests, at least two years before it was ever used by anyone else in this country.

S. S. Scott: The fabrication of heavy frames by the use of resistance welding known commercially by the term "flash welding" as described by Mr. Thomson is used extensively by the automobile body builders, especially for welding 20 gage sheets together in 120-in. widths, thus affording a large saving by purchasing same in small sizes.

The initial cost of these resistance welders is high but in quantity lots the fabricated product can readily pay off the first cost within a short period. For instance, one automobile body builder purchased a 1,500-kva. flash welder to take in 120-in. width sheets of 20 gage material for $75,000 and spent $35,000 on developing same and the following year purchased a similar machine for $75,000 and spent $20,000 for development.

Attention is called to Mr. Thomson's description of the flash welder under discussion, which is a 750-kva. water-cooled transformer and the open-circuit voltage at welding jaws is 12 volts. We had considerable experience in my former company with a 500-kva. flash welder which had an open-circuit voltage of 7 volts at the welding jaws. This machine had a capacity rating of 12 sq. in. maximum, cross section. By using this machine for flash welding large cylindrical rings, 12 sq. in. in cross section, we were able to reduce the cost of welding 80 per cent.

Experiments have proven the fact that transformers designed for flash welding are usually designed for one particular job as it is proven that a transformer designed for flash welding of steel is not suitable for fast upsetting and one designed for steel not suitable for welding alloys of copper or aluminum.

Mr. Samuel Martin, Jr.'s paper is of particular importance to the welding field today as it brings forth the efficiency to which the portable electric resistance brazing machines have been perfected.

In visiting a number of plants throughout the country in the past few years, it was of interest to see how this phase of brazing has been advanced, especially so amongst the small industrial companies.

Mr. Martin refers to a comparison of the three different silver alloys used as a brazing medium, which is of interest to the field and I wish to add the following:

a. Where the design of the joint will allow, tinned surfaces are brazed together using no flux. This applies to many types of joints, tinned tap straps to winding conductors, etc.

b. For butt brazing use the standard silver alloys.

c. For brazing cable, strips, and braids to flat terminals use standard silver alloys.

d. For all bar work or where joints from raw copper can be made without considering insulation use 10 per cent silver alloy.

e. Use alloy consisting of 90 per cent lead and 10 per cent tin, 0.005 in. thick for solidifying braids and similar work.

With reference to flux used for brazing, we might add:

a. For lead 90 per cent, tin 10 per cent—no flux is required. For pure tin leaf use lactic acid. For silver alloys use fused borax.

Sometimes it is beneficial to use borax dry or as a paste with vaseline, according to the nature of the work.

It is of interest to note in Mr. Martin's paper that for every ampere in the primary lines there are 18.4 amperes in the secondary.

The load, which is the resistance of the carbon blocks plus that of the piece being brazed, has a great deal to do in affecting the power factor. Thus, for heavy brazing units, carbon having resistance of 0.0007 ohms per cu. in. should be used. On large joints time is required for the uniform distribution of heat. High-resistance carbon blocks heat much faster than low-resistance carbon and care is required to prevent the burning of the joint surfaces.

The cost data for brazing, as shown by Mr. Martin, are very interesting and verify tests made recently, as the time required to braze a joint is in practically every case less than that required to solder or rivet and solder.

Many types of joints now standard practise reduce the time formerly required for leading from 50 to 75 per cent. With proper equipment and well trained operators brazed joints can invariably be made for less cost than the original type of leaded joint.

J. C. Lincoln: Mr. Bergman in Fig. 2 of his paper shows a method of obtaining a voltage control for the welding machine by reducing, and in some cases, reversing the flux in the horizontal path in the magnetic circuit. In other words, the path marked *R C*. At the same time he saturates the part of the magnetic circuit which he calls *R M* so that the armature reactance does not increase the flux in this magnetic circuit.

Some 25 or 30 years ago I used the same scheme in a somewhat similar form to obtain a compounding effect for a plating machine with the use of a shunt winding only. As is well known, the use of a series coil on a plating machine is objectionable because there is a chance that the counter electromotive force of the material in the plating vats may discharge through the series coil of the plating generator and reverse the polarity. This possibility, which is a real objection to a compound-wound generator does not exist in a shunt-wound generator.

In Fig. 1, the direction of rotation is indicated, also the polarity of the main pole of the bi-polar structure. The polarity of the field from the armature ampere turns is also shown. There is a slot in the middle of the pole faces as indicated. The shunt field instead of being connected between the positive and negative brushes as usual, is connected between the third brush and the positive brush. As the load comes on armature reaction tends to increase the flux into the armature on the upper part of the north pole in the field and the lower part of the south pole in the field.

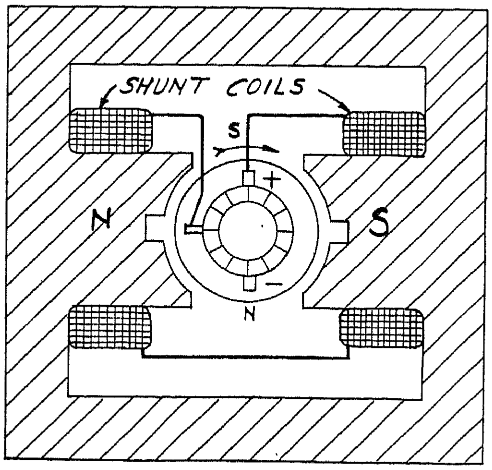

Fig. 1

Magnetic circuit was designed so that this increase of flux could take place and as a result the voltage between the third brush and the positive brush increases as the load comes on, the increase being proportional to the load. By this means a compound effect was obtained similar to what would be obtained with a series coil. This machine and others like it were put into, and so far as I know, are still in commercial operation.

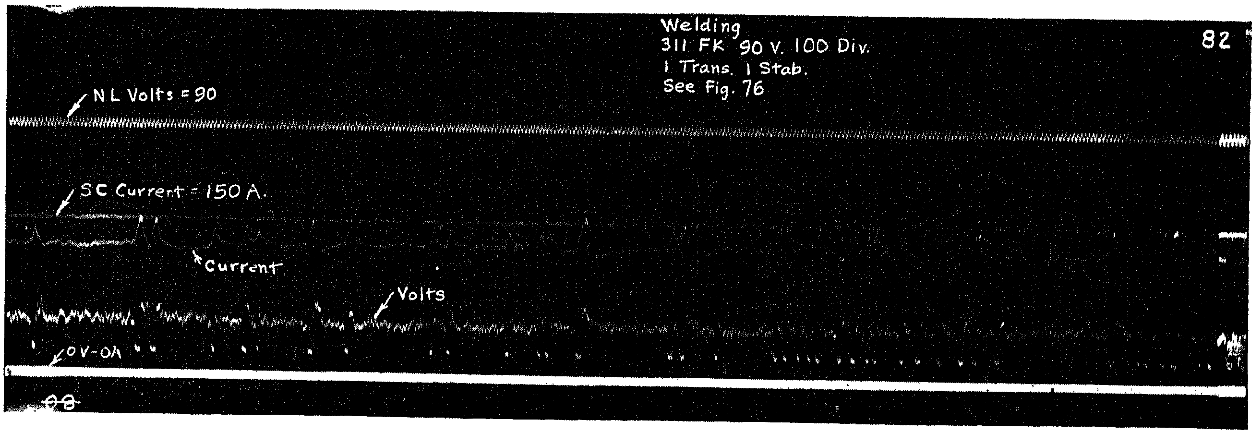

Fig. 2

The statement is made by Mr. Blankenbuehler that a deposited metal is free from porosity when the welding machine is so built there is no overshoot in the current. The question of eliminating porosity in welding is of great importance and I am asking what evidence Mr. Blankenbuehler has for the statement (3) in his summary.

In this connection I would call attention to the fact that a great deal of welding is done on street railways using a resistance connected to the trolley wire. In this case the supply of voltage is approximately 500 volts, the arc voltage is 15 or 20 volts so there is no possible chance of overshooting in welding made by apparatus of this sort, but it also is true that as a general thing, welds made with apparatus of this sort are poor. Therefore, it seems to me that it is clear that the elimination of overshooting does not necessarily produce a high-grade weld.

These three papers show different types of welding machines which have been designed to use ordinary bare wire and which satisfactorily meet the conditions which they were designed to take care of.

Fig. 2 is a typical oscillogram when bare wire or thinly coated wire is used on the electrode. It is here seen that very frequently the voltage drops to zero and this occurs whenever the arc is short-circuited by the globule of metal dropping off the electrode to the work. It is this condition of continuous and frequent short circuit which the welding machine of the present day has to take care of. It is further true that welds made with bare wire or thinly coated wire have satisfactory tensile strength but do not have ductility which compares with the ductility of the parent metal.

Recently there have been introduced electrodes with heavy coatings in which the weld metal has a tensile strength equal to or higher than the tensile strength of welds made with bare wire and in which the ductility of the weld metal approaches that of the parent metal. This high ductility of the weld metal makes possible the distortion of the welds without breaking; makes possible a slight stretch of the welds in order to relieve any locked up strains and in general presents the advantages of ductile steel as compared with non ductile cast iron in any structure.

A very interesting thing to note is that the oscillogram of a weld made using heavy-coated electrode is materially different from that using bare wire. (Fig. 3.) It will be noted in this oscillogram that there are no periods of short circuit, and the metal apparently goes across the arc in small particles. It certainly indicates the fact that the arc is not short-circuited during the ordinary welding operation.

I am calling attention to this characteristic, because as I see it the welding machines of the future will have to meet the conditions shown in oscillogram No. 37 rather than the conditions shown in oscillogram No. 82, and the machines which are discussed at the conventions five years from now will be machines designed to meet the conditions shown on oscillogram No. 37 rather than the conditions shown on oscillogram No. 82.

Mr. Thomson in his paper makes the following statement: "A predetermined amount is flashed off, averaging slightly less than an inch total." The cuts seem to indicate that the rings are bent up so that it would not be necessary to flash off an inch of metal in order to get the full cross section and the question I am asking is—why is it necessary to flash off an inch of metal when apparently half of this amount would be just as effective?

Again on page 684 it is stated that the electric furnace is advantageous in that it gives a strain anneal. I would assume that a weld of this kind made in soft steel would not have locked up strains of such an amount set up so that strain anneal would be necessary. The question is whether the furnace would be used for strain anneal if it were not necessary to size the rings.

J. H. Blankenbuehler: My answers to Mr. J. C. Lincoln's questions follow.

Laboratory tests, in which current was passed through a molten pool of metal at such a value as to keep the pool molten, then caused to increase instantly to a higher value, and then turned off, definitely showed that current surges caused the deposit to be porous.

The welding done on street railways using a resistor connected to the trolley wire is not usually of the best quality because of two factors; first, the polarity is the reverse of that recommended for good welding practise and, second, with such a high-voltage supply the operator finds it easier to hold a longer arc than is consistent with good quality. The presence or absence of current surges in this case are not determining factors.

F. Creedy: It is very gratifying to me to note that engineers are beginning to appreciate the importance to good welding of eliminating transients. When I first drew the attention of American firms to this in 1927 (based on work done in 1924, 5, 6), I found practically no one who appreciated the point except Mr. Ludwig of the Westinghouse Company and Professor C. A. Adams. I note the remarks of Mr. J. F. Lincoln in the discussion, but I certainly got no favorable reaction from his firm at that time.

Like Mr. Churchward I do not see why single arc welders should have a much higher open-circuit voltage than 60 volts although I note that several makers advocate 70, 80, and 90 volts.

I quite agree with Mr. D. W. Ver Planck that the study of welding transients is only begun and there are a great many problems which have not yet been cleared up. Perhaps at a later date he will make a mathematical contribution. In making this analysis, however, it is a mistake to limit oneself to the use of the transforming reactor either of the Bergman or Blankenbuehler form. The essential reason for these transients is that magnetic energy has to flow in or out whenever the fluxes increase or decrease. To design a machine on conventional lines with very high-stored magnetic energy and then use a powerful transforming reactor to produce a forced flow of energy can hardly, one would think, be the final solution. With regard to the remarks of those who claim to have used such a transformer at an early date, it should be pointed out, that it is not enough merely to place a series transformer with primary in the welding circuit and secondary opposing the transient voltages in the exciting circuit but this transformer *must be so proportioned* that it changes the constants of the circuit, so that the solutions of the differential equations on which the current depends become approximately logarithmic instead of oscillatory. If wrongly proportioned it might actually make matters worse instead of better.

M. Thomson: Mr. Lincoln has asked the question as to why we flash off as much as an inch of stock. The answer is that in forming of rings for magnet frames there is a sort of a "V" at the joint, and also sheared stock is rather irregular, so it is therefore necessary to flash off considerable metal to make proper fit between the faces to be welded. It is also necessary to heat the stock sufficiently to insure a sound weld. We find that with perfectly fitted faces and very high-current density the amount of flash can be reduced by at least one-half, but cost of such a joint is higher than the one we use.

The other question asked by Mr. Lincoln, was whether furnace would be used for strain anneal, if it were not necessary to size the rings. While the rings are heated to 1,400 deg. fahr. primarily for sizing operation, we make it a practise to give a strain anneal to all our fabricated motors, whether or not sized, the practise having been extended because the strain annealed steel has been found to be better magnetically. Not all the strains in the stock are from welding. The rolling and forming of the plate itself leave strains which it is desirable to relieve.

As to when it pays to invest in a flash welder; how many pieces must one make to justify the expense? The whole question is one of economics and must be figured for each job. The original cost of equipment, depreciation, power, labor costs, number of pieces made and all such factors enter into the problem. However, on frames of the type we build, I would estimate that about two hundred frames a week would justify the expense of resistance welding equipment.

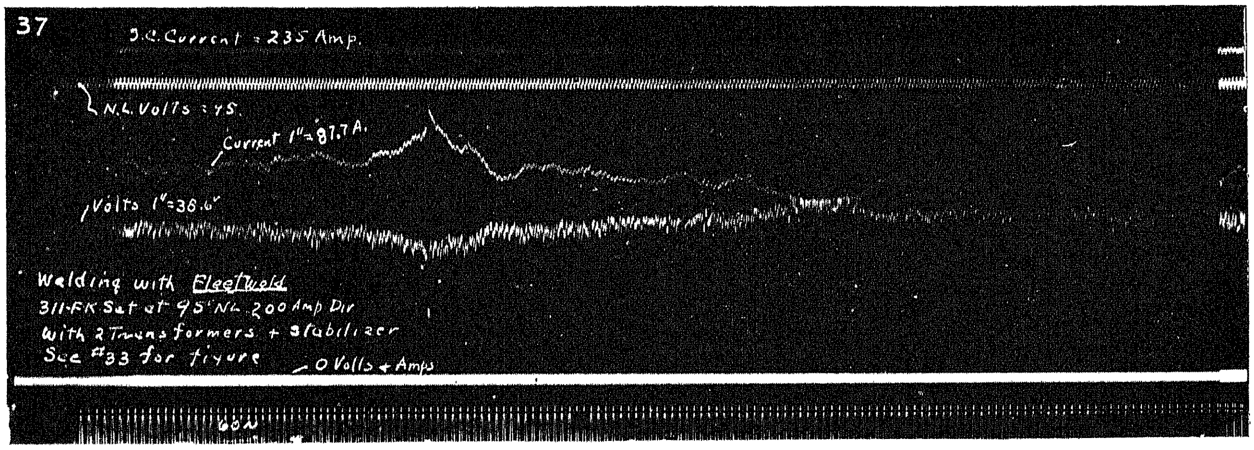

Fig. 3

The Conductivity of Insulating Oils—II

BY J. B. WHITEHEAD*
Fellow, A. I. E. E.

INTRODUCTION

THE conductivity of poorly conducting or "insulating" liquids, although long recognized as ionic in character,[1] is nevertheless highly anomalous and is in no case subject to exact expression in terms of empirical laws. On the first application of a constant difference of continuous potential the current has a high initial value which decreases slowly with time, reaching a constant value only after hours or even days. There is no discharge curve of corresponding magnitude. The phenomenon suggests the somewhat similar dielectric absorption in solids, but is obviously of different character and origin. The decaying current-time curve is characteristic of all liquid dielectrics. By various measures of refinement and purification it is possible to reduce greatly both the initial and the final conductivities[2, 3, 4] but even under the most careful conditions of preparation and experiment the phenomenon is present in pronounced degree. In fact, it tends to disappear only in liquids of higher conductivity, being then masked by conduction of more familiar types.

The final constant current often shows saturation characteristics similar to those of ionized gases. This, through the work of Jaffé,[3] Schröder[4] and Mie,[5] has led to the generally accepted conclusion that conduction in such liquids obeys the corresponding laws of gases. The value unity, of the Langevin[6] constant, necessary for agreement with experiment, leads Jaffé to conclude that hexane, pentane and certain weak solutions in these liquids, behave like very dense gases. It is also usually assumed that this type of agreement with gas theory is limited to highly refined liquids, as studied by Jaffé and Schröder under repeated distillation and careful elimination of influence from the electrodes, walls of containing vessels, etc. The saturation characteristic of the final current obtains also, however, for transformer oil,[7] and other refined insulating oils of engineering practise. The range of values of final conductivities with saturation characteristic is thus from 10^{-18}, the lowest value given by Jaffé, to 10^{-13} mhos per cu. cm. corresponding to a none too good transformer oil.[6] The ionic mobilities in these cases are of the same order of magnitude, ranging from 5×10^{-4} in hexane[3] to 1×10^{-4} in transformer oil,[7] these values being thus of the same order as those of electrolytic ions in aqueous solutions. The source of the ionization in these residual conducting states is not clearly defined. It has been variously attributed to external radio active influence,[1] to gases adsorbed in the electrodes,[4, 8] and to electrolytic impurities for the simplest liquids. Walden[9] has suggested an unrecognized type of dissociation arising in the complexity of the organic molecular structure, which seems more probable in the case of the insulating oils.

Little attention has been given the *initial* high values of conductivity other than to record the decay of its value with time. This decay is generally attributed to the presence of impurities or dissolved electrolytes which are being swept out by the applied electric field. This cannot be the complete explanation, however, since the effect is conspicuous in the most carefully refined liquids, and in all cases the liquid soon resumes its initial condition, permitting repetition. In fact there is much evidence that this decaying conductivity is superposed on that due to dissolved impurities, the clean-up process being only initial and partial, because the ratio of initial to final conductivity is increased on purification, and becomes less in the presence of increased extraneous conductivity (see below).

The early high value of conductivity is of great practical importance in the case of insulating oils in its bearing on the energy loss under alternating electric stress. These carefully refined oils have remarkably good insulating properties; their final conductivities are in the range 10^{-13} to 10^{-15} mhos per cu. cm., being better in this particular than many of the liquids used in the experiments referred to above, and in most cases showing pronounced residual saturation characteristics. The initial values of conductivity are from three to ten or more times the final values. The initial values determine the alternating loss at commercial power frequencies. Tank,[10] using the Helmholz pendulum in a point-by-point method, measured the initial conductivity of several commercial oils over the first 0.25 sec. following the application of potential difference. Within this range he found the conductivity to be constant and of proper value to account for the loss at 50 cycles. We have found a similar result for a poor oil, but a quite different result for a carefully purified oil. We have in fact shown that a brief initial dielectric absorption may be present and that by taking due account of this the alternating losses are completely accounted for by the characteristics as observed under continuous potential.

THE EXPERIMENTS

Continuous current-time records, beginning within 0.01 sec. after the application of voltage have already been reported by the author,[7] using the string galvanometer. They revealed pronounced variations in the shapes of the curves for different oils, also saturation, unequal potential gradient, and other anomalous properties. In the present work we have extended the studies to two other oils and have made corresponding observations with continuous and alternating voltage, measuring the losses, thus permitting correlation of properties under the two conditions.

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1. For references see Bibliography.

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The two oils now studied were two samples of the same insulating oil prepared by the refiner especially for this work. One sample (B-4) had suffered slight deterioration by heating to 80 deg. cent. in the presence of air. The other was freed of air by slow spraying into a 0.1 mm. vacuum. The measurements were made on films 1 mm. thick between coaxial cylindrical electrodes equipped with guard rings and protective screening, and immersed in constant temperature vacuum tanks.[16] Observations were made within the temperature range 30 to 60 deg. and at 500, 1,000, and 1,500 volts, continuous and alternating; frequency, 60 cycles. The short-time current curves were obtained with the amplifier-oscillograph and rapid action switch as developed by Waldorf[11] for this work. The high sensitivity Schering bridge for the alternating loss measurements is due to W. B. Kouwenhoven and A. Banos, Jr.[12] It measures energy loss to within 1 × 10^{-6} watts, at all values down to 1 × 10^{-6} watts, and at power factors down to 0.00007 ± 0.000005.

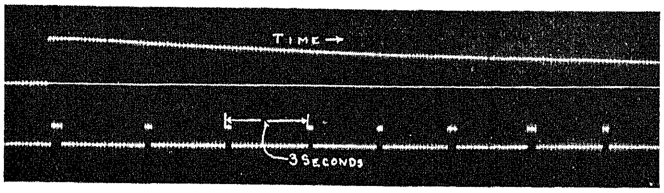

FIG. 1—TYPICAL DECAYING CHARGING CURRENT OF INSULATING OILS

Interval: 0.002—24 seconds

THE RESULTS

Fig. 1 shows the decay in current in oil B-4 at 60 deg. during the first 24 sec. following the application of 1,500 volts. This curve is generally typical of the oils of this class. They differ, however, as to the absolute values and also as to the initial rates of decay. Fig. 2 shows the first 0.03 sec. after the application of voltage; in this record the initial low peak is the trace, at greatly reduced sensitivity, of the geometric charging current and its merging into the subsequent constant conduction current. The complete history in semi-logarithmic coordinates of the change of current in the same oil from an initial value sensibly constant over a period of one sec., is shown in Fig. 3 followed by a slow decay to an approximately constant value reached only after 40 min. The left hand portion of the curve of Fig. 3 was taken photographically and the right hand portion was obtained with a high sensitivity Weston d'Arsonval galvanometer. The initial constant current of 77.5 × 10^{-8} amperes corresponds to a conductivity of 15 × 10^{-14} mhos. per cu. cm., and the final constant current of 18.8 × 10^{-8} amperes to a conductivity of 3.6 × 10^{-14} mhos. per cu. cm.

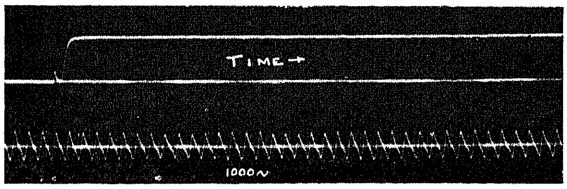

FIG. 2—INITIAL CONSTANT CONDUCTION CURRENT IN CERTAIN OILS

Interval: 0.002—0.03 second

As among different oils there is a noticeable difference in the type of variation of the initial current with the electric intensity. Fig. 4 taken from the earlier paper,[7] shows this relation for a pure transformer oil (A) (final conductivity 3 × 10^{-16} mhos per cu. cm.) and for the same oil slightly deteriorated (B) (final conductivity 2.4 × 10^{-15} mhos per cu. cm.), as result of standing 6 months exposed to air. In the former the initial current increases more rapidly than the applied voltage, in the latter in accordance with Ohm's law, for the range 500 to 1,500 volts. The final constant current in this oil showed clear saturation characteristics. Similar results for the oils B-4 and B-6 of the present studies are shown in Figs. 5 and 6, in which the tendency to saturation for the final current is evident for both oils.

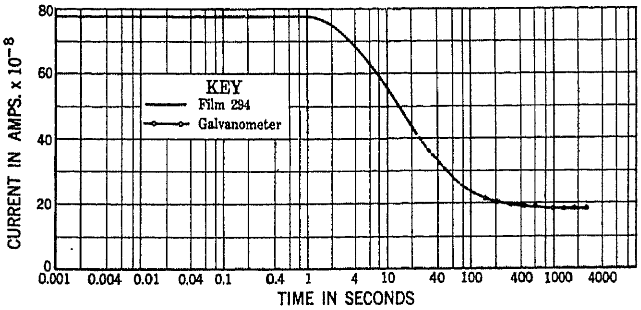

FIG. 3—CHARGE CURRENT CURVE SPECIMEN *B* 4 AT 1,500 VOLTS AND 59.7 DEG. CENT.

The influence of temperature on initial and final currents is seen in Figs. 7 and 8. The increase with temperature in both oils is very much greater for the final than for the initial conductivities. In Figs. 9 and 10 the ratio of the initial to final current is shown as related to applied voltage and to temperature. In both oils this ratio increases with the applied voltage and more rapidly for the purer oil. In both oils at constant voltage the ratio decreases with increasing temperature, and more slowly in the purer oil B-6. These facts are accounted for by the initial-current-voltage relation; by the tendency to saturation described above, and particularly by the increase of conductivity with increase of temperature, in its tendency to mask the decay of current with time. There is here clear indication that the initial and final conductivities are of different origin.

In Table I is given a series of values of energy loss measurements with corresponding values of power factor, for oils B-4 and B-6 at 1,500 volts, 60 cycles, and at temperatures 30 deg., 45 deg., and 60 deg. cent. Corresponding values of the initial constant values of currents as taken from the oscillograms are also given, and in the last column the value of the energy loss as computed from the initial conductivity, *i. e.*, assuming that the loss is all of $I^2 r$ type. It is seen that there is excellent agreement for oil B-4 at all temperatures between the loss as measured and as so computed. For oil B-6 the loss as measured is higher at each temperature than as computed as due to initial d-c. conductivity. At 60 deg. the loss as computed accounts for 84 per cent, at 45 deg. for 80 per cent and at 30 deg. for only 67 per cent of the measured loss. In other words, in each case there must be some other type of loss present than that due to conduction alone. In order to investigate this question further additional current time curves were taken at 30 and 45 deg. increasing the sensitivity of the amplifier oscillograph to the maximum, and reducing the time interval between voltage application and exposure to the minimum, as permitted by the apparatus. The results at 30 deg. and 1,500 volts for both charge and discharge, as plotted from the oscillogram, are shown in Fig. 11. A record at 45 deg. showing brief initial decay followed by constant value is seen in Fig. 12. These curves show that within 0.004 sec. following the application of voltage, or of short circuit following charge, there is a brief initial condition of decay of both charge and discharge currents of approximately the same type and magnitude and so suggesting the phenomenon of dielectric absorption in solids. It is not accounted for by the transient due to the normal circuit constants for this is negligible when the record begins. Further it is greater at 30 deg. than at 45 deg. and it does not appear on the records at 60 deg., indicating its low value and brief duration. It is thus apparently related to the increasing viscosity of the oil at lower temperatures. In Figs. 13 and 14 these initial quasi-absorption currents are shown, as plotted from the records, in their relation to the subsequent longer, but still brief, constant values, followed by slow decay to much lower ultimate possibly constant values. As seen the initial absorption is more pronounced for 30 deg. than for 45 deg. These curves should be compared with Fig. 3 for an oil in which the absorption is completely obscured by high conductivity.

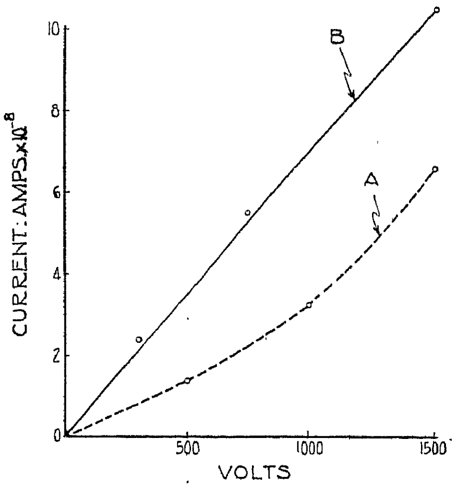

FIG. 4—VARIATION OF INITIAL CURRENT AT 22 DEG. CENT. WITH VOLTAGE TRANSFORMER OILS "A" AND "B"

TABLE I—OIL SPECIMEN B 4

Temperature (deg. cent.)	Loss in microwatts 1,500 v., 60 ~	Power factor	Initial constant current in amperes 1,500 volts, d-c.	Computed d-c. loss microwatts
30	94	0.00015	6.5×10^{-8}	97
45	407	0.00065	26.9×10^{-8}	404
60	1031	0.00165	68.3×10^{-8}	1025

There remains the question whether the brief initial absorption shown in Figs. 11 to 14 is sufficient to account for the excess of the measured a-c. loss over that computed from the relatively long period of the initial constant conductivity following the brief initial absorption. The answer is that it is sufficient. The absorption portions of both charge and discharge curves obey very closely a negative exponential relation with the time, as shown by the empirical equations given in Fig. 11. While the values of the ordinates of the curves in the oscillograms are small they nevertheless permit the reproduction of Fig. 11 to within an accuracy of

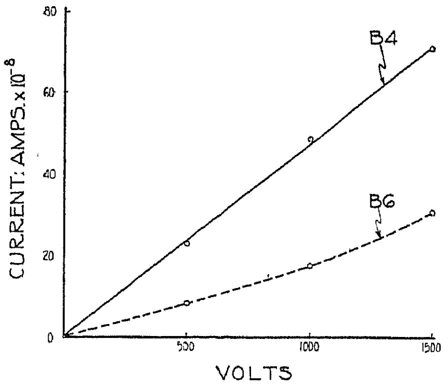

FIG. 5—VARIATION OF INITIAL CONSTANT CURRENT AT 60 DEG. CENT. WITH VOLTAGE

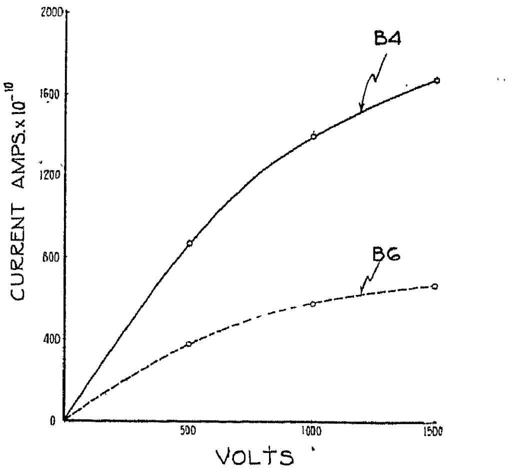

FIG. 6—VARIATION OF 40-MIN. CURRENT AT 60 DEG. CENT. WITH VOLTAGE

about five per cent, for those taken at 30 and 45 deg.; reproduction was not attempted for the record at 60 deg., owing to the extremely small values of current. If now using the method of von Schweidler[13,14] the total loss be computed as the sum of the losses due to absorption and to subsequent constant conductivity, we find the very excellent agreement between the losses as measured under alternating stress, and as computed from the d-c. absorption and conductivity given in Table II. The constant conductivity following the absorption, while of only brief duration, say about one sec. or more, is nevertheless of relatively quite long duration as compared with the period of the alternating circuit, and so it may be considered as the final constant value of von Schweidler's analysis. The initial absorption is absent in specimen B-4, being apparently masked by the greater initial conductivity which in this case is sufficient to account for all loss.

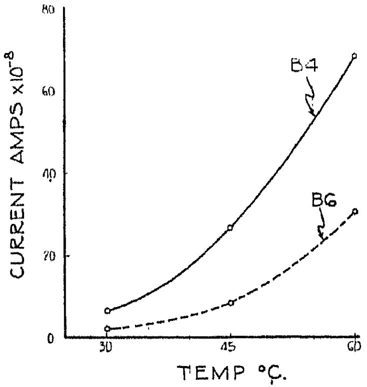

FIG. 7—VARIATION OF INITIAL CONSTANT CURRENT AT 1,500 VOLTS WITH TEMPERATURE

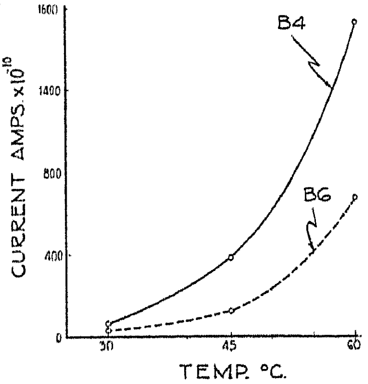

FIG. 8—VARIATION OF 40-MIN. CURRENT AT 1,500 VOLTS WITH TEMPERATURE

It has been suggested that the oscillations of polar molecules may account for a portion of dielectric loss even at frequencies as low as 60 cycles. Evidence to the contrary has recently been presented by Hamburger,[15] who concludes that for normal temperatures and materials this type of loss is negligible for frequencies less than 10,000 cycles. The present experiments are in accord with this view since the entire loss is accounted for by the anomalous conductivity as observed with continuous voltage.

DISCUSSION

The fact that the initial absorption and conductivity as measured determine dielectric loss under alternating stress gives to the short-time characteristics great practical importance. For this means that they are probably also related to the commonly observed rate of deterioration of the insulating properties of these oils. A considerable proportion of the initial conductivity as ordinarily found is apparently electrolytic and due to impurities, to the products of slow oxidation, to inert suspended matter, and to residual moisture. All of these effects may be largely removed by filtration, distillation, and evacuation, and the prolonged application of electric stress further contributes to the clean up. However, even after the most careful purification by the methods mentioned, an initial absorption and a conductivity much greater than the final conductivity remain.

The question then arises, can the ultimate application of the most refined methods of purification yield a liquid which, in accordance with the idea of Kohlrausch, shows no conductivity, or even a liquid with a minimum conductivity which is constant from the first application of electric stress? The answer to this question seems to be definitely, no. First as regards the final steady current: While Jaffé concludes from his studies on hexane and pentane, that in these liquids there is no dissociation and inherent conductivity, he was nevertheless unable to eliminate all conductivity, and only able to account for a part of that remaining as due to radio active influence. Schröder[4] concludes that in

TABLE II—OIL SPECIMEN B 6

Temp. (deg.) cent.)	Loss in microwatts 1,500 volts 60 ~	Power factor	Initial constant current in amperes 1,500 volts d-c.	Computed losses in microwatts: From initial constant current	Computed losses in microwatts: From absorption	Total
30	47	0.00007	2.10×10^{-8}	31.5	15.2	46.7
45	165	0.00024	8.83×10^{-8}	132.5	32.8	165.3
60	544	0.00079	30.6×10^{-8}	459.0		

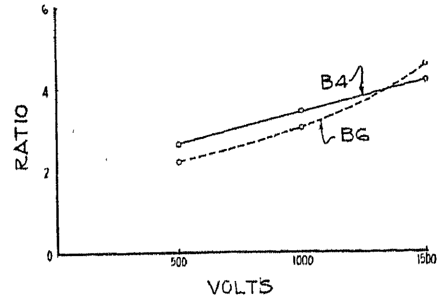

FIG. 9—RATIO OF CURRENTS AT 60 DEG. CENT. INITIAL CONSTANT TO 40 MIN. VS. VOLTAGE

ethyl ether there is a residual volume ionization. This however is questioned by Fassbinder,[8] who concludes that ethyl ether has no dissociation, but that the conductivity observed is the result of absorption at the electrodes, with resulting dissociation. Carvallo[9] concludes that sulfureous anhydride has no inherent dissociation and that the conductivity which can not be eliminated is due to photoelectric influence of physical, rather than chemical type. Thus all of these investigations in which the utmost care was taken to free the liquid from impurities indicate the impossibility of completely suppressing electric conductivity, which in each case is attributed to the influence of some extraneous cause impossible to eliminate.

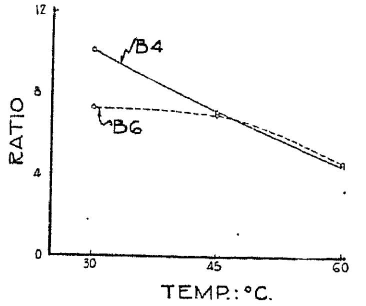

FIG. 10—RATIO OF CURRENTS AT 1,500 VOLTS: INITIAL CONSTANT TO 40 MIN. VS. TEMPERATURE

A somewhat more satisfactory explanation is offered by P. Walden[9] who points out that the purest sulfuric acid conducts better than some water salt solutions and that there thus must be no question of the possible dissociation and conductivity of a pure liquid. Further, that sulfur dioxide, while subject to electric purification as regards conductivity, is nevertheless decomposed, with resulting increase of conductivity at moderate values of electric stress. Regarding this as evidence of chemical instability, he notes that in a number of dissociating liquids containing several CO-, OH-, and CN- groups, the values of mean conductivity are in the same sequence as the dissociating power. The same is true of solvents which form multi-valued molecules and which tend to polymerization and dissociation. Mixtures of pure liquids sometimes show higher and sometimes lower conductivities, than those of the compounds. Some conducting liquids yield with difficulty to a lowering of conductivity by electric clean-up, distillation, etc., and return promptly to their initial condition. Vant Hoff[9] assumes that perfectly pure water possesses a conductivity due to dissociation of its molecules into H and OH. All of these matters point to a real lack of homogeneity even in the purest liquids and to molecular groups, probably transient, other than the simple grouping by which the liquid is usually recognized. Granted the possibility of such a departure from molecular stability, it is not difficult to picture a transient state of ionization resulting in conductivity.

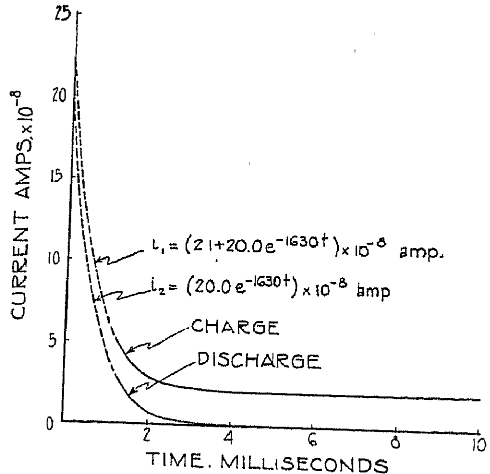

FIG. 11—CHARGE AND DISCHARGE CURRENTS SPECIMEN B 6 AT 1,500 VOLTS AND 30.3 DEG. CENT.

As regards the initial conductivity, as stated, its excess over the final value is accentuated in the purest liquids. While generally attributed to electrolytic impurities, there is direct evidence that this view is scarcely tenable in the prompt recovery of conductivity following its removal by continuous electric stress. A first assumption naturally is that this conductivity is due to an accumulation of ions up to an equilibrium condition as determined by the rate of formation. This view is shown to be not sufficient by Schröder in that the initial and final saturation current values indicate that the type of ionization in these cases would be swept out far more rapidly than is indicated by the experimental current curves. It appears more likely that the initial conductivity is due to the action of the electric field on the complex molecular groupings suggested by Walden, drawing them off more slowly than the simpler type of ion, owing to their larger size. Whitehead and Marvin[7] have shown the accumulation of space charge of a very sluggish type throughout the volume of transformer oil under electric stress, and it appears probable that the resulting polarization also contributes to the slow decay from the initial conductivity. This view is also supported by the initial current-voltage curves of the present experiments in which the current increases more than in proportion to the voltage, indicating a cumulative or secondary ionization effect. It is probable that with increasing field strength a greater and greater proportion of the complex molecular groups are split up into ions.

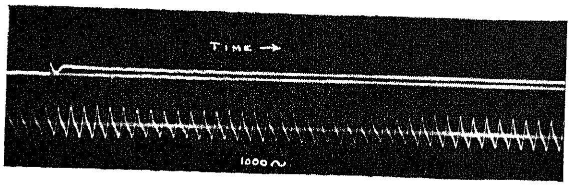

FIG. 12—DIELECTRIC ABSORPTION IN OIL

Insulating oils such as described in this work may clearly be included in the picture suggested above. Their final conductivities compare favorably with those of the purest simple liquid. Their molecular complexity is well known and hence a large initial current is to be expected. As regards the question of losses, since they are due to the initial absorption and conductivity, improvement in insulating properties and probably in the life of such oils is to be looked for first by careful purification, but generally by the experimental selection of oils having the most stable molecular structure; for stability in this regard should mean lower initial conductivity, lower loss, slower deterioration, and longer life.

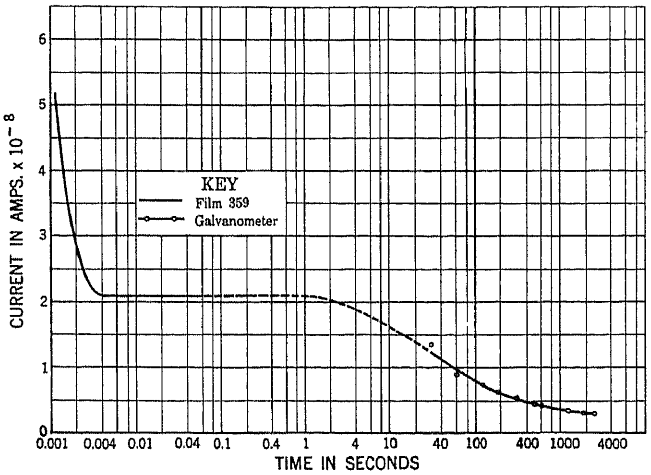

Fig. 13—Charge Current Curve Specimen *B* 6 at 1,500 Volts and 30.0 Deg. Cent.

Little can be said as to the dielectric absorption noted, owing to our ignorance of the fundamental nature of this phenomenon. The picture presented however is of a number of ions always present in the free state. They have a certain latitude of movement, restricted, however, in some way by a viscous structure of the liquid. On the application of electric stress there is a general motion, limited, but resulting in some type of permanent polarization, which is the origin of the current on discharge.

Conclusions

1. Certain high-grade insulating oils have electrical characteristics similar in type and comparable in magnitude to those of the carefully refined poorly conducting liquids, used in physical research. Conductivities of the order 10^{-15} mhos/cu. cm. are common.

2. In the purest state they show a definite though brief dielectric absorption. There are in fact three definite stages of conductivity: A. An initial brief, rapidly descending value, with corresponding curves for charge and discharge current, thus indicating the presence of dielectric absorption. The duration of this phase is of the order of 0.004 sec. B. An approximately constant value of conductivity within the interval 0.004 to 1 sec. C. A very slowly decaying conductivity, reaching a constant value only after 30 min. or more.

3. The initial dielectric absorption increases with decrease of temperature and is thus associated with the viscosity. It is difficult to measure at 60 deg. or in an oil slightly deteriorated. In these cases the absorption, if present, is obscured by the greater value of conductivity of phase B.

4. The dielectric loss under 60 cycles alternating stress is definitely determined by the continuous current characteristics within the interval 0 to 0.1 sec. The loss at the higher temperatures and for oils having high initial conductivity is completely accounted for by the conductivity of phase B. For the purest and best oils below 60 deg. the alternating loss is completely accounted for by the conductivity of phase B plus the loss computed from the curve of dielectric absorption pertaining to phase A. At 30 deg. this second component may represent 35 per cent of the total loss.

5. The initial absorption, conductivity, and subsequent decay, are not completely accounted for by residual impurities, as frequently stated. It is highly probable that they are in part due to the variety of molecular complexes and groupings occurring in such hydrocarbons, resulting in a heavy slow moving type of ion, which is subject to wide variation in properties with the value of the electric intensity and with other external influence. Thus these ions which give rise to the initial stages of conductivity are probably quite different from those involved in the final conductivity, and which have been used by Jaffé and others for drawing analogies between the conductivity of a liquid and of a gas. There is in fact no immediate evidence that this latter type of ion is involved in the alternating loss at commercial frequencies.

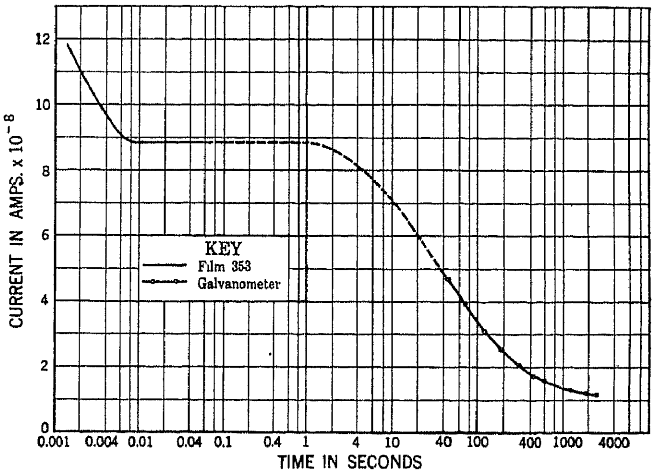

Fig. 14—Charge Current Curve Specimen *B* 6 at 1,500 Volts and 45.0 Deg. Cent.

The experimental results reported have been taken partly from a research on the fundamental properties of insulating oils, supported by the Engineering Founda-

tion, and partly from a research on the properties of impregnated paper, supported by the Utilities Research Commission of Illinois, both in the Johns Hopkins University. Grateful acknowledgment is made of the interest and careful work of Professor W. B. Kouwenhoven, and Drs. R. H. Marvin and S. K. Waldorf, and Mr. A. Banos, Jr.

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Discussion

For discussion of this paper see page 705.

Fundamental Properties of Impregnated Paper

BY J. B. WHITEHEAD* and W. B. KOUWENHOVEN†
Fellow, A. I. E. E. Member, A. I. E. E.

INTRODUCTION

THE excellent insulating properties of impregnated paper and their adaptability to the special requirements of the insulation of high-voltage underground cables are well known. However, the complexity of the component parts and of the processes involved in their combination has so far prevented a complete understanding of underlying electrical processes, and also complete control of the characteristics of the assembled material. The immediate purpose of this paper is to report the first results of a study of the separate electrical behavior of the paper, the impregnating compound, and of their combination, and of such indications as may be present of the relations of fundamental to final characteristics. The ultimate purpose of this study and those to follow is the hope to point out methods of control and improvement.

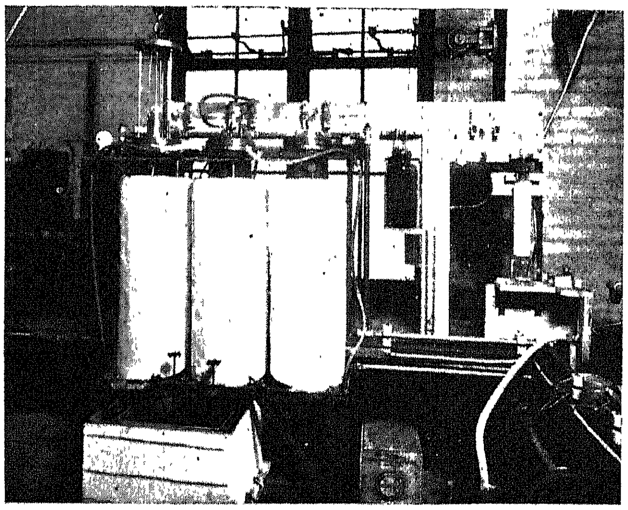

FIG. 1—CONSTANT TEMPERATURE DRYING, IMPREGNATING, AND MEASURING TANKS

EXPERIMENTAL METHOD

In the equipment shown in Fig. 1, three tanks were provided for the test specimens. In one of these the paper was studied, in another the oil, and the third used for studies of impregnated paper. The temperature of the test specimen in each tank was determined by resistance thermometers and was automatically controlled for each specimen independently of the others to within 0.1 deg. cent. The method of temperature control is described in a paper by Kouwenhoven and Waldorf.[1]

Electrical measurements were made under both alternating and continuous voltage. Dielectric loss, power factor and capacitance, as related to temperature between 30 deg. and 60 deg. cent. were measured at alternating, 60-cycle voltage of 500, 1,000, and 1,500 volts effective. These quantities were determined with a modified Schering bridge, whose phase angle sensitivity is $\pm 5 \times 10^{-6}$ and which has permitted measurements of power factor as low as 0.00007. The special features of this bridge, its shielding and other conditions necessary to obtain this degree of accuracy are described in a separate paper by Kouwenhoven and Banos.[2]

The measurements under continuous voltage were the short and long time values of charge and discharge currents and the dielectric absorption, irreversible absorption, and final conductivity deducible therefrom. The ranges of voltage and temperature were the same as those explored in the alternating measurements. For the short time values of charge and discharge current beginning within one millisecond of the application of voltage, or of short circuit, the amplifier oscillograph as developed by Waldorf[3] was employed. This instrument increases the normal sensitivity of the electro-magnetic oscillograph 10^5 times, giving a deflection on the photographic film of 1 mm. per 4×10^{-8} amperes. The measurements with this instrument have been of great value in a study of the correlation between the continuous and alternating characteristics. Dielectric loss is completely accounted for by conductivity and dielectric absorption as read under continuous voltage. The further development of the amplifier oscillograph, as applied to this work, is described by S. K. Waldorf[4] in a separate paper.

TEST SPECIMENS

The test specimens were in the form of cylindrical capacitors. The central member, the high-voltage electrode, was a brass tube 5.08 cm. outside diameter, 61 cm. long, on which the paper was spiralled as in impregnated paper cables, with butt joints between tapes and a registration of one-quarter of the width. The outer electrodes were of 0.4 mm. sheet lead; the main electrode had a length of 25.4 cm. with a guard electrode of 7.6 cm. at each end. The main or measuring electrode and its lead were completely shielded (see Fig. 2). The entire test capacitor was suspended from the cover of the vacuum tank and all connections were brought through this cover.

A second capacitor of approximately equal dimensions was used for the oil. In this case, however, the inner

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1. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

electrode and guards were rigidly attached to the central cylinder with bakelite spacing rings. The nominal capacitor dimensions were the same for both types, as follows: outside diameter of inner cylinder 5.08 cm., inside diameter of outer or measuring electrode 5.385 cm., total thickness of dielectric of oil, of paper, or of impregnated paper 1.52 mm. After separate measurements on paper and oil the oil was drawn into the heated vacuum chamber containing the paper sample, impregnating the same.

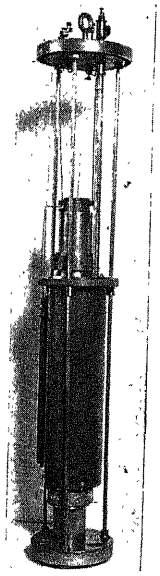


Fig. 2—Specimen Mounted for Test

Materials and Preparation

A high-grade wood pulp paper 0.1 mm. thick and 2.54 cm. wide was used, 15 layers making up the specimen. The samples were dried and evacuated at 0.1 mm. Hg. (105 deg. cent.). They were brought to the same condition as regards dryness, as indicated by a standard value of final conductivity of 1.6×10^{-15} mhos per cu. cm. (105 deg. cent.). The two specimens used in the present tests, designated as A-2 and A-3, proved to have closely similar characteristics throughout all tests.

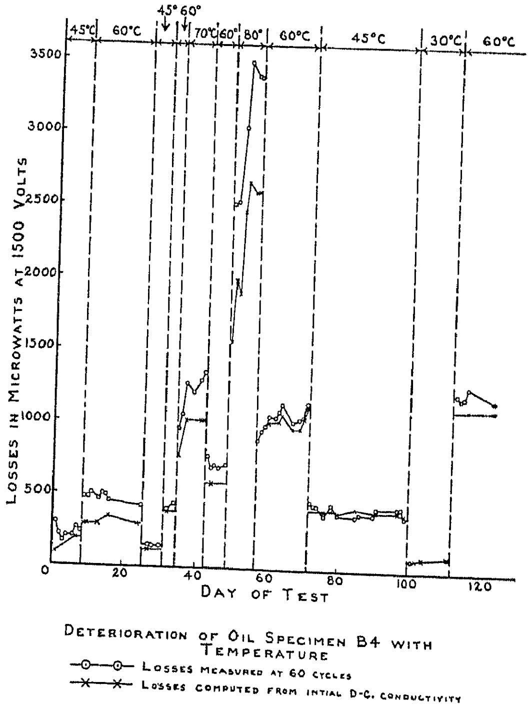
Oils

Two samples of high-grade cable oil, which will be designated as B-4 and B-6 were used. Both were from the same manufacturer, and from the same batch. The oil was received in air tight containers; nevertheless, it contained considerable air.

Oil B-4 was allowed to deteriorate slightly by raising its temperature to 70 and 80 deg. for the time intervals indicated in Fig. 3, showing the history of this treatment and its results. The entrapped air was largely though not completely removed before heating, and the deterioration, therefore, was doubtless due to oxidation. It will be noted that the results of this deterioration were a substantial increase in dielectric loss and in final conductivity. The final long time conductivity of B-4 at 45 deg. cent. was 0.70×10^{-14} mho. As shown below its value at four milliseconds after application of voltage was 4.84×10^{-14} mho. Oil B-4 was used to impregnate paper sample A-2 and the finished product was called C-1. The pressure at the time of impregnation was less than 1 mm. Hg. at 60 deg. cent. The impregnation of the paper sample A-3 took place at 0.5 mm. pressure (60 deg. cent.). The final conductivity of oil specimen B-6 at 45 deg. cent. was 0.21×10^{-14} mho, and at four milliseconds its value was 1.43×10^{-18} mho.

Experimental Results

The results are represented in a series of curves as follows: The alternating current results give the loss-voltage, loss-temperature, power-factor-voltage and capacitance-temperature relations for the specimens B-4, A-2, and C-1 in Figs. 4 and 5 and for B-6, A-3, and C-2 in Figs. 6 and 7. The comparison of the two oil samples as regards continuous current-temperature and current-voltage relations is found in Figs. 8 and 9 respectively. Figs. 10, 11, 12, and 13 give the continuous current-temperature and current-voltage data for the specimens A-3 and C-2.



Deterioration of Oil Specimen B4 with Temperature

Fig. 3

Oils: A-C. Behavior. As between the two oils, as a result of the deterioration the loss and power factor in B-4 are about twice those in B-6. This ratio also extends to the observations under continuous voltage, as shown in Figs. 8 and 9. In each case there is a sharp rise of loss with increasing temperature. The power-factor-voltage curves are flat at 30 and 45 deg. but show a slow linear rise at 60 deg., being much the same in both oils. As regards apparent capacitance, determined by alternating measurements, B-4 shows a slightly higher value and both show a capacitance decreasing with increasing temperature.

Papers: A-C. Behavior. Both papers, A-2 and A-3, show much the same characteristics throughout. The losses, closely proportional to the square of the voltage, decrease with increasing temperature. The power-factor-voltage curves are flat throughout, decreasing with increasing temperature. The capacitance increases with temperature. Thus the variation of both loss and capacitance with temperature is in opposite sense to that found in the oils. The decrease of loss in paper with increasing temperature may be ascribed to a decrease in dielectric absorption. The high resistance conducting films of residual moisture take on increased conductivity with increasing temperature (Evershed effect). There is thus a lower $I^2 r$ loss in the passage over them of the charging current of the main dielectric structure. At the same time there is a corresponding increase in number and of surface of these conducting filaments which means an increased effective capacitance as shown in the curves. Note, also, in support of this suggestion the decrease of both four milliseconds and eight milliseconds charging current readings with increasing temperature, (Fig. 12) with corresponding increase of final values of charging current, the former indicating decreased absorption and the latter increased conductivity.

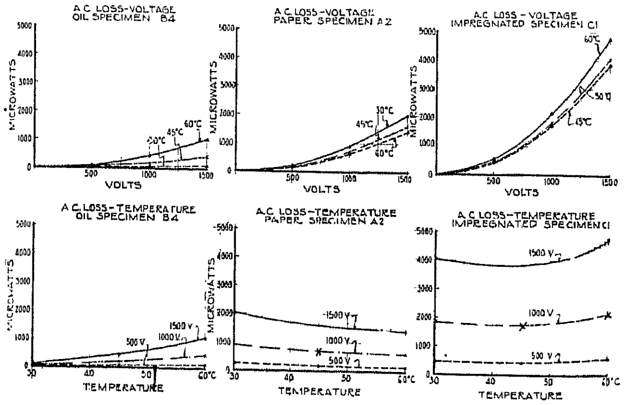


FIG. 4

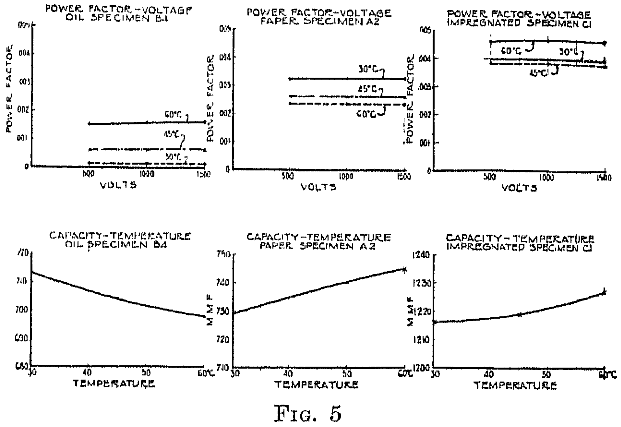


FIG. 5

Impregnated Paper: A-C. Behavior. As between the two impregnated specimens C-1 and C-2 there are some noteworthy differences. In each the power factor is constant with respect to the voltage and the losses increase approximately as the voltage squared. In C-1, impregnated with deteriorated oil, the loss decreases with increasing temperature from 30 deg. up to 40 deg. but increases thereafter, (Fig. 4) and the loss temperature curves have a "U" shape at all three voltages. In C-2, on the other hand, the loss decreases with increasing temperature at all voltages. In both oils the losses increase rapidly with increase in temperature, (Figs. 4 and 6) but in B-4 the greater value of the loss is sufficient to offset the corresponding decrease in loss in the paper. In B-6 the magnitude of the loss is not sufficient to offset the opposite type of variation in the paper over the temperature range studied.

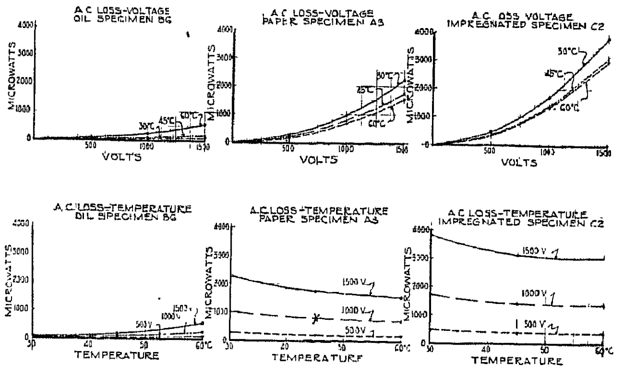


FIG. 6

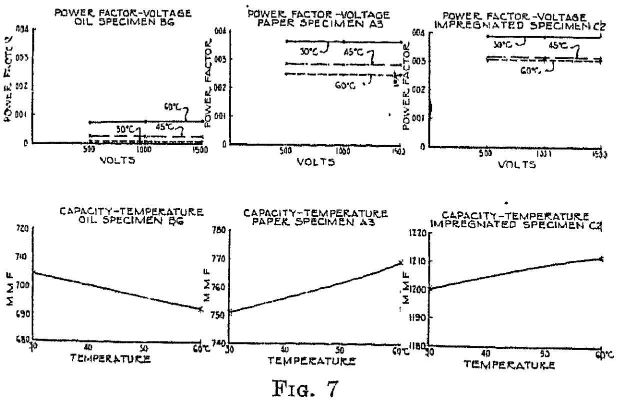


FIG. 7

It is interesting to note that the measured loss in the impregnated specimen C-1 is 1.9 times the sum of the losses in the separate samples of the oil B-4 in paper A-2, throughout the voltage and temperature range studied. In sample C-2, the ratio varies from 1.4 to 1.58. It is evident that the increase in loss following impregnation is due to the large increase in the dielectric absorption resulting in the combination of the two dielectrics, oil and paper. In this combination the oil penetrates the interstices between the paper fibers, and as the oil has some conductivity it constitutes an electrode surface and conducting filaments thereto, for the deeper lying dielectric structure of the fibers, thus increasing the absorption current. For example, the value of the current at four milliseconds after the application of 1,500 volts on paper sample A-2 at 30 deg. was 55×10^{-8} amperes, on impregnated specimen C-1, 130×10^{-8} amperes, on paper sample A-3, 54×10^{-8} amperes, and on impregnated sample C-2, 96×10^{-8} amperes (see Figs. 10 and 11). Thus the charging current in each case is increased by impregnation, and considerably more so in C-1 than in C-2, owing to the higher conductivity of the oil in the former. The values of these short time currents, as taken with the amplifier oscillograph, are taken in connection with each loss measurement. Typical examples are given in Figs. 14 and 15.

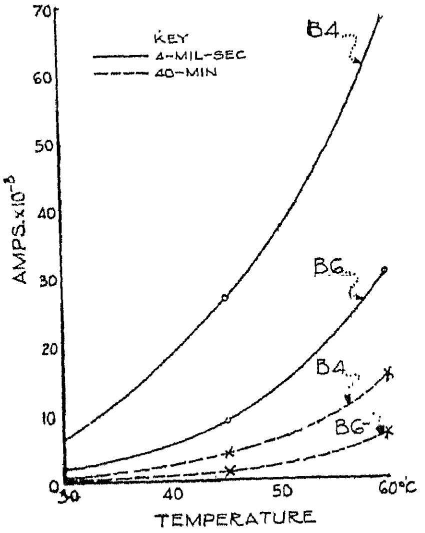


FIG. 8—COMPARISON AT 1,500 VOLTS OF 4-MILLISECOND AND 40-MIN. CHARGE CURRENT—TEMPERATURE RELATIONS FOR OIL SPECIMENS B 4 AND B 6

The phenomenon is still further clarified if the irreversible conduction current is deducted from the total charging current. The irreversible current is a true conduction current and is obtained by subtracting the ordinates of the curve of discharge current from the corresponding ordinates of the curve of charging current. Its value may vary with the time elapsed since the application of voltage and it merges into the final conduction current. Since it is a conduction current, having no corresponding discharge current, the irreversible current contributes to the loss, but plays no part in dielectric absorption. Deducting the irreversible current in each of the above cases, the four millisecond and eight millisecond charging currents on samples C-1 were 83×10^{-8} and 48×10^{-8} amperes with ratio 1.73, giving a rough measure of the slope of the current-time curve. The corresponding figures for C-2 were 50 and 23, with ratio of 2.17. These values obtain for 60 deg. cent. and 1,000 volts. We note from these figures that in impregnated specimen C-2 the short time absorption current is not only lower in value, but also decreases more rapidly than in the case of C-1. Therefore the dielectric absorption is less in C-2. Thus the well-known fact that the higher conductivity in an oil leads to greater values of dielectric loss is attributable in part to an increase in dielectric absorption, and in part to increased irreversible conduction current.

A further result of the penetration of the conducting oil into the structure of the paper is found in the large increase in capacitance. As shown in Figs. 5 and 7, the capacitance as measured with alternating voltage is approximately of the same value for both oil and paper singly. In combination, however, the capacitance is substantially increased in both impregnated specimens. This increase in capacitance after impregnation is due largely to the filling of the interstices in the paper with a medium whose dielectric constant is greater than that of air, but also in part to dielectric absorption. The increase in C-1 is greater since the conductivity of the oil in this specimen is higher, and permits a more rapid passage of charge to the insulating oils of the wood fiber structure.

THE LOSSES IN IMPREGNATED PAPER

The measured values of loss in all the samples studied whether of oil, of paper, or of impregnated paper, may be computed in each case from the fundamental absorption and conduction characteristics of the respective specimens, as determined from the short time charge and discharge currents measured with continuous potential. The accuracy of this agreement between alternating current measured and direct current computed values for the impregnated specimen C-2 is indicated in Table I. The method of analysis leading to this agreement has already been briefly described in a paper by one of the authors.[5] It is proposed to discuss the method in greater detail as related to the present work in a separate paper.

TABLE I—CORRELATION OF D-C. AND A-C. MEASUREMENTS IMPREGNATED PAPER SPECIMEN C-2

Temperature (deg. cent.)	Volts	Losses in microwatts Computed from d-c. charge "current-time" relation	Observed at 60 cycle
60	1,500	3,003	3,046
	1,000	1,290	1,341
	500	335	333
45	1,500	2,925	3,107
	1,000	1,388	1,378
	500	356	344
30	1,500	3,690	3,816
	1,000	1,770	1,691
	500	448	424

The general nature of the relation between direct current and alternating current characteristics may be briefly stated as follows: The total alternating loss in insulation showing dielectric absorption is due to three factors, namely, reversible dielectric absorption, irreversible absorption, and true conductivity. The relative importance of these three quantities as determining alternating loss may vary widely. Dielectric absorption, with the sustained values of charging current which it causes, results in a component of alternating current in phase with the electromotive force, thus causing loss. The magnitude of this component depends on the initial value of the continuous current curve and on the rate at which it decreases. Both these quantities depend in an indeterminate way on the ratios of resistivity to specific inductive capacity in the two media oil and paper. The irreversible absorption current has no corresponding component on discharge and under alternating stress thus results in a true alternating conduction current and loss. The amount of this component of the loss may be quite considerable, and inasmuch as the irreversible current may be much higher for short time intervals than for long intervals, it is the short time interval, as related to the period of the alternating circuit, which is important in determining the loss. The irreversible absorption current merges eventually into the final conduction current. The amount of loss due to this final value is usually very small and its contribution to the loss negligible in the better grades of insulation. (See Table II.)

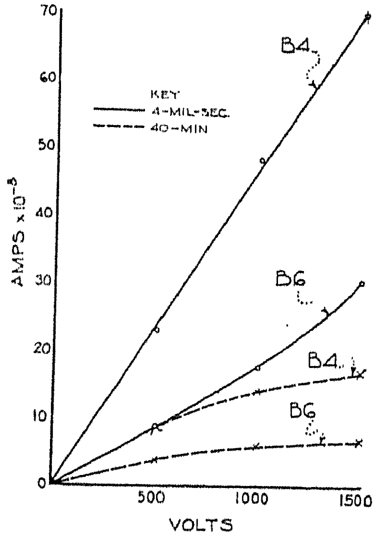


FIG. 9—COMPARISON OF 4-MILLISECOND AND 40-MIN. CHARGE CURRENTS OIL SPECIMENS B 4 AND B 6 VARIATION WITH VOLTAGE AT 60 DEG. CENT.

TABLE II—MEASURED A-C. LOSSES AND CALCULATED D-C. LOSSES IN MICROWATTS
1500 VOLTS—60 DEG. CENT.

		Losses computed from continuous current records			
Specimen	Measured a-c. losses at 60 cycles	Reversible absorption	Initial Irreversible	Final conductivity	Total
Oil B-4	1031	6*	1025	229	1031
Paper A-2	1465	1410	80	0.59	1490
Impregnated paper C-1	4892	3451	1350	3.4	4801
Oil B-6	544	85*	459	100	544
Paper A-3	1555	1410	75	0.65	1485
Impregnated paper C-2	3046	2478	525	2.1	3003

*Estimated.

As an example of the relative values of the losses due to these several causes, refer to Table II, in which the losses at 1,500 volts and 60 deg. due to the several causes are separated for the oil alone, the paper alone, and for the impregnated paper, and for both the oils studied.

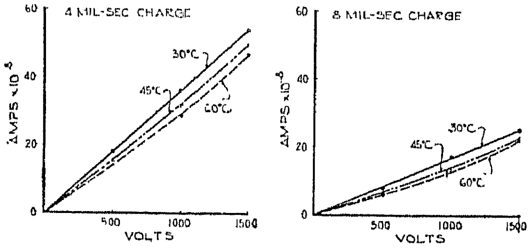


FIG. 10—CHARGE CURRENT—VOLTAGE RELATIONS PAPER SPECIMEN A 3

Note particularly that in the oil B-4 99.5 per cent of the total losses are conduction losses, due to the initial value of the irreversible charging current. There is practically no absorption, and final conductivity is submerged in initial conductivity, which is over four times as great as the final value. In the case of the papers the loss is almost entirely due to reversible absorption, that due to the initial irreversible current being only of the order of 0.5 per cent, with still smaller values for the loss due to final conductivity. After impregnation, however, the loss due to irreversible current rises from 0.5 per cent to about 30 per cent in specimen C-1, impregnated with the oil B-4 of higher conductivity, and from 0.5 per cent to 17.4 per cent in specimen C-2, impregnated with the better oil, B-6.

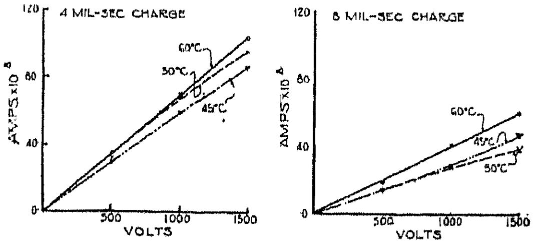


FIG. 11—CHARGE CURRENT—VOLTAGE RELATIONS IMPREGNATED PAPER SPECIMEN C 2

The remaining components of the losses in the impregnated samples are due to absorption, that in C-1 being greater than in C-2, for reasons already described above.

D-C. BEHAVIOR

The d-c. characteristics of oil, paper, and impregnated paper, and their separate influence, have already been described in some measure. Figs. 8 to 13 also present several additional matters of interest. The short time characteristics of the two oils, as related to temperature, are shown in Fig. 8. As indicated the higher final conductivity of B-4 is reflected in about the same ratio, in the values of charging current at 4 milliseconds after the application of voltage. The variation with voltage as shown in Fig. 9 reveals a straight linear relation between current and voltage for the oil B-4, but with a tendency for a greater rate of increase for oil B-60. We have noticed this tendency also in other oils, and this is ascribed to some type of secondary ionization, which is masked as the conductivity is higher, as in B-4. Note that the long time values of charging current in both oils show pronounced approach to saturation values conforming to the behavior of the purest dielectric liquids reported in the literature of physical research.

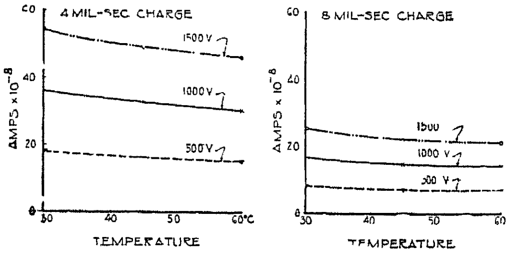


Fig. 12—Charge Current—Temperature Relations Paper Specimen A 3

As regards the paper, it may be noted (see Figs. 10 and 12) that the initial values of current decrease slowly with increasing temperature, while the opposite is the case of the final conductivity of dry paper.

For the impregnated paper (see Figs. 11 and 13) it is of interest to note the greater influence of temperature on the eight millisecond values as compared with the four millisecond values. The later values increase steadily with increasing temperature. This is due to the greater influence after the longer interval of the temperature increase of the conductivity of the oil. The importance of the early values of charging current in causing the loss is clearly evident because for C-2 we find the loss decreasing with increasing temperature.

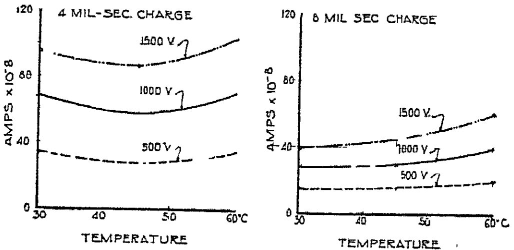


Fig. 13—Charge Current—Temperature Relations Impregnated Paper Specimen C 2

Acknowledgments

The authors wish to acknowledge the help given by Messrs. S. K. Waldorf and A. Banos, Jr., in all phases of the work. Grateful appreciation is also expressed to the Utilities Research Commission of Illinois, whose financial support has made possible the initiation and active prosecution of the project.

Conclusions

1. The total dielectric losses in impregnated paper have been separated into three components, absorption, irreversible, and final conductivities, due to fundamental characteristics, which have been studied by means of continuous current measurements.

2. The relative importance of the several components has been studied for two grades of impregnating oil as applied to the same paper.

3. The importance of the initial conductivity of the oil, as distinguished from the final conductivity, is emphasized and higher conductivity is reflected in increased losses due to both reversible and irreversible dielectric absorption.

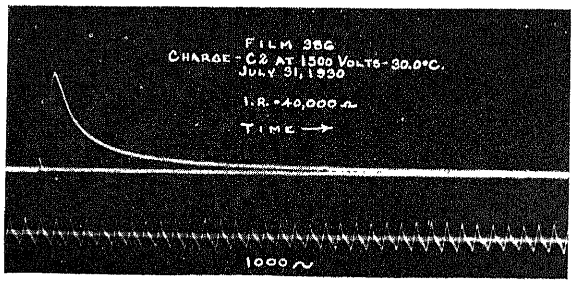


Fig. 14

4. The importance of the irreversible absorption current in its early stages, as being a true conduction current, is emphasized. This loss may be as high as 30 per cent of the total loss in an oil of excellent insulating properties.

5. Further studies of this character should clarify the relative influence on loss and on power factor characteristics of the density, air resistance, thickness, and other properties of the paper, and of such chemical and physical properties of oil as may be correlated with conductivity.

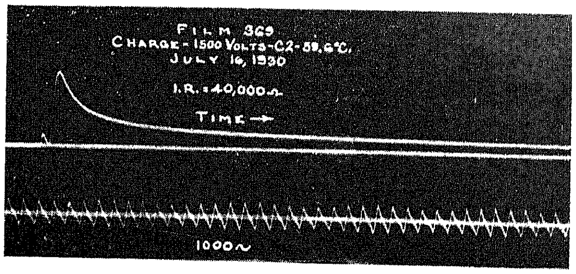


Fig. 15

The work is being continued.

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Discussion

THE CONDUCTIVITY OF INSULATING OILS—II

(WHITEHEAD)

FUNDAMENTAL PROPERTIES OF IMPREGNATED PAPER

(WHITEHEAD & KOUWENHOVEN)

H. H. Race: The main conclusions of Dr. Whitehead's paper are based upon the results given in Table II which shows measurements for one oil at three temperatures. The second column gives the dielectric loss as measured with a 60-cycle alternating voltage. The fifth column gives the loss as calculated from the first horizontal portion of the due current-time curve. Except at the highest temperature these two calculations do not agree and the author states that the difference is accounted for by an initial exponential characteristic of the d-c. current-time curve during a very short time after the application of potential. I should like to ask Dr. Whitehead whether he has any physical picture of a mechanism which would explain the characteristic that he has observed.

Because of the high resistivity of good oils, the currents to be measured are very small and the author deserves considerable credit for developing sensitive apparatus for determining the short-time d-c. characteristics. His conclusions, if correct, will have a far reaching effect in the development of the theory of liquid dielectrics.

For this very reason, the data deserve very critical analysis. In studying the oscillograms presented in the paper, the question arose as to whether the initial exponential portion might not be an oscillograph error rather than a characteristic of the oil. The reasons for thinking this might be true are as follows:

1. The d-c. data at 45 deg. cent. are shown by the oscillogram of Fig. 12. The measured current is so small that the deflection is only about three times the width of the line. The record for 30 deg. cent. is not shown but the deflection would be only about 25 per cent of that shown in Fig. 12 due to the increased resistivity of the oil and would be correspondingly difficult to measure accurately.

2. The transient term of the deflection of a standard galvanometer element as a function of time is of the form

$$S = (A\ \epsilon^{iat} + B\ \epsilon^{-iat})\ \epsilon^{\frac{-\rho t}{2 m}}$$

where

$$a = \sqrt{\frac{k}{m} - \left(\frac{\rho}{2\,m}\right)^2}$$

ρ = frictional damping constant
m = mass of moving element
k = restoring force

For critical damping, $a = 0$ so that the transient term is a pure exponential, of the form

$$S = (A + B)\ \epsilon^{\frac{-\rho t}{2 m}}$$

which has a time constant, $(\rho/2\,m)$.

When such an element is underdamped the transient term is oscillatory with a frequency of $(a/2\,\pi)$. If very slightly underdamped (a) might be very near zero so that the element might overshoot once and then approach the final value from above. This seems to be at least a possible explanation for the observed curve since the oscillogram shows that the element required about 0.5 millisecond to reach a maximum deflection after the low sensitivity shunt was removed and the exponential portion of the curve lasted only for about another millisecond.

3. Again, the natural frequency of a high sensitivity oscillograph galvanometer varies from 1,000 to 3,000 cycles per second which is of the same order as the time constant of 1,630 which was calculated from the observed curves.

4. If the oil used by Dr. Whitehead is similar to that tested by us, the data shown in Fig. 19 of my paper (*Some Electrical Characteristics of Cable Oils*) indicate that the 60-cycle a-c. loss would be greater than the d-c. loss at low temperatures. A 60-cycle curve would be similar to our curve (1) but below it and would probably depart from the d-c. locus at about 50 deg. cent. This offers a possible alternative explanation of the differences between a-c. and d-c. loss observed by Dr. Whitehead.

The short-time d-c. characteristic can be used to distinguish between the mechanisms of space charge formation and polar molecule orientation as explanations of dielectric loss. Therefore, it would be very valuable if Dr. Whitehead could extend both the frequency and temperature ranges of his measurements.

Wm. A. Del Mar: In a paper[1] which I presented to the A. I. E. E. in 1922, in collaboration with Mr. C. F. Hanson, a connection was shown between the resistivity of petrolatum compound and the power factor of paper impregnated therewith. This curve was plotted through 18 experimental points and connected them quite satisfactorily. In the discussion of this paper Mr. Hanson presented a similar curve with over 40 points, showing a similar correspondence between resistivity and power factor. Subsequent experimental work enabled us to obtain curves with several hundred points lying very close to them. The curves were plotted from a formula based on the assumption that the entire dielectric loss was ohmic in character. The resistivities were all measured with direct current after one minute electrification.

Dr. Whitehead now tells us that we should not use the resistivity measured after one minute, but after about 1/10th second electrification. If his conclusions are correct, there must be a definite relation between the resistivity after 1/10th second and after one minute electrification for a given oil. This, however, is not borne out by the data on oils, *B*-4, and *B*-6, in the Whitehead-Kouwenhoven paper which are said to be identical except in regard to aging.

The resistivity-power factor relation which holds so well for petrolatum does not hold for cylinder-oil-rosin type compound. Possibly this may be due to different ratio between the 1/10th second and one minute electrification resistivities. Whatever the explanation, it is an undoubted fact that to obtain a given power factor the resistivity of this type of compound does not have to be as high as where petrolatum is used. As resistivity measurements made after 1/10th second electrification are difficult to make, whereas those made after one minute electrification present no difficulty, Dr. Whitehead would be doing a service to the industry if he would correlate these tests with the object of improving power-factor control.

C. L. Dawes: The paper by Messrs. Whitehead and Kouwenhoven is particularly valuable in that it gives a definite explanation, substantiated by experiment, of the so-called *V*- or *U*-curves of power loss in impregnated-paper insulation as a function of temperature. As is well known, these characteristic curves are frequently obtained with impregnated paper, either when in test samples or in cables. In this type of characteristic the minimum value of power occurs usually in the region between 30 deg. cent. and 50 deg. cent. Many explanations of this characteristic have been advanced. Hochstädter probably came nearest to giving the correct explanation, in view of the experimental results obtained by the authors of this paper. (See London *Electrician*, Vol. 89, August 18, 1922, page 180.) He attributes the *U* or *V* curves to the combination of a component having a drooping loss-temperature characteristic and a component having a rising loss-temperature characteristic just as the authors do. However, instead of attributing the drooping characteristic to the paper alone as has been done by the authors, he attributes it to hysteresis loss in the insulation when it is in the more viscous or the solid state which occurs at the lower temperatures. When the insulating compound becomes more fluid at the higher temperatures, the resistance loss predominates and this increases rapidly with the temperature. Hochstädter found that *V* curves were obtained with a mixture of beeswax and rosin alone; the oils alone tested by the authors do not give *V* curves, but have continually rising power-temperature characteristics such as Hochstädter obtained with beeswax and rosin at higher temperatures. The difference at the lower temperatures between Hochstadter's results and those of the authors is probably due to the much greater viscosity of the beeswax and rosin at these lower temperatures. Also, Hochstädter attributed the rising characteristic almost entirely to resistance or conduction losses and this also appears to agree with the results obtained by the authors, as may be seen by a study of their Table II. Thus in general the authors and Hochstädter appear to be in rather close agreement. The only difference of any moment arises from Hochstadter's attributing the drooping hysteresis loss characteristic to the more viscous condition of the impregnating compound while the authors find it to be due to the loss in the paper alone.

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[1] *Effects of the Composite Structure of Impregnated Paper Insulation on its Electrical Properties*, A. I. E. E. TRANS., Vol. XLI 1922, p. 563.

It follows that to obtain the *U* curve, the impregnating oil must have low loss relative to that of the paper. Thus it may be inferred that when impregnated paper has a *U* curve of power loss, a relatively low-loss impregnating compound is implied.

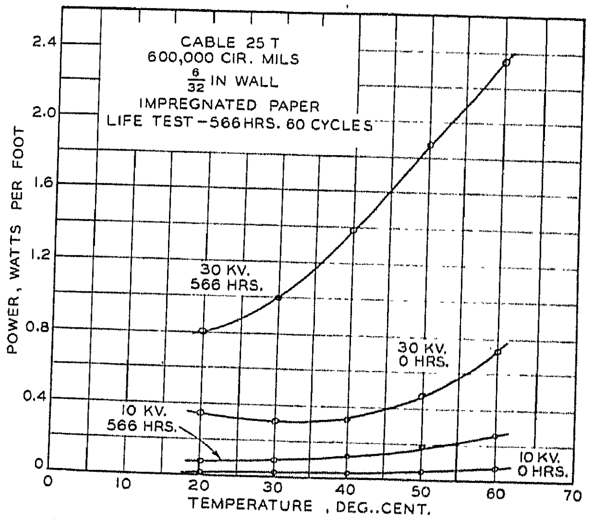

FIG. 1—EFFECT OF LIFE TEST ON POWER-TEMPERATURE CHARACTERISTIC OF CABLE

In a number of tests of impregnated paper samples which has been conducted at the Harvard Engineering School, this *U*-shaped curve has not as yet been obtained. That is, the power loss has always increased with temperature. In order to make comparison with the samples tested by the authors, I have assumed that the loss in their Figs. 5 and 7 increases as the square of the voltage gradient. Under those conditions our samples showed slightly less loss for any given temperature than those of the authors. This might indicate that a *U* curve should have been obtained with our samples, but such was not the case. Perhaps the power loss in our paper was very low.

Frequently, however, we have obtained these *U* curves with cables. Fig. 1, herewith, shows four power-loss-temperature curves taken at 60 cycles for a 600,000-cir. mil, 6/32-in. (1.76-mm.) wall paper single-conductor cable. Two of the curves were taken at 10 and 30 kv. before the cable was put on test. Two more curves were obtained at these same two voltages after the cable had been subjected to 566 hours of accelerated life tests with weekly cycles of voltage gradient variation (from zero to 225 volts per mil average) and temperature variation (20 deg. cent. to 60 deg. cent.).

The curves taken at 10 kv. show no indication of the *U* characteristic. The initial curve at 30 kv., however, has a decided *U* characteristic, whereas the curve taken at the conclusion of the life test rises rapidly and has no *U* characteristic. This undoubtedly indicates that under the time and cyclic application of stress and temperature the impregnating compound underwent deterioration. This resulted in the compound having higher relative loss than the paper and thus eliminating the *U* curve obtained with it initially, and a continually rising characteristic resulted. It would have been interesting to know the extent to which the characteristic of the paper itself had been affected by the life test.

It is well known that at the higher temperatures the power-factor curve, when plotted as a function of voltage, usually is not flat. It may increase or it may decrease with voltage gradient. Such results have been obtained by many investigators and also at Johns Hopkins University. (For example see Whitehead and Hamburger, *Impregnated Paper-Insulation*, TRANS. A. I. E. E., January 1928, Vol. 47, p. 311.) However, the authors state that for both specimens of impregnated paper the power factor is constant with respect to the voltage. This is substantiated by their Figs. 5 and 7 to the accuracy which can be obtained from their very small scale. I am asking the authors if they have any explanation as to why their values of power factor at 45 deg. and 60 deg. cent. are constant, as this is not generally true.

I believe that the data and the curves which the authors present would be much more useful if they were more fundamental in character and did not involve the geometry of their apparatus. For example, microwatts per cubic cm. is much more fundamental and much more useful for comparative purposes than the total microwatts of their particular samples, which they give in the paper; likewise, dielectric constant is much more fundamental and useful than the capacitance of their particular samples.

The authors have, however, made a distinct contribution to our knowledge of the effects of the components of the dielectric loss in impregnated-paper insulation, such for example as the determination of the direct and alternating current losses in the paper and oil individually and when in combination.

S. K. Waldorf: Dr. Race has suggested that the small decay in the charge current records on the oscillograms, an example of which is given in Fig. 12 of Dr. Whitehead's paper, may be due to an over-travel of the oscillograph vibrator, rather than due to absorption in the oil. By recording the response of the apparatus to d-c. impulses, it has been shown that the oscillograph vibrator used was over-damped rather than under-damped, and did not over-travel. Additional tests with an air condenser at high voltage, and with a mica condenser and resistance at low voltage, have proved the reliability of the amplifier-oscillograph. The tests are described in greater detail in a paper by the writer, which is to be presented at a meeting of the Institute in the near future.

Further, discharge records on oils have been obtained indicating the same decaying current on discharge, verifying the presence of reversible dielectric absorption in the oils.

Dr. Race has also raised the question as to the accuracy of the measurement of oscillograms. It is our practise to measure recorded deflections to a tenth of a millimeter, which gives a value of current accurate to within about 5 per cent or better on oscillograms having the smaller deflections. With larger deflections, the accuracy of such measurements is of the order of 1 per cent and better. It has been found by taking additional oscillograms that the results are reproducible, and the probable accuracy of the measurements thereby improved.

J. B. Whitehead: Dr. Race has asked what is meant by "dielectric absorption" in connection with these studies. Ever since the discovery of the residual charge of a Leyden jar physicists, and more recently engineers, have speculated as to the nature of this phenomenon. The term dielectric absorption is commonly used to describe it, evidently in the idea that the residual charge is in some way absorbed in the dielectric. Maxwell has shown beyond question that a mixture of different solid dielectrics will account for the phenomenon of absorption. He also recognizes that a similar phenomenon might arise in other causes. This has been clearly shown in many experiments since his time, as for example, in the anomalous conductivity of liquids, *i. e.*, a conductivity which may vary with the time and show departures from Ohm's law. Solid complex dielectrics often show evidence of this type of conductivity, which causes them to show departures as regards dielectric absorption from the simple conception of Maxwell.

While all liquids, even the purest, show anomalous conductivity, such as mentioned, it has always been supposed that they do not show the phenomenon of residual charge or dielectric absorption. We have shown, however, in our paper the presence of a small and short-lived dielectric absorption in an insulating oil, which is sufficient to account for the excess of dielectric loss, as measured, over the maximum loss which may be accounted for by conduction. My answer to Dr. Race, therefore, is that in our experiments we have encountered in an oil the same phenomenon that in the case of solids is usually described as dielectric absorption.

There has been a noticeable tendency recently to attribute the excess of measured loss under alternating stress, over that which may be accounted for by conduction, to a molecular frictional loss in terms of the Debye theory of the oscillation of dipolar molecules. There is no question that this theory offers a convincing picture and appears to account satisfactorily for dielectric losses within the upper ranges of frequency and for highly viscous media. For low frequencies, say 60 cycles, and for viscosities ordinarily encountered in insulating oils, no convincing evidence has been presented that the observed losses contain any appreciable component of the Debye type. It is the suggestion of my paper that for low frequencies and low viscosities, the greater portion, if not all, the loss may be accounted for by the initial or short time conductivity, including dielectric absorption.

Answering Mr. Del Mar's comments, we have found as a usual thing that as the conductivity of an oil increases, say due to oxidation or the presence of impurities, the difference between the initial short-time conductivity and the long-time conductivity becomes less and less. The permanent or final conductivity then masks the effect of the initial condition, which apparently is due to different causes. In such a high conductivity oil, the initial and final conductivity being the same, the loss and hence the power factor is completely accounted for by the conductivity measured at one minute or later.

It is difficult to compare Mr. Del Mar's results on petrolatum with our studies in oil, because he has correlated the conductivity of the petrolatum with the power factor of the impregnated paper. The separate influences of both oil and paper on the impregnated product are so complex that the facts before us do not seem to warrant discussion. I may say, however, that so far we have not been able to find any direct correlation between the one-minute conductivity of an oil and the subsequent properties of impregnated paper. It is our hope that through the more intimate study of the fundamental properties of both oil and paper that we may be able eventually to indicate those fundamental properties which do have most important bearing on the final behavior.

W. B. Kouwenhoven: Professor Dawes data in regard to the *U* loss-temperature characteristics of impregnated paper are interesting. In Fig. 4 of our paper we show a *U*-shaped curve for specimen C-1. Specimen C-2 has a falling loss-temperature characteristic for the range plotted in Fig. 6. However, we made loss measurements of this specimen at temperatures above 60 deg. cent. We found that the minimum loss occurred at a temperature between 60 deg. and 75 deg. cent. and, therefore, this specimen also has a *U* characteristic when the temperature range is extended. In general we believe that for a given sample of paper the minimum loss occurs at higher and higher temperatures as the quality (lowness of loss) of the impregnating oil is improved.

In the impregnated specimens C-1 and C-2 the power-factor voltage relations are so nearly constant that they appear in Figs. 5 and 7 as straight lines. The actual values of power factor at various voltages and temperatures from which these curves were plotted are given in the table below:

Voltage	Specimen C-1 at temperature cent.			Specimen C-2 at temperature cent.		
	30 deg.	45 deg.	60 deg.	30 deg.	45 deg.	60 deg.
500	0.00399	0.00385	0.00461	0.00389	0.00315	0.00305
1,000	0.00397	0.00382	0.00465	0.00389	0.00315	0.00306
1,500	0.00394	0.00377	0.00463	0.00388	0.00315	0.00306

A study of this table indicates that the values in most cases vary slightly with voltage. It must also be kept in mind that the voltage gradient in our specimens at 1,500 volts applied is 25 volts per mil. At low gradients there is in good impregnated specimens very little change of power factor with voltage.

Traffic Control by Light Beams

For Major-Minor Intersections

BY R. C. HITCHCOCK[1]
Member, A. I. E. E.

Synopsis.—The object of this traffic controller is to keep the signal green on the major street, and to give the green signal to the minor street only when requested. The minor street vehicle can always get the green signal after a short wait, while the major street receives green light for as much extra time as the minor street traffic permits.

Two experimental horizontal beam installations, which accomplish the above object, are now in operation in Wilkinsburg, Pennsylvania. The focussed light beam, phototube unit, relay panels, and associated circuits are described in this paper.

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THE installations described were developed by the Westinghouse Research Laboratories to obtain working information on traffic control systems and field experience on phototubes illuminated at a distance from light sources. They are experimental and the equipment is not available in commercial form.

When traveling on a major highway, it is annoying, as well as wasteful of time, to stop at a red traffic signal, when there is no car waiting to use the green light which has been flashed on the minor highway. To keep the light green all the time on the major, and give the right-of-way to the minor street only when necessary[2] a light beam can be focussed on a phototube across the minor street, so that when a car intercepts the beam the traffic light would be flashed green on the minor street.

The first installation was in East Liberty, Pennsylvania, at the corner of Stanton and Highland Avenues. The second installation was at the corner of Coal Street and South Avenue, the latter being parallel to the William Penn and Lincoln Highways, in Wilkinsburg, Pennsylvania. South Avenue carries a fraction of the heavy main highway traffic. This installation is illustrated in Fig. 1.

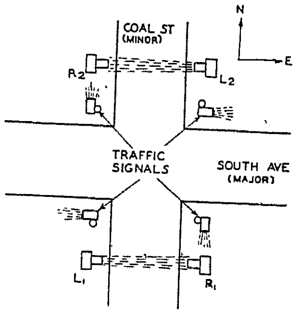

Fig. 1—Second Installation South Ave., and Coal St.

The next step was to make an installation on the main highway itself. This third controller is at the corner of Penn and Trenton Avenues, Wilkinsburg, Pennsylvania, Penn Avenue at this point carrying all the traffic of the William Penn and Lincoln Highways. This intersection was selected because of the heavy traffic on Penn Avenue, the density often reaching one thousand cars per hour.

In some localities having wide boulevards, independent and combining streams of traffic can often be allowed to move continuously. For instance, a right turn is sometimes permitted on a red signal, when the turn is made into a street having four traffic lanes. The turning vehicle keeps to a curb lane, and the through vehicle takes the adjacent lane. The majority of Pittsburgh streets are narrow, consequently all turns are made only on the green signal.

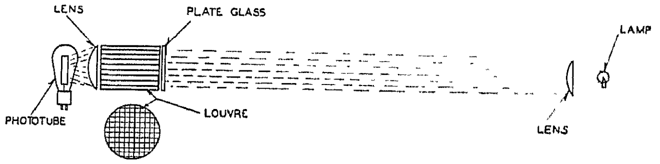

Fig. 2—Optical System, Lenses and Second Installation Louver

The system to be described is designed to handle only cross traffic on major-minor intersections. One reason for following this idea is the large number of potential installations. Probably there are more major-minor intersections than there are of the major-major type. Another strong reason is that the installation is fairly foolproof, a regular timer motor being provided to control the intersection if anything should go wrong in the automatic control.

Devices are in use which are actuated by the traffic on both intersecting streets, and no specified program is used. Such equipment has to be very carefully designed and should be inspected frequently, for if failure occurs in any unit, the signals may remain as they were at the time of failure. With the major-minor device to be described, the failure of any of several units causes the vehicle controlled device to be automatically disconnected and the timer motor runs on the predetermined cycle.

Typical Traffic Cycle

Using a one minute cycle, a typical timing for a

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[1] Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[2] The first work on this device was done by Dr. P. Thomas of the Westinghouse Research Laboratories.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

major-minor street is 42 sec. major street green, 3 sec. amber, 12 sec. minor green, and 3 sec. amber. Thus the ratio of green light for major and minor streets is 42/12 = 3.5:1, yet this timing is frequently used at an intersection where the traffic ratio is from 10:1 to 50:1, where 1,000 cars per hour and more are passing on the major street. The reason for the short major cycle is to allow the minor street cars to enter after a maximum waiting time of 45 sec. This is done with the new automatic controller, while allowing the major street as much more than a 42-sec. green signal period as the minor traffic permits. The object of the major-minor street traffic controller is to keep the major street traffic moving with a continuous green signal, and, following the cycle just mentioned, to allow minor street traffic to gain a green signal after a maximum wait of 45 sec.

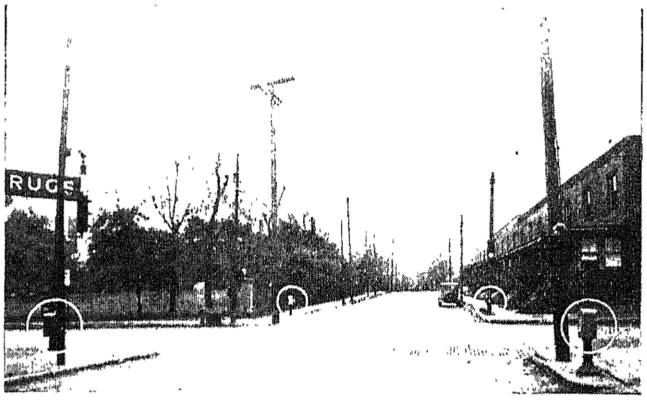

Fig. 3—Third Installation, Wm. Penn and Lincoln Highways, Wilkinsburg, Pa.

This has worked out quite well in practise—sometimes the major street gets a green signal for as much as five minutes, but after the first 45 sec., the controller allows a minor street vehicle to get the right of way in 5 sec., or whatever time delay is set to allow for major street turns.

Recently a movement has been made in Pennsylvania to eliminate the use of traffic lights on main highways, especially where they pass through small towns. The regular cycle timer undoubtedly works a hardship on the major traffic lane by flashing a red signal when there is no need for it. However, the use of a minor street controlled traffic signal always gives the major street a certain minimum green signal, and at the end of this minimum time the signal is not changed until a minor street vehicle requests it by intercepting the light beam.

Light Beam Direction

The first installation used vertical light beams, but the grease and dirt which fell on the part of the system placed below street level[3] suggested a change in light beam direction. In the second and third installations, the light beams are horizontal and the cleaning problem is therefore decreased. Since the second installation was put in, several details have been improved and up to the present time it has been working for over five months with only three minor interruptions. The third installation has been operating for over a month, with no interruptions.

One interesting problem presented by the horizontal device, was that cars turning from the main street would cross the minor street light beam, and if no provision were made to allow for this, the light would be flashed green on the side street, with no cars waiting for it. The solution was to introduce a time delay, requiring a car to stop in the beam for several seconds before any action took place. This was so timed that cars making the turn would not stay long enough in the beam to cause operation. It is usual for a driver to stop as he drives up to a red light, and it is felt that it is only right for a minor street driver to wait for his turn of the green light. The usual time that the car must wait is 3 to 5 sec.

Reliability

Probably the main advantage of both the original and revised forms of the controller is its reliability. If any of several minor accidents should happen, the regular predetermined program period is followed. Some of these possible minor accidents are; burning out of the lamp sending the light beam across the street, a blown fuse in any of the four control boxes, a collision between a car and one of the control boxes, or dirt collecting on any of the glass parts which transmit the light from the lamp to the receiving phototubes.

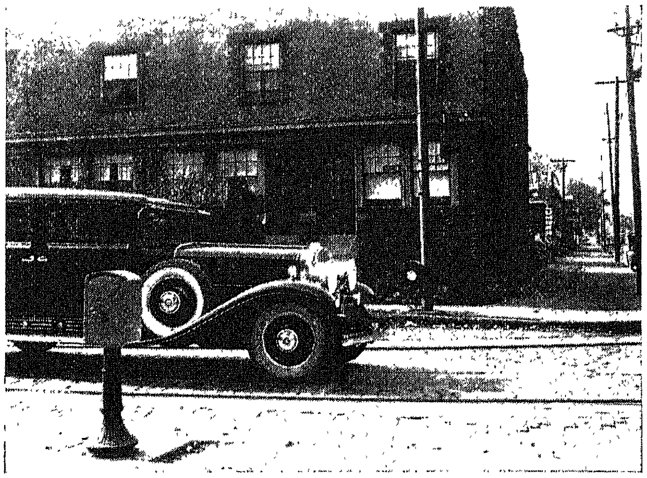

Fig. 4—Car Intercepting Light Beam Third Installation

If any of the above things should happen, the major and minor streets divide the cycle and the control is the same as a regular program traffic timer. When the device is operating properly, the major street receives as much extra green light as the minor street traffic allows.

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[3] For this reason the first installation has been discontinued.

After five months' operation only three interruptions have occurred—two of these due to dirty lenses and resulting from the apparatus going on "program" operation; and the third due to a loose connection which resulted in one relay box being out of service for about two hours.

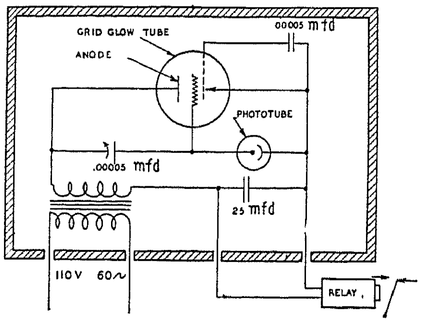

Fig. 5—Phototube-Grid Glow Tube Circuit

One of the lamps burned out after 4⅛ months, and after 4½ months one of the original glow tubes was replaced. As will be shown in explaining the next installation, data are now available on getting more light across at the lower filament voltages, which will increase the life. Glow tubes with longer life are now being made. Just before a grid glow tube fails, its upper or lower operating limit, as shown by the variable condenser setting, increases or lowers, respectively. Grid glow tubes sometimes fail glowing, or fail to glow, depending on the construction. For the circuit shown in Fig. 5 it is an advantage to have the tube fail glowing, so that the program goes on the regular program cycle, and this type of tube is used. However, by making inspections and recording the upper and lower condenser limit, the failure can be predicted a few days before it happens. It is, of course, quite easy to replace tubes.

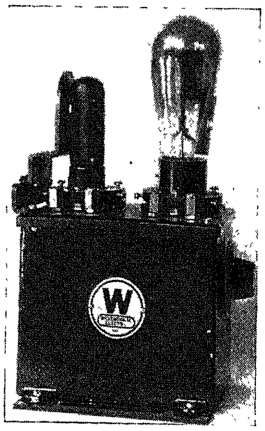

Fig. 6—Mounted Phototube, and Grid Glow Tube with Adjusting Condenser

Third Installation

Penn Avenue carries through traffic on U. S. routes 22 and 30, the William Penn and Lincoln Highways. At the intersection of Penn Avenue with Trenton Avenue the third installation was put in working order on September 16, 1930. An illustration of the intersection, showing by circles the location of the control boxes, is given in Fig. 3. Fig. 4 shows one light unit, and its receiver, with a car intercepting the beam. The centerline of the light beam was placed low enough to work with any car now on the market.

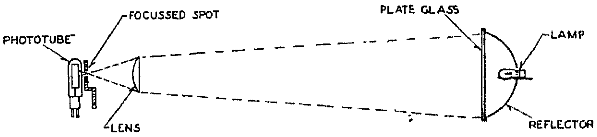

Fig. 7—Optical System, Reflector Lens and Focused-Spot Third Installation

The grid glow phototube unit is shown in Fig. 5.[4] The use of paraffin-impregnated cork washers and a clamping device[5] was found to keep the tube bases free from moisture and dirt. The wiring, as shown in Fig. 5 is inside a small casting, the upper panel being of moldarta, and screwed to the casting with a paraffin-cork gasket to keep it tight. The transformer and

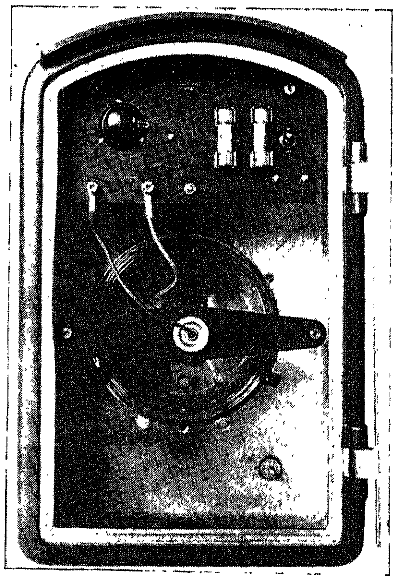

Fig. 8—Rear View of Reflector Unit, Lamp Mounting, Rheostat and Switch

condenser are enclosed. The no-load loss of the transformer provides sufficient heat to prevent condensation.

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[4] The theory and operation of grid glow tube circuits has been covered by articles by Mr. D. D. Knowles in: *Elec. Journal*, Apr. 1928, p. 176, *Elec. Journal*, Feb. 1930, p. 116; *Elec. Jour.* Apr. 1930, p. 232; *Electronics*, July 1930, p. 183.

[5] E. H. Vedder, *Elec. Jour.*, Apr. 1930, p. 224.

Tests were made of the phototube-grid glow tube unit, in a humidifier, and even with the humidity at 100 per cent the unit ran satisfactorily for several hours. In getting 100 per cent humidity water was sprayed over the unit, at times, and still the paraffin-cork washers and gaskets held their seal.

The glass of the phototube is painted black with the exception of the front. This keeps daylight out of the tube so that adjustments may readily be made with the back of the control box open. A shield in front of the phototube, has a small hole on which the light source is focussed. Most of the extraneous light is kept out, by this focussed hole. In the second installation, a louver was used to keep out the extraneous light. The complete optical system of the third installation is shown in Fig. 7.

The light system used in the second installation is shown in Fig. 2. A lens focusses an image of the lamp filament on a plate glass disk, the light rays being practically parallel. To prevent light from any other source from entering the phototube, a louver is provided. This is a square sectioned honeycomb, which passes light parallel to its axis, but stops light which enters at an angle of a few degrees. For convenience in mounting the louver, the lens in front of the phototube is placed with its plane side facing the lamp. The phototube plate is covered by light from the lens, space being saved by moving the tube closer to the lens than would be possible if the light were brought to a spot on the phototube plate.

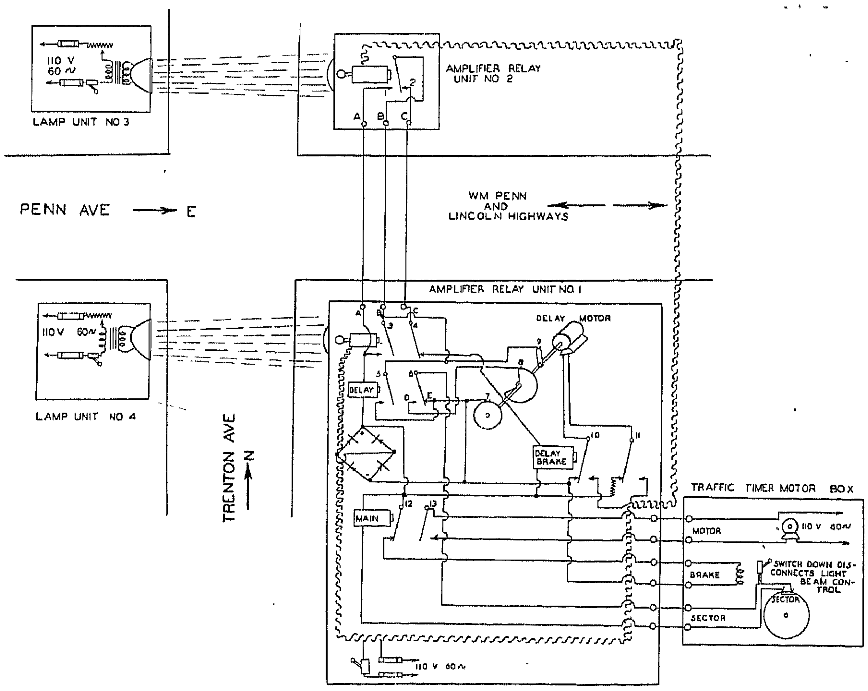

Fig. 10—Detailed Wiring, Third Installation, Penn and Trenton Avenues

It is difficult to figure the efficiency of these optical systems, but experimental results show that a fairly good parabolic reflector is better than any reasonably priced lens of similar diameter. The rear view of the lamp unit box, showing the reflector, is given in Fig. 8. The lamp socket has two knurled clamp nuts which allow motion of the lamp socket in and out, and a large hole in the support permits lateral motion. A resistor is provided in series with the primary of the transformer so that the lamp can be run at the desired voltage.

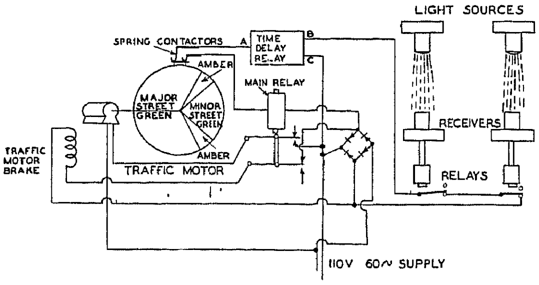

Fig. 9—Schematic Diagram Third Installation

The schematic wiring of the third controller is shown in Fig. 9 and the detailed wiring in Fig. 10. As shown in Fig. 9 the major street signal is green, the main relay is pulled up, power is off the motor, and the brake is on. When a car crosses the light beam momentarily as in turning from the major street into the minor, the light controlled relays pull up, and the circuit wires marked *A* and *B* of the time delay relay are open circuited. However, unless the car remains in the beam for the specified time delay there is no other action in the controller. If, though, the car does wait in the beam for three seconds, circuit *A B* is opened and then the traffic motor starts up, and the cycle begins, giving amber then green to the minor street.

In Fig. 10, the traffic motor is shown at the right, and has the usual sector and brake. Fig. 11 shows a simplex traffic timer motor as adapted for use in the automatic controller. The brake coil is shown to the left of Fig. 11, when voltage is applied to this coil, the eddy currents induced in the disk cause it to slow down. A small panel contains the brake, motor, and sector binding posts, and also a small toggle switch. In case the regular program is desired, this toggle switch is pressed down, opening the sector circuit, see Fig. 10, so that the motor traffic timer motor runs continuously.

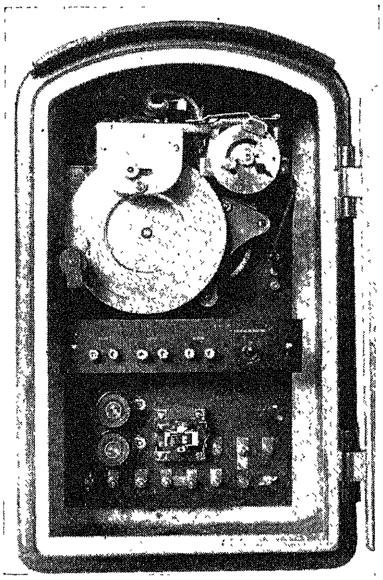

FIG. 11—TRAFFIC MOTOR SHOWING BRAKE AND SECTOR

The main relay panel mounted on its pedestal is shown in Fig. 12, the rectox fins can be seen projecting from the back. At the top of the panel in Fig. 12 is the time delay motor, next in order come the main delay, delay motor brake, main, and amplifier (light-sensitive) relays. The other phototube-grid glow tube unit and single-light sensitive-relay panel is not shown.

OPERATION OF CONTROLLER

When there are no cars waiting in the light beam, the traffic timer motor is stopped by the main relay, which takes off the power and puts on the brake. The a-c. delay motor, too, is stopped by the delay brake relay which takes off the alternating current and puts on direct current.

If either light relay is pulled up due to a car intercepting the beam, two connections are made; (1). the delay relay pulls up, transferring the main relay control to segment 8 of the delay motor; (2). the delay brake relay opens, releasing the delay motor brake and putting alternating current on the delay motor. If a car stops in the beam, segment 8 makes contact for 5 seconds, and as it opens the main relay also opens, takes off the brake and starts up the traffic timer motor.

If the car does not wait in the beam, but just intercepts the beam momentarily, as in making a turn, the delay relay opens, the main relay connection is transferred from the delay motor segment 8 and delay relay contact 6 *D* to the delay relay contact 6 *E*, and the timer motor is not started up.

Control boxes 1 and 2 have only one fuse system, that of box No. 1. If the power goes off, or a fuse blows, the main relay has no power to pull up, and when it is in the released position the traffic timer motor is supplied with voltage, and runs on the regular cycle.

RECEPTION BY PUBLIC

A yellow line was painted on the street, and in the third installation the right hand control box was painted yellow and an illuminated sign reading, FOR GREEN LIGHT STOP OPPOSITE YELLOW BOX, was provided. For those drivers who never seem to stop for yellow lines on the street, or read signs, there soon appeared a group of self-appointed "traffic directors" who, living in the neighborhood, knew how the device operated—and the motorists were instructed to pull up opposite the yellow box.

Due to the newness of the device, traffic during certain hours of the day, has increased on Trenton Avenue. Drivers soon realized that they could get a green signal on the minor street by driving up and intercepting the light beam. Even so, the major street always gets its regular quota of the green signal.

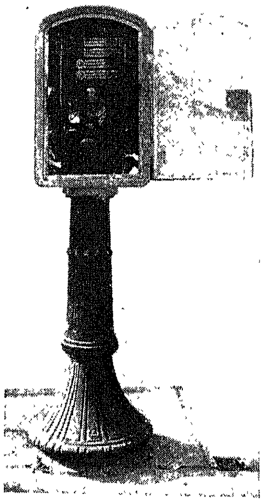

FIG. 12—CONTROL PANEL NO. 1 WITH GS UNIT ON PEDESTAL

Short time statistical studies indicate that the time of red light flashed on Penn Avenue using the automatic controller, is from 50 to 70 per cent as much as was flashed when using the regular cycle timer.

ACKNOWLEDGMENTS

The writer wishes to express his appreciation of the cooperation of the Wilkinsburg officials, especially Burgess W. M. Turner and Chief of Police W. R. Bishop.

In the Westinghouse Research Laboratories, much credit is due to Dr. P. Thomas for his original work on the system. Thanks are due to Mr. D. D. Knowles and Dr. E. D. Wilson for furnishing appropriate grid glow and phototubes. The writer also wishes to express his thanks to Mr. E. F. Critchlow for his valuable assistance in this work.

Discussion

F. Hamburger, Jr.: I have been very much interested in this method of traffic control for major—minor intersections. Particularly the presentation showing the time saved in this method as compared to straight automatic control in a predetermined cycle. I notice however that the paper states that a 12-second green is allowed for minor street traffic. For light traffic this is, I think, still too long a time; possibly 6 seconds would normally be sufficient and a saving of time would result.

I should like to ask whether it is possible with this control to set the time to, for example, 6 seconds for the minor green and provide a step-up feature or time extension to permit a longer minor street green in the event that several cars follow each other on the minor street. This system would still further decrease the wasted time.

R. C. Hitchcock: The installation described is at a corner where street cars make turns. The 12-second allowance permits an automobile, following a street car, to pass before the light flashes red again. If no street cars were present, a shorter fixed green time could undoubtedly be used to advantage. Experimental circuits are being devised to proportion the green time to the vehicle density.

Recently a statistical study was made of the relative time lost when using different traffic light cycles.* The curves in the figure show the time wasted for two fixed program controllers, the theoretical best controller, and the present automatic (light beam actuated) controller. The abscissa is the traffic ratio, and the ordinate is the time wasted in car seconds per hour.

The average automatic cycle wastes 62 per cent as much time as the old 40-18 cycle (40 = seconds major green; 18 = seconds minor green), and 80 per cent as much as a theoretically perfect cycle. In the present installation, at a traffic density ratio of 15:1 the average saving, over the original cycle, is 3,100 car seconds per hour.

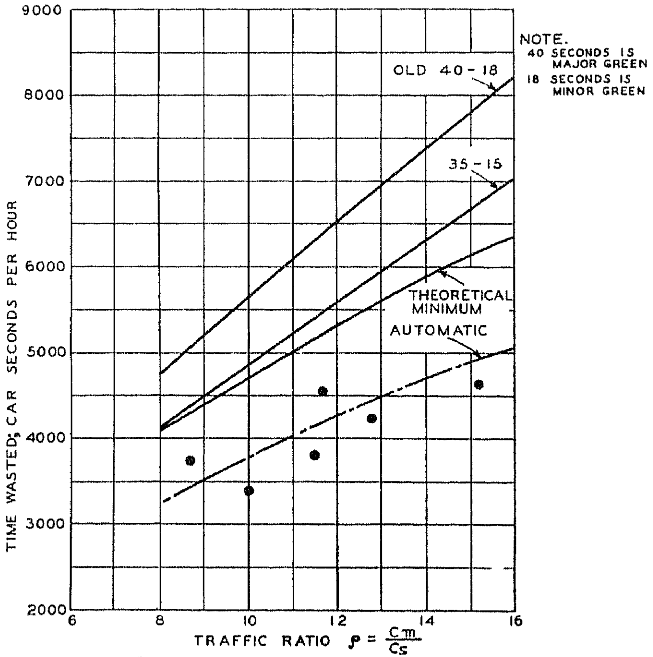

FIG. 1—COMPARISON OF TIME WASTED BY DIFFERENT CONTROLLERS

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*Electric Journal, May 1931, page 318.

Commutation and Current Collection in Hydrogen

BY ROBERT M. BAKER[1]
Associate, A. I. E. E.

Synopsis.—*This paper describes a number of tests on commutator machines and slip-rings operating in hydrogen. The results seem to warrant the following conclusions: (1) A commutator machine designed and adjusted for good commutation in air will operate satisfactorily and give good brush life in hydrogen. (2) If a commutator machine must spark in hydrogen, the brush life may be increased many times by keeping the relative humidity below ten per cent. (3) The contact resistance of carbon brushes on a commutator may be lowered materially by the introduction of hydrogen. (4) Carbon or graphite brushes cannot be operated satisfactorily on plain tool-steel slip-rings running in hydrogen, and (5) The contact drop between a carbon or graphite brush and a brass slip-ring may be ten times as high when the ring is running in air as when it is running in hydrogen.*

INTRODUCTION

THE first electrical machine to be operated in hydrogen was the synchronous condenser, chosen largely because of the ease with which it could be covered by a gas tight housing. There being no need for a shaft extending through the housing, the difficulties and expense of a gland seal were at once eliminated. Also, the machine was simple electrically. By using a separate exciter, the necessity of running a commutator machine in hydrogen was eliminated and the only sliding contact or weak link in the electrical system was that between the brushes and the slip-rings which carry current for the rotor. The next advance will probably be the application of a direct-connected exciter to one of these machines and this will require the exciter machine to be operated in hydrogen. If it is proved that commutation can be satisfactorily accomplished in hydrogen it may not be long before the hydrogen-cooled synchronous converter will make its appearance. Therefore it would seem that much depends upon the accumulation of knowledge on the general subject of commutation and current collection in hydrogen. It represents a field which until just recently had received no attention and whether or not there will arise any new and serious difficulties is yet to be seen.

This paper is not intended as a complete treatise of the subject but is only a beginning. It describes a few tests made on commutator machines and slip-rings running in hydrogen. Although not extensive these tests establish a few facts quite conclusively.

COMMUTATION TESTS

The set-up used for these tests is shown in Fig. 1. The machines are respectively;—a one-hp. d-c. driving motor and a five-hp. d-c. generator which was operated with its armature short-circuited. Electrical connections through the bed-plate facilitate metering, and make it possible to regulate the degree of commutation on the generator by shunting current from the commutating pole winding. The tank, shown suspended, fits into a mercury seal in the bed-plate, and has a window in each end for viewing the commutators of the respective machines. The gas connections are also in the bed-plate, the gas being carried in at one corner and out at another. Upon starting each new test it was necessary to pass considerable hydrogen through the tank to wash out the air, but after a good hydrogen purity was once established it was sufficient to merely maintain a small positive pressure in the tank. No quantitative data were taken on the brushes of the driving motor and the commutation on this machine was maintained good throughout the tests.

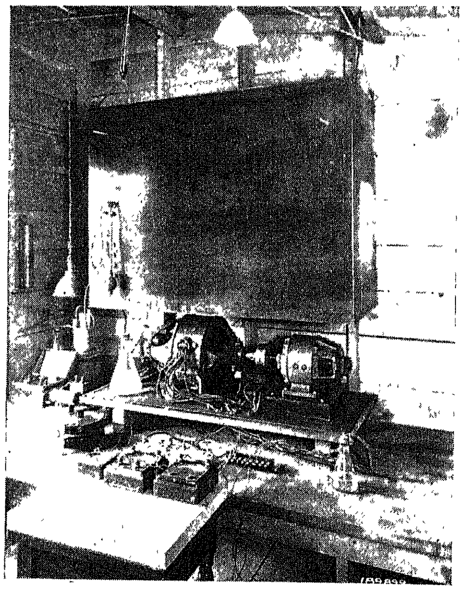

FIG. 1—COMMUTATION TEST SET

A hygrometer installed in the tank after the first two tests showed the existence of some rather surprising conditions. For instance, it was thought, at the time the first two tests were made, that the machines were running in a dry hydrogen atmosphere. The hygrometer showed that this was not true. The upper curve of Fig. 2 shows how the humidity in the tank varied during one test which was run under full load and good com-

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[1] Research Engr., Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

mutating conditions. Hydrogen was passed into the tank during this test at the rate of about 10 or 15 cu. ft. a day. The gas in the supply line was found to be quite dry and it apparently absorbed moisture after passing into the tank. It was found that by leaving the inside of the tank and the machines exposed to the air for only a short time during a shutdown, it was possible to reduce quite materially the initial value of relative humidity in the tank. The lower curve of Fig. 2 was taken after an exposure of only a few hours.

TABLE I—CALCULATED BRUSH LIFE OF GENERATOR BRUSHES

			Calculated brush life in days			
Test No.	Duration of test—days	Atmosphere	Brush No. 1	Brush No. 2	Brush No. 3	Brush No. 4
1	15.0	Air	588.0	229.0	812.0	362.0
2	6.41	Hydrogen	94.4	9.2	128.0	10.6
3	6.78	"	222.0	205.0	282.0	161.0
4	7.0	"	43.7	59.9	69.3	45.0
5	15.0	"	1,875.0	1,770.0	2,000.0	2,310.0

Brush life is calculated on the assumption of one inch allowable brush wear. Brushes Nos. 2 and 4 are the two positive brushes of the generator and are on opposite brush arms.

Table I gives the results of five brush life tests made on the short-circuited generator. All tests were made with full-load current in the brushes but the generator was not always adjusted for the same grade of commutation. Tests, Nos. 1, 2, 3 and 4 were made with 30 per cent of the current shunted from the commutating pole winding of the generator. The sparking at the trailing edge of the brushes was quite severe, and naturally resulted in rather short brush life even under the most favorable conditions. Test No. 5 was made with almost sparkless commutation.

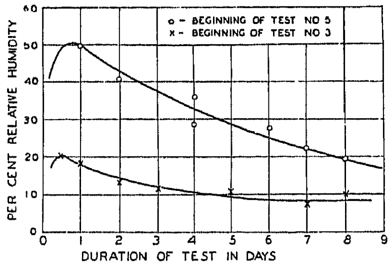

Fig. 2—Humidity of Gas in Tank vs. Duration of Test

The most interesting comparison is that between tests Nos. 1 and 2, which were made under identical conditions except that the former was made in air and the latter in hydrogen. In air the minimum calculated brush life was almost a year whereas in hydrogen it was only a little more than a week. Furthermore the damage to the commutator during the test in hydrogen was considerable. The trailing edges of the commutator bars were burned away, leaving a bright metallic surface. In some cases the depression was as much as 1/32 in. This first test in hydrogen was quite unique in that it showed such a great difference in wear between the positive and negative brushes. Both tests described were made before the hygrometer was installed and consequently the relative humidity of the medium in which they were run is not known. It is very likely however, that the test in hydrogen was started with a relative humidity of about 50 per cent and ended with a relative humidity of about 20 per cent.

Tests Nos. 3 and 4 were made for the purpose of determining whether or not the unusually bad behavior of the brushes and the commutator in test No. 2 could be attributed to the high relative humidity which was suspected during this test. These tests were made with 30 per cent of the current shunted from the commutating pole winding of the generator and the only difference in the two tests was in the relative humidity of the hydrogen atmosphere in which they were run. During test No. 3 the relative humidity was maintained at less than 10 per cent; whereas during test No. 4 the relative humidity was maintained practically constant at 50 per cent. In order to obtain the high humidity in the latter test it was necessary to bubble the dry hydrogen from the supply line through a flask of water placed inside the tank. Table I shows that the wear at the higher humidity was approximately four times as great as it was at the lower humidity. An inspection of the commutator gave further evidence of improved performance during the low-humidity test. Upon shutting down test No. 3 the commutator appeared dark and uniformly smutted, but apparently no damage had been done. A light application of fine sand paper showed the commutator to be in perfect condition. This was not true of the commutator at the end of test No. 4. It was apparent immediately upon shutting down this test, that the commutator had been badly burned. The bright copper was showing on the trailing edge of the commutator bars just as it was after test No. 2, and the roughness could not be removed by any reasonable amount of sanding. It is interesting to note that brushes Nos. 1 and 4 which were on the side of the machine nearest the moisture supply, show somewhat shorter life than brushes Nos. 2 and 3 which were on the other side of the machine. In conclusion it may be said that if a commutator machine is running in hydrogen and sparking badly, the presence of moisture is detrimental to both the brushes and the commutator.

So far, all the tests described were made with the generator adjusted to give severe sparking at the brushes, the idea being to determine what would happen if this condition should arise in service. The various troubles encountered under this severe condition should not lead the reader to believe that commutation in hydrogen is especially difficult or that it will usually be associated with troubles such as roughened commutator and rapid brush wear. To show that operation in hydrogen may be quite satisfactory when the commutating conditions are normal it is only necessary to refer to test No. 5, the results of which are tabulated in the last line of Table I. This test was made while the generator was delivering full load current, but the shunt had been removed from the commutating pole winding. It was during this test that the data for the upper curve of Fig. 2 were taken. The minimum brush life of almost five years and the good condition of the commutator at the end of this test would seem to indicate that as long as a machine is designed and adjusted for good commutation in air, it will behave quite satisfactorily in hydrogen. This last test was repeated several times and the same good results were always obtained.

SLIP-RING TESTS

The apparatus used in these tests consisted of a collector or set of slip-rings driven by an adjustable speed d-c. motor and so arranged that the space around the rings and motor could be filled with hydrogen. The arrangement was similar to that shown in Fig. 1 except that the d-c. generator was replaced by a set of slip-rings. Each ring was ⅞ in. wide and 9½ in. in diameter and was trimmed with one brush ⅝ in. wide by 1¾ in. thick by 2 in. long. Direct current was supplied to the brushes and rings by an arc-welding set. Electrical leads, brought out through the bed-plate made it possible to measure the contact drop of any brush.

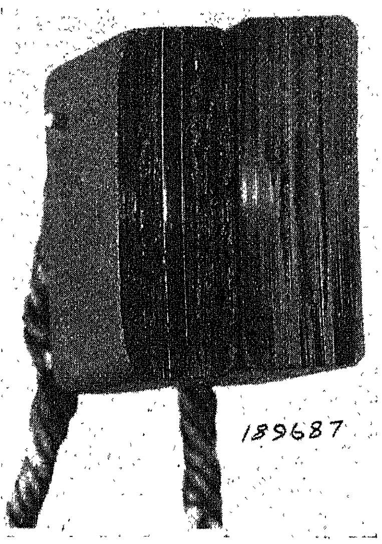

FIG. 3—BRUSH FACES SHOWING DAMAGE DONE BY THE FORMATION OF HARD SPOTS

A test consisted simply of allowing a brush to operate on a slip-ring in air or in hydrogen and observing the results. Most of the tests were made with a current density of about 40 amperes per sq. in. and a peripheral ring speed of 2,500 ft. per min. The duration of a test was determined entirely by the behavior of the brush and ring. Some tests lasted only a day whereas others were continued for more than a week.

TESTS ON STEEL RING

The first tests were made on a tool steel ring trimmed with different grades of carbon and graphite brushes. The results of these tests were always the same. Lumps of hard material, sometimes 0.02 or 0.03 in. in diameter, were formed in the brush face, and immediately began to score the ring. Upon analysis, the hard material proved to be cementite (Fe_3C) which had apparently been formed by a direct combination of iron and carbon in the presence of localized high temperatures in the contact. Fig. 3 shows two brushes, each of which was operated for about 24 hours with a density of 40 amperes per sq. in. on a tool-steel ring in hydrogen. The brush on the right is a fairly hard carbon brush whereas the one on the left is a very soft graphitic brush. Some of the hard particles still remain in the face of the soft graphitic brush although most of them have fallen out. The difference in the appearance of these two brush faces is undoubtedly due to the relative ease with which the hard particles could burrow through the material of the respective brushes.

In an attempt to prevent the formation of hard spots in the brush face, a helical groove[2] was cut in the face of the tool steel ring. With this change it was found that a number of brush grades which failed on the plain ring could be operated without difficulty. The formation of hard particles in the brush face was eliminated and the ring acquired a good polish.

TESTS ON BRASS RING

These tests were quite similar to those made on the tool-steel ring and again the results were almost independent of the grade of brush used. All grades of carbon and graphite brushes showed a marked decrease in contact drop when the air surrounding the rings was replaced by hydrogen. In fact, the ratio of the contact drop in air to that in hydrogen was usually greater than ten to one. For most grades of brushes this reduction was permanent although other grades, after running for a while, began to smut the ring, and when this occurred the contact drop increased to about the same value it would have in air. A tabulation of contact drop values in air and in hydrogen for three grades of brushes is given in Table II. The time required for the contact drop to change from the high to the low value or vice versa was quite variable. In some tests an hour or more was required for the change, whereas in others the change was almost instantaneous.

TABLE II—CONTACT DROPS IN AIR AND IN HYDROGEN

	Contact drop in air		Contact drop in hydrogen	
Brush grade	Brush pos.	Brush neg.	Brush pos.	Brush neg.
A	1.34	0.88	0.18	0.17
B	1.30	0.92	0.08	0.11
C	1.18	1.05	0.08	0.10

Contact drops given in volts.
Grade A—Graphitic.
Grade B—Similar to A.
Grade C—Carbon brush slightly graphetized.

No attempt will be made to explain the phenomenon disclosed by the above tests, but it was noticed that the results were peculiar to the particular brass ring on which the tests were made. Similar tests were made on the brass ring of another test set and it was impossible to even approach the low values of contact drop obtained in the first brass-ring tests. The only explanation which can be found for this disagreement in behaviour is the difference in the compositions of the two brass rings. This would also explain the fact that the steel slip-ring in the earlier tests showed only a slight decrease in contact drop when hydrogen was introduced.

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[2] This device is described in *Application of the Helical Groove to Slip-Rings and Commutators*, by G. M. Little, see page 718.

When it was first discovered that a very low contact drop could exist between a carbon brush and a brass slip-ring, it was realized that such an occurrence on the commutator of a d-c. machine might prove disastrous. As a result the contact drop of one of the brushes on the generator used in the commutation tests was measured both in air and in hydrogen. The average contact drop in hydrogen was found to be only about twenty per cent less than the contact drop in air, and on the small machine this was not enough to make any visible change in the grade of commutation obtained.

CONCLUSIONS

It is realized that the tests reported in this paper are not very complete but from the results of the tests described the following tentative conclusions may be made:

1. A commutator machine designed and adjusted for good commutation in air will operate satisfactorily and give good brush life in hydrogen.
2. If a commutator machine must spark in hydrogen the brush life may be increased many times by keeping the relative humidity below 10 per cent. This will also prevent damage to the commutator.
3. The contact resistance of carbon brushes on a commutator may be materially lowered by the introduction of hydrogen.
4. Carbon or graphite brushes cannot be operated satisfactorily on plain tool-steel slip-rings running in hydrogen.
5. The contact drop between a carbon or graphite brush and a brass slip-ring may be ten times as high when the ring is running in air as when it is running in hydrogen.

Discussion

For discussion of this paper see page 722.

The Application of the Helical Groove to Slip-Rings and Commutators

BY G. M. LITTLE[1]
Member, A. I. E. E.

Synopsis.—An outline is given of experiments showing that a sliding-brush-slipring contact, even with well seated brushes, only makes electrical contact at a comparatively small number of minute spots at any given instant.

Differences are shown in the properties of the contact film formed on a grooved and on a plain slip ring.

Commerical applications of the helical grooves are mentioned.

It is shown that a helical groove cut in the face of a slip-ring or commutator has the following effects:

(1) It provides a means for forcing a more uniform distribution of current over the brush face, and between parallel connected brushes.

(2) It permits the escape of air trapped under a brush.

(3) It prevents local spot heating and glowing under overload conditions.

(4) It raises the contact drop of the brush, producing better commutation and cooler operation.

(5) In some cases, an increased brush life.

* * * * *

THE performance of brushes on slip-rings and commutators is dependent on many more or less obscure conditions. Some of these are fairly well understood and are under control, so that for the most part it is pretty well known what current density, spring pressure, peripheral speed and grade of brush are best for a given brush application. There are many purely mechanical conditions that are undesirable such as loose bars, worn bearings, poorly balanced armatures, rough tracks (in traction service), friction, vibration, and loose connections, and there are dirt, oil, acid fumes and many other obstacles to good brush performance.

Even though all of the above mentioned external conditions are satisfactory, there are still difficulties in transmitting current to and from the stationary brushes and the rotating slip-ring or commutator. It is usually found that the current is not divided evenly between parallel connected brushes and this overloads some of them.

The present article is the result of an investigation of some of the causes of trouble that exists right at the sliding contact between a brush and a rotating slip-ring or commutator.

Much is said in this article about overloads and the concentration of current, at the brush face, into one or more spots which get very hot. It is a most plausible supposition that under the *normal* running condition of a brush, which is well fitted to a slip-ring (or commutator), that the current flows across the sliding contact *at all times* through a multitude of tiny contacts distributed over the *entire polished face of the brush.* There is good evidence that this is only partly so. It is shown that with a well fitting brush riding on a smooth slip-ring, there is only a comparatively small number of minute contact points or groups of points concerned in carrying the current, *at any given instant.* Under light pressure there may be from 3 to 6 active current carrying spots, while under increased pressure, 50 to 100 spots may be shown to be active.

The demonstration consisted in discharging a large condenser across the contact between the brush and the slip ring. This discharge, estimated to be at least 3,000 amperes, was over in only a few millionths of a second, and was so rapid that, though the slip-ring was revolving at 1,500 r. p. m. the pattern burned on the face of the ring and on the face of the brush consisted of a group of well defined minute spots, and the distribution of these spots over the brush face was found never to be the same in successive experiments. These spots constitute less than 3 per cent of the total area of the polished brush face. (Fig. 1).

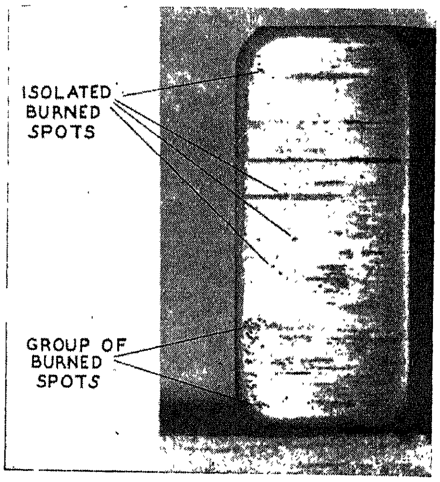

FIG. 1—SPOTS PRODUCED ON BRUSH FACE BY THE DISCHARGE OF A LARGE CONDENSER (3,000 AMPERES)

Apparently only a very minute portion of the brush face area is carrying current at any given instant

It is probable that under normal running operating conditions the current at each successive instant of time finds a different group of contact spots to carry it across the sliding contact. These groups shift around with incredible rapidity as was shown by a method suggested by Mr. R. M. Baker, who proposed the use of a double filament oscillograph to record simultaneously but separately the current flowing through the two halves of a divided brush. In the oscillogram (Fig. 2) the

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[1] Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

current is seen in some instances to drop to zero in one half while at the same instant the other half is seen to be carrying all of the load (oscillogram made by Mr. Mebbs and the writer).

It was observed that if the current density was very high, a brush was likely to glow and throw off white hot streamers. This glowing manifested itself as a small stationary or slow moving white hot spot in the polished surface of the brush. This of course could not be seen directly, but it heated the surrounding brush material for 1/8 of an in. in all directions to a good red color and if the small white hot spot was near to the edge of the brush the red glow of the surrounding brush material was readily seen.

A study of this glowing, confirmed in the writer's mind the correctness of the usual explanation of the cause which was based on the negative temperature coefficient of carbon. If a brush is overloaded and one part of the brush face should by chance be a little warmer than another part, the current will tend to crowd into this warmer area raising its temperature more and more until a low-resistance hot spot is developed, which then carries substantially all of the current.

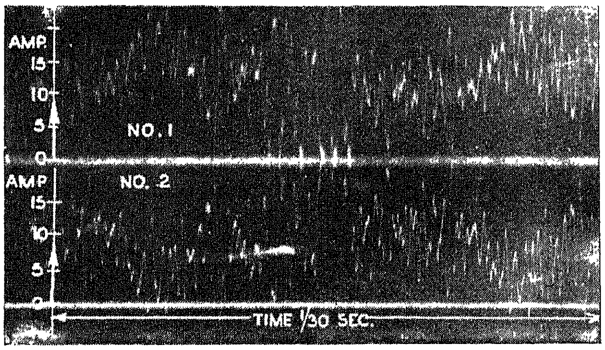

Fig. 2—Oscillogram Showing Fluctuations in the Current between the Leading (No. 1) and the Trailing (No. 2) Halves of a Divided Brush

It was observed that a hot spot of this kind will remain in one location on the brush face for many hundred revolutions of a commutator or slip-ring, during which time all sparking of the brush will be stopped. This tendency of a spot to take all the current and to hold to one position on the brush face is further promoted by the local expansion of the hot carbon. This expansion causes the hot spot to become slightly raised above the polished level surface and the brush rides on this "hill" until it is worn down. During this time most of the spring pressure is concentrated on the minute raised area.

The local temperature rise due to friction is thus probably a large contributing cause for bringing about the stationary hot-spot condition.

A very convincing observation of this action was made on a badly overloaded glowing brush riding on a slip-ring. The expansion at the hot spot was so great that it raised the brush off of the slip-ring noticeably and it was possible to see through the gaps on either side of the hill.

Fig. 3 shows the surface of a carbon brush that had been glowing. The light colored spots, around which are seen semi-circular cracks, show where the current had concentrated. This had raised the local temperature so high that it had changed the dark colored carbon into the lighter colored graphite.

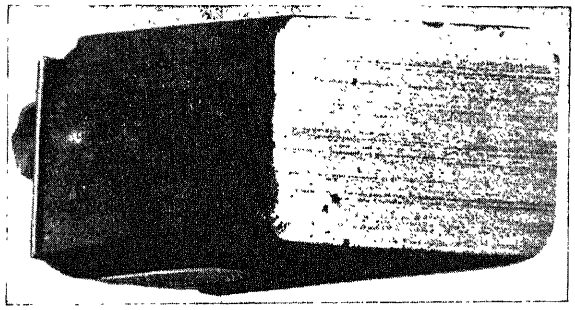

Fig. 3—Face of a Brush That Had Been "Glowing" and Throwing off "Streamers"

Note the light colored graphite spots, the semi-circular cracks, and the black cavities left when a graphitized spot falls out

When the cracks go clear around, a little white hot semi-spherical particle falls out and is thrown off by the rapid rotation as a brilliant streamer.

Narrow crosswise grooves in the surface of a rotating commutator, produced by undercutting the mica, are not able to break this up, because a hot spot has sufficient area that it can bridge the small gaps without interference to the current flow. Also if the current was broken, there would not be enough time for a spot to cool appreciably before it again made contact. When a brush is badly overloaded the hot-spot temperatures may rise to 2,000 deg. cent., sufficient to graphitize the minute areas affected.

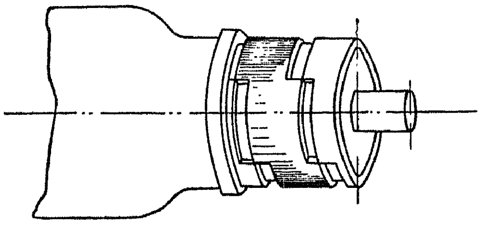

Fig. 4—The "Straight Around" Offset Groove

With these observations in mind it occurred to the writer that if the current passing through the hot spot could be blocked and some other portion of the brush face, or even another parallel connected brush, could be forced to take the load for a portion of each revolution of the commutator, during which period the hot spot would have an appreciable time to cool; perhaps a more uniform distribution of current under the brush face and between parallel connected brushes would result, together with an absence of local spot heating and glowing.

Two schemes were evolved, (1) a row of side-by-side, straight around, offset grooves (see Fig. 4) and (2) the helical groove, (see Fig. 5). Both schemes were tried and gave similar results but the helical groove was easier and cheaper to make and could, if necessary, be cleaned more readily. In both schemes a given spot on the face of the brush can carry current and be subjected to frictional heating for only a portion of each revolution of the slip-ring or commutator, during which time some other spot or group of spots is forced to carry the load. Any air trapped under the brush face which is a contributing factor in uneven current distribution is permitted to escape as are any loose abrasive, solid particles of brush or ring material. Some years ago Messrs. Shand, Wagner, and Eschholz demonstrated that, in the case of parallel-connected metal graphite brushes running on a slip-ring, air was trapped under the brush face. This was illustrated by drilling a hole down through the brush and connecting it to a manometer tube. Eight or ten in. of water pressure was sometimes found. Crosswise grooves were placed in the face of the slip ring to let this air escape and it was found that more uniform distribution of current followed. These experiments did not reach a commercial success because of the tendency of the grooves to fill with dirt.

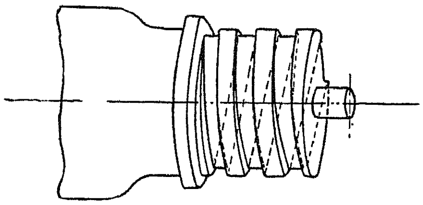

Fig. 5—The Helical Groove

It forces a more uniform current distribution between parallel connected brushes and diminishes any tendency to glow

Tests showed that the helical groove improved the performance of brushes on a slip-ring very much. It is not at all unusual on a commercial slip ring to find that practically all of the current is being carried, by only one or two brushes out of a set of five or more connected in parallel. These two brushes will carry this overload for some hours, then become idle and one or two of the others will take on the load for a time. This treatment works a serious injury to the flexible copper connections, the shunts, oxidizing them and raising the resistance of the joint between the shunt and the brush. The overloaded brushes waste away much faster than those which are carrying their correct rated load.

Tests were then made to determine what effect grooving a slip-ring had on the contact resistance. Use was made of two copper slip-rings, one plain and one grooved, mounted on the same shaft. Each was equipped with one positive and one negative brush and the rings were so arranged that they could be operated either in series or in parallel.

At the start with clean sand-papered rings and brushes, the contact resistance on both the rings was extremely low but the grooved ring brush contact was almost always much lower than that of the plain ring. The resistance of both rings then slowly rose, until finally the resistance of the grooved ring and brushes became about 20 per cent higher than that of the plain ring and its brushes; this sometimes took 20 or 30 hours.

At the beginning of this test, it would appear that no film or polish having a high resistance was present and the bare surface of the grooved ring had the lower contact resistance even though the available contact area was much smaller, because the pressure per square inch exerted by the spring on this smaller area was raised, in this particular instance, to about three times that on the plain ring. As the test proceeded, there was apparently a very slowly formed high-resistance film or polish, on the sliding surfaces, and this film on the grooved ring and its brushes finally became higher in resistance than that which formed on the plain ring and its brushes. With certain brushes, (such as Stackpole L21) running on a plain and a grooved ring connected in parallel, and supplied with a constant current it was usual to find, after the day's run, that the warm grooved ring had the higher resistance and was taking only 5 amperes while the warm plain ring was taking 10 amperes. On shutting down for the night and starting again in the morning, cold, both rings were found to have acquired a very high resistance, and usually the grooved ring was initially the lower of the two. This grooved ring would start off with ten amperes while the plain one took only five. This condition often lasted for hours though finally a return to the former state always occurred as the rings became warm.

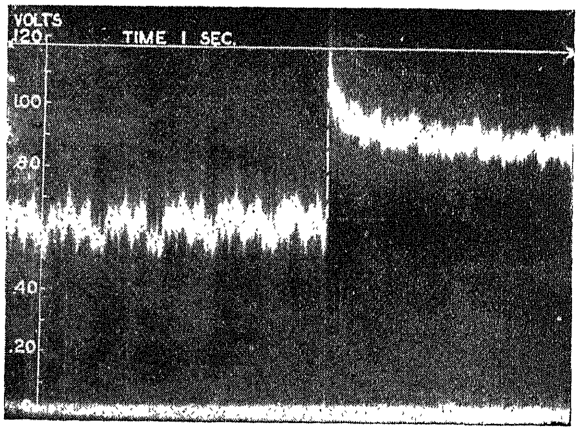

Fig. 6—Voltage Change on a Plain Slip-Ring as Current is Suddenly Stepped Up from 1 to 7 Amperes

Note that the voltage is very high during the first fiftieth of a second. By means of the helical groove, the current carrying time of a given spot can be limited to less than 1/50 of a second. This keeps the temperature down and the resistance up, giving a desirable higher contact drop.

While the difference in the character, between the slowly formed film of the grooved ring, from that on a plain ring is probably the main reason for the difference in the contact resistance (the 20 per cent increase), there is another well demonstrated cause:

An oscillograph shows that on a plain ring a sudden forced increase in current density at the contact between a brush and a ring causes a large, but rapidly diminishing, increase in the contact resistance (see Fig. 6). It would appear that with a suddenly applied current an appreciable time is required for the temperatures of the current carrying contact spots to rise, during which time the resistance (owing to the negative temperature coefficient) remains comparatively high. The helical groove scheme furnishes means of control whereby the duration of time that any given spot on a brush face can carry a heating current and maintain frictional contact may be limited to 0.02 of a second or less; and from an inspection of the oscillogram as given in Fig. 6 it appears reasonable to believe that this should result in an increase in the contact resistance.

This temperature lag effect on the resistance of the sliding brush contact is shown if a 25-cycle a-c. current is used. The repeated changes from 0 to the maximum current value and back occur so quickly that the temperature of the current carrying contact points does not change appreciably and as a consequence the voltage and current are in phase and the resistance remains constant, as is shown by an oscillogram in Fig. 7. This is further confirmed by slowing down the frequency so much, (to 3 cycles per minute) that a very appreciable period of time lapses while the current is passing through one cycle. The oscillogram then shows that the voltage curve is distorted and the resistance changes enormously, as in Fig. 8, as the current values slowly change.

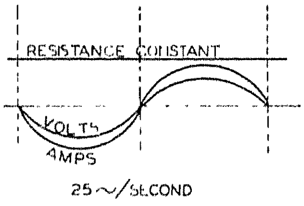

FIG. 7

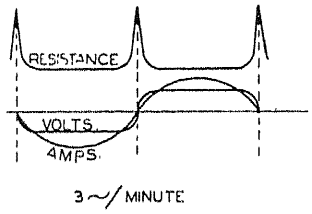

FIG. 8

FIG. 7 CURVE (FROM OSCILLOGRAM) SHOWING THAT VERY RAPID RECURRING FLUCTUATIONS IN CURRENT DENSITY ARE NOT FOLLOWED BY FLUCTUATIONS IN TEMPERATURE AND RESISTANCE

FIG. 8 CURVE (FROM OSCILLOGRAM) SHOWING THAT SLOW RECURRING FLUCTUATIONS IN CURRENT DENSITY ARE FOLLOWED BY CORRESPONDING CHANGES IN TEMPERATURE AND RESISTANCE

LABORATORY TESTS ON SLIP-RINGS

A test[2] of five parallel-connected, metal-graphite brushes on a plain bronze slip-ring gave frequent exhibitions of one or two brushes taking all of the current, while later the same brushes run on a grooved ring were free from this trouble.

Another test showed that with a grooved and a plain ring in series, and with only one brush on each ring, the plain ring brush sparked badly, the grooved ring brush operated perfectly, while the set was subjected to a heavy overload.

Still another test was made on plain steel rings operating in an atmosphere of hydrogen. Concentration of current and local overheating caused pitting of the brush face and the formation of extremely hard particles (apparently iron carbide) which became imbedded in the face of the brush and rapidly scored the surface of the ring. Grooving this same ring did away with the scoring of the ring and the coarse pitting of the brush face.

LABORATORY TESTS OF THE HELICAL GROOVE ON COMMUTATORS

The laboratory investigation of the application of a helical groove to a commutator has disclosed some interesting information.

One of the first trials was made on a defective d-c. 2,000-ampere, 500-volt generator.

This machine would carry $\frac{1}{2}$ or $\frac{3}{4}$ load very satisfactorily, but on full load trouble would develop in a few hours time, and on load and a half, the glowing, sparking, and streamers would develop within ten minutes.

After the commutator was grooved $\frac{3}{8}$ in. pitch, $\frac{1}{8}$ in. groove, the machine carried full load indefinitely (over a week) with no glowing and stood the 50 per cent overload for 3 hours; also it was noted the grooved commutator ran 10 deg. cent. cooler than the plain commutator. Later this same grooved commutator machine was operated at full load and 8 out of the 44 brushes were removed; no distress was observed. This suggested the possibility of building shorter and less expensive commutators.

A long (200 hr.) test on a plain, and later, grooved $\frac{1}{8}$-in. ridge, $\frac{1}{8}$-in. groove d-c. traction motor, 40-amperes, 500-volts, gave an improvement in sparking, a 7 deg. lower commutator operating temperature, and left the commutator in better condition at the end of the test. The current distribution between the parallel connected brushes was more even.

A short test on a large a-c. single-phase railway motor showed a more uniform distribution of current between the brushes on the same arm.

One odd and possibly useful feature of the helical groove is, that if sparking occurs at one spot only in the revolution of the commutator, *e. g.*, at a high or a low bar or a flat, the fact is shown by the formation of a peculiar pattern burnt on the face of the brush and this can serve to locate the offending bar by rotating the commutator until the burned pattern rests accurately on the ridge. This is illustrated in Fig. 9.

It appeared from the above data that grooving a commutator probably does much more than simply prevent the formation of hot spots.

The cooler operation suggested that the wasteful circulating current that occurs under a brush face during ordinary commutation had probably been reduced, and this could only be explained by supposing that in some way the contact resistance had been raised.

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[2] Taken at the laboratory of the Stackpole Carbon Co.

The observed reduction in the sparking at the trailing edge of brushes on a grooved commutator, may be explained by the increased contact resistance, which served to cut down the volume of circulating current which the spark had to handle, but there is another explanation that may be even more plausible.

It is common belief that an arc (a spark) can be formed in air, only, if the temperature of the separating contacts is high enough to ionize the air and make it a conductor.

As the helical groove on a commutator prevents any large rise in temperature of contact spots under the brush face, it is very probable that the small arc-starting spots at the trailing edge of a brush and under the brush face are, to some extent, prevented from reaching the temperature necessary to ionize the air and start the spark.

The depth of the groove has little or no effect on the performance, though it is reasonable to expect that a deep groove might promote cooler operating due to the increased radiating surface and the larger amount of cooling air that is dragged across the brush face.

The width of the groove and the pitch of the thread are being studied and it would appear that to secure the best results in different kinds of service, these dimensions will be varied considerably.

When a helical groove is cut in a commutator or slip ring the available sliding contact area is reduced, and with no change in the spring pressure, the pressure per unit area is raised.

If this increase in the unit area pressure is prevented by lowering the spring pressure on the grooved ring brush, a rise in the contact resistance is more easily and surely shown. It is possible to lower the spring pressure on a grooved ring to a greater extent than can be done on a plain ring without causing sparking but this must not be done if there is any vibration.

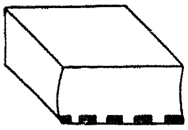

Fig. 9—Pattern Burned on a Brush Face if There is Sparking at One Spot on a Grooved Commutator, *e. g.*, at a High Bar

The appearance presented by a grooved commutator while it is rotating, conveys an impression that there must be a powerful action tending to drag the brushes toward one end of the shaft, but no such tendency exists as the brushes slide on the tops of the ridges.

When grooving a commutator, it is very desirable to stone the surface after the groove has been cut in and the edges slightly rounded, as any burr left is disastrous to brush life and sand papering alone is not effective. For subsequent polishing, a very fine sand paper backed by a wooden block, is best.

Commercial Applications to Slip-Rings

The above slip-ring tests were so convincing that when the information was given to one of the public service companies, they decided to cooperate in an experiment on the rings of one of their 13,000-ampere rotary converters. The operation was so satisfactory that they are now having another machine grooved and put into service. Another public service company is putting in operation, a large synchronous condenser equipped with hydrogen enclosed steel slip-rings with helical grooved surfaces.

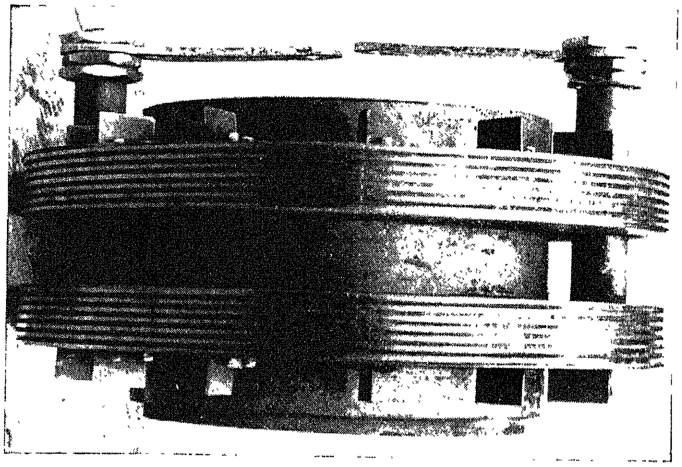

Fig. 10—Grooved Slip-Rings on Two 15,000-Kva. Synchronous Condensers (Hydrogen Enclosed)

Fig. 10 shows a 1/8-in. wide groove 1/8-in. wide ridge, 1/2-in. pitch ring for a 15,000-kva. synchronous condenser.

Discussion

COMMUTATION AND CURRENT COLLECTION IN HYDROGEN

(Baker)

THE APPLICATION OF THE HELICAL GROOVE TO SLIP-RINGS AND COMMUTATORS

(Little)

T. T. Hambleton: It will be of interest in connection with Mr. Baker's paper to comment briefly on similar work the General Electric Company has done.

In the summer of 1929 the General Electric Co. had undertaken to equip a 3,000-kw., 10,000-ampere, 300-volt, 60-cycle converter with a gas tight enclosure for a trial of operation in hydrogen. This machine was erected in the customer's plant early in the year 1930. But for various reasons, among them being load requirements, operation in hydrogen was not tried till November 1930. Several trial runs each of several days duration, have since been made in hydrogen. The results show a reduction of windage loss of approximately 25 kilowatts, or eight-tenths of one per cent increase in efficiency at full load and increased cooling efficiency.

Means were provided for observing from outside the enclosure, the temperatures of main and commutating field coils and commutator. The results obtained as compared with operation in air check fairly well with what would be expected based on the thermal characteristics of hydrogen.

It was found when operating in hydrogen, that the rating of the converter could be increased 25 to 30 per cent with approximately the same ultimate temperatures in current carrying parts as were reached at the lower rating when operating in air. The same quantity and temperature of cooling water was maintained in these tests.

The reduction in commutator temperature was much greater than in the other parts. Its temperature rise at full load in hydrogen was just 50 per cent of that measured when running in air. This is accounted for by the large reduction in brush contact voltage drop and possibly some reduction in friction together with the higher efficiency of cooling medium. The appearance of the commutator was much better than is customarily seen in air; the surface being highly polished and entirely free from any trace of film or deposit. Commutation was very good both in air and hydrogen. No material difference could be noted.

The one condition which will determine whether or not hydrogen cooling can be applied to large converters relates to a-c. brush wear. A certain rate of a-c. brush wear has been accepted in the past where machines have been serviced and cleaned on an average of once per week. But when a converter is enclosed in hydrogen, this running period must be lengthened to two or three months in order that the cost scavenging gas and replacement of hydrogen may be warranted.

It is probable that the rate of a-c. brush wear acceptable with open converters will not be permissible with this greatly lengthened period between cleanings. The problem of a-c. brush dusting is therefore the principal one to be solved. Its solution may be found in some entirely different grade of brush; in some arrangement of gas filter to remove the dust; in isolating and providing suitable insulation for the a-c. brush gear; or in devising some arrangement for blowing out and cleaning the surface of the insulation by means of a hydrogen blast which can be applied without the necessity of opening the enclosure.

W. C. Kalb: Mr. Little has presented a very valuable contribution to the literature on current collection in rotating electrical machinery. His conclusions appear sound from a theoretical viewpoint and are well supported by laboratory tests. It will be interesting to note the results obtained from more extensive application under operating conditions, especially on commutators.

The fact that the current passing between a brush and the slip-ring or commutator is carried at a given instant by a few points of small area, has been well established. It is probable, however, that many engineers have not realized the extremely high-current density momentarily existing at these points, nor the current density which must be reached before glowing is obtained. A test made by the writer a few years ago indicates that the local current density when glowing occurs, must be well over 1,000 amperes per sq. in.

Two graphitized brushes of approximately half and half lamp-black and petroleum coke composition were mounted on a bronze slip-ring $12\frac{3}{4}$ in. in diameter and driven at a surface speed of 2,430 ft. per min. The brushes were 2½ in. by 1 in. by 1 in. with the contact face reduced to an area 1 in. by ¼ in. To insure firm contact, a tension of 18 lb. per sq. in. was applied to this reduced area. Contact drop of positive plus negative brushes was measured with varying current density and the results obtained are indicated by Fig. 1. It will be noted that this curve is of conventional form up to 360 amperes per sq. in., when the rate at which contact drop increases began to accelerate. The curve, however, continues regular in form up to 780 amperes per sq. in., when it took a sudden drop and readings from that point on through still higher current densities were very irregular. While visible glowing was observed in only four instances, at current densities of 820, 830, 850 and 860 amperes per sq. in., it seems probable that above 780 amperes per sq. in. glowing was taking place under the brush face practically all the time. This accounts for the erratic character of the readings at the very high-current densities.

The increasing increment in contact drop above 360 amperes per sq. in., contrary to the commonly accepted theory that contact drop tends to reach a fixed upper level, is probably due to the difference in coefficient of expansion of the bronze ring and the carbon brush. On this particular test the arc of contact was not large enough to observe such an effect but on another test with a larger arc of contact it was observed that at the very high-current densities, when the rate of increase in contact drop began to accelerate, a line of light could be seen between the ring and the central portion of the brush face. The brush was bearing only at the extreme leading and trailing edges, indicating that the expansion of the ring had given it a longer radius than the brush face and probably accounting for the unusual increase in contact drop.

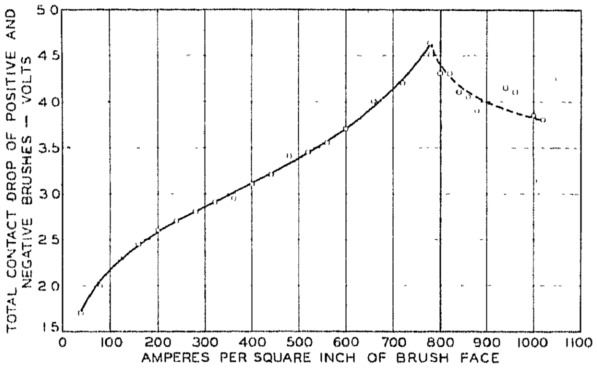

Fig. 1

Robert M. Baker: I wish to thank Mr. Hambleton for his discussion which I feel is a valuable addition to my paper. Mr. Hambleton does not specify the actual decrease in contact-drop of the commutator brushes in hydrogen, although in view of the results of slip-ring tests reported in my paper this decrease may have been as great as 90 per cent.

It is my belief that the problem of slip-ring brush dusting can be solved most easily by resorting to an entirely different grade of brush.

Studies in Non-Linear Circuits

BY CHAUNCEY GUY SUITS*

Non-member

Synopsis.—*The series circuit consisting of an inductance L, capacitance C, resistance R, and a source of alternating electromotive force, where the circuit elements are all linear, is regarded as a special case of a more general type of circuit in which L, C, and R may be functions of the current or other variables. The practical importance of these non-linear circuits is noted and some of their properties are reported. In the series circuit with a linear resistance, capacitance, and an iron core inductance it is shown that the abrupt rise in current at a certain critical voltage is characterized by resonance in the sense that*

$$L \frac{d\,i}{d\,t} = -\frac{1}{c} \int i\,d\,t.$$

In this non-linear resonance however the above condition is satisfied in but a limited portion of the cycle. During this portion of the cycle the peak current may be calculated from Ohm's law.

The series circuit with linear resistance, capacitance, and an iron core reactor with superimposed steady magnetizing force, is shown to exhibit a pulsation oscillation which is explained on the basis of the positive impedance-current characteristic of such reactors.

Parallel resonance for the non-linear case of an iron core reactor is reported, and it is shown that in conjunction with series resonance the unique characteristic is obtained wherein the r. m. s. current decreases with increasing applied voltage.

The abrupt volt-ampere characteristic of one of these circuits is combined with the power amplifying property of the saturable core reactor to produce a sensitive relay of unique actuating characteristics.

AMONG the most interesting fundamental electrical circuits are those exhibiting simple series and parallel resonance. When these well-known experiments are performed for non-linear[1] circuit elements, many new and unique characteristics appear. It is the object of this paper to describe some of these characteristics.

Consider the series circuit consisting of an inductance, a capacitance, a resistance, and a source of alternating electromotive force. The differential equation for this circuit may assume the form[2]

$$L\,(i) \frac{d\,i}{d\,t} + \frac{1}{C\,(i)} \int i\,d\,t + R\,(i)\,i = E \sin\,(\omega\,t + \psi)$$

where the applied voltage is sinusoidal. Thus in general the coefficients $L\,(i)$, $C\,(i)$, and $R\,(i)$ may be functions of the current of any form whatsoever.

The familiar simple series circuit where the inductance, capacitance, and resistance are essentially independent of current is in this sense a special case of a much more generalized type of phenomena described by the equation above. The linear circuit, partly because it lends itself to a simple mathematical treatment capable of adequately representing the physical functions encountered in many practical cases, has received more thorough study. The mathematical impedimenta need not however intimidate the experimental approach, for many of the non-linear circuits may be demonstrated with the simplicity characteristic of the more familiar cases.

The importance of the study of these circuits rests to some degree upon the fact that a great many practical cases, as for example power circuits in which ferro-magnetic materials are used, are by no means well approximated by linear circuits and behave under certain conditions in a manner which by comparison is regarded as anomalous, when in reality they are a fundamentally different and unfamiliar phenomena. The large number of cases that have been reported in recent years[3] of various types of instability in power circuits incorporating capacitances can probably be analyzed in terms of the elementary circuit types here described, although that has not been attempted in this paper.

Many non-linear circuit elements are familiar; for example, resistances are available which increase or decrease with increasing current, instantaneously or with thermal lag, or have abrupt discontinuities; inductive impedances are available which increase or decrease with increasing current, while capacitances are known with very complicated volt-ampere characteristics. In the interests of clarity only one non-linear circuit will be described in detail; this is the simple series circuit consisting of a linear resistance, linear capacitance, and iron core inductance.[4] The special properties of other fundamentally important non-linear circuits will be summarized, and a single technical application will be noted.

The impedance-voltage characteristic for an iron core inductance obtained for sinusoidal applied voltage has the general form of Fig. 1, where r. m. s. values are plotted. When the current is very small, the "effective

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*Research Laboratory, General Electric Co., Schenectady, N. Y.

[1] "Non-linear" will be used throughout to indicate a non-linear volt-ampere characteristic for alternating current.

[2] The $L\,(i)$, $C\,(i)$, and $R\,(i)$ may be given explicit expression where experimental data are available. In the case of the closed iron-core inductance, for example, this has been done in the appendix.

[3] K. E. Gould, A. I. E. E. JOURNAL, November 1927, p. 1160.
L. F. Blume, A. I. E. E. TRANS., 1914, p. 735.
G. Faccioli, A. I. E. E. JOURNAL, 1922, p. 351.
O. G. C. Dahl, A. I. E. E. TRANS., 1925, p. 792.
R. C. Clinker, A. I. E. E. TRANS., 1914, p. 723.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

[4] This case was probably first discovered by Martienssen, (*Phys. Z.*, 1910, p. 448), and was later studied by Biermanns (*Arch. f. El.*, 1915, p. 345) and Margand (*Rev. Gen. d'El.*, 1921, p. 635).

31-37

impedance"[5] increases with increasing current; for a larger value of current the impedance is largely independent of current, until saturation effects appear where the impedance decreases as the current increases. In the simple series circuit here described the current will always be held to values which are sufficiently large to cause the impedance to be either independent of current or decreasing with increasing current.

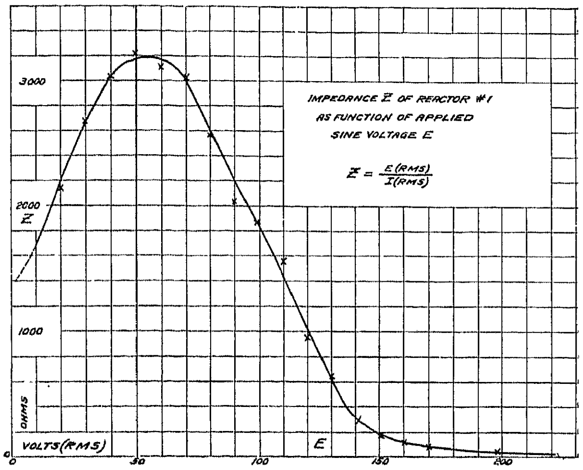

Fig. 1

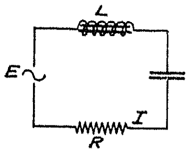

Fig. 2

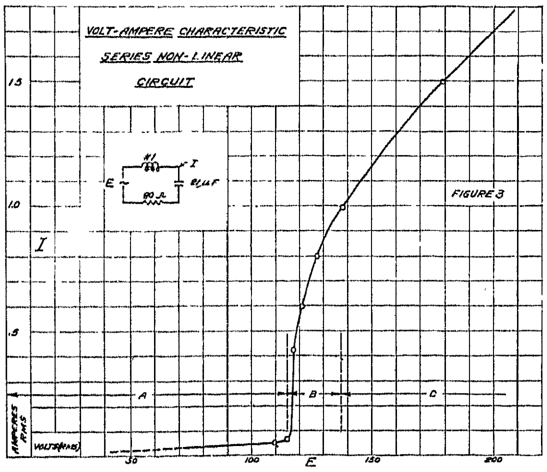

Fig. 3

If a sinusoidal alternating voltage is applied to the series resistance-capacitance-inductance circuit of Fig. 2, the interesting volt-ampere characteristic of Fig. 3 may be obtained. It may be seen that the current in the circuit rises very rapidly at a certain voltage, and continues to rise with increasing voltages, although less rapidly. This sequence of events is shown strikingly by the film record of Fig. 3, where the current I in the circuit is shown with the applied voltage E. The current rises many fold, in response to a voltage change which is imperceptible on the film. The film further illustrates the voltage E_c across the capacitance and the voltage E_L across the inductance when the current is building up. The voltages and current in the circuit may be studied more closely from the oscillograms numbered from 1 to 9, which were taken for points of the curve of Fig. 3. For the region (*a*), Fig. 3, of small applied voltage, the current through the inductance is not sufficient for saturation, and because of proper choice of circuit constants the inductive impedance is large compared to the capacitative or resistive impedances. Nearly all of the applied voltage is across the inductance; the current and voltage relations are approximately those of an inductive circuit. The oscillograms numbered 1 to 3 inclusive are characteristic of this region. For numbers 1 to 3 are shown the applied voltage E, current I, inductive voltage drop E_L, and capacitative drop E_c for $E = 110$ volts r. m. s. It may be seen that the current lags behind the applied voltage E (approximately in phase with E_L) by about 90 deg., and that the capacitative drop is small compared to the inductive drop for all portions of the cycle. As the voltage is further increased, saturation effects appear, the effective inductance decreases, approaching a high-current condition. This further increases the current, decreasing the inductance, resulting in the abrupt increase in current with voltage, characteristic of the region (*b*). The oscillograms numbered 4, 5, and 6 have been taken for the applied voltage $E = 118$ volts r. m. s.; this is the steepest portion of the volt-ampere characteristic. Of particular interest is the oscillogram numbered 6. It may be seen that the capacitative voltage E_c is equal and opposite to the inductive voltage drop E_L throughout the interval

TABLE I

E r. m. s.	I peak amperes	E volts	R ohms
118	0.90	95.3	105
121	1.30	123	95
127	1.64	158	97
138	2.0	187	93
179	2.90	258	89

D-c. resistance for the circuit, exclusive of capacitance = 88 ohms.

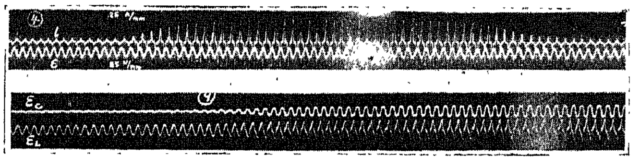

Fig. 3-a

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[5] See Appendix.

indicated by the arrows. (Shown in phase, for convenience in comparison.) Moreover, the peak of current occurs at this point in the cycle.

Let us examine the condition for resonance in the analogous linear circuit. The differential equation is:

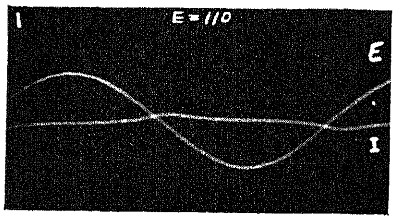
$$L\frac{d\,i}{d\,t} + \frac{1}{c}\int i\,d\,t + R\,i = E\sin(\omega\,t + \psi) \qquad (1)$$

For resonance

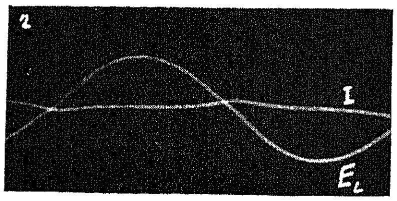
$$L\frac{d\,i}{d\,t} = -\frac{1}{c}\int i\,d\,t \qquad (2)$$

that is, the inductive term is equal and opposite to the capacitance term. When this is true the current i is limited only by resistance and is given by

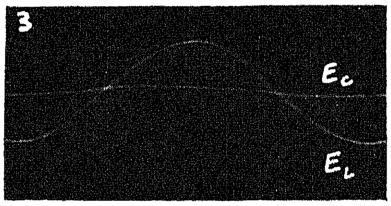
$$i = \frac{E}{R}\sin\omega\,t \qquad (3)$$



Oscillogram No. 1



Oscillogram No. 2



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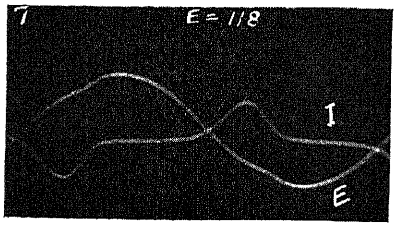
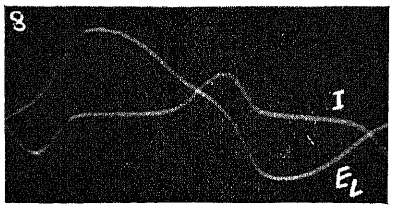
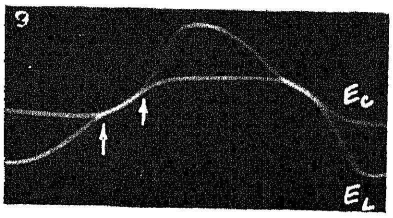


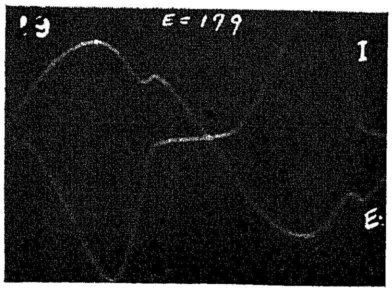
Oscillogram No. 4



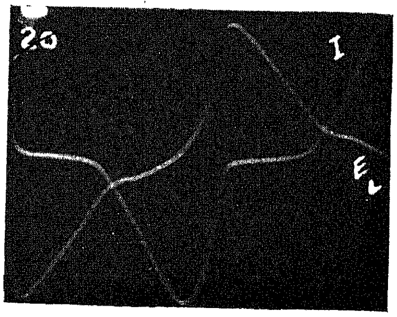
Oscillogram No. 5



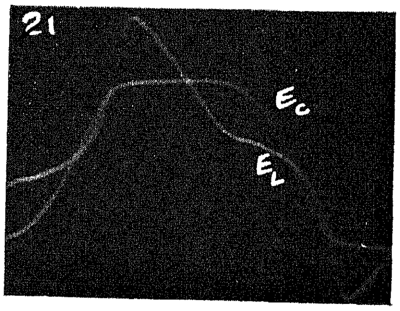
Oscillogram No. 6



Oscillogram No. 7



Oscillogram No. 8



Oscillogram No. 9

From oscillogram No. 6 it may be seen that for the *non-linear* circuit the condition (2) is indeed satisfied, but for a limited portion of the cycle. When the circuit resistance is calculated from the instantaneous voltage and the current peak (7) good agreement with the measured values of resistance may be obtained. In Table I are shown the values of current and voltage measured from the oscillograms (for five points of the curve of Fig. 3), and the calculated resistance at the peak of current. The measured d-c. resistance for the circuit exclusive of capacitance was 88 ohms. The agreement may be seen to be very good at high flux densities where the hysteresis loss, which has been neglected, is a factor of less importance.

From this evidence it is concluded therefore that for the non-linear series circuit the abrupt increase in current with voltage is characterized by resonance in the sense that

$$\frac{1}{c}\int i\,d\,t = -L\frac{d\,i}{d\,t} = -K\,N\frac{d\,\phi}{d\,i}\,\frac{d\,i}{d\,t} \qquad (4)$$

for a limited portion of the cycle. When the condition (4) is satisfied the instantaneous value of the current (the peak of current) may be calculated from Ohm's law and is limited only by resistance. Resonance in non-linear circuits may thus be regarded as partial; because it is characteristic of these circuits, the terminology "*non-linear resonance*" is suggested.

If the applied voltage is now further increased the current continues to increase (region c), although less rapidly. Oscillograms 7, 8, and 9 characterize this region. If the voltage is now decreased, this volt-ampere curve may be retraced exactly. Through the series of events described the peak of current has advanced from a position of lagging 90 deg. (referred to the peak of voltage) to a position of leading by approximately 15 deg., and may with further increase in applied voltage lead by a larger angle.

For some values of applied voltage the non-linear circuit is analogous to the linear simple series resonance in that the fundamental components of voltage and current have the same angular position. Incidentally the r. m. s. values of the inductive and capacitative voltage drops are of the same order of magnitude.

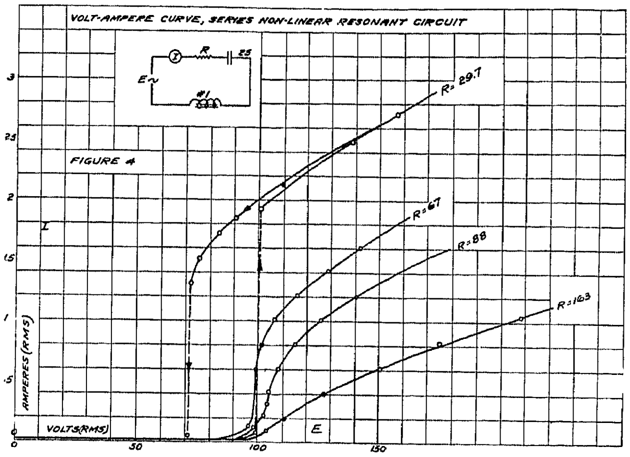


Fig. 4

A complete family of curves for different values of resistance is given in Fig. 4. It may be seen that the non-linear properties of the circuit increase as the series resistance decreases, but that there is a critical value beyond which a hysteresis effect appears. The critical value of resistance is 67 ohms for the circuit for which these data were obtained.

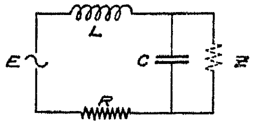


Fig. 5

An interesting classical experiment in linear resonant circuits is known as the constant current circuit. Thus in Fig. 5 the branch L, R, C is in resonance with the applied voltage E. For the case in which the resistance R is small, the current in any element (capacitative, inductive, or resistive), shunting L or C, is independent of the magnitude of the shunt impedance Z. The phase of the current in the shunt branch is displaced 90 deg. referred to the applied voltage E.

An experiment which is interesting because of analogy with the above is that of the constant current shunt to a non-linear resonance circuit. Thus in Fig. 6 is shown a variation of current with resistance when the resistance is shunting the capacitance of a non-linear circuit. It may be seen that for the range of resistance between 0 and 40 ohms the current decreases with increasing resistance; for values of resistance between 40 and 140 ohms the current increases approximately linearly with increasing resistance; and for values of resistance higher than 140 ohms the current decreases with increasing resistance.

It would be a great convenience for many purposes to be able to calculate the "critical voltage" for which the rapid rise in current takes place. There is at present no exact means of making this computation. An empirical method will be given however for determining the critical voltage for non-linear resonance. Reference to the family of characteristics of Fig. 4 will show that this quantity is essentially independent of the resistance in the circuit. In Fig. 7 is given the family of volt-ampere characteristics for the case of changing capacitance. The slight dependence of critical voltage on capacitance may be observed.

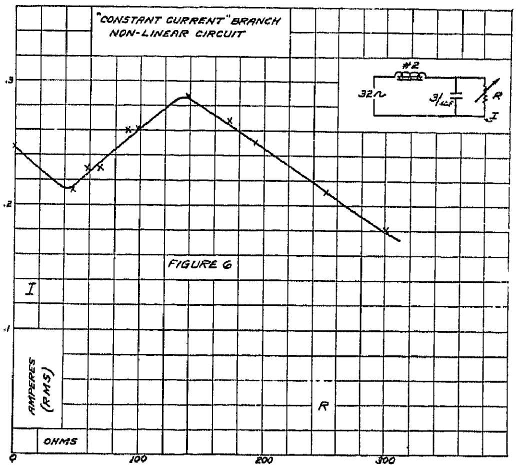


Fig. 6

This voltage therefore depends primarily upon the characteristics of the iron core reactor. The essential correctness of this conclusion is made plausible by a consideration of the enormous range of inductance in reactors of this type. The maximum and minimum values of $\frac{d\,\phi}{d\,i}$ (proportional to instantaneous inductance, where ϕ is magnetic flux and i is current) differ by a factor of several hundred, in practical cases where the saturation portion of the magnetization curve is included. One may expect therefore that several-fold changes in capacitance or resistance might be comparatively unimportant, as is indeed true. It is found experimentally that the critical voltage for resonance is such that the peak ampere-turns reach approximately the knee or saturation portion, of the (alternating) B-H characteristic for the iron. Since commercial transformers are usually designed so that the peak ampere-turns, for the specified voltage, reach the knee of the magnetization curve, the critical voltage for a series circuit using this transformer winding is approximately the rated voltage of the winding. This makes the practical design of a series resonant circuit comparatively simple; a 110-volt transformer coil will resonate at approximately 110 volts when used in this circuit.

An important characteristic of the non-linear circuit is its response to changing frequency, in particular, the dependence of the critical voltage on frequency. Experimentally the curves of Fig. 8 may be obtained where a volt-ampere characteristic is given for the frequencies 35.2, 43.1, 55, and 61 periods per sec. The critical voltage is plotted against frequency in Fig. 9, for which the data were taken from Fig. 8 above. An approximately linear variation with frequency may be observed. That this must be so may be inferred from the following consideration: It has been shown that the critical voltage for resonance depends, to the practical exclusion of all other factors, upon the properties of the iron, and in particular upon producing a peak magnetizing force (proportional to the current) for which saturation becomes rapid. The maximum value E_L of the voltage across a coil is given by

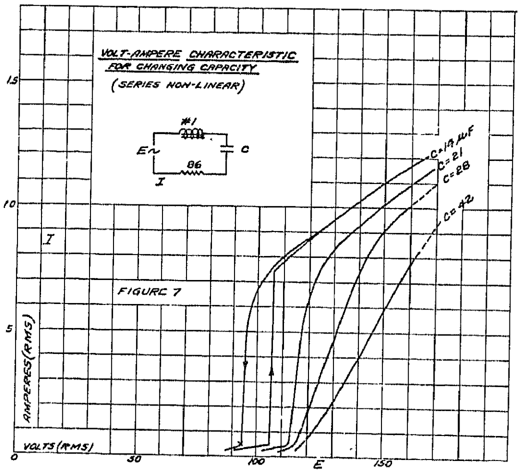


Fig. 7

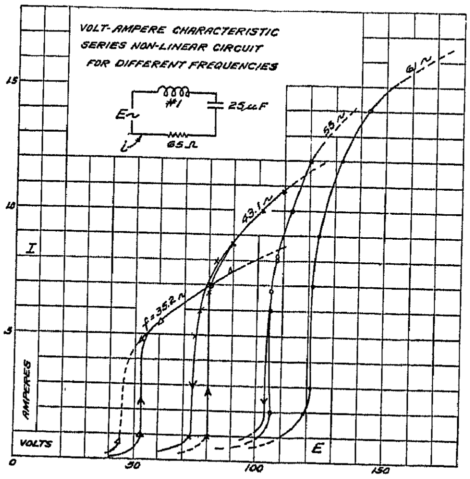


Fig. 8

$E_L = K\,\mu f I$ K = geometric factor

$\mu = \dfrac{d\,\phi}{d\,i}$, ϕ flux, i current

f = frequency

I = current (maximum)

For the special case of μ constant it is clear that to maintain I at any fixed value one must decrease voltage in proportion to the frequency. For the case of distorted current this is approximately true.

Series Non-Linear Circuit with Inductance of Positive Impedance-Current Characteristic

The characteristic behavior of the simple series non-linear circuit for the case of an iron core inductance has been described. Another interesting example of the series circuit is that in which the iron core reactor has superimposed a steady magnetizing force. Such reactors[6] have as a distinguishing feature a positive impedance-current characteristic for some values of alternating current, as shown in Fig. 10. Experimentally one finds that for proper values of steady magnetizing force, applied voltage, capacitance, and resistance for the circuit, a type of low-frequency oscillation appears. The film record of Fig. 11 illustrates this condition. The topmost tracing for each film is that of the voltage applied to the circuit, while the lower is that of the current. The voltage applied to the circuit is being continuously increased throughout the record. The pulsation, which begins at approximately 20 volts peak, persists with but small change in pulsation frequency until the applied voltage has risen to 70 volts peak, where a permanent type of oscillation sets in. This behavior may be described as a type of resonance, in which the rising resonant current increases the impedance of the reactor to the point where the circuit is detuned from its resonant condition. The

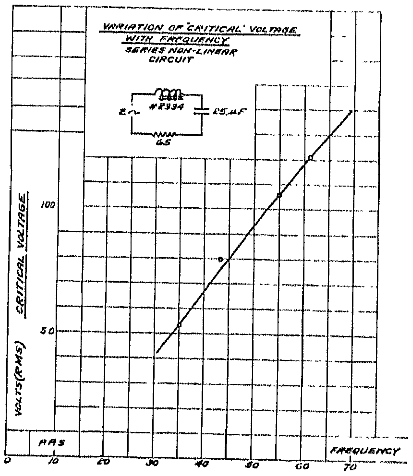


Fig. 9

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[6] A. Boyajian, A. I. E. E. Journal, 1924, p. 958.

current then drops to a low value and this sequence of operations repeats itself. The frequency of pulsation of the r. m. s. current depends to the greatest degree upon the steady magnetizing force in the reactor and the voltage applied to the circuit, and as before, the capacitance and resistance in the circuit are factors of smaller importance.

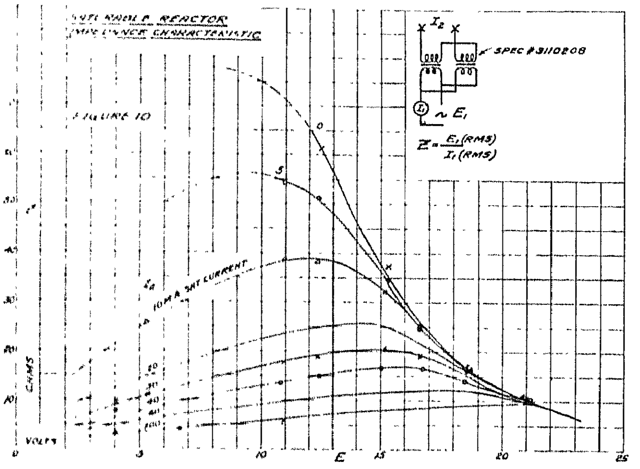

Fig. 10

Series-Parallel Non-Linear Circuit

Another type of resonant circuit with non-linear characteristics, which represents many interesting features, is the case of parallel resonance. Parallel resonance is difficult to demonstrate alone for non-linear elements, but lends itself very well to an experiment in which parallel and series resonance are combined. In the circuit of Fig. 12 are shown resistance R_1 in series with the reactor L_1 in series with the shunt branch, including the reactor L_2, resistance R_2, and capacitance C. When the voltage applied to this circuit is increased continuously the volt-ampere characteristic of Fig. 13 may be obtained. We will presume that the reactor L_2 is made up of more turns than the reactor L_1; that is, it is designed so that a higher voltage is necessary to produce saturation effects. For low values of applied voltage the impedance of the reactor L_1 is high. All the voltage to a good approximation is across the reactor L_1, the voltage across the capacitance is small, and the reactor L_2 is unsaturated and its impedance is high. The portion of the volt-ampere characteristic (a) is thus traversed, which is entirely analogous to the simple series resonance circuit first described. When, however, non-linear resonance is initiated, the voltage across the capacitance rises and eventually becomes sufficiently great to saturate the reactor L_2. Thus the shunt branch R_2, L_2, C becomes resonant and, analogous to linear resonance, the impedance between its terminals becomes high, causing the reactor L_1 to desaturate and the series current I to fall.

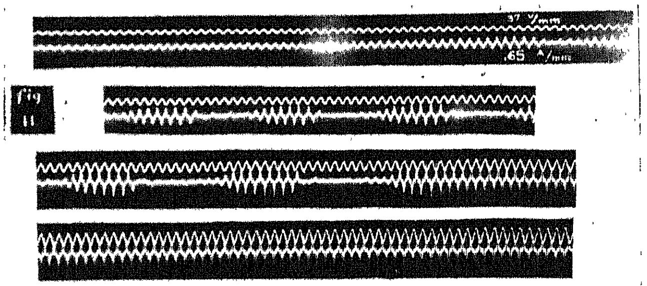

Fig. 11

This behavior is characteristic of region (b) in Fig. 13, where the r. m. s. current decreases with increasing applied voltage. This is the only circuit known to the author, with the exception of thermionic devices, that has this property. If the voltage applied to the circuit is further increased, the point will eventually be reached where the reactor L_1 again saturates, resulting in the second rise in the volt-ampere characteristic shown by the portion (c) of Fig. 13.

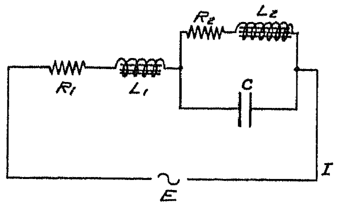

Fig. 12

A film record, Fig. 14, illustrates the sequence of events just described. The topmost tracing is that of voltage applied to the circuit, while the lower tracing is for the current in the series branch. The interesting portions of the record have been indicated on the film. At (a) the series circuit becomes non-linear resonant characterized by the high peak current, resistance limited. The current wave form is symmetrical to the point (b), where a phase shift of harmonics occurs. At (c) the current is very unsymmetrical with a single large peak and two small peaks, indicating the presence

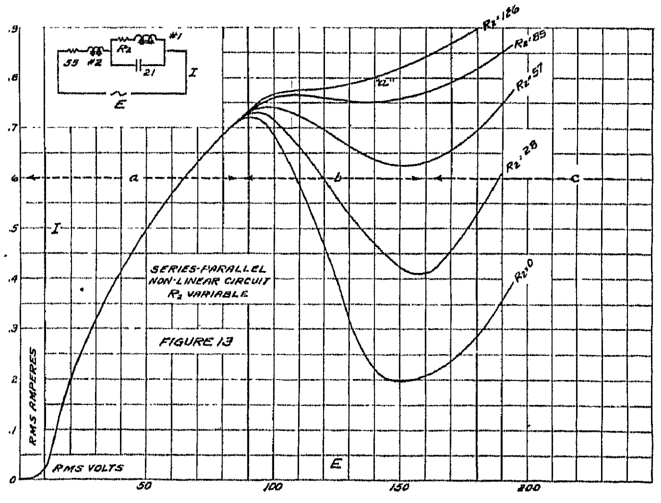

Fig. 13

of even harmonics. Between (*c*) and (*d*) the r. m. s. current is decreasing with increasing applied voltage, reaching a minimum at (*d*); the parallel branch is resonant here, with the current (not shown) reaching a maximum at (*d*). At (*d*) the series reactor L_1 again saturates, resulting in the second rise in r. m. s. current shown between (*d*), (*e*), and following points. In the circuit for which this record was taken, $R_1 = 0$ (nominally), $R_2 = 30$ ohms, $C = 21\ \mu F$, L_1 = reactor No. 1, L_2 = reactor No. 2.

When the series resistance for the parallel circuit is removed (all constants as above except $R_2 = 30$ ohms (nominal)) the current minimum shown at (*d*), Fig. 14, is more marked, and the wave form distortion is more extreme. Film Fig. 15 was taken in the same manner as Fig. 14 with the exception that $R_2 = 0$. The minimum current at (*d*) is very striking. At (*e*) the r. m. s. current has increased but slightly over the value at (*d*). At (*f*) the series reactor again saturates, the series current rises to a high value and is predominantly third harmonic. No adequate theory of this behavior is yet available. The harmonic content is changing between (*f*) and (*g*), tending toward a large fundamental at (*g*).

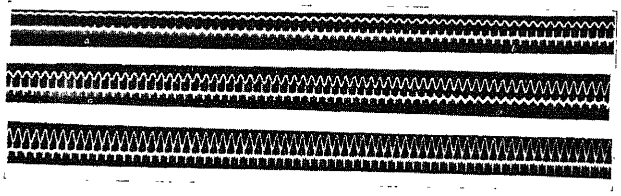

FIG. 14

When the resistance R_2 is varied the complete family of curves shown in Fig. 13 may be obtained. An interesting case is the curve marked "*a*" for which through a certain range of voltage, which in practical cases may be as large as 30 per cent, the r. m. s. current is independent of the applied voltage. It is also possible to so adjust the constants of the circuit that the peak current in the circuit is similarly independent of voltage, and constant in magnitude and phase throughout wide limits. The oscillograms of Fig. 15A illustrate the latter case. Here the applied voltage is shown as the topmost tracing, while the series current is shown in the lower tracing. The circuit constants for this case are: $R_1 = 12.9$ ohms, $R_2 = 164$ ohms, L_1 = reactor No. 2, L_2 = reactor No. 1, and $C = 21\ \mu f$.

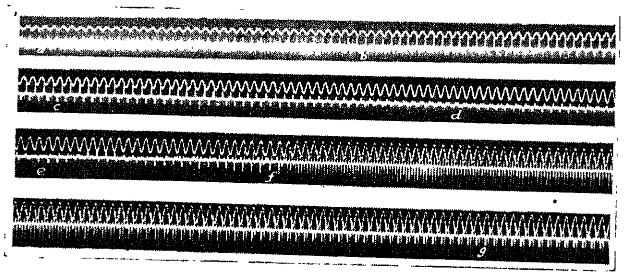

FIG. 15

When the natural frequencies of the parallel branch R_2, L_2, C lie above the frequency of the applied voltage E, the series parallel circuit operates in the manner analogous to the arc circuit for the production of high frequency oscillations. The oscillograms of Fig. 16 illustrate applied voltage and high frequency current through the capacitance for this condition. The fundamental frequency of the current for one case (marked "condenser current") is 1,750 periods per sec. The condenser charges up to the line voltage vectorially less the drop across the saturated impedance of L_1 and discharges in damped oscillations during each half cycle. The fundamental frequency of the damped oscillations may be varied through wide limits, and in the laboratory frequencies as high as 3,000 periods per sec. have been produced from a 60-cycle source.

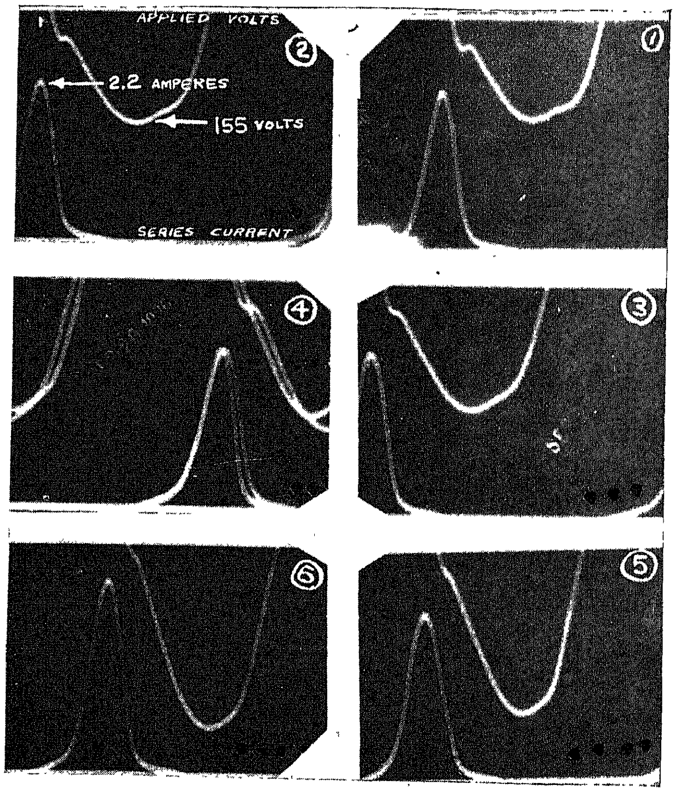

FIG. 15-A

ALTERNATING CURRENT RELAYS USING NON-LINEAR CIRCUITS

A relay has in general two characteristic properties: (a) a non-linear actuating characteristic and (b) capability of power amplification. Practical mechanical relays have in addition a hysteresis cycle, which is very undesirable in many applications. Thus a mechanical relay which is designed to actuate at two volts will not release until the applied potential drops to approximately one volt. The use of the series non-linear circuit as a relay is suggested by the similarity of the volt-ampere curve to the characteristic of an ideal relay. The resonant circuit has no power amplifying properties of its own, but the saturable core reactor[7] is available for this purpose, and the two may be combined as shown in Fig. 17. In this circuit the voltage across the capacitance in the resonant circuit is used to actuate the saturating reactor, which in turn controls a load through its alternating current circuit. In this resonant relay the non-linear actuating characteristic is contributed by the resonance circuit, while the power amplification results from the use of the saturable reactor. The curves of Fig. 18 illustrate a sequence of voltage and current changes in the different branches of this circuit during operation. The current in the resonant branch R_1, X_1, X_2 is indicated by I_1; the direct current in the primary of the saturating reactor is indicated by I_2; and the load current in the a-c. side of the saturating reactor is indicated by I_3. The over-all operating characteristics of the device as a relay are indicated by the curve $I_3 E_1$. These curves are a very close approximation to the characteristics of an ideal relay. It may be seen that the ratio of the "on" to "off" load current may be made as large as desired and is a function only of the design of the saturating reactor and is governed by the same considerations as in all circuits where these types of reactors are used to control power.

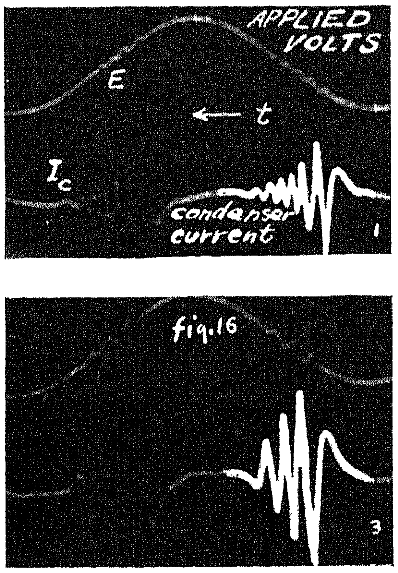

Fig. 16

The *amount of power* which may be controlled in a relay of this type is of importance in certain applications. For a saturable core reactor the power amplification factor is a function of the maximum and minimum values of impedance required, or in terms of load current, the on to off current ratio. Some theoretical considerations indicate that the power amplification of a saturable reactor is quite unlimited; for reactors which have been produced thus far however, an amplification of 1,000 for an incandescent lamp load has not been exceeded. This present limit is easily removed by cascading the saturable reactors. The method is illustrated in Fig. 19, where three stages of amplification are shown. It should be emphasized that there is no limit to the number of stages that may be cascaded in this manner, and hence no limit to the power amplification that may be obtained. In the laboratory experiments thus far it has not been necessary to use more than two stages. A power amplification of 10,000 times is easily obtainable with very inefficient equipment in two stages. If the power amplification factor for the reactor, and the efficiency of the rectifier, are known, it is easily possible to calculate the total amplification that may be obtained in cascade. Let us assume for the reactors of Fig. 19 an amplification factor of 100 for

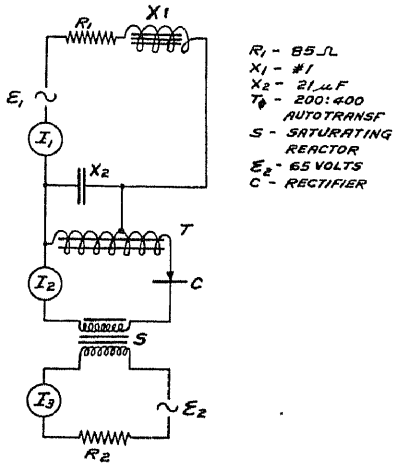

Fig. 17

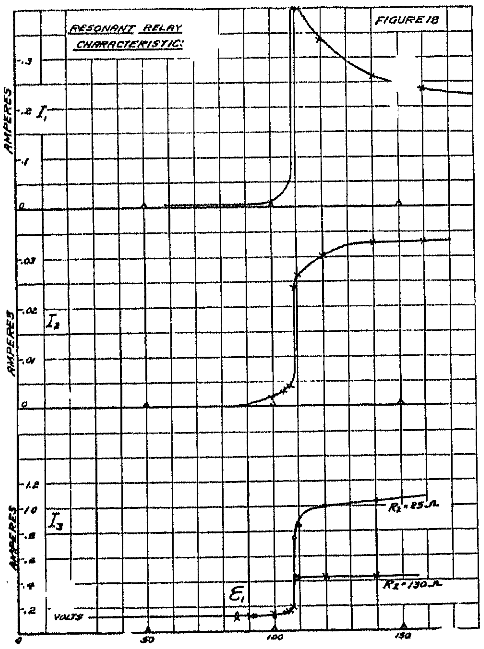

Fig. 18

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[7] A. Boyajian, *loc. cit.*

a given $\left(\frac{\text{on}}{\text{off}}\right)$ impedance ratio, and that the efficiency of the rectifier is 50 per cent for the load we require. Then

$$\text{Total amplification} = (A_1)\left(\frac{A_2}{2}\right)\left(\frac{A_3}{2}\right) = \frac{10^6}{4}$$

where (A_1) is the amplification factor for the first stage, $\left(\frac{A_2}{2}\right)$ the factor for the second reactor plus rectifier, and $\left(\frac{A_3}{2}\right)$ the same for the third stage.

The *sensitivity* of resonant relays is limited by the characteristics of the resonant circuit plus the associated saturable reactor circuit. If but a small amount of power is taken from the resonant circuit the most sensitive relay action is obtained, and it is found that in practical relays a one per cent change in voltage is sufficient to determine the "on" or "off" condition. If the frequency is not constant an uncertainty will exist from this cause, which at 60 cycles will be about one per cent per cycle. There are means available for obtaining still greater sensitivity, but thus far no application has required it.

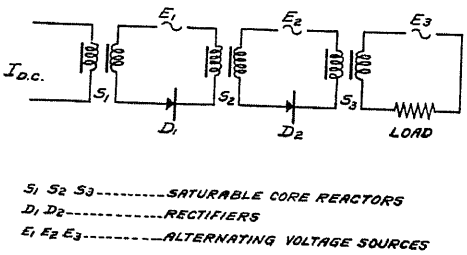
E_1
E_2
E_3
$I_{D.C.}$
S_1
S_2
S_3
D_1
D_2
LOAD
S_1 S_2 S_3----------SATURABLE CORE REACTORS
D_1 D_2-------------RECTIFIERS
E_1 E_2 E_3----------ALTERNATING VOLTAGE SOURCES

FIG. 19

Appendix

Design data on the iron core reactors used in these experiments are here given.

Core:	1.75 in. stack 0.025 in. punchings T. P. 41 *CX*-5 steel Mean length magnetic circuit = 7.4 in. Center leg area 1.3 in.
Primary winding:	480 turns 0.02 in. enameled wire
Secondary winding:	132 turns 0.04 in. enameled wire Tap at 44th turn
Rating:	110-volt 100-watt 50 to 140-cycle 10/20/30-volt secondary

The iron core reactors used in these experiments have the following specifications:

Reactor No. 1: 110-volt primary winding (480 turns)
Reactor No. 2: 30-volt secondary (132 turns)

The magnetization curve for the iron used in this type of transformer is included in Fig. 20.

IMPEDANCE

The term "impedance" or "effective impedance" must be properly defined for this special case. The instantaneous value of the potential difference e between the terminals of a coil is given by

$$e = K N \frac{d\phi}{dt} = K N \frac{d\phi}{di}\frac{di}{dt} = L(i)\frac{di}{dt}$$

where K is a constant, ϕ is the magnetic flux, N the number of turns, and i is the current. Leakage flux and ohmic resistance will be neglected. The term $\frac{d\phi}{di}$ varies with the magnitude of the current in a complicated manner and has a hysteresis cycle; it is similar to permeability. The inductance defined by

$$L(i) = K N \frac{d\phi}{di}$$

varies in a similar manner with the current. The value of the "impedance" Z defined by

$$Z = \frac{E}{i}$$

where E is the r. m. s. alternating voltage between the terminals of a coil and i is the r. m. s. current through the coil, will therefore depend on the wave forms of E and i. In the data that are included here Z has been measured with applied sinusoidal 60-cycle voltage and distorted current; the difference between this value and Z measured for other wave forms may be quite large.

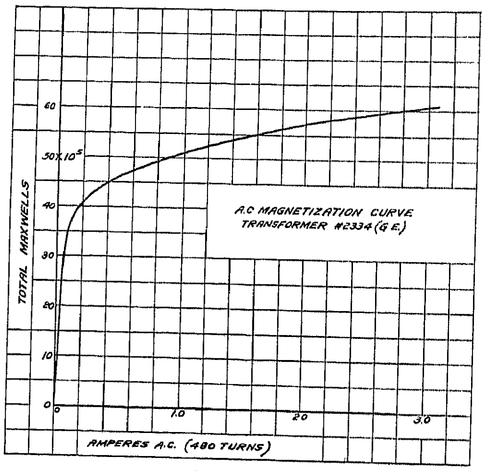

FIG. 20

Discussion

Ernst Weber: The author considers the phenomena in a simple non-linear circuit having resistance, capacitance, and an iron-core inductance in series. Oscillograms show distortions in the waves of the current and the voltage drops across the reactor and capacitance, and on page 726 the author suggests the new terminology "non-linear resonance" to account for these irregularities. However, a fundamental analysis will show that this phenomenon is quite in accord with ordinary series resonance of the double energy linear circuit, and differs only in that it exists for only a small part of the period.

Instead of giving "effective" magnetizing curves with a-c. excitation, it would have been more advantageous to give the actual hysteresis curve for one cycle of the magnetizing current. For example, in the sketch in Fig. 1 let flux Φ be plotted against the magnetizing current i_m and let the arrows indicate the directional variation of the flux Φ with the increase and decrease of i_m.

A well-known method is to replace the characteristic of a non-linear inductance, as in the case of iron cores, by a series of broken lines as indicated in Fig. 2. During the increase of i_m in a section, approximately given by a straight line characteristic, the flux Φ is proportional to i_m, so that we may write

$$\Phi_\nu = K_\nu i_m + C_\nu \qquad (1)$$

Then the induced e. m. f. equals

$$e_{L\nu} = \frac{d\,\Phi_\nu}{d\,t} = -K_\nu \frac{d\;i_m}{d\,t} \qquad (2)$$

and from a comparison with the usual equation for the self-inductive e. m. f.,

$$e_L = -L\frac{d\,i}{d\,t}$$

we note that

$$L_\nu = K_\nu \qquad (3)$$

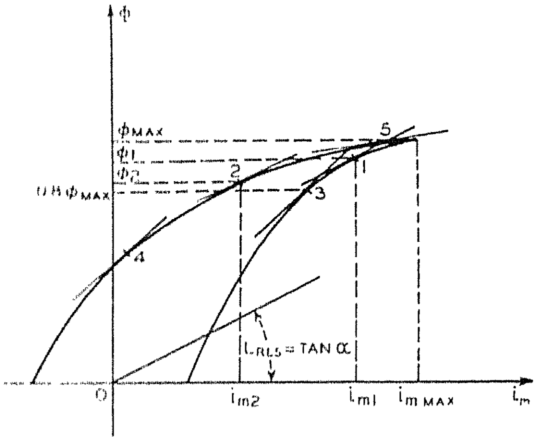

FIG. 1

K_ν is therefore defined as the inductance of this section, and is evidently a constant parameter of this section.

In the case of a double-energy linear circuit, the resonance condition is given by

$$\omega L_{res} = \frac{1}{\omega C}, \qquad L_{res} = \frac{1}{\omega^2 C} \qquad (4)$$

If C and ω are constants, there is only one definite value of L_{res} which satisfies this condition. Instead of changing the inductance mechanically or externally in order to obtain resonance as in the linear double-energy circuit, this variation of inductance in the non-linear circuit is performed by the action of the iron core itself. It is apparent that any one of these sectionally defined inductances L_ν might coincide with the resonance inductance L_{res} according to equation (4), and that this coincidence would then hold for only a small part of the period. Furthermore, the increase and decrease of i_m produces two different values of Φ for the same i_m, so that for a sinusoidal alternating voltage as used by the author in taking the oscillograms, the increase and decrease of voltage produces two different values of i_m at the same voltage. Hence we should expect to find two different resonance regions in the current curve and this is borne out by the oscillograms on page 726 of the author's paper. The fact that this resonance condition exists for a short period of time rather than for just an instant (corresponding to an exact mathematical solution) is due partly to the inaccuracy of the oscillogram recorder, and partly due to the form of the resonance curve itself as indicated in Fig. 3, where the change in current is plotted against the variation of inductance. Thus this so-called "partial resonance" is only another aspect of the well-known phenomenon of series resonance. At the same time the above analysis accounts for the author's successful computation of the resistance from the oscillogram data.

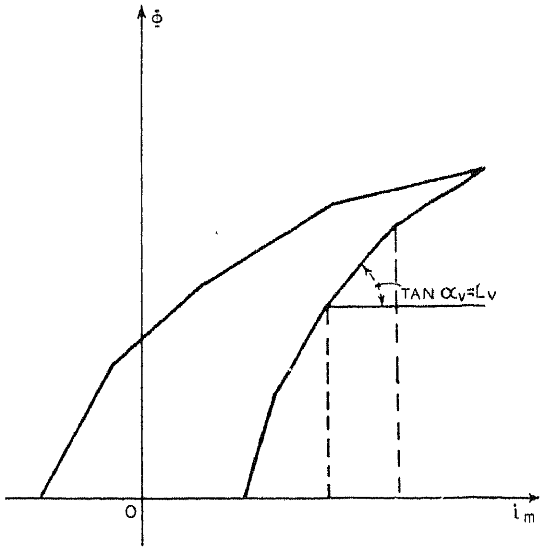

FIG. 2

If the author states that no exact means are available to compute the so-called critical voltage at which the current rise is very rapid, suggesting the presence of a third harmonic, he is right in that an exact theory of ferromagnetic materials has never as yet been advanced. But with good engineering accuracy we can compute this critical voltage, since we know that for series resonance the equation

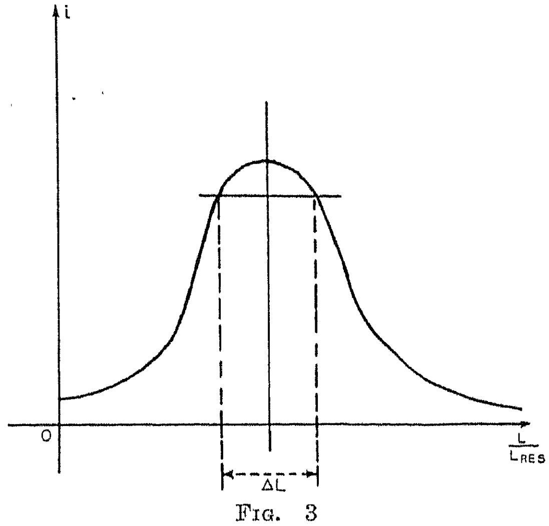

FIG. 3

$$L_\nu = \frac{1}{\omega^2 C} \qquad (4a)$$

must be satisfied. Considering L_ν as the slope of a straight line in Fig. 1 the parallel tangents on the hysteresis loop give the two positive values of Φ and i_m at which resonance occurs, (points 1 and 2 in Fig. 1). Knowing Φ it is not difficult to calculate the "critical voltage."

The author finds that the critical voltage is in the region of the "saturation" portion of the magnetization curve. This is natural, since it is in this region that the greatest change of L occurs and therefore the most probable region for resonance under any condition. Furthermore, the author finds a dependence on the frequency, and this is also evident from equation (**4a**), its influence being partially compensated for by the non-linear characteristic of the iron core.

It may be shown now, that the oscillograms on page 726 of the author's paper verify the analysis made above. In oscillogram number 1 the current is so small that no resonance occurs, since the hysteresis loop does not have a "critical" tangent. In oscillogram number 4 the current peak occurs at about 80 per cent of the maximum value of the voltage on the ascending part of the wave. This corresponds to about point 3 in Fig. 1 (of this discussion), and the parallel tangent on the descending portion of Φ indicates 4 as the corresponding point. According to the hysteresis loop, the current at 4 is about zero and hence no second maximum can occur. In oscillogram number 7 the peak of the current occurs shortly after the voltage has passed its maximum value, giving the point 5 in Fig. 1. It is clear that this is a single point since the parallel tangent on the ascending portion of the hysteresis loop is "imaginary" and not real.

Quite the same explanations hold in the case of parallel resonance; I cannot find any new difficulty.

Before concluding I would like to ask the following questions:

1. In the measurement of the effective impedance and effective current, was proper account taken of the fact that in circuits with iron-core inductances the transients are of long duration, sometimes several minutes?

2. What is meant by the following statement on page 727 ". . . but there is a critical value of resistance beyond which a hysteresis effect appears." It would seem that the hysteresis effect appears at any value of the resistance, the amount of the hysteresis effect being dependent upon the maximum value of the current i_m, which in turn depends upon the resistance.

3. What is the exact significance of a phase angle between voltage and current of two different non-sinusoidal wave shapes, when the accepted definition of the phase angle in such a case has no physical meaning?

4. It must be assumed from various remarks in the paper (see statements on page 732 giving reactor No. 1 as the primary and reactor No. 2 as the secondary winding) that the two non-linear circuits given in Fig. 12 involve L_1 and L_2 as transformer windings on the same core and not as independent reactors. Furthermore, the asymmetric currents in these combined circuits, as shown in oscillograms 14 and 15, indicate very clearly transients, characteristic of a saturated transformer. The decrease of these transients seems to be compensated for by the increase of the applied voltage. Assuming then that L_1 and L_2 are involved as transformer windings why was the mutual inductance neglected as a possible explanation of the behavior of this series—parallel circuit?

5. In Fig. 18 are the ideal curves of an ideal relay readily obtained or is there some dependence on the saturation curves of the different transformers?

In conclusion, may I suggest as a reference for the graphical treatment of non-linear double-energy circuits the work by R. Rüdenberg entitled "Elektrische Schaltvorgänge" (*Electrical Switching Phenomena*), published by J. Springer, Berlin.

B. L. Robertson: The study of very elementary systems in mechanics or electrical engineering usually involves the solution of the linear type differential equation, either by differential equation processes or by means of the operational calculus. A great number of such problems has been worked out but the cases treated have been mostly those in which the coefficients have been constant to start with, or are assumed constant in order to simplify the labor in obtaining the answer. Such an assumption, if its limitations are properly known, may be entirely satisfactory since the answer is understood as an approximation and can be altered later to suit actual conditions.

The vast majority of transient systems, however, contain variable parameters and hence must be represented by the general form of differential equation given by Mr. Suits in his second paragraph of the paper, the coefficients of the derivatives being functions of the dependent variable. The first of such transient systems which comes to mind is that discussed in the paper, the series electrical circuit with considerable iron as part of the magnetic path.

Inductance is usually defined and treated as bearing a linear relation to the current flowing through the circuit, and most textbooks make no mention of the fact that circuit constants are not always constant but really are variable. One notable exception is the discussion in the chapter on Networks With Variable Parameters given in Dr. V. Bush's book Operational Circuit Analysis. The treatment of variable circuit quantities would appear of next interest in the field of transient studies and the paper by Mr. Suits is therefore quite welcome.

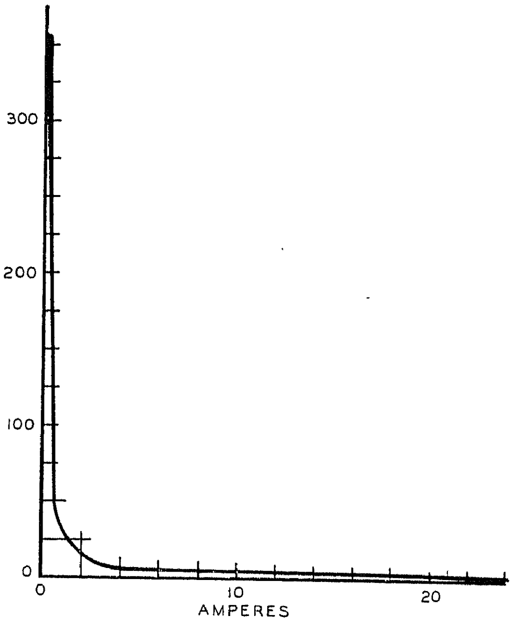

Fig. 4—Inductance Curve from the Magnetization Curve of Fig. 20

To show the variation in inductance for the transformer used in the analysis which the paper describes, an inductance curve was plotted from the magnetization curve given by Fig. 20 of the paper. The inductance is expressed as

$$L\,(i) = K\,N\,\frac{d\,\phi}{d\,i}$$

$$= 480 \times 10^{-8}\,\frac{d\,\phi}{d\,i} \text{ henrys}$$

This equation assumes that all of the flux links all of the turns, which is of course not strictly true but is a pretty good approximation for a transformer. $\frac{d\,\phi}{d\,i}$ is the slope of the saturation curve. The variation in inductance is shown by Fig. 4 of this discussion, the range being from 360 to 1.8 henrys.

The inductance can be expressed to a very good degree of accuracy in terms of the current and this relation substituted in the original differential equation for the circuit. That is, this particular problem, being one of the simplest of those containing variable circuit quantities, is capable of solution by methods not too involved.

The impedance-voltage curve, Fig. 1, for the transformer, is of considerable interest. It was taken experimentally and hence actually shows what the transformer will do under different impressed voltages, but the first (rising) portion of the curve does not appear to check with the magnetization curve. The inductance, if following that given by Fig. 4 of the discussion, would show the greatest impedance at zero voltage, the impedance curve from then on being of the same shape as that illustrated in Fig. 20.

The answer probably lies in the fact that the magnetization curves for iron do not rise as abruptly as shown in Fig. 20 but have a kink in them at the start. Such a kink would immediately explain the curve presented in the paper because the inductance would begin at a lower value than 360 henrys, rise to a value considerably higher than that, and then fall away as before. The impedance curve would follow it although not proportional to it. I would like to ask Mr. Suits if this is not the case with the iron he has been using.

It might be pointed out that although the characteristics for permeability, inductance, and impedance for such a magnetic circuit as a transformer possess the same general shape, these quantities are distinct physically, and analytically do not bear a linear relation to each other.

C. G. Suits: I wish to make the following comments on the interesting discussion by Mr. B. L. Robertson. Linear circuits may be regarded as understood, in the sense that given sufficient time and patience one may predict the transient and steady-state performance of such systems to a good approximation. The comparative simplicity of the analysis of such cases has doubtless been an important incentive to the attainment of the present high state of perfection of the science. Non-linear circuits are on the contrary in a much less happy state; except for simple cases they are not understood in the sense alluded to above. A case of particular importance is that of the ferromagnetic inductance, and for this element it is even impossible, except by graphical means, at present to predict the current, knowing the applied voltage and the characteristics of the magnetic material. I therefore heartily agree with Mr. Robertson when he states that "The treatment of variable circuit quantities would appear of next interest in the field . . ."

The case of the inductance of an iron-core reactor which Mr. Robertson has discussed is particularly interesting. Consider the hysteresis loops of Fig. 5, (this discussion) which are reproduced from a paper by Lehde, *Rev. Sci. Inst.*, Jan. 1931. The slope of the hysteresis curve is proportional to the incremental (or instantaneous) value of the inductance. The alternating *B-H* characteristic (similar to Fig. 20 of the paper) may be obtained by drawing a curve through the tips of the hysteresis loops. It is true that the initial portion of the *alternating B-H* characteristic has the "kink" noted by Mr. Robertson. However, this initial inflection is never traversed in a-c. circuits except during the first cycle for a completely demagnetized specimen. The rising portion of the impedance-voltage curve of Fig. 1 (of the paper) can therefore not be ascribed to this cause. The reason for this effect will be apparent from a consideration of the *shape* of the hysteresis loops. Thus, at sufficiently small magnetizing forces the loop (as loop I) comprises steep portions (of high incremental inductance) and less steep portions (of relatively low incremental inductance) of approximately like extent. The effective inductance *averaged* over a cycle will therefore assume a mean value lying between these limits. For greater magnetizing forces the *shape* of the loop changes (as loop II) in such a manner that the effective inductance which is similarly averaged over a cycle now comprises a larger proportion of incremental values of high inductance.

Several conclusions may be drawn from the situation thus presented. The effective impedance of an iron-core reactor operating in the hysteresis region is determined not only by the maximum value of the incremental inductance, which is given closely by the alternating *B-H* curve, but by the *shape* of the hysteresis loops, which cannot be inferred from the alternating *B-H* characteristic. It is interesting to note that a type of saturation and hence low incremental inductance occurs at low as well as high magnetizing forces; the change in $\frac{d\,\Phi}{d\,i}$ takes place discontinuously in the hysteresis region and smoothly at large values of H.

Mr. Weber points out that when a hysteresis loop is traversed there are in general two values of incremental inductance which might be available for non-linear resonance. This is true for the hysteresis region but is not true for the saturation region where the flux is a single valued function of the magnetizing force.

It is further stated that "with good engineering accuracy we can compute this critical voltage . . .," and, "Knowing Φ it is not difficult to calculate the critical voltage."

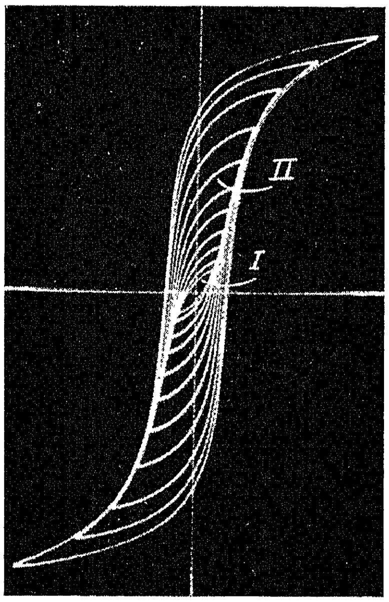

Fig. 5

The important result of the experiment reported in the paper is that quasi-resonance is identified with the region of rapid rise in current with voltage in the sense that the incremental value of the inductance becomes equal to the capacitative impedance for a limited portion of the cycle. But this knowledge is not sufficient to allow this critical voltage to be calculated. For to calculate the resonant value of the incremental inductance it is necessary to know the current $\left(\text{assuming } \frac{d\,\Phi}{d\,i} \text{ to be related to the current in a known analytic manner}\right)$ in the circuit. The current, however, depends upon the capacitance, the frequency, the resistance, the magnetic constants, and the applied voltage E in a manner which expressed analytically is a solution of

$$L\,(i)\,\frac{d\,i}{d\,t} + \frac{1}{c}\int i\,d\,t + R\,i = E\sin\left(\omega\,t + \psi\right) \qquad (1)$$

Solutions of (1) for the case of the closed iron-core inductance have never appeared. That is not surprising in view of the fact that the simpler case where $c = \infty$ is yet unsolved. I therefore cannot agree with Mr. Weber's statements quoted above.

In the conclusion of his discussion, Mr. Weber asks five questions which are herewith answered.

1. Yes.

2. The hysteresis effect referred to in the quotation is the hysteresis which appears in the volt-ampere curve of Fig. 4 (of the paper) and not the hysteresis loops of the *B-H* characteristic as assumed by Mr. Weber. In view of the confusion which exists it would doubtless be desirable to give a different name to the former effect. Possibly the statement: "but there is a critical value of resistance beyond which the effective current becomes a double valued function of the applied voltage," removes the ambiguity.

3. There are at least two ways in which the conventional concept of a phase angle as used for sinusoidal wave forms may be used for non-sinusoidal cases. One may without ambiguity refer to the "phase of the zero of voltage," or the "phase of the peak of voltage," see page 726 of the paper. It is equally useful to refer to the phase of the fundamental components of non-sinusoidal wave forms always described in the paper.

4. The reactors of Fig. 12 of the paper have separate magnetic circuits as shown in the diagram. I find no basis either in Fig. 12 or in the appendix for the assumption that L_1 and L_2 have a common magnetic path.

5. The considerations which govern the design of the transformer (by no means a necessary part of the circuit) in the resonant relay of Fig. 17 (of the paper) are essentially no different from those found in any power circuit. Thus for the case in point the magnetizing current of the transformer *T* should be a predetermined small percentage of the maximum (ohmic) load current.

Magnetic-Circuit Units as Adopted by the I. E. C.

BY ARTHUR E. KENNELLY*
Fellow and Past President, A. I. E. E.

Synopsis.—*The decisions of the International Electrotechnical Commission (I. E. C.) at Oslo in July 1930, with respect to international magnetic units and unit names, are briefly reviewed.*

So far as concerns the c. g. s. magnetic units, the decisions appear likely to meet with general favor; but the series is incomplete and might advantageously be supplemented by at least one name—that for the c. g. s. unit of reluctance.

The series of units in the practical or volt-ampere-ohm system is also incomplete; but apparently cannot be completed satisfactorily without first deciding two questions: (a) whether the system should be rationalised and (b) whether the m. k. s. or the c. g. s. s. system should be endorsed. It is recommended that these questions be left for the present undecided, until they have been widely discussed, and illustrative literature developed in aid of international decision.

C. G. S. MAGNETIC UNITS

At its Plenary Meeting in Oslo, last July, the International Electrotechnical Commission (I. E. C.) adopted resolutions concerning c. g. s. magnetic units, which should enable various ambiguities and inconsistencies to be eliminated in future from international electrotechnical literature of the magnetic circuit.

Firstly, it reaffirmed the *maxwell* as the name of the c. g. s. magnetic unit of flux ϕ, as adopted at Paris in 1900.

Secondly, it unanimously supported the hypothesis that, for electrotechnical purposes, permeability μ_0 of free space is a quantity having certain physical dimensions, although numerically equal to unity in the c. g. s. magnetic system. Hence, even in a vacuum, or free space, magnetic flux density $\mathcal{B}$ is a different physical quantity from magnetising force $\mathcal{H}$; so that there is both need and justification for their two quantity names (magnetising force and flux density), their two c. g. s. unit names (*oersted* and *gauss*) and their two symbols ($\mathcal{H}$ and $\mathcal{B}$). It was decided to apply the name *oersted* to the c. g. s. magnetic unit of $\mathcal{H}$ and the name *gauss* to the c. g. s. magnetic unit of $\mathcal{B}$. The absolute permeability μ of a magnetisable substance was decided to have the same unknown dimensions as space permeability μ_0; but the relative permeability of the substance μ/μ_0 was held to be nondimensional, or a mere numeric.

Thirdly, it adopted the name *gilbert* for the c. g. s. unit of magnetomotive force $\mathcal{F}$.

It may not be amiss to consider the changes that these I. E. C. resolutions probably import into electrotechnical usage.

By far the most important result of the new I. E. C. resolutions is that, if followed, the indefiniteness and ambiguity which have often characterised the literature of the magnetic circuit, on questions of $\mathcal{H}$ and $\mathcal{B}$, can be eliminated. The term *gauss* for instance, can only be used for flux density $\mathcal{B}$.

I. E. C. RESOLUTIONS IN ENGINEERING WORK WITH C. G. S. MAGNETIC UNITS

In order to conform with the new I. E. C. rulings concerning c. g. s. magnetic units, it seems to be indicated that:

1. When $\mathcal{H}$-$\mathcal{B}$ curves, or "saturation curves," are drawn in the c. g. s. system, for a sample of magnetic material, such as dynamo steel, the abscissas of $\mathcal{H}$ should be in *oersteds* and the ordinates of $\mathcal{B}$ in *gausses*.
2. When $\mathcal{H}$-μ curves, or "force-permeability" curves are drawn in the c. g. s. system, for a sample of magnetic material, the abscissas should be in *oersteds* and the ordinates in units of *absolute permeability* μ. These units of μ have no name at present, and their physical dimensions in terms of length, mass and time, have not been assigned. If however, the same curves are to be interpreted as *relative permeability* curves, the ordinates become expressible in μ/μ_0, and are mere numbers.
3. When $\mathcal{F}$-ϕ curves, or "excitation curves," are drawn in the c. g. s. system, for any given magnetic circuit, the abscissas of $\mathcal{F}$ should be in *gilberts* and the ordinates of ϕ should be in *maxwells*.

I. E. C. RESOLUTIONS AND TEXT-BOOK LITERATURE

The equation for magnetisation of materials in the current literature of various countries is well known to be:

$$\mathcal{B} = \mathcal{H} + 4\pi\mathcal{I} \qquad \text{gausses} \quad (1)$$

where $\mathcal{H}$ is the uniform magnetising force acting upon uniform isotropic material, $\mathcal{I}$ is the uniform intensity of magnetisation thereby established in the material along the direction of $\mathcal{H}$, and $\mathcal{B}$ is the uniform flux density, also established in that direction. From a logical standpoint, and in conformity with the I. E. C. ruling, the above equation should be interpreted as:

$$\mathcal{B} = \mu_0\mathcal{H} + 4\pi\mathcal{I} \qquad \text{gausses} \quad (2)$$

where μ_0 is the permeability of free space (*gausses per oersted*) and is numerically equal to unity in the C. G. S. magnetic system.

If this interpretation is not read into equation (1), we have for the case of non-magnetic material, such as a vacuum,

$$\mathcal{I} = 0 \qquad (3)$$

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

and

$$\mathcal{B} = \mathcal{H} \qquad (4)$$

or flux density $\mathcal{B}$ is physically, as well as numerically, equal to the magnetising force $\mathcal{H}$. If that were the case, one of the two quantities magnetising force and flux density would be superfluous, and either $\mathcal{B}$ or $\mathcal{H}$ could serve for both. The dislocation in electrotechnical literature involved by such a hypothesis would apparently be much greater than that entailed by these new I. E. C. resolutions.

If, however, the world's physicists should determine and declare at some future time, that the permeability μ_0 of free space is a mere numeric without dimensions; so that $\mathcal{B}$ and $\mathcal{H}$ are physically identical, it would be incumbent upon electrotechnicians to reconsider these I. E. C. resolutions.

THE I. E. C. RESOLUTIONS AND MAGNETIC OBSERVATORY MEASUREMENTS

In the determination and recording of the earth's magnetic field at the various magnetic observatories of the world, measurements are ordinarily made of the "field intensity," in its vertical and horizontal components. Since these measurements are made in air, which is a very feebly magnetic gaseous medium, it may be claimed that the quantity measured is a magnetic flux density $\mathcal{B}$ and not a magnetising force $\mathcal{H}$; although the numerical difference between the two is for ordinary purposes insignificant.

These component flux-density measurements are commonly expressed in *gammas* or γ; where $1\ \gamma = 10^{-5}$ *gauss*. The Greek letter gamma seems to have been chosen[1] on account of its resemblance to G, the initial letter of the name Gauss. The I. E. C. endorsement of the name *gauss* as the c. g. s. flux-density unit, thus conforms with this magnetic observatory usage.

STEPS NECESSARY TO COMPLETE THE SERIES OF NAMES INTERNATIONALLY ASSIGNED TO THE C. G. S. MAGNETIC UNITS

Although the I. E. C. Resolutions of 1930, in pursuance of the Paris Congress Resolutions of 1900, provide names for the most important units of the C. G. S. magnetic system; yet there remain certain gaps in the terminology of the series, as the accompanying Table shows.

The question arises as to whether new internationally selected names are desirable to fill in the blanks of the Table. Opinions differ, not merely in different countries; but also in one and the same country. They vary over the whole range from the one extreme that all should be named, to the other extreme that none should be named; but that in each case the phrase c. g. s. unit of . . . should be used. It should, however, be pointed out that in the latter case, since there are two c. g. s. systems, the magnetic (with $\mu_0 = 1$ and $\kappa_0 = 1/v^2$) and the electric (with $\kappa_0 = 1$ and $\mu_0 = 1/v^2$), it is desirable to remove ambiguity, by using the somewhat lengthy phrase—c. g. s. magnetic unit of . . . .

TABLE I
C. G. S. MAGNETIC UNITS AND THEIR NAMES

	Quantity	Symbol	Name	Date of International Adoption
1	Magnetomotive force	$\mathcal{F}$	gilbert	1930
2	Magnetising force	$\mathcal{H}$	oersted	1930
3	Magnetic flux	ϕ	maxwell	1900 and 1930
4	Magnetic flux density	$\mathcal{B}$	gauss	1900 and 1930
5	Magnetic permeance	$\mathcal{P}$		
6	Permeability (absolute)	μ		
7	Magnetic reluctance	$\mathcal{R}$		
8	Magnetic reluctivity	ν		
9	Magnetic pole strength	m		
10	Intensity of magnetisation	$\mathfrak{I}$		

In the writer's opinion, at least one of the blanks in Table I should be filled in, by the international adoption of the name for the c. g. s. unit of magnetic reluctance $\mathcal{R}$. It is often desirable to develop the complete expression for magnetic Ohm's law $\phi = \mathcal{F}/\mathcal{R}$, with names for all three units. With the name of the reluctance unit adopted, it would not be necessary to adopt a special name for the unit of reluctivity ν. Just as the c. g. s. unit of resistance R under the provisional name *abohm*, gives rise to the name for the c. g. s. unit of resistivity ρ under the derivation *abohm-cm.*, so any name for the c. g. s. magnetic unit of reluctivity can be derived by adding (– cm.) to the adopted c. g. s. magnetic unit of reluctance.

Since reluctance is magnetomotive force per unit of flux, a temporary name for the unit of reluctance, until international action may have been taken, is the *gilbert per maxwell.* In the same way, those who for any reason may prefer not to use the I. E. C. name of *oersted* for $\mathcal{H}$ may correctly employ the longer term—*gilbert per cm.*, and likewise those who prefer not to use the I. E. C. name *gauss* for $\mathcal{B}$ may correctly employ the longer term—*maxwell per sq. cm.*

THE PRACTICAL SYSTEM OF MAGNETIC UNITS

By its resolution to adopt the "practical" unit of magnetic flux, (10^8 *maxwells* or a *maxwell-eight*), under the title *pramaxwell*, together with the availability of the prefix *pra* for converting c. g. s. magnetic unit names into corresponding practical unit names, the I. E. C. has sanctioned the use of practical magnetic units, in response to a proposition from the Italian National Committee, at Bellagio, in 1927.

On this basis, a conducting loop has one volt induced in it, when it is threading magnetic flux uniformly at the rate of one *pramaxwell per second.*

It is important to consider the possibilities presented by this international recognition of a practical magnetic unit. As early as 1891, a committee of the American

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[1] Bibliography 24.

Institute of Electrical Engineers recommended for international consideration a series of magnetic units in the practical system; but the recorded opposition of the Chicago Congress, in 1893, to the suggestion, caused the plan to be set aside in favor of international magnetic units in the c. g. s. magnetic system. After nearly forty years, the I. E. C. action of 1930 has restored a practical magnetic unit to good standing, if it can justify itself in the opinion of electrotechnicians throughout the world. The immediate advantage of the *pramaxwell* is, of course, that in the fairly numerous working formulas of electrical engineering connecting induced voltages with magnetic flux, the numerical coefficient 10^8 can ordinarily be eliminated and dispensed with by expressing the flux in *pramaxwells* instead of in *maxwells*. Some consider that this is a sufficient justification for the adoption of the *pramaxwell*, without extending the practical system any further. On the other hand, a single isolated magnetic unit in the practical system has less chance of proving its utility, than if it forms a member of a practical series.

What advantages might be secured from the adoption of a complete series of magnetic units in the practical system?

As an answer to this question, it has been pointed out by various writers, that if the magnetic units in the practical system were suitably adopted, the practical system would be converted into an absolute system independent of the parent c. g. s. system. Instead of defining the *volt* as 10^8 c. g. s. magnetic units of e. m. f., the *ohm* as 10^9 c. g. s. m. u. of resistance, and so on, these various practical units would be the respective fundamental units of e. m. f., resistance, etc., in the comprehensive electrodynamical system to which they all belonged. At present, this desirable condition does not hold; because the *volt*, *ohm* and *ampere*, etc., may be regarded as belonging to any one of a series of systems, such as the quadrant-eleventh-gram-second system of Maxwell, the meter-kilogram-second system of Giorgi, or the cm-gram-seven-second system[2] of Dellinger, Bennett, Karapetoff, and Mie. In each of these cases, the *watt* is the mechanical power required to accelerate the unit of mass with the unit of acceleration: *i. e.*, the eleventh-gram accelerated one quadrant per second^2, or the kilogram one meter per second^2, or the gram-seven(ten metric tons)one centimeter per second^2. Consequently if the *pragauss*, as the practical unit of flux density were internationally adopted as one *pramaxwell per square quadrant*, (10^{-10} gauss) the Maxwell system would be endorsed, if per square meter (10^4 gauss), the Giorgi system would be endorsed, and if per square centimeter (10^8 gauss), the Dellinger-Bennett system would be endorsed. It is evident that while the adoption of the *pramaxwell* is a relatively simple matter, and does not involve entanglement with any particular dynamical system, the adoption of the *pragauss* as the *pramaxwell* per unit area, would logically involve association with a particular dynamical system.

The adoption of the *pragilbert* as the practical magnetic unit of magnetomotive force involves another question; namely, whether the practical system should be "rationalised" or not?

RATIONALISED UNITS

Up to the present point of development, the practical volt-ampere-ohm system of units is neutral, being neither rationalised nor unrationalised; but the adoption of either the practical unit of m. m. f. or of the practical unit of flux density, would logically determine which of the two conditions should prevail. It was Heaviside who first drew attention[3] to the fact that the units in the c. g. s. system were so defined and adopted that a large number of formulas dealing with electrogeometrical problems of the rectilinear type, such as the capacitance of a slab condenser, prominently included the spherical factor 4 π, whereas corresponding problems of a spherical type, such as the capacitance of a spherical condenser, wherein such a factor might be expected to occur, did not include it. He proposed to rectify this inconsistency by revising and rationalising the fundamental definitions of the units and their dependent formulas; but the method which he advocated for doing this would have involved changing the magnitudes of the units. It would have been necessary to recalibrate all or nearly all working standard ohms, voltmeters, ammeters, etc.—a manifestly forbidding proposition. It has since been pointed out, however, by various writers,[4] that the advantages of rationalisation can be secured without changing any existing practical units or standards, by taking the factor 4 π out of the unit definition of m. m. f. and inserting it into the unit definitions of permeance and reluctance, thereby keeping the unit of flux unchanged. This would, however, involve other corresponding changes in fundamental formulas.

DOUBTFUL ADVANTAGE OF ATTEMPTING TO RATIONALISE THE C. G. S. SYSTEM NOW

At the present time, it seems very doubtful whether attempts could usefully be made to rationalise the c. g. s. system by taking the 4 π out of m. m. f. and putting it into permeance. The Paris Congress of 1900 opposed the plan, and the I. E. C. Conference of 1930 endorsed the *gilbert* as the simple unrationalised c. g. s. unit of m. m. f. at the value of the 4 πth of an abampere-turn (0.07958), or the $10/(4\pi)$ of an ampere-turn (0.7958). These are good reasons for leaving the c. g. s. system in its classical unrationalised state.

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[2] Bibliography 8, 10, 13, 14, 15, 17, 18.

[3] Bibliography 3.

[4] See discussion by Bennett of bibliography 25, TRANS. A. I. E. E., April 1930, Vol. 49, p. 497.

REASONS FOR RATIONALISING THE PRACTICAL MAGNETIC UNITS

Although the c. g. s. magnetic units remain unrationalised, there is no reason why the practical magnetic units should not be rationalised if it were agreed to be advisable to do so internationally.

Table II presents a few magnetic-circuit formulas, under both unrationalised and rationalised forms, in parallel columns, for comparison. It will be seen that in case No. 1, which is the fundamental formula of the c. g. s. system, representing the repulsive force between two equal positive poles, the unrationalised formula is simpler than the rationalised formula. In all the other cases, however, the rationalised formula is the simpler and more logical.

TABLE II
MAGNETIC—CIRCUIT UNIT SYSTEMS

	Unrationalised	Rationalised	
1. The force between two like poles, each of strength m at distance r, in a medium of permeability μ, is:	$F = \frac{m^2}{\mu r^2}$	$F = \frac{m^2}{4 \pi \mu r^2}$	(5)
2. The magnetomotive force of a coil of N turns, carrying a current I, is:	$F = 4 \pi N I$	$F = N I$	(6)
3. A magnetic pole of strength m would emit a magnetic flux ϕ of:	$\phi = 4 \pi m$	$\phi = m$	(7)
4. The mechanical work done by an active coil of steady m. m. f. $\mathfrak{F}$ during a movement which changes by ϕ units the total extraneous flux linked with it, is:	$W = \frac{\mathfrak{F} \phi}{4 \pi}$	$W = \mathfrak{F} \phi$	(8)
5. The work done in carrying a magnetic pole of strength m once around a current of strength I, is:	$W = 4 \pi m I$	$W = m I$	(9)
6. The tension or tractive force between opposed parallel pole faces per unit of active surface area, carrying uniform flux density $\mathcal{B}$, if μ_0 is the permeability of free space, is:	$f = \frac{\mathcal{B}^2}{8 \pi \mu_0}$	$f = \frac{\mathcal{B}^2}{2 \mu_0}$	(10)
7. The magnetic energy per unit volume of a medium of permeability μ, carrying uniform flux density $\mathcal{B}$, is:	$w = \frac{\mathcal{B}^2}{8 \pi \mu}$	$w = \frac{\mathcal{B}^2}{2 \mu}$	(11)
8. The hysteretic work per unit of volume and per cycle, of a magnetisable material subjected to cyclic magnetisation, is:	The area of the Ewing loop in $\mathcal{H}$ $\mathcal{B}$ units, divided by 4π. $w = \frac{1}{4 \pi} \int \mathcal{H} . d\mathcal{B}$	The area of the Ewing loop in $\mathcal{H}$ $\mathcal{B}$ units. $w = \int \mathcal{H} . d\mathcal{B}$	(12)

A great advantage of the rational series is that its unit of m. m. f. is the current-turn itself and not the 4 πth part, or 0.0795775th of a current-turn in any system. In the practical volt-ampere-ohm system, the rationalised unit of m. m. f. becomes the ampere-turn, which is self explanatory and for which no special name is needed. Otherwise a *pragilbert* based on the gilbert, would be the 4 πth part of one ampere-turn and would be unrationalised. If therefore the ampere-turn came to be adopted as the practical unit of m. m. f., the practical magnetic system would logically become rationalised. In view of what has been shown by Heaviside, Lorentz, and other authors, as to the advantages of rationalised units, it is the opinion of the present writer that they should meet with general favor, and ultimately become internationally adopted. It appears that this would also involve the rationalisation of the electrostatic units, as an automatic and equally advantageous consequence. Thus, the total electrostatic flux in a condenser would become numerically equal to its charge in *coulombs*.

It may be noted that in the books and papers that have already been published in the m. k. s. and c. c. g. s. systems, their units are rationalised.

NEED FOR DISSEMINATION OF LITERATURE ON SERVICEABILITY OF PRACTICAL MAGNETIC UNITS; BUT NO NEED FOR ACTION BEING TAKEN ON THEM IN NEAR FUTURE

Table III presents a series of rationalised magnetic units in the m. k. s. and c. g. s. s. practical systems, columns IV and VI give provisional names of the units; while columns V and VII give the number of the basic c. g. s. magnetic units in each practical unit. Thus there are 10^4 c. g. s. units of flux density or gausses in the *pramaxwell per square meter*, and 10^8 gausses in the *pramaxwell per square centimeter*.

Before attempting to secure international action leading to a choice between the m. k. s. and c. g. s. s. practical systems of magnetic units, it is desirable that their relative advantages should be generally discussed and more widely known. For this purpose, it would be useful to have papers or primers published indicating the formulas and treatment of practical problems in each. In the writer's opinion, their respective advantages are fairly well balanced, so long as the formulas and problems relate to simple magnetic and electrostatic circuits. In more general cases, however, the unit of mass of 10^7 grams in the c. g. s. s. system places it at a great disadvantage compared with the unit mass of 10^3 grams or one kilogram in the m. k. s. system.

For the present, it seems desirable to allow time for the practical units to develop along the lines of individual preference. It is evident from Tables I and III that magnetic-circuit computations can continue to be carried on without difficulty in the classical c. g. s. system, especially if an international unit of reluctance is added to the list. Writers desiring to use either of the two rival practical systems have sufficient units already available for ordinary use, although some of the names are lengthy, provisional, and open to criticism.

TABLE III
MAGNETIC UNITS IN THE RATIONALISED M. K. S. AND C. G. S. S. SYSTEMS

I	Unit II	III Symbol	$L = 1$ m M. K. S. (Giorgi) IV	$M = 10^3$ gm. V C. G. S. U.	$L = 1$ cm. C. G. S. S. (Dellinger-Bennett) VI	$M = 10^7$ gm. VII C. G. S. U.
1	Magnetomotive force	$\mathcal{F}$	ampere-turn	$4\pi/10$	ampere-turn	$4\pi/10$
2	Flux	ϕ	pramaxwell	10^8	pramaxwell	10^8
3	Magnetising force	$\mathcal{H}$	ampere-turn/m	$4\pi/10^3$	ampere-turn/cm	$4\pi/10$
4	Flux density	$\mathcal{B}$	pramaxwell/sq. m	10^4	pramaxwell/sq. cm	10^8
5	Space permeability	μ_0	henry/m	$10^7/4\pi$	henry/cm	$10^9/4\pi$
6	Permeance	$\mathcal{P}$	henry	$10^9/4\pi$	henry	$10^9/4\pi$
7	Space reluctivity	ν_0	yrneh-m	$4\pi/10^7$	yrneh-cm	$4\pi/10^9$
8	Reluctance	$\mathcal{R}$	yrneh	$4\pi/10^9$	yrneh	$4\pi/10^9$
9	Inductance	$\mathcal{L}$	henry	10^9	henry	10^9
10	Magnetisation	$\mathfrak{J}$	pramaxwell/sq. m	$10^4/4\pi$	pramaxwell/sq. cm	$10^8/4\pi$
11	Pole strength	m	pramaxwell	$10^8/4\pi$	pramaxwell	$10^8/4\pi$
12	Magnetic moment	$\mathfrak{M}$	pramaxwell-m	$10^{10}/4\pi$	pramaxwell-cm	$10^8/4\pi$

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Discussion

S. L. Gokhale: The paper under discussion divides itself naturally into two parts.

1. A summary of what the I. E. C. decided during the last meeting at Oslo, July 1930.

2. The author's suggestions and recommendations as to what should be done next.

With reference to the first item, it is gratifying to note, that in supporting the hypothesis, that the permeability of space is a dimensional quantity, the I. E. C. has qualified the recommendation with the limiting clause, that the hypothesis is accepted for "electrotechnical purposes." To the proposition thus stated, there can be no objection even by one who believes space permeability to be non-dimensional. It is, in my opinion, the best possible compromise under the circumstances.

For many years, the scientists dealing with electromagnetic phenomena have been divided into two groups, according to their conception of the nature of magnetic field. According to magnetostatic conception, which is generally accepted by physicists, permeability is a non-dimensional quantity.

According to the magnetodynamic conception which is almost unanimously accepted by the electrical engineers, permeability is a dimensional quantity. No crucial test to judge the validity of these two conceptions has yet been formulated, and no rational decision is possible for the present. A technical decision based on majority of opinion could have easily been reached, but such a decision would have been of no more value than that of the classical decision against the Copernican theory. The resolution of the I. E. C. has carefully avoided a similar error by accepting the magnetodynamic conception "for electrotechnical purposes," leaving physicists free to retain their faith in the opposite conception until a rational decision can be reached.

According to the author, the I. E. C. resolution carries with it certain implications, some of which have been explicitly stated in the paper. There is one implication, however, which seems to have been overlooked by the author and which might well be stated in explicit terms. According to the magnetodynamic conception, B_0, that is induction in non-ferro-magnetic space, is physically not identical with H though the two quantities might be made numerically equal by choosing such units for B and H as will secure the equality. According to the magnetostatic conception, B_0 and H are physically identical and cannot be made unequal except by an arbitrary fiat, such as by dislocation of the 4π from the equation $F = 4\pi N I/10$ to the equation $\mu_0 = 4\pi/10$. The I. E. C. has not made this change, which if it had made it, would have been contrary to the spirit of compromise which has guided its decision regarding the dimensional character of permeability.

The question of the change of position 4π, naturally leads to the question of reasons for and against the change. The reasons favoring the change are:

a. That the 4π in the equation $F = 4\pi N I/10 L$, is irrational.

b. That the transference of the 4π from the above equation to the equation for space permeability $\mu_0 = 4\pi/10$ leads to simplification of computation of magnetic circuits.

The reasons opposing the change are:

c. That the 4π in the equation $F = 4\pi N I/10 L$ is rational; on the contrary its presence in the equation $B_0 = 4\pi H/10$ would be irrational.

d. That the transference of the 4π from the equations for F and H to the equations for B_0 and μ_0, does not lead to any simplification in computation except in the few cases where the magnetic circuit is purely ferromagnetic, not partly or wholly non-magnetic. In practical work the magnetic circuits under computation are generally partly iron and partly air; the transference of the 4π cannot therefore lead to the desired simplification to an extent sufficient to justify the change.

e. The insertion of the 4π into the equation $B_0 = H$, so as to change it to the form $B_0 = 4\pi H/10$ is a direct repudiation of the alternative hypothesis that permeability is a non-dimensional quantity. Such repudiation is contrary to the spirit of compromise which has thus far guided the decision of the I. E. C., and is unjustifiable until the dimensional character of permeability is conclusively proved.

As to the arguments (a) and (c), it is easy to ascertain which of the two views is rational. The equation for m. m. f. as a function of ampere-turns expresses the relation known by its appropriate name, "the law of circuitation of magnetic force." Circuitation is a process of going round a circuit, and involves the factor π in some form. The presence of the π in the equation $F = 4\pi N I/10$ is therefore perfectly rational. It is the equation $B_0 = \mu_0 H = 4\pi\,.\,H/10$, that the π is really irrational for there is no circuitation involved in the relation of B_0 to H, even when B_0 and H are supposed to be quantities of dissimilar kinds.

As to simplification, as one of the expected results of the rationalization, it is easy to see that much of the simplification claimed by the advocates of the rationalized units is an illusion. For example, in Table II, item 6, the two equations are:

"Unrationalized" "Rationalized"

No. 6.......... $f = B^2/8\pi\mu_0$.... $f = B^2/2\mu_0$

Apparently the factor 4π has been dropped out of the rationalized equation; it is necessary to note, however, that the value of μ_0 is not the same in the two cases; on substituting the two values of μ_0 we have

"Unrationalized" "Rationalized"

No. 6a......... $f = B^2/8\pi_x 1$.... $f = B^2/2_x 4\pi$

so that when it comes to numerical computation, there is no advantage gained by the use of the rationalized formulas. One type of computation which confronts a practical designer is the number of ampere-turns necessary to produce a specified flux, in a specified region of the magnetic circuit which is partly iron and partly air, such as the field of an electric motor or generator. In such cases, if the usual "unrationalized" formula be used, the 4π enters into the computation of the ampere-turns for the iron part, but not for the air part; if on the contrary the rationalized formula be used, the same factor 4π enters into the computation for the air part. The only important case within my scope in which the 4π is effectively suppressed is that of a transformer without leakage. The advantage so gained in this one case, is certainly not worth the dislocation of the 4π and the inconvenience it involves, irrespective of the theoretical reasons for and against this change.

One part of the author's argument is not clear to me, and might be equally confusing to others besides myself. A little further explanation will be appreciated. Referring to the equation

$$B = H + 4\pi J \tag{1}$$

the author says

"from a logical standpoint, and in confirmity with the I. E. C. ruling, the above equation should be interpreted as

$$B = \mu_0 H + 4\pi J \tag{2}$$

"If this interpretation is not read into equation (1), we have for the case of non-magnetic material, such as air or vacuum,

$$B = H \tag{4}$$

or flux density B is physically as well as numerically equal to the magnetizing force H."

As I understand the situation, the interpretation suggested is perfectly legitimate provided its use is limited to "electrotechnical purpose." That is all that is implied in the I. E. C. ruling, and nothing more. The ruling does call on the advocates of magnetostatic conception to abandon that conception as if it were conclusively disproved, and to accept the doctrine that permeability of space is a dimensional quantity for all purposes as if it were proved.

Physicists who believe that B and H are physically identical—and of these I am one—prefer to use separate symbols and names of units, because, as they believe, B and H are two different aspects of the same physical entity, *viz.*, magnetic force. In magnetic material this force cannot be measured directly, that is, statically, by the magnitude of the mechanical force on a magnet pole. It has, therefore, to be measured indirectly for which there are two practical methods available namely, the kinetic and the kinetostatic. In the kinetic method a pole is supposed to have traveled along the line of the force, the amount of work done during this displacement being used as a measure of the force

$$\text{force} = d\,w/d\,l.$$

When thus measured, the force is called magnetizing force expressed symbolically by H, and measured in terms of the two quantities in terms of which it is measured, that is gilberts (work) per centimeter (movement). This method gives the value, not of the magnetic force at a point, but the mean value of the force for all points along a short length ($d\,l$) of the path.

Another indirect method of measuring the force is kinetostatic, that is in terms of the static force (electrostatic force) generated by kinetic action (translatory movement) of the magnetic lines of force. The e. m. f. generated by the translation of the lines is a measure of the rate of translation of the lines

$$E = d\,\phi/d\,t$$

or

$$B = \frac{d\,\phi}{d\,s} = \frac{E\,.\,d\,t}{d\,s}$$

Thus we have a second method of measurement and, therefore, also a second unit. This method gives also, not the value of the force at a point, but the mean value of the force for all points of a surface transverse to the line of the force. It is just as we measure grain by the bushel or by the pound. The mere fact that we find it convenient and in a sense necessary to use two methods of measurement does not prove that the entity measured and expressed in two ways is therefore not identical in its physical nature. Such is my understanding. If there be an error I would highly appreciate correction.

C. C. Whipple: In equation (1) $\beta = H + 4\pi J$ which has been used for sometime past, the ideas presented are that the total magnetic density in a body is partly supplied directly by the magnetizing force (H) and partly by the so-called intensity of magnetization, within the material. The difficulty with equation (1) is not with the ideas presented, but with the mathematical exactness of the expression. In equations such as this, it is usually accepted that the units on each side of the equation mark, must be the same. Here is where the difficulty has been with equation (1).

If the "hypothesis" suggested by the I. E. C. is generally agreed to and adopted, as it will no doubt be, then "Permeability μ_0 of free space is a quantity having certain physical dimensions, although numerically equal to unity in the c. g. s. magnetic system." There then follows the equation (2) $\beta = \mu_0 H + 4\pi J$ in which the difficulty referred to above does not exist, provided the dimensions of μ_0 are so defined that the product $\mu_0 H$ has the same dimensions as β.

Certain workers in magnetics have in the past written equation (1) as $B = H + \beta$. This simplification is very useful in discussing magnetic properties. In this equation β was referred to as the intrinsic or metallic flux density. This symbol has been used by Dr. Steinmetz and Evershed.

In this connection equation (2) will now be used and it would be advisable at this time for the A. I. E. E. to consider a symbol for the two right terms of the equation, in order that the literature of the future may be uniform in this respect. If the Greek β is retained for the left hand member, I should like to suggest the following symbols which if agreed upon should be adopted for future use:

$$\beta = \beta_0 + \beta_m \quad \text{or} \quad \beta = \beta_0 + \beta_i$$

In this case β_0 is associated with μ_0—free space—and β_m or β_i with the magnetic material present, as the intrinsic or metallic flux density.

To some this point may seem of little importance, but when one considers the confusion which has resulted from the lack of proper consideration to nomenclature in the past, it would seem at once advisable at this stage to adopt some uniform symbols for the terms referred to. If no uniform symbols are adopted by the A. I. E. E., the individual workers will adopt their own, with resultant confusion in the literature.

Personally I would favor the symbol β_m for the term $4\pi J$. If at some later date the units are rationalized the 4π will drop out and J will stand alone.

A. E. Kennelly: It is gratifying to observe that the I. E. C. decision to accept the working hypothesis of dimensions for permeability meets with Mr. Gokhale's approval, although he confesses the opinion that in reality "B and H are physically identical." It is through friendly acquiescences in a convention of this kind that we electrotechnicians can hope to secure international agreement. As he points out, the convention that μ has dimensions involves a change from equation (1) to equation (2) if we are to avoid confusion in electrotechnical books.

As regards the question of "rationalization' in practical magnetic units, that is a matter for discussion and international decision. In the author's opinion, the question is important and should be carefully considered before taking a decision which may commit electrotechnicians for many decades. Such papers and textbooks as have already appeared in the Giorgi and Dellinger-Bennett systems have, however, employed rationalized magnetic units, and there are strong arguments for them.

There is much to be said in favor of Mr. Whipple's plea for a special symbol of metallic flux density.

Mercury-Arc Rectifier Research

BY ALBERT W. HULL* and HERBERT D. BROWN†
Non-member Associate, A. I. E. E.

1. INTRODUCTION

EXPERIMENTAL research during the last three years has removed much of the mystery which shrouded the mercury-arc rectifier, and made it a piece of standard electric apparatus comparable to the motor and transformer. This paper gives a brief account of some of these investigations.

The method has been experimental, guided by modern theory. The effect of different factors upon the operation of the rectifier has been observed, and the results interpreted in the light of theory. Much remains to be learned, especially in the field of high current and high voltage; but the boundary line between knowledge and ignorance is fairly well marked.

These investigations have greatly expanded the range of current and voltage that can be handled by a rectifier and have indicated the path for still further progress. No theoretical limit to either current or voltage is in sight at present.

2. OPERATING CONDITIONS

Frequent references to normal structure and operating conditions will be necessary. Figs. 1 and 2 show, for purposes of illustration, typical General Electric rectifiers of early and more modern types respectively. The differences are mostly in the neighborhood of the anodes, and will be discussed in the following pages. One conspicuous change, however, may be dismissed with a very brief statement, *viz.*, the change from mercury-cooled iron anodes supported by porcelain bushings, to graphite anodes with micalex bushings. This change is an important mechanical simplification, but has no great operating significance, under present conditions, except the more certain maintenance of good vacuum. The micalex seal, a section of which is shown in Fig. 2C, is made from a mixture of mica and lead borate glass, fused and molded under high pressure, yielding a vacuum-tight seal of high mechanical and dielectric strength. This allows the clamped seal (Fig. 2C) to be metal-metal instead of metal-porcelain, thus avoiding differential lateral expansion. In the latest experimental models this metal-metal seal is welded.

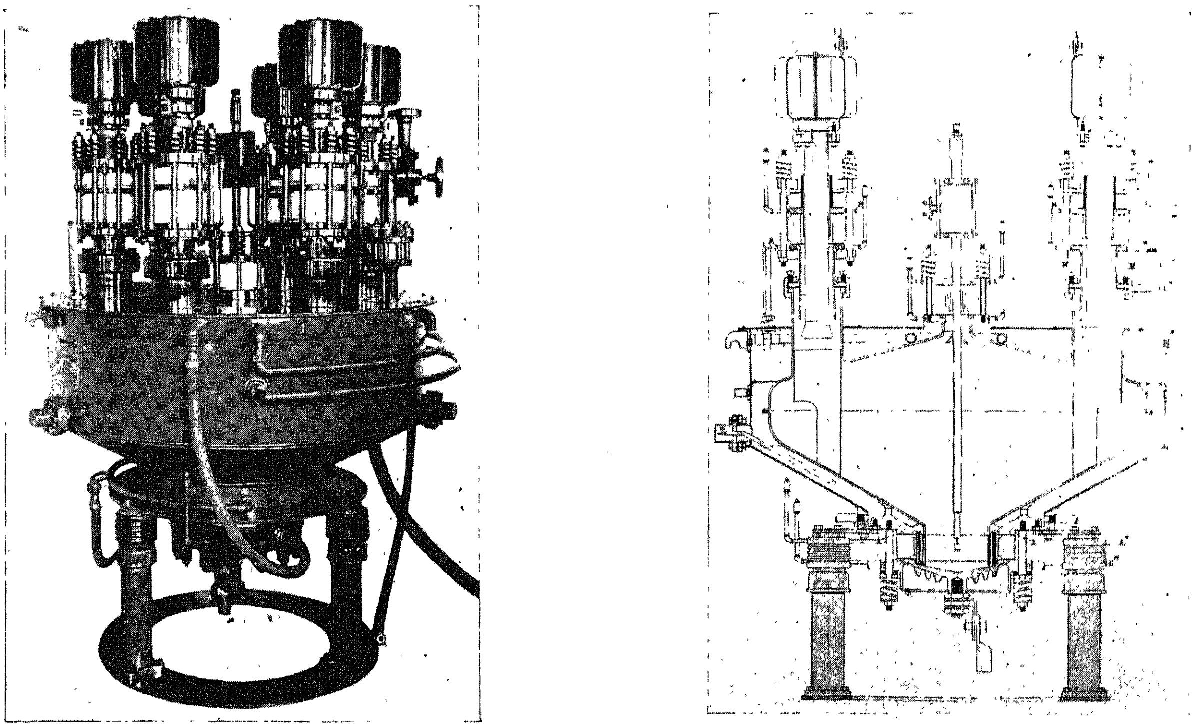

FIG. 1—TYPICAL IRON-ANODE RECTIFIER

(A) assembled, (B) section

The normal operating requirements of a rectifier are very simple:

1. It must *start easily*, *i. e.*, it must begin rectifying

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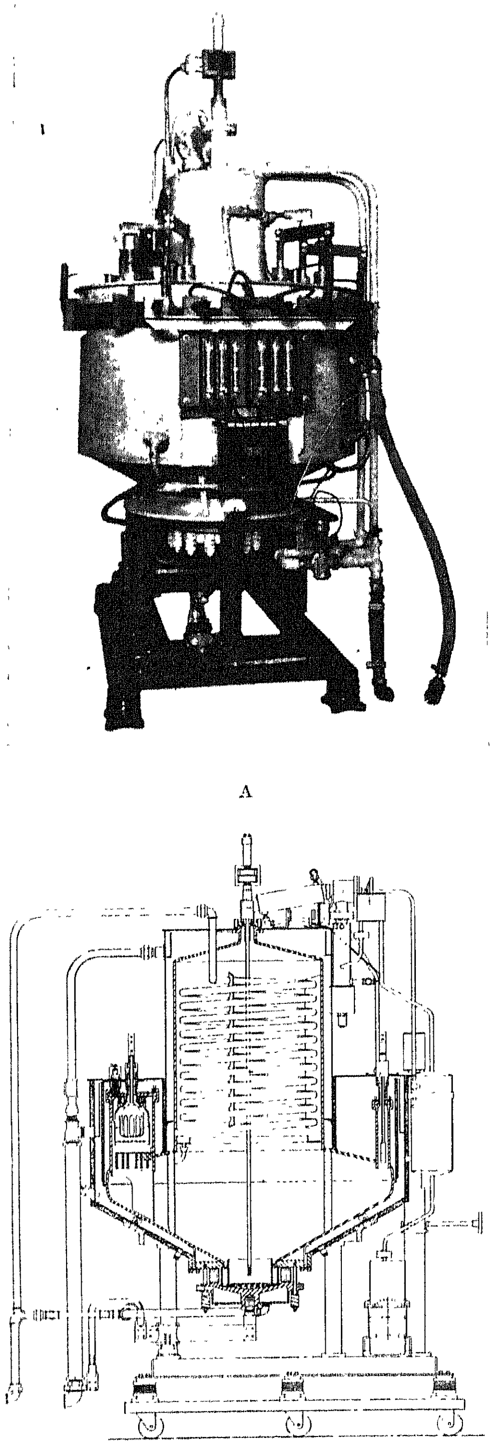
whenever the a-c. crest voltage exceeds the d-c. line voltage by a small margin, of the order of 20 volts.

2. It must *not arc back*, *i. e.*, allow current to pass in the reverse direction.

3. The *voltage drop* between anode and cathode, when current is flowing, must be low enough to meet the efficiency requirements. The practical limits of voltage drop, which are approximately 20 volts minimum and 40 volts maximum, are generally satisfactory for line voltages of 500 or more, but are too high for 250-volt operation. The hot-cathode rectifier promises to fill this gap (see section 10).

4. *Voltage regulation* will be required in the future. This is a circuit function rather than a rectifier function in the case of a simple rectifier, and can be accomplished by transformer compounding. But grid-controlled rectifiers of the type shown in Fig. 2, called thyratrons, can be made to give any desired regulation without compounding. This is discussed in section 9.

The arc-back requirement has received most attention up to the present, because the greatest electrical sin of today is interruption of service. When a rectifier arcs back it is a two-way conductor instead of one-way, and hence constitutes a short circuit of the a-c. supply line as well as the d-c. line or network which it feeds. The harm done by such short circuits, assuming they are cleared by circuit-breakers, is mainly interruption of service. Rectifiers are built to withstand arc-backs of less than one-second duration without injury, the only symptoms being a slight pitting of the anode tip which has performed the forbidden function of acting as cathode, and a slight evolution of gas resulting from this melting. Figs. 3A and 3B show photographs of iron and graphite anodes respectively that have been oper-



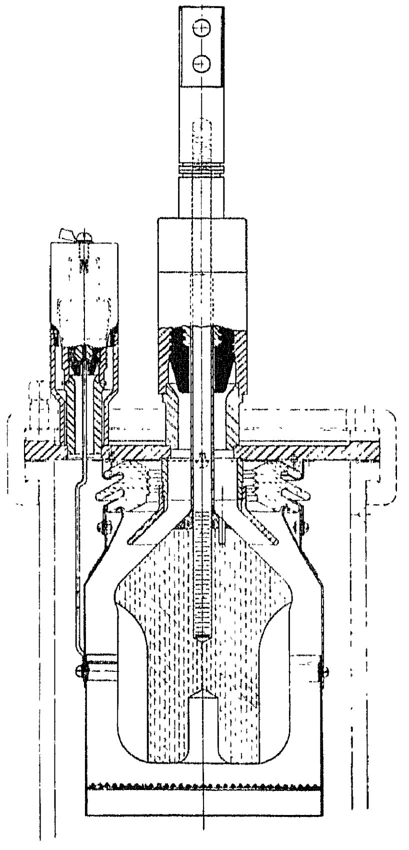


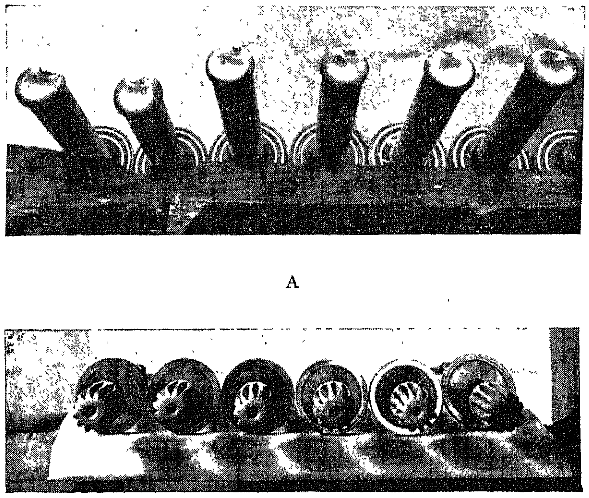
FIG. 2—TYPICAL RECTIFIER OF RECENT DESIGN, WITH GRAPHITE ANODES, SUBDIVIDING GRILLES, AND CONTROL GRIDS

(A) assembled, (B) section

(C) Section of anode housing in (B), showing micalex seal and graphite anode ated for long periods. The iron anodes are seen to be somewhat pitted, but the graphite anodes show scarcely any roughening.

The gas evolution accompanying arc-back, if excessive, delays resumption of service until the pumps can restore the necessary vacuum. Fast pumps and well-degassed anodes have reduced this time to the order of one minute, and experiments now in progress are expected to reduce it to one second or less.

This brief discussion of what arc-back is and does has been introduced here because of the important role it has played in the history of rectifiers. Any misimpression that arc-back is to be looked upon with tolerance as a normal accompaniment of mercury rectifier operation will, it is hoped, be corrected in the following pages. Arc-back may be expected in the future under the same conditions as failure of other electrical apparatus, *viz.*, when the rectifier is loaded beyond its capacity, or in case of mechanical failure of some of its parts, such as vacuum pumps or water-conditioning apparatus. An important conclusion from the above discussion is that when such overloads or failures do cause arc-back, the rectifier as a sealed-up unit is not injured, and only the offending mechanical part needs repair.



A

B

FIG. 3—TYPICAL ANODES AFTER LONG OPERATION, SHOWING PITTING DUE TO ARC-BACK

(A) Iron anodes; (B) carbon anodes

3. CONTROL OF MERCURY VAPOR PRESSURE

The most important single factor in mercury-arc rectifier operation is the control of the mercury vapor pressure. This pressure depends on the temperature of the cooling water. The problem for investigation divides itself into two parts: (a) What is the best cooling-water temperature, assumed uniform, and the permissible temperature limits? (b) If the water temperature is not uniform, what parts should be controlled?

(*a*) *Optimum Temperature and Temperature Range.* The optimum temperature may be determined empirically, by gradually increasing the load current, at a given water temperature, until arc-back occurs, then repeating at a higher temperature. Eventually a limiting temperature is reached, above which the rectifier will not operate at any load. Uniform temperature of the cooling water is maintained by rapid circulation. In this way a load-temperature curve is obtained. Fig. 4 gives the load-temperature curve of a 6-anode rectifier similar to that shown in Fig. 2, with 15-cm. diameter arms, at 3,000 volts. The ordinates represent the maximum safe load for reliable short time operation at each temperature. The important fact to be noted is that, while operation is possible at any temperature below a definite maximum, the load that can be carried is small at low temperature and increases uniformly with temperature. Hence: (1) *The best operating temperature is as near the maximum as can safely be maintained.* (2) *For a given load there is a definite operating range of temperature.* For example, for the rectifier represented in Fig. 4 the operating range for 1,500 amperes load is between 42 and 60 deg. cent.

It is seen that there are two limitations to the range of reliable operation at a given voltage. One is high temperature, *i. e.*, high vapor pressure of mercury; the other is high current.

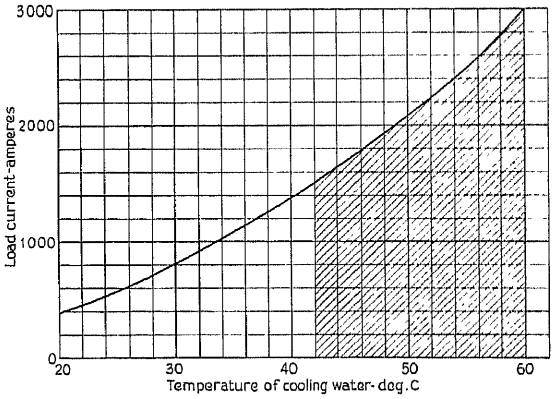


FIG. 4—LOAD VS. TEMPERATURE CURVE OF SIX-ANODE RECTIFIER SIMILAR TO FIG. 2, WITH 15-CM. DIAMETER ARMS, AT 3,000 VOLTS, FOR LOADS OF 1 MIN. DURATION (OF FIG. 7.)

The operating range for 1,500 amperes is indicated by the shaded area

The maximum *vapor pressure* for a given operating voltage may be expected to follow closely the relation between vapor pressure and sparking potential; in fact, the only difference to be expected between arc-back potential and sparking potential is the change in the surface condition of the negative electrode, due to the presence, in and just below the surface, of gas molecules, which have penetrated the surface in the form of positive ions. Such a composite surface is known to emit electrons more easily than a pure metal surface, and may be expected to allow a spark discharge at lower voltage. Hence the arc-back voltage should be very close to the sparking potential between electrodes of low electron work-function.

This conclusion appears to be justified by the results of a large number of tests which we have made of sparking potentials. Typical results are shown in Figs. 5 and 6. Fig. 5 gives the sparking potential of an idle iron anode in a rectifier similar to Fig. 1, having 15-cm. diameter arms, with and without current to an adjacent anode. It is seen that the presence of ions due to current in the adjacent arm lowered the sparking potential by 50 per cent for the same vapor pressure; or, conversely, lowered the vapor pressure for the same sparking potential by 50 per cent (vapor pressure doubles for each 10 deg. temperature).

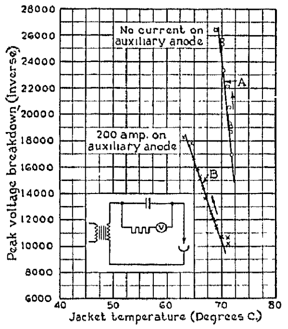


Fig. 5—Sparking Potential as Function of Mercury Vapor Pressure, for an Idle Anode in a Rectifier Similar to Fig. 1

Note that the sparking potential is lowered by the flow of current in an adjacent arm

Fig. 6 shows the sparking potentials of a hot-cathode mercury-vapor rectifier, with the cold barium-coated cathode used, first, as anode (curve *A*), and then as cathode (curve *B*), showing the effect of cathode material. For comparison, the curve of actual arc-back voltage of a similar rectifier with 15-ampere load is shown in curve *C*. The fact that the arc-back voltages are slightly higher than the cold sparking potentials may be attributed to a difference in the cleanness of the anode surfaces in the two tubes. There is no evidence in the case of curve *C* of a lowering of spark potential due to positive ions.

The points to be observed in Figs. 5 and 6 are, (1) that there is a fairly sharp limit to operating vapor-pressure, as expressed by temperature of cooling water, above which spark discharge will take place between the electrodes even when idle; (2) the evidence available indicates that the operating arc-back potential, for light loads, does not differ greatly from the sparking potential but that with the intense ionization accompanying heavy loads it may fall to about half the sparking potential.

The *current limitation* is of a different nature. It will be shown later (section 6) that the limiting load current at any temperature can be predicted from the volt-ampere characteristic. This characteristic changes from negative to positive as the load increases, and at a definite load rises abruptly. It is at this point that arc-back occurs.[1] The steep rise is associated with the approach to complete ionization of all the gas in the anode arms. The load-temperature curve thus acquires a rational meaning, *viz.*, the temperature determines the vapor pressure of mercury in the anode arms, and *the maximum load current is that which utilizes all the molecules of this vapor as positive ions to neutralize the electron space charge.* This fundamental limitation to the current-carrying capacity of an arc was predicted by Langmuir and Mott-Smith,[2] and verified experimentally by them for a small mercury arc in a quartz capillary tube. It has been found to hold for all the rectifiers tested.

The vapor pressure in the anode arms is not completely determined by the temperature of the cooling water in the case of continued heavy loads, because of the temperature gradient in the iron wall and the iron-water boundary, and under some conditions a pressure gradient in the mercury vapor due to convection of mercury spray and its vaporization in the arc stream. Differences as great as 10 deg. cent. between water temperature and equivalent vapor pressure have been observed and even greater differences may exist. Hence, when the water temperature is adjusted to give the maximum safe vapor pressure in order to carry the maximum load, if this load is maintained the pressure will rise above the safe value. This results in a load-time characteristic such as is shown in Fig. 7, which represents the behavior of the same rectifier, the characteristics of which were shown in Fig. 4, at 60 deg. cent. and 3,000 volts.

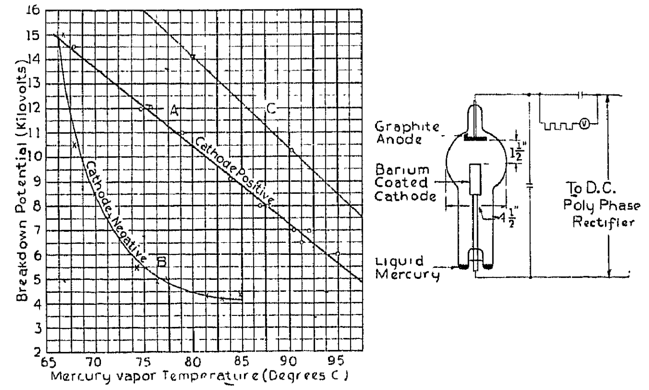


Fig. 6—Sparking Potential and Arc-Back Potential in a Hot-Cathode Mercury-Vapor Rectifier, Showing Effect of Electrode Material

Curve A, sparking potential, carbon negative; curve B, sparking potential, barium-coated cathode negative; curve C, arc-back potential of similar rectifier with 15-ampere load

(*b*) *Vapor Pressure and "Control Point."* What part of the cooling water determines the vapor pressure in the anode arm? To answer this question a series of simultaneous measurements was made of vapor pressure in the anode arms and temperature in different parts of the tank, during normal low-voltage operation. The vapor pressure was measured by a method originally suggested by Langmuir. A copper block containing a thermometer was pressed against the outer wall of the glass anode insulator, and slowly cooled until, at a temperature t_1, a thin film of condensed mercury appeared on the inner glass wall. The block was then slowly heated until the film disappeared at t_2. The mean of the "dew-points" t_1 and t_2 was taken to represent the pressure in the arm. Since the dew-point readings require time, only the steady-state condition for each load was observed.

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[1] The mechanism of this arc-back is discussed in section 6.

[2] Langmuir and Mott-Smith, *G. E. Rev.*, *27*, 770, 1924.

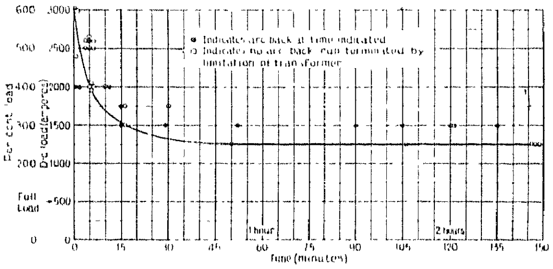

FIG. 7—LOAD-TIME CURVE AT 3,000 VOLTS OF SAME RECTIFIER AS IN FIG. 4, AT 60 DEG. CENT.

Abscissas represent observations of the length of time each load could be carried without arc-back

It was found that, with light loads, *the dew-point corresponds very closely to the temperature of the part just beneath the anode arm*, and is nearly independent of the temperature of the rest of the rectifier. This region below the anode arm is the nearest mercury-wetted portion to the anode, and the above result means that the vapor pressure in the anode arm is in equilibrium with the liquid mercury below it.

For heavy loads the vapor pressure in the arm, as measured by the dew-point, rises above that corresponding to the water at the bottom of the anode arm, as shown in Fig. 8, by as much as 10 deg. cent. This is about the difference to be expected between the water and the iron, for the existing heat flux of approximately 40 watts per sq. cm. of iron surface, and slowly moving water.

4. ANODE HEATERS

The limits of vapor pressure are almost certain to be momentarily exceeded if liquid mercury comes in contact with hot anodes, or any hot surfaces near the anodes. In some of the early models it was observed, when they were opened, that the anodes and their housings were thickly coated with small drops of mercury, which had condensed as the rectifier cooled. These rectifiers were known to arc back frequently within the period between 20 min. and 2 hr. after starting, which was about the range of time required, under varying loads, for the anodes and housings to become hot enough to vaporize mercury rapidly, and thus lead to excessive local pressure, with resultant arc-back. Heaters were therefore installed in the anodes, as shown in Fig. 9A, to maintain the anodes and housings at a temperature higher than the body of the rectifier, so that condensation could not occur.

The result was the entire disappearance of the 20-min. to 2-hr. arc-backs, and, in fact, of nearly all arc-backs. Fig. 10 shows the load record (interruptions indicated by small black squares) of a typical station containing four rectifiers, for several months preceding and following the installation of anode heaters. It is seen that, under similar load conditions, the number of arc-backs during seven months following installation was five as compared to 200 in the two months preceding installation. Three of these five occurred in the first three days after installation, before the mercury had completely evaporated from the anodes and housings.

The anode heaters shown in Fig. 9A were awkward and had to be energized by insulated transformers. In the newer rectifiers the heaters are around the anode necks, as shown in Fig. 9B, and are thus at tank potential and may be operated in series or parallel. The heaters may perhaps be omitted entirely if, as indicated in Fig. 2, the level of the cooling water is above the top of the anode chambers, and its volume and flow is so controlled that the hottest water is always on top, and loss of heat from the anode terminals is prevented.

5. DIMENSIONS VS. CURRENT-CAPACITY

Since the current-capacity depends but little upon temperatures other than those near the anode arms, it may be anticipated that the important dimensions will be those of the arms. This has been found to be true. The first tests, however, gave the surprising result that the capacity is nearly independent of the dimensions of the arms, for geometrically similar plain cylindrical arms like Fig. 1. Typical results of these tests are shown in Fig. 11. The reason that the current does not increase with diameter is that the maximum safe operating temperature decreases as the diameter of the arm increases. Thus the total amount of vapor in the arm, and hence the current capacity, should remain about constant, in agreement with observation.

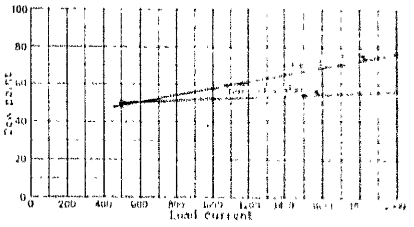

FIG. 8—DEW-POINT OF MERCURY IN ANODE ARM, AS FUNCTION OF LOAD

Note that for small loads the dew-point coincides with the temperature of the water just below the anode arm, but rises above it with heavy load

The behavior is very different when the anode arm is subdivided so as to be equivalent to a group of small arms. If the part of the arm just below the anode is divided by metal vanes into narrow parallel channels, the limiting vapor pressure is determined by the dimensions of these channels and their distance from the anode, and is independent of the diameter of the arm. Under these conditions *the current capacity is proportional to the square of the diameter, i. e., to the cross section of the anode arm.* This is shown in Fig. 12, which gives the observed maximum loads, of one minute duration, for arms of different diameters, similar to that shown in Fig. 2. The mode of operation of these subdivided arms will be discussed in Section 8.

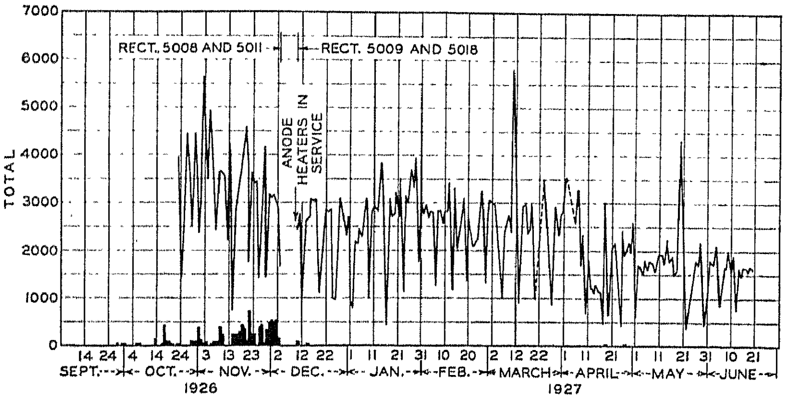

Fig. 10—Load Record of Typical Station Containing Four Rectifiers Before and After the Installation of Anode Heaters

Each small black square indicates an arc-back. The time of installation is indicated by arrow. Note that the number of arc-backs fell from 100 per month to essentially zero

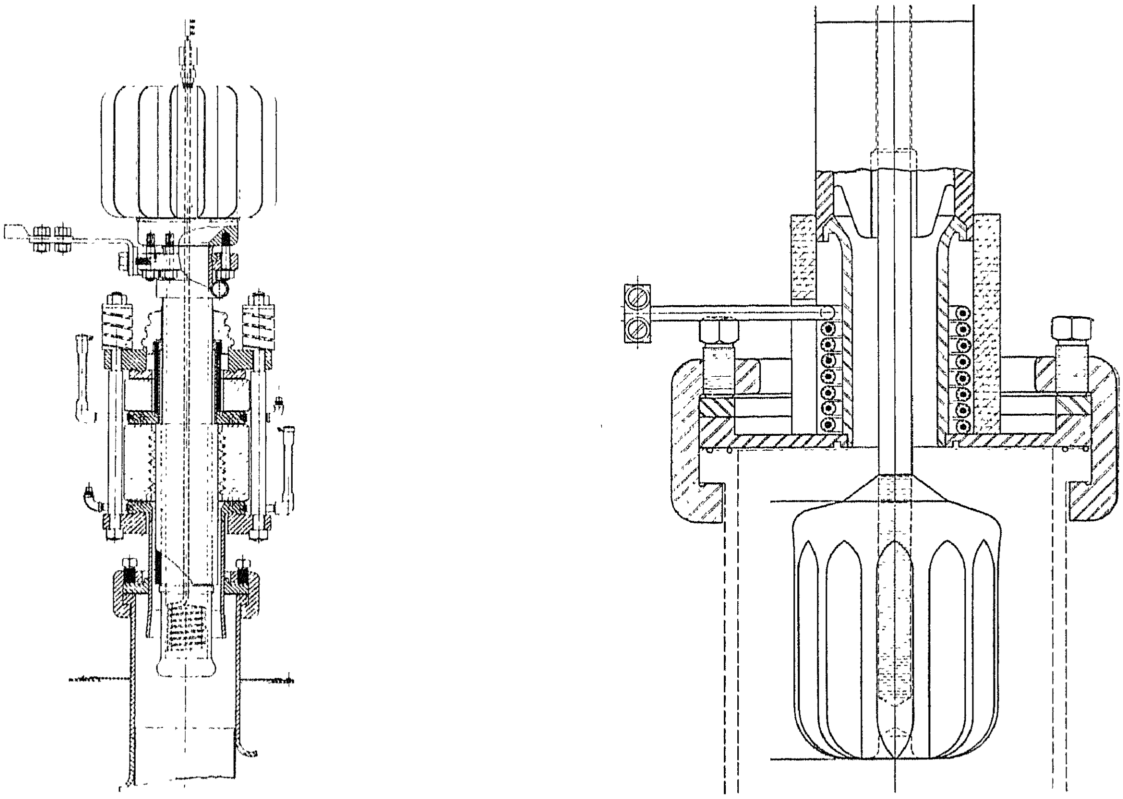
A
B

Fig. 9 (A) Internal Immersion Type Heaters Installed in the Iron Anodes of Early Rectifiers
(B) External Sheath-Wire Heaters Around Anode Necks

6. Voltage Drop. Positive Resistance

The pulse of a rectifier is the voltage drop between cathode and anode when the anode is positive. It is a valuable index of health, especially in studying or conditioning a new rectifier. For example, an exceptionally low voltage-drop tells that the vapor pressure is abnormally high, and arc-back may be expected with continued load; a high voltage-drop indicates excessive load, which must be reduced or the vapor pressure raised if reliable operation is to continue. If one anode

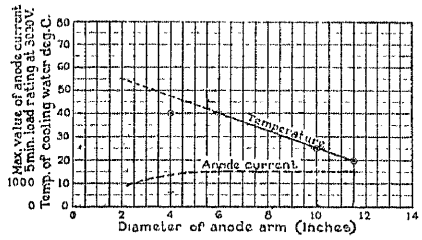

Fig. 11—Maximum Current Capacity and Maximum Operating Temperature as a Function of Diameter of Anode Arms

For plain cylindrical arms of type shown in Fig. 1

has a higher voltage-drop than the others, it contains foreign gas, due either to leakage or incomplete exhaust; if lower, its grid has probably been injured during assembly or exhaust.

The voltage drop has been studied both with the oscillograph, using low alternating voltage so as to accentuate the positive part of the characteristic, and with the peak-reading vacuum tube voltmeter. Fig. 13 shows a typical characteristic taken with an oscillograph. The positive portion is practically a straight line, showing that the voltage drop at constant current, under normal conditions, is constant and not a function of time.[3] It is, however, a function of current. If characteristics similar to Fig. 13 are taken with different values of load current, the voltage drop is found to vary with current as shown in Fig. 14. The slope or "resistance" of this characteristic is negative for small currents, zero for moderate currents, and positive for large currents.

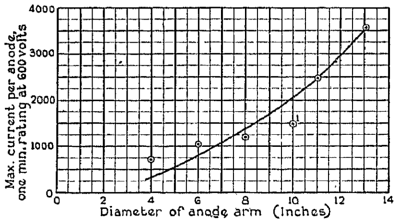

FIG. 12—CURRENT CAPACITY VS. DIAMETER OF ANODE ARMS FOR SUBDIVIDED ARMS OF TYPE SHOWN IN FIG. 2

The solid line represents the theoretical limiting value of 4 amperes per cm.^{2} (The point marked (1) is from a test at 3,000 volts)

The negative portion of the characteristic is well known. The positive portion is observable only with low-pressure arcs, and is due to the approach to complete utilization of all the gas. This effect was first observed by Langmuir and Mott-Smith,[4] who pointed out that the number of ions needed to neutralize space charge is proportional to the current; and that the maximum number of ions cannot exceed the number of molecules of the gas. When all the gas has been ionized, a further increase in current can be obtained only by increasing the speed of the electrons, which means increased arc-drop. The voltage may be expected to begin to rise before this state is reached. In our tests the maximum degree of ionization for stable operation was about 4 per cent. At this point a new phenomenon appeared, *viz.*, a rapid reduction of pressure in the anode arm, which appears to be due to a pumping action not thoroughly understood at present. The pressure falls during the current cycle, causing the voltage drop to increase, often to very high values.

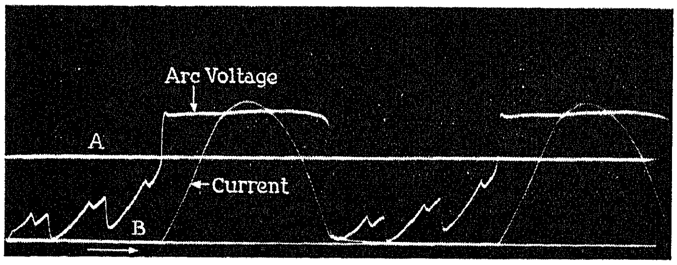

FIG. 13—TYPICAL OSCILLOGRAM OF VOLTAGE BETWEEN ANODE AND CATHODE IN SINGLE-PHASE RECTIFIER WITH LOW ALTERNATING VOLTAGE

The voltage-drop is essentially independent of time but is a function of current as shown in the next figure

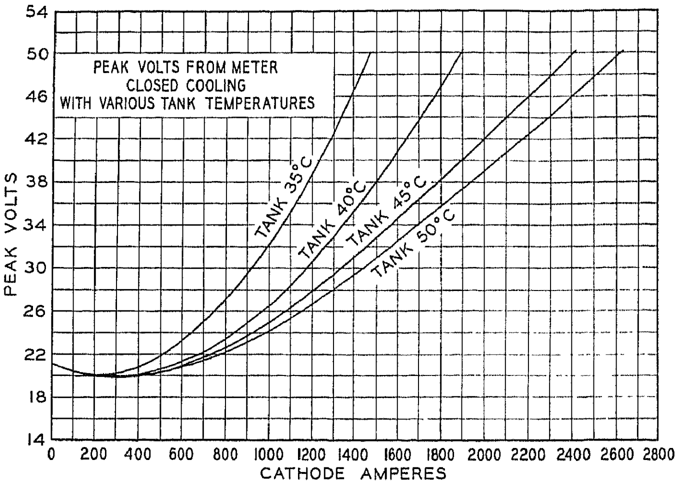

FIG. 14—VOLTAGE DROP IN MERCURY ARC AS FUNCTION OF CURRENT

The resistance is positive for currents above a certain value

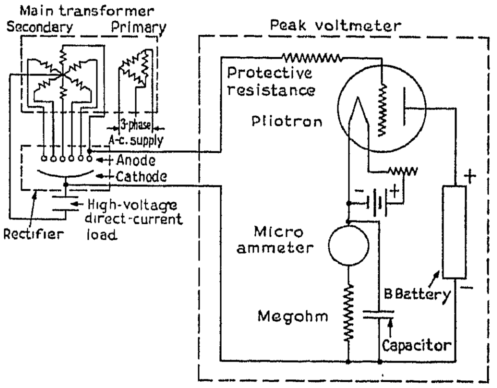

FIG. 15—CIRCUIT SHOWING SELF-ADJUSTING PEAK VOLTMETER CONNECTED TO MERCURY-ARC RECTIFIER

The microammeter reads peak volts directly

When volt-ampere characteristics are obtained for different vapor pressures, it is found that the current at which the characteristic becomes positive is proportional to the vapor pressure, as may be seen in Fig. 14. This is to be expected from the point of view expressed above that positive resistance is due to full utilization of the mercury vapor.

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[3] Under the *abnormal* conditions of extreme overload the vapor pressure, and with it the voltage drop, may vary with time during the current cycle.

[4] Langmuir and Mott-Smith, loc. cit.

At operating voltages oscillograms are inconvenient and the voltage-drop is too small to be read accurately. In order to observe the voltage drop under operating conditions, a peak-reading vacuum-tube voltmeter was adapted to be self adjusting and direct reading, by the insertion of a high resistance in multiple with a condenser in the cathode circuit (Fig. 15), and was used in all tests. It was found that *arc-back usually occurs when the voltage drop exceeds about 50 volts.* Hence the maximum load which a rectifier can carry may be predicted from the intersection of its volt-ampere characteristic with the 50-volt line (see Fig. 14).

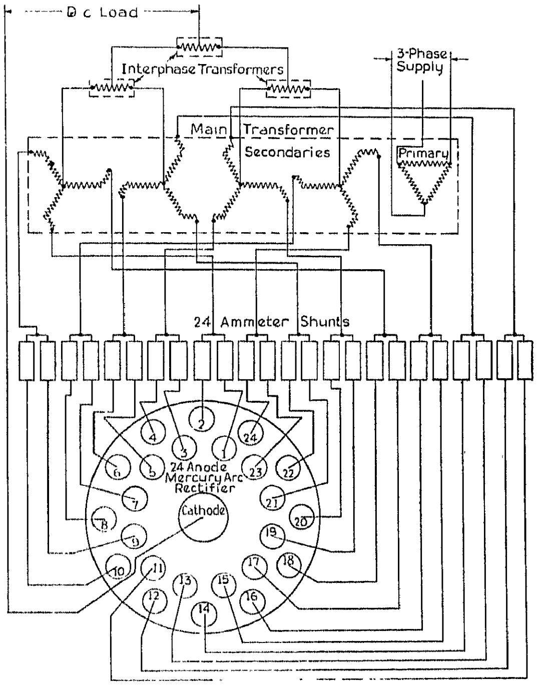

FIG. 16—CONNECTIONS FOR TESTING DIVISION OF CURRENT BETWEEN ANODES OPERATING IN PARALLEL

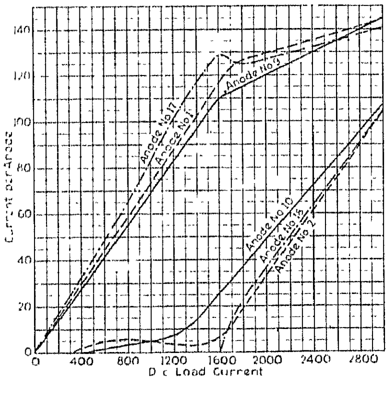

FIG. 17—DIVISION OF CURRENT BETWEEN INNER AND OUTER ANODE FOR THREE TYPICAL PAIRS OF PARALLEL-CONNECTED ANODES

The reason for these high current or *high arc-drop* arc-backs is not fully understood, but is believed to be high local gas pressure, caused by one or perhaps both of the following series of events:

(a) Excessive voltage drop is usually accompanied by surges which excite oscillations of the transformer system, sometimes of sufficient voltage to arc over insulation. Such oscillations may be seen in the oscillograms of Figs. 18 and 19. It seems probable that the surges are caused by rapid variations of vapor pressure, due to the following mechanism:

Ions which strike the metal walls with energy greater than 22 volts[5] penetrate the walls, and remain occluded for a considerable time, constituting a large reservoir of stored gas that diffuses out slowly. The disappearance of vapor in this way may take place so rapidly that the reduction in pressure cannot immediately be compensated by an inflow of vapor. This causes the voltage drop to rise still higher, until a point is reached where

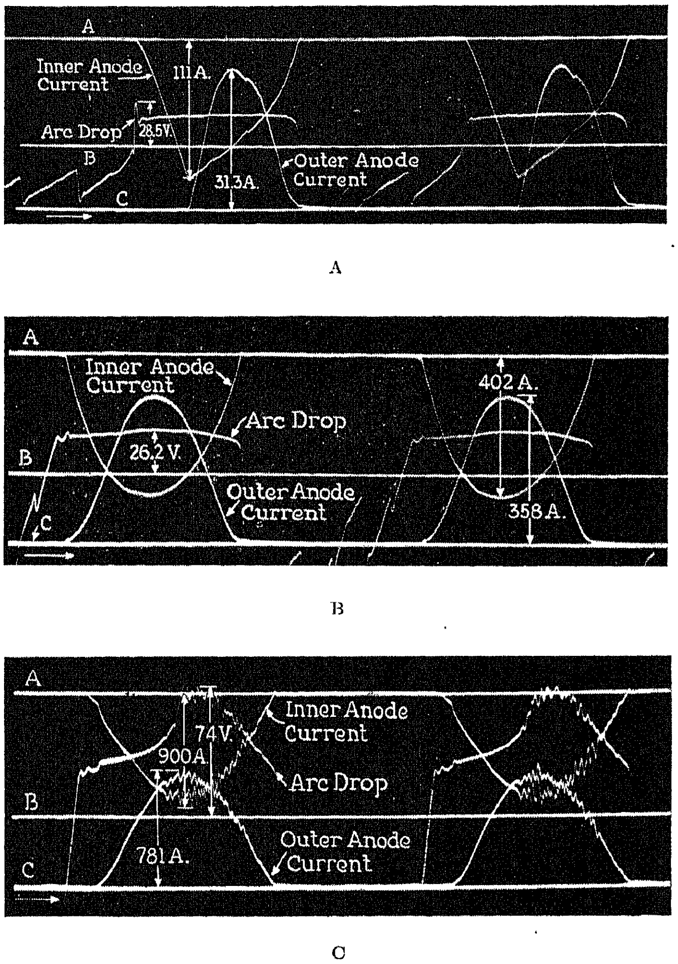

FIG. 18—OSCILLOGRAMS SHOWING THE CURRENT TO THE TWO ANODES (DEFLECTION DOWN FOR INNER ANODE, UP FOR OUTER) AND THE VOLTAGE DROP, FOR A PAIR OF PARALLEL ANODES

Trace A = current to inner anode
Trace C = Current to outer anode
Trace B = voltage between cathode and anode

(A) Total load 250 amperes; (B) total load 1,500 amperes; (C) load 3,000 amperes. Note that the anodes share the current equally at large load. The voltage drop is normal with 250 and 1,500-ampere load, but very high at 3,000-ampere load the stored gas becomes unstable. The increasing energy of impact of the ions begins to heat the walls sufficiently to liberate the stored gas, which quickly restores the vapor pressure, probably to a higher value than normal. The cycle is then repeated. There is evidence from other experiments that the amount of gas thus stored and liberated may far exceed the normal quantity of vapor in the anode arm. Thus if the vapor pressure were previously near the maximum safe value, it will be raised momentarily above this value, and arc-back will result.

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[5] Hull, A. I. E. E. TRANS., Vol. 47, 1928, p. 755.

(b) A sudden evolution of gas in the anode arm, sufficient to cause arc-back, may result from the formation of a cathode spot on the metal wall near the anode. There is abundant evidence that such cathode spots do develop when, and only when, the voltage-drop exceeds about 40 volts.

For example, if a rectifier is "baked out" by current under such conditions that the voltage-drop is high, the walls are found to be scored by numerous bright, narrow channels, frequently forked, running from the bottom of the anode arm to a point opposite the anode tip. Each of these channels represents the path of a rapidly moving cathode spot. If the voltage has been very high, the housing will be found melted where the cathode spot has stood still opposite the anode tip.

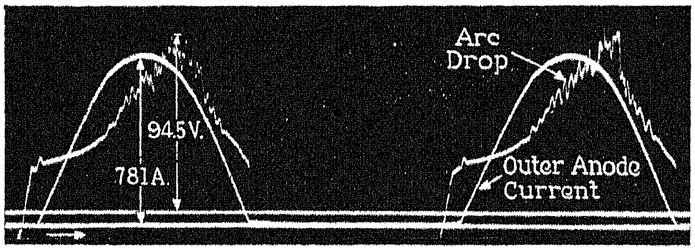

FIG. 19—OSCILLOGRAM OF CURRENT AND VOLTAGE OF ONE SET OF SINGLE ANODES, IN THE SAME CIRCUIT AS THE PAIRS OF ANODES OF FIG. 18

Total load 1,500 amperes. Note excessive voltage drop, and compare with Fig. 18, B and C.

As a second example, a special hot-cathode metal rectifier was made, the central portion of which consisted of an iron tube 2.5 cm. in diameter and 30 cm. long interposed between cathode and anode. The tube was so constructed that the formation of a cathode spot near the anode could be observed visually. It was found that such a spot invariably formed when the voltage between cathode and anode exceeded 40 volts.

An example of the absence of such cathode spots, when the voltage-drop is kept low, is furnished by a new mercury-arc rectifier which was exhausted with great care. The current and temperature were so regulated that the voltage-drop, as continuously observed by a peak-reading voltmeter, never exceeded 30 volts. It was then opened and found to be entirely free from scorings.

The formation of a cathode spot on the wall of the rectifier requires the maintenance of two arcs in series. The insulated metal tank must be anode for the mercury arc starting from the cathode, which requires that it shall be at least 10 volts positive with respect to the cathode. The current then passes by conduction through the iron to a point near the anode, where an iron arc plays from wall to anode, which requires about 30 volts. The minimum voltage for the two arcs in series is therefore about 40 volts, in agreement with observation.

7. PARALLEL OPERATION IN POSITIVE RESISTANCE RANGE

If the positive resistance shown in Fig. 14 represents a true instantaneous characteristic of the arc, it should be possible to operate such arcs in multiple, for large currents, without stabilizing resistance or reactance. This was tested in a 24-anode rectifier similar to the 12-anode rectifier illustrated in Fig. 1, except that the anode arms (15 cm. diameter) were 30 cm. longer than shown in Fig. 1. The anodes were arranged in two concentric rings of twelve each. They were connected in pairs to the terminals of a 12-phase transformer, each pair consisting of one anode of the outer group and one of the inner group, as shown in Fig. 16. The length of the arc path to the outer anode was 15 cm. longer than that to the inner, the total lengths of the arms, from the main body of the rectifier to the anode tip, being 45 cm. and 60 cm. for the inner and outer arms respectively. This seemed a sufficiently unfavorable condition for division of current between the anodes to constitute a crucial test. The individual anode leads contained no reactance or resistance except a 0.002-ohm ammeter shunt for measuring the current.

From the characteristic given in Fig. 14 we should expect one anode of each pair to carry all the current for light loads, for which the characteristic is negative, and for a small portion of the positive range, up to the current value (cf. Fig. 14) at which the voltage-drop is equal to the voltage-drop for zero current of the other anode of the pair; here the current will start to flow to the second anode. At loads greater than this both anodes of each pair will carry current, and will share it more and more equally as the load increases.

Fig. 17 shows the currents actually observed for three pairs of anodes, which are typical of all. They conform completely to the values predicted from the characteristic arc-drop curve. Up to a certain load one anode of each pair carries all the current. This limiting load represents about half the load that one anode alone can safely carry, so that there is no danger or disadvantage in having one anode do all the work. Above this limiting load both anodes of each pair share the current. It will be noted that the inner anode of each pair picked up the load first in every case, and carried slightly more current at all loads than the outer one, corresponding to its lower resistance, the two currents being approximately in the inverse ratio, 45/60, of the lengths of their individual arc paths. Thus the current divides according to resistance as in any ohmic circuit.

As additional evidence of the approximately equal division of current, the rectifier was tested for current-capacity, first with one set of anodes alone and then with the two sets in parallel. *It was found that the rectifier would carry twice as much current with the parallel anode connection as with either set of 12 anodes alone.*

The division of current between parallel anodes was also observed oscillographically. Six pairs of anodes were used, each pair connected as in Fig. 16, in a standard six-phase double wye circuit. Fig. 18A shows the current and voltage for one pair when the total load was 250 amperes. Current starts first to the inner anode (trace *A*, deflection down) while that to the outer anode (trace *C*, deflection up) does not start until the middle of the cycle. In Fig. 18B, where the total load is 1,500 amperes, current starts in the two anode arms almost simultaneously, and they share the load in about the inverse ratio of their lengths. The low voltage-drop of 26.2 volts maximum shows that the arms are not overloaded. With 3,000 amperes (Fig. 18C) the two anodes share the current almost equally. But the voltage-drop rises during the cycle to a maximum value of 74 volts, and exhibits the oscillations characteristic of overload.

For comparison, a similar test was made with a single set of six anodes, the outer row of anodes being used. Fig. 19 shows the current and voltage drop for one of these anodes at a total load of 1,500 amperes. The voltage-drop shows the same features as in Fig. 18C, rising to an ever higher value, *viz.*, to 94.5 volts. It is to be noted that this high voltage drop occurred at 3,000 amperes in the case of parallel anodes (Fig. 18C) whereas it occurs at 1,500 amperes for the single set of anodes (Fig. 19). The voltage drop is normal at 1,500 amperes (Fig. 18B) in the parallel case, *i. e.*, when two anodes instead of one are connected to each transformer terminal. This evidence, therefore, agrees with that of the load limit tests in showing that the *load capacity is doubled by doubling the number of anodes and using them in pairs, each pair being in multiple without stabilizing reactance.*

8. SUBDIVISION OF ANODE ARMS

The tests described in the last section indicate that if any number n of adjacent anodes are grouped in parallel as a unit, without stabilizing resistance or reactance, the current which they can safely carry is n times that of a single anode. Combining this result with that of section 5, where it was shown that the current capacity of a small anode in a small cylindrical housing is equal to that of a large one, we have a means of increasing the capacity of a rectifier almost without limit, *viz.*, by equipping it with a large number of small anodes, and using for each phase as many of these anodes in parallel as are needed to supply the required load.

The group of parallel-connected anodes may obviously be connected together inside the rectifier instead of outside. This is equivalent to combining them into one large anode, with a number of parallel paths leading to it, each path similar in dimensions to one of the single anode arms. This is the principle of subdivided arms referred to in section 5. The parallel paths are most conveniently formed by subdividing the arm by partitions. Fig. 2B shows one type of subdivision, in which the partitions are concentric cylinders.

This principle of subdivided arms has been very carefully tested, using partitions of different forms and dimensions, and anode arms from 15 to 33 cm. diameter. The results show that with a given "mesh" of subdividing partition the current that can be carried is proportional to the total cross section of the arm, as it should be. The maximum operating vapor pressure, and hence the current per unit cross section, depend upon the dimensions of the partitions.

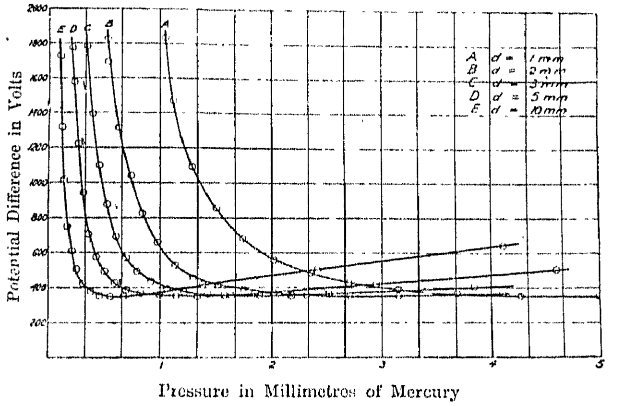

FIG. 20—SPARKING POTENTIAL VS. PRESSURE FOR AIR, AT DIFFERENT ELECTRODE DISTANCES *d*,

Note that for low pressures the sparking potential is higher the lower the pressure and the smaller the distance

Grid and partitions somewhat similar to those described here have been used in rectifiers in the past, but their function and mode of operation have never been clearly demonstrated. It is believed that the tests described above constitute a firm basis for the use of such partitions. Their mode of operation is described in the following section.

9. GRIDS AND THEIR USES

Grids may serve two purposes:

(a) Grids or grilles like those described in the last section, consisting of parallel passages longer than they are wide, may perform the function described above of enabling a rectifier to operate at higher vapor pressure than would be possible without these grilles. To call them anti-arc-back devices is not quite appropriate, since the rectifier will operate as reliably without as with them, but at a lower temperature and correspondingly smaller current. It is not the purpose of this article to discuss the mechanism of arc-back, many points of which are still obscure, but the following brief picture will make clear the function of the grille in increasing the possible operating pressure. For this purpose arc-back potential will be identified with sparking potential of the gas, which appears to be justified by the tests described in section 3.

The sparking potential of a gas decreases with decreasing pressure, as is well known, to a minimum value, and then increases rapidly with further decrease of pressure. Fig. 20 shows typical voltage-pressure curves for air. The minimum sparking potential occurs at pressures between 1 mm. and 1 cm. for the different gases, at electrode distances less than 1 cm., and its value varies from about 100 to 400 volts. The "tungar" rectifier, which operates near this minimum, has an arc-back potential of 175 volts, and hence a maximum operating d-c. potential of about 75 volts.

It will be noted in Fig. 20 that, at pressures above that corresponding to the minimum, the sparking potential is higher the greater the distance. The tungar operates on this side of the minimum, *viz.*, at 5 cm. pressure of argon. It might seem that advantage could be taken of this fact to increase the arc-back voltage of the tungar by increasing the distance between electrodes; for example, at 2-cm. distance the sparking potential is 1,800 volts. The answer is that it is impossible to maintain this distance under rectifying conditions. For the ionized gas that is left between the electrodes at the end of the rectifying cycle forms a conducting sheath around the positive electrode, building it out until its distance from the negative electrode is that corresponding to minimum sparking potential. This applies to all pressures higher than the "minimum" and all reasonable electrode distances.

It is evident therefore that high-voltage rectification can be accomplished, if at all, only in the range of pressure below the minimum point. In this range *the sparking potential is higher the smaller the distance between electrodes.* Here also the ionized gas tends to distort the electric field, in such manner as to *lengthen* the distance between electrodes and thus lower the sparking potential. But the possible lengthening is limited to the dimensions of the space around the anode. Thus with an anode arm 30 cm. in diameter the effective sparking distance between anode and wall is of the order of 60 cm. At this distance the pressure of mercury vapor corresponding to a sparking potential of 6,000 volts is approximately 0.003 mm. *The function of the grille is to decrease this distance* and thus allow a higher operating pressure. For example, by placing a grille 5 cm. below the anode the distance is decreased from 60 cm. to 5 cm., and the pressure thus increased from 0.003 mm. to about 0.040 mm.

The form of the grid is important, if it is to perform this function of shortening the distance between anode and body of the rectifier. Tests have shown that wire grids, unless their mesh is very fine, are not effective in shortening the distance, since the ions diffuse freely through the holes. On the other hand, partitioned passages or grilles like those described above are effective because they do not allow ions to diffuse through, provided the length of the passages is large compared to their width. Schottky[6] has shown that for a circular passage of radius r the fraction of ions which reach a distance l from the mouth is $\exp \frac{-2.40\,l}{r}$. Hence if the length of the passage is twice its diameter, only e^{-10} or $\frac{1}{22000}$ of the ions that enter will get through.

It is thus seen that a grille of this type, while allowing the arc to pass through, will not permit ions from the main body of the rectifier to diffuse through it during the time when the anode is negative. Under these conditions the distance which determines the operating vapor pressure is that between grille and anode. This theory is in agreement with our experiments.

(b) Grids may be used to control the starting of an arc. Rectifiers equipped with such grids are called *thyratrons.* Any type of grid can perform this function, provided it is insulated, and the voltage that must be applied to it to allow or prevent starting of the arc depends upon its location and fineness. For example, with the combination shown in Fig. 2c, of an insulated wire grid close to the anode and an uninsulated grille below, the arc will not start at all at practical operating voltages unless the grid is made positive with respect to the body of the rectifier. On the other hand, grids may be made so coarse that the arc will start easily with zero or positive grid voltage, but may be prevented from starting by making the grid sufficiently negative.

The characteristics and applications of thyratrons have been dealt with elsewhere.[7] One application, however, which is especially useful in rectifier operation, should be described here, *viz.*, the use of the grids for obtaining automatic voltage regulation. The principle of this regulation is that the *phase* of the voltage applied to the grids is automatically advanced as the load increases. Thus with light load the phase of the grid voltage lags considerably behind that of the corresponding anode, so that the starting of the anode current, which cannot take place until the grid becomes positive, occurs relatively late in each cycle. As the load increases the phase of the grid voltage is automatically advanced, causing the current to start earlier in the cycle and *flow for a longer time in each cycle,* thus compensating for its *lower average value* resulting from the lower line voltage.

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[6] W. Schottky, *Zeit. f. Phys.*, *31*, 181, 1924.

[7] A. W. Hull, "Hot-Cathode Thyratrons," *G. E. Rev.*, *32*, 213-23, 390-99, 1929; D. C. Prince, "The Inverter Using Thyratron Tubes," *G. E. Rev. 31*, 347, 1928; O. W. Pike, "Grid-Regulated Heavy-Current Tubes," *Electronics*, May, 1930; C. A. Sabbah, "The Series-Type Inverter," *G. E. Rev.* (to appear soon).

The circuit for accomplishing this phase advance is shown in Fig. 21 and the resulting regulation in Fig. 22. The grid voltage is derived from a current transformer in the line, so connected that its variable output voltage is added to a 90-deg. lagging constant voltage derived from a selsyn transformer. The constants are so adjusted that at light load the anode current starts late in the cycle, advancing with load until at the desired overload it starts at the earliest possible starting time, namely, the time at which the anode voltage rises above the rectified d-c. line voltage. A further advance in phase cannot further increase the time of current flow per cycle, and therefore the output voltage must fall with increase of load beyond this value.

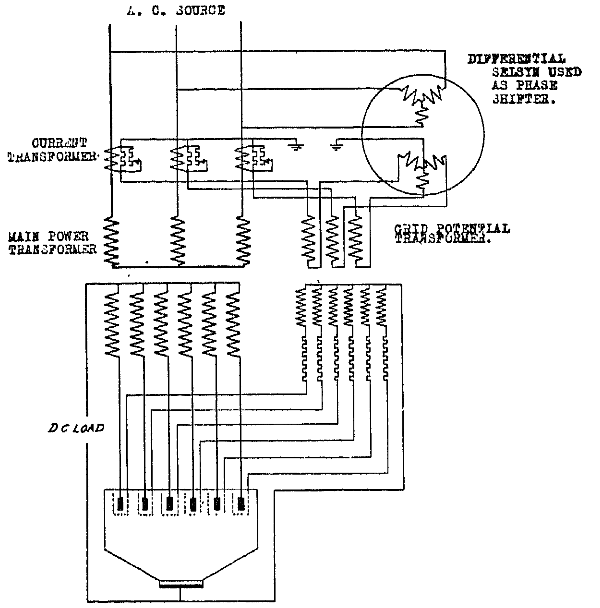
A. C. SOURCE
DIFFERENTIAL SELSYN USED AS PHASE SHIFTER.
CURRENT TRANSFORMER
MAIN POWER TRANSFORMER
GRID POTENTIAL TRANSFORMER.
D C LOAD

Fig. 21—Circuit for Obtaining Voltage Regulation by Advancing the Phase of Grid Voltage

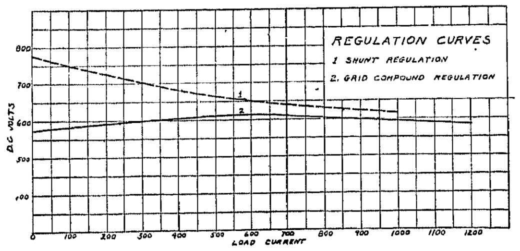

Fig. 22—Voltage Regulation Obtained with the Circuit Shown in Fig. 21

The unregulated characteristic is shown by the broken line, Curve 1

10. Hot-Cathode Rectifiers

An extended discussion of this subject would be premature, since the largest hot-cathode installations at the time of writing are about 1,000 kw. Hot-cathode rectifiers and thyratrons are available in capacities up to 400 kw. at 8,000 volts. Fig. 23 shows a group of these tubes, in glass bulbs, and Fig. 24 a developmental model of hot-cathode metal thyratron for moderate voltage, with current rating of 100 amperes average, 600 amperes maximum.

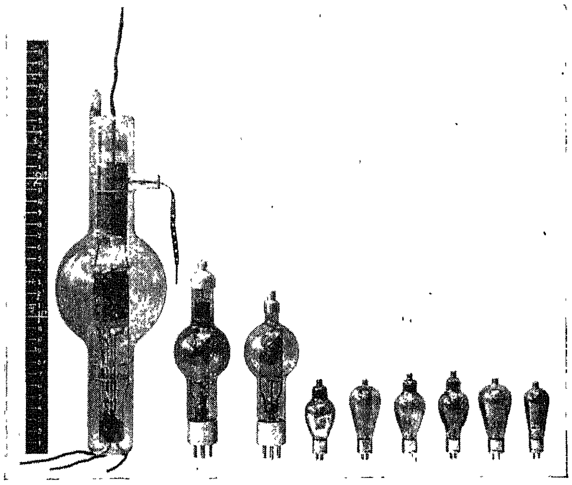

Fig. 23—Typical Hot-Cathode Mercury-Vapor Tubes

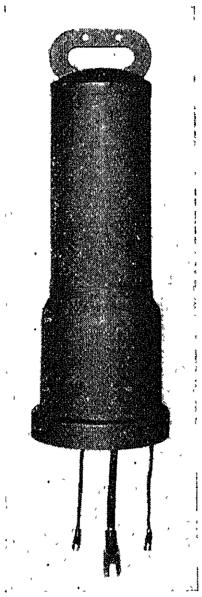

Fig. 24—Hot-Cathode Metal Thyratron, Rated 100 Amperes Average, 600 Amperes Maximum

There are many features which make hot-cathode rectifiers attractive, two of which deserve special mention. The first is ease of control of the mercury-vapor pressure. A single drop of mercury suffices to furnish the required amount of vapor. If this drop is located in *the coolest part of the tube,* which may be remote from the arc, then the temperature of this part will completely determine the vapor pressure throughout the tube. The rest of the walls may be as hot as desired, since no liquid mercury ever comes in contact with them. For example, the rectifier shown in Fig. 24 operates satisfactorily with its walls at 400 deg. cent. This makes it possible to build hot-cathode mercury-vapor rectifiers much smaller than mercury-arc rectifiers of similar capacity.

A second advantage of hot-cathode rectifiers is low voltage drop, with resulting high efficiency at low operating voltage. Three factors contribute to this low voltage drop:

(a) *Short distance* between cathode and anode, made possible by the absence of mercury spray. In the mercury-arc rectifier the anodes must be shielded from this spray, which means a long arc path.

(b) *"Efficient" cathode material.* A mercury arc has a 10-volt cathode drop, which means a power consumption of 10 watts per ampere of arc-current. A barium-coated hot cathode, properly constructed to avoid loss of heat by radiation, requires but 2 watts per ampere.

(c) *Choice of gas.* A hot-cathode rectifier may utilize any monatomic gas or metal vapor whose pressure can be maintained at the proper value. There are many practical vapors such as cadmium, sodium, caesium, with ionizing potentials lower than mercury, which may be expected to give a lower voltage drop for the same length of arc. For example, the voltage drop in the metal tube shown in Fig. 24, which is 10 volts at 100 amperes load with mercury vapor of 0.030-mm. pressure, may possibly be reduced to half this value by a proper choice of vapor and vapor pressure.

Thus the hot-cathode rectifiers promise to fulfill the future requirements of low-voltage applications, with an efficiency of 95 per cent or better at 250 volts, and well over 90 per cent at 100 volts.

Induction Motor Slot Combinations

Rules to Predetermine Crawling, Vibration, Noise and Hooks in the Speed-Torque Curve

BY GABRIEL KRON[1]

Associate, A. I. E. E.

Synopsis.—*A theory of coexisting stationary and rotating slot
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magnitude and speed of the parasitic fluxes due to the slot
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Three rules are developed for the determination of destructive vibrations and noise, eight rules for establishing crawling speeds, and eight rules for finding hooks in the speed-torque curves of induction motors.

INTRODUCTION

INCE induction motors were first built it has been
noticed that some motors behave erratically;
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In the meantime engineers arrived at certain empiri-
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determine by actual tests which of these rotors was
e best.

In spite of the importance of the problem and the
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In the following pages the writer investigates the causes of the erratic behavior of these and other motors tested by him or by others and sets up rules to predetermine them. Many of the noisy regions, hooks and crawlings had been foretold by him from these rules and those were all found at the preassigned speeds without exceptions.

The slot openings are analyzed by a new method. The idea of "revolving permeances" is believed to be novel. By assuming a two pole flux as the first harmonic and the fundamental flux with p pairs of poles as the pth harmonic, the confusing use of "subharmonics" and "fractional harmonics" is avoided. With this notation and with the use of the general term of Fourier's series the rules and the formulas for speeds are expressed in simple forms easily remembered.

I—Harmonic Fluxes Due to Slot Openings

I—ASSUMPTIONS

As a first approximation, the theory of induction motors assumes a fundamental sine wave current-density distribution in space and in time. This is assumed to produce a sine wave magnetomotive force which in conjunction with a uniform permeance along the air-gap produces a sine wave flux-density distribution in time and in space. Refinements are usually introduced by assuming non-sinusoidal distribution of current density in time (time harmonics) or in space (space harmonics), still assuming constant permeance along the air-gap.

In the following an investigation will be made to determine (1) what fluxes exist in the presence of a fundamental sinusoidal current-density and a non-

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sinusoidal air-gap permeance and (2) how these fluxes interact.

The following assumptions are made besides the usual ones:

1. The slots are replaced by *effective* slots having rectangular shape. (Fig. 1a)
2. The lines of flux are parallel in the air.

II—THE REVOLVING PERMEANCES

First it is assumed that both the stator and rotor teeth are removed. The permeance of the resulting gap $\mathcal{P}_c$ is constant and is represented on Fig. 1b by unit height.

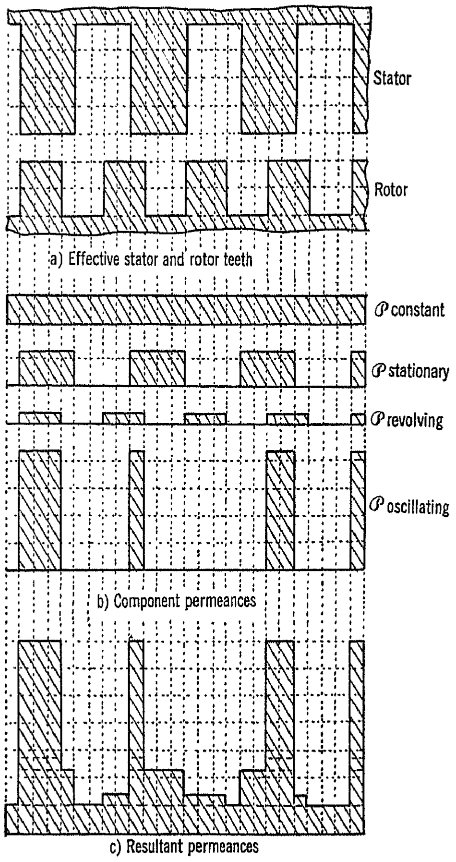

FIG. 1—PERMEANCES DUE TO SLOT OPENINGS

a. Effective stator and rotor teeth
b. Component permeances
c. Resultant permeance

Let the stator teeth be introduced. At certain places the length of the gap is decreased and the permeance increased. This change of permeance can be considered as if an additional permeance $\mathcal{P}_s$ consisting of rectangles had been introduced with the stator teeth. If the number of stator teeth is G_1, this permeance $\mathcal{P}_s$ can be analyzed into a constant term and a series of harmonic permeances, $\pm A_k \cos k G_1 x$ (See Appendix), all being the integer multiples of G_1. The largest of these is $A \cos G_1 x$, the harmonic having G_1 pairs of poles.

Let now the stator teeth be removed and the revolving rotor teeth, G_2 in number, be introduced. This is equivalent to introducing an additional revolving permeance $\mathcal{P}_r = \Sigma B_k \cos (k G_2 x - v k G_2 \omega t)$ where v is the ratio of the rotor speed to the two pole synchronous speed S. (S is 3,600 with 60 cycles.) The speed of each of these permeances is $v k G_2 / k G_2 = v$. The largest permeance is $B \cos (G_2 x - v G_2 \omega t)$, the one with G_2 pairs of poles.

If the stator teeth are *not* removed when the rotor teeth are introduced, at those points along the circumference where no stator teeth exist, the additional permeance due to the rotor teeth is $\mathcal{P}_r$, same as above, but at those points where stator teeth do exist, the additional permeance is larger than $\mathcal{P}_r$ by an amount $\mathcal{P}_o$. This permeance $\mathcal{P}_o$, due to the coexistence of slots, is much larger than $\mathcal{P}_s$ or $\mathcal{P}_r$. In decreasing the air-gap, $\mathcal{P}_s$ or $\mathcal{P}_r$ change very slightly, but $\mathcal{P}_o$ increases very rapidly.

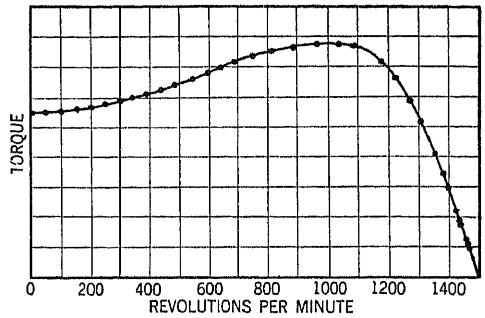

FIG. 2—SLOT RATIO 24/18—4-POLE—THREE-PHASE (STIEL)

Other smooth speed-torque curves tested:
Slot ratio 24/18—4, and 8 pole—two and three phase
Slot ratio 24/19—4, 6 and 8 pole—two and three phase
Slot ratio 24/20—6, and 8 pole—two and three phase
Slot ratio 24/22—4, 6 and 8 pole—two and three phase
Slot ratio 24/28—8 pole—three phase

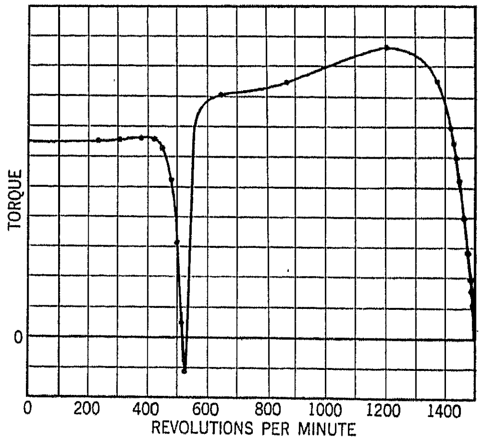

FIG. 3—SLOT RATIO 24/19—4-POLE—THREE-PHASE (STIEL)

As the rotor revolves, each part of $\mathcal{P}_o$ describes a peculiar cycle. The width of each rectangle increases from zero to a definite value, then it moves along the periphery, soon decreases and finally disappears. In spite of its complicated behavior it can easily be expressed in a Fourier's series by noting that if two series representing $\mathcal{P}_s$ and $\mathcal{P}_r$ are multiplied together the product represents the oscillating permeance $\mathcal{P}_o$. Hence

$$\mathcal{P}_o = \Sigma C_k \cos[(k_1 G_2 \pm k_2 G_2) x \pm v k_2 G_2 \omega t]$$

The largest among these revolving double-infinity harmonics are

$$\mathcal{P}_o = C_1 \cos[(G_1 - G_2) x + v G_2 \omega t] + C_2 \cos G_1 x + C_3 \cos(G_2 x - v G_2 \omega t)$$

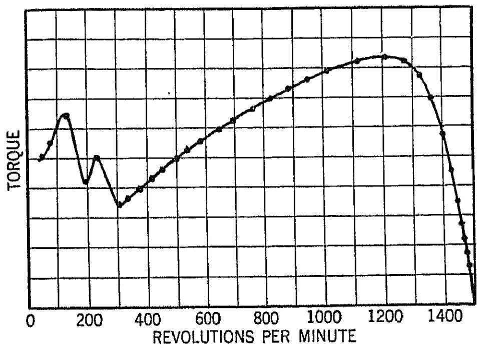

FIG. 4—SLOT RATIO 24/20—4-POLE—THREE-PHASE (STIEL)

With decreasing air-gap $\mathcal{P}_o$ increases rapidly and it may reach a value of the same order of magnitude as the constant gap. It is this very permeance which causes all the trouble.

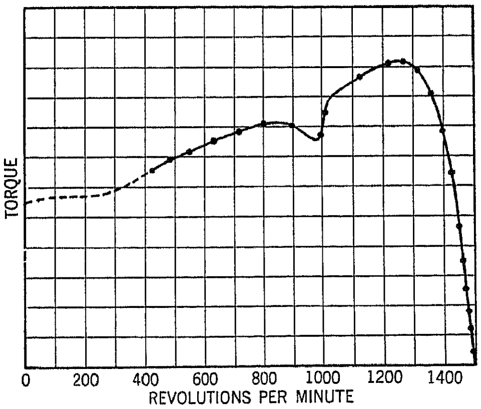

FIG. 5—SLOT RATIO 24/27—4-POLE—THREE-PHASE (STIEL)

Among the permeances only the largest ones, $\mathcal{P}_o$ with $(G_1 - G_2) = \pm D$ pairs of poles, $\mathcal{P}_s$ with G_1 and $\mathcal{P}_r$ with G_2 pairs of poles will be considered.

III—FLUXES

It is shown in the Appendix that whenever a revolving current density with "p_1" pairs of poles exists along an air-gap having a constant permeance plus a revolving permeance with "p_2" pairs of poles, it produces a revolving m. m. f. space distribution with "p_1" pairs of poles. This m. m. f. produces with the permeance "p_2" two revolving fluxes having $(p_1 + p_2)$ and $(p_1 - p_2)$ pairs of poles.

If $p_2 = p_1 = p$, the m. m. f. has in addition an oscillating term (under the assumptions). One revolving flux is producd with $2p$ pairs of poles and a single-phase flux with p pairs of poles in addition to the revolving fundamental flux.

Among the fluxes only those produced by $\mathcal{P}_o$ can effectively interact with the fundamental. The fundamental m. m. f., $M \cos(p x - \omega t)$ and $\mathcal{P}_o = C \cos(D x + v G_2 \omega t)$ produce fluxes $F \cos[(p \pm D)x \pm (v G_2 \pm 1) \omega t]$ with $(p \pm D)$ pairs of poles revolving with a speed $\pm S (v G_2 \pm 1)/(p \pm D)$. When $p \pm D = p$ or $p \pm 1$, powerful disturbing effects occur.

It should be noted that no assumptions had been made as to the number of phases or types of winding, hence all the rules in part I are independent of them.

IV—FIRST RULE

If the difference of slots is 1 or $2p \pm 1$, a flux is produced differing by two poles from the fundamental flux. An unbalanced force moves with a speed $(G_1 v) S$ or $[(G_1 \pm 2p) v \pm 2] S$ with respect to the rotor. When the speed of this unbalanced force approaches the critical speed of the rotor, noise and vibration occur accompanied by a diminution of torque or even crawling.

It has been shown by Fritze that whenever two fluxes differing by two poles exist, an unbalanced force moves with the difference of their frequencies. In these motors one of the fluxes is the fundamental, the other is a flux of the same order of magnitude as the fundamental, consequently the noise at the critical speed is so objectionable that whenever it occurs in the working range, the motor is usually useless for all purposes. The chances that the unbalanced force (which revolves sometimes G_1 times as fast as the rotor) reaches the critical speed before the rotor reaches its working speed, are greater with smaller p, larger G_1 and larger

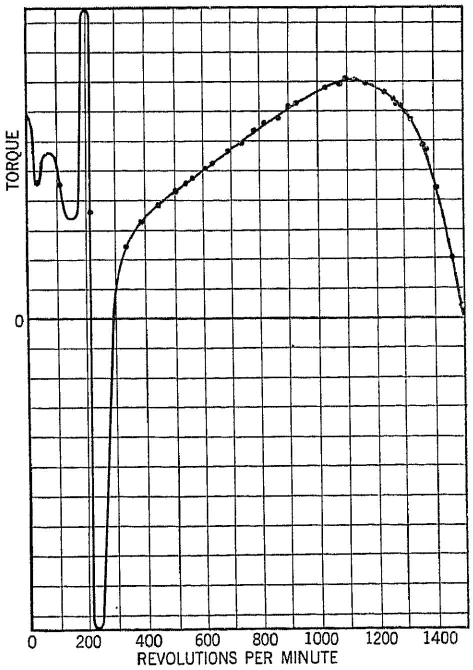

FIG. 6—SLOT RATIO 24/28—4-POLE—THREE-PHASE—UNSKEWED (STIEL)

rotors. Experiments also show that the severeness of the vibrations and the diminution of torque are proportional to the rotor current.

Experimental evidence

1. $G_1 = 24$, $G_2 = 19$, four-pole, three-phase, 50 cycle. Stiel shows its speed-torque curve on Fig. 3. At 500 r. p. m. its noise was an "indescribable wild roar." It also crawled at that speed.

The writer tested a similar motor on 60 cycles and the shrill noise and vibration occured in the neighborhood of 1,000 r. p. m. With decreasing frequency the speed of vibration slightly increased, just as required by the formula. The diminution of torque was not noticeable.

This same motor was tested on two-phase and single-phase showing exactly the same behavior as on three-phase.

As the closed rotor slots were gradually opened no change in the intensity of the noise was noticeable. This fact clearly shows that the assumption of rectangular rotor slots is fully justified, even with closed rotor slots.

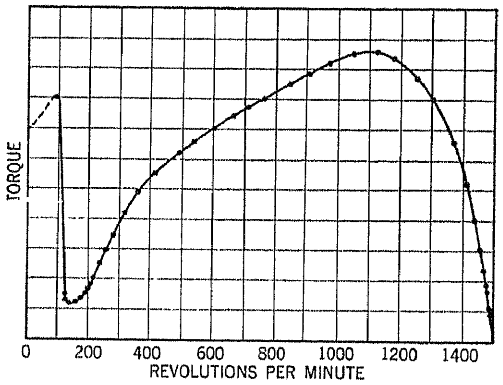

FIG. 7—SLOT RATIO 24/42—4-POLE—THREE-PHASE (STIEL)

2. $G_1 = 24$, $G_2 = 19$, six-pole. This was noisiest again in the neighborhood of 1,000 r. p. m. In both four-pole and six-pole motors the frequency of the backward revolving parasitic fluxes is the same, $(1 + 22\,v)$, hence the speed of the unbalanced force is also the same $(2 + 20\,v)$. (However in the four-pole motor it revolves backward, in the six-pole it revolves forward.) The same rotor on 8 poles was quiet.

3. $G_1 = 24$, $G_2 = 25$, four-pole, 50 cycle. Stiel describes two noisy regions, at 750 (or 700) and 900 r. p. m. Here a two-pole and a six-pole flux are produced. The frequency of the forward revolving six-pole flux is $(1 + 22\,v)$ and of the fundamental $(1 - 2\,v)$, hence the frequency of the unbalanced force is $24\,v$. The frequency of the forward revolving two-pole flux with respect of the rotor is $(1 - 26\,v)$, hence the frequency of the unbalanced force is again $24\,v$. The two unbalanced forces are identical. The motor did not crawl like the 19-slot rotor at 500 r. p. m.

4. $G_1 = 24$, $G_2 = 27$, four-pole. Stiel gives its speed-torque curve as Fig. 5. Noise and diminution of torque occurred at 1,000 r. p. m.

5. $G_1 = 24$, $G_2 = 29$, four-pole. Stiel found it useless due to noises.

6. $G_1 = 54$, $G_2 = 53$, six-pole. Stiel mentions that this motor is known to behave as the 24 25 combination.

7. $G_1 = 72$, $G_2 = 57$, 16-pole. It was used on a two-speed motor. When it was thrown over to low speed it became very noisy until it reached low speed. Then it ran quietly. It is to be expected that with motors having a large number of poles the noise would occur only if the rotor is moving above synchronous speed.

8. $G_1 = 36$, $G_2 = 29$, six-pole. It has been known to be very noisy.

9. $G_1 = 72$, $G_2 = 79$, eight-pole. It was very noisy.

10. $G_1 = 36$, $G_2 = 45$, eight-pole. It has been known to be noisy on various frames.

11. $G_1 = 48$, $G_2 = 43$, four-pole. Heubach mentions that it was noisy and vibrated badly.

12. $G_1 = 46$, $G_2 = 41$, four-pole, single-phase. (Heubach.) Noisy.

13. $G_1 = 60$, $G_2 = 65$, six-pole, two-phase. (Punga.) Noisy.

14. $G_1 = 54$, $G_2 = 47$, six-pole. (Punga.) Noisy.

15. $G_1 = 54$, $G_2 = 59$, six-pole. (Punga.) Noisy.

16. $G_1 = 36$, $G_2 = 41$, six-pole. Noisy.

17. $G_1 = 36$, $G_2 = 43$, six-pole. Noisy.

V—SECOND RULE

If the difference of slots is $2\,p$, a fundamental parasitic flux is set up revolving with a speed $s\,(1 \pm v\,G_2)/p$ rev. per min. A single-phase torque is set up with a frequency $v\,G_2$ or $(2 \pm v\,G_2)$. When the frequency of this torque is the same as the critical frequency of the rotor for torsional vibrations, noise and vibration is produced.

Rumbling noise is also set up when the speed of the parasitic flux is low. The cause of this rumbling noise requires more investigation.

Experimental evidence

1. $G_1 = 24$, $G_2 = 18$, six-pole. It was quiet below synchronous speed. When driven backward, it had a rumbling noise in the neighborhood of 200 r. p. m. When driven beyond synchronous speed as a generator, it became very noisy around double synchronous speed.

2. $G_1 = 24$, $G_2 = 28$, four-pole. It was noisy at all speeds, with the strongest rumbling noise on both sides of 140 r. p. m. It was tested on one-, two-, and three-phase.

3. $G_1 = 48$, $G_2 = 60$, 12-pole. It was very noisy at 95 r. p. m.

4. $G_1 = 36$, $G_2 = 30$, six-pole. It was known to be noisy.

5. $G_1 = 24$, $G_2 = 22$, two-pole. It was very noisy at low speeds.

VI—THIRD RULE

If the difference of slots is p, a single-phase flux with p pairs of poles exists (see Appendix) which sets up a single-phase torque with a frequency $2\,v\,G_2$ or $(2 \pm 2\,v\,G_2)$. When the frequency of this torque is the same as the critical frequency of the rotor for torsional vibrations, noise, and vibrations are produced.

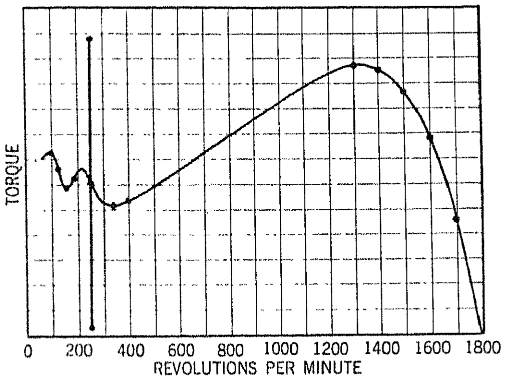

FIG. 8—SLOT RATIO 24/28—4-POLE—THREE-PHASE—SKEWED

Experimental evidence

1. $G_1 = 48$, $G_2 = 46$, four-pole. It was very noisy at all speeds.

2. $G_1 = 36$, $G_2 = 34$, four-pole. It was noisiest around 1,000 r. p. m. showing also signs of vibration.

3. $G_1 = 24$, $G_2 = 22$, four-pole. Stiel found this motor quiet. The writer also found a similar motor quiet, but when it was driven above synchronous speed it became very noisy in the neighborhood of 2,400 r. p. m. This combination has been known to be noisy below synchronous speed also.

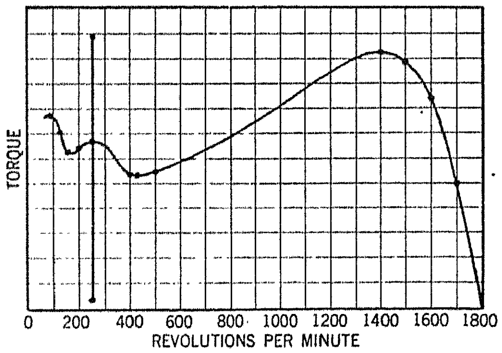

FIG. 9—SLOT RATIO 24/28—4-POLE—TWO-PHASE—SKEWED

4. $G_1 = 24$, $G_2 = 28$, eight-pole. Severe vibrations occurred above 1,800 r. p. m.

VII—FOURTH RULE

If the difference of slots is $2\,p$, at a rotor speed $(2/G_2)\,S$ the speed of the parasitic flux with p pairs of poles becomes that of the fundamental, the motor crawls and behaves just like a reaction machine. When G_2 is greater than G_1 this locking occurs in the working range; when smaller, the locking occurs at a negative speed.

Experimental evidence

1. $G_1 = 24$, $G_2 = 28$, four-pole, one-, two-, and three-phase. Figs. 8 and 9. The rotor was skewed one slot pitch, still the variation of torque while locked was considerable. (The synchronous speed was watched in all examples through a stroboscope.) Stiel did not know of the possibility of the existence of a superimposed reaction machine, so he plotted the vertical line as a hook in the speed-torque curve (Fig. 6). His motor was unskewed.

2. $G_1 = 48$, $G_2 = 60$, 12-pole (Fig. 10). The crawling occurred at $(2/60)\ 3{,}600 = 120$ r. p. m. It was skewed.

3. $G_1 = 36$, $G_2 = 42$, six-pole. Stiel mentions that this motor behaves as the 24/28 combinations. In both cases locking occurs at one-seventh synchronous speed and this fact gave the erroneous idea that they were caused by a powerful seventh harmonic of the stator m. m. f. wave.

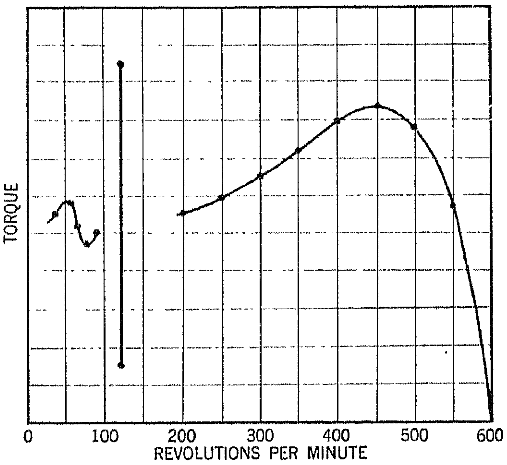

FIG. 10—SLOT RATIO 48/60—12-POLE—THREE-PHASE—SKEWED

4. $G_1 = 24$, $G_2 = 20$, four-pole. When driven backward or when the leads were reversed under load, the motor crawled at $(2/20)\ 3{,}600 = 360$ r. p. m.

5. $G_1 = 24$, $G_2 = 18$, six-pole. When driven backward it crawled at $(2/18)\ 3{,}600 = 400$ r. p. m.

6. $G_1 = 48$, $G_2 = 58$, 10-pole (Fig. 11). It crawled at $(2/58)\ 3{,}600 = 124$ r. p. m.

7. Any reaction machine is a particular case where $G_1 = 0$, $G_2 = 2\,p$, number of poles $= 2\,p$. The parasitic flux has $p - 2\,p = -\,p$ pairs of poles. When the speed of the rotor is $(2/2\,p)\,S = S/p$ r. p. m., the fundamental and the parasitic flux lock.

VIII—FIFTH RULE

When the rotor has more than $(G_1 + p)$ slots, the flux due to the stator slot opening produces a hook at $S/(G_1 + p)$ r. p. m. When the slots are more than $(G_1 - p)$, a hook occurs at $S/(G_1 - p)$ r. p. m. if driven backward.

The fundamental m. m. f. $A \cos\,(p\,x - \omega\,t)$ with p

pairs of poles and the stator slot opening $\mathcal{P}_s = B \cos G_1 x$ with G_1 pairs of poles produce two fluxes $C \cos [(G_1 + p)x - \omega t] + D \cos [(G_1 - p)x + \omega t]$ with $(G_1 + p)$ and $(G_1 - p)$ pairs of poles, the first revolving forward with a uniform speed $S/(G_1 + p)$ the second revolving backward with a uniform speed $S/(G_1 - p)$. The hook in the speed-torque curve is due to induction motor effect.

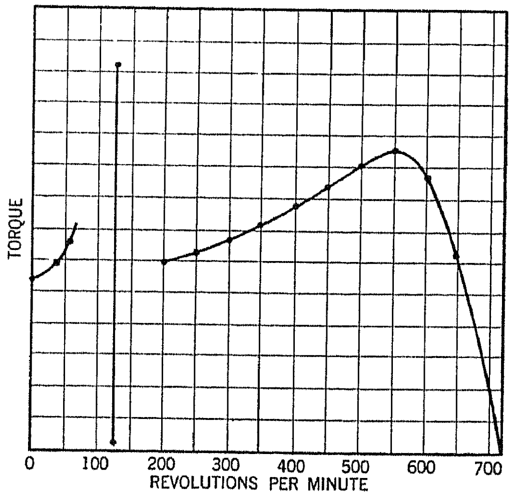

Fig. 11—Slot Ratio 48/58—10-Pole—Three-Phase—Skewed

When the harmonic flux has as many pairs of poles as there are rotor bars, no current can flow in the rotor, since the e. m. fs. add up to zero. As the difference increases, the rotor current and the resulting hook increases proportionally. When the difference between G_2 and $(G_1 + p)$ [or $(G_1 - p)$] is large, say 80 per cent, the hook is so great that the motor is useless, unless the motor is skewed. As the skew approachse $p/(p + G_1)$ times the circumference, the hook disappears proportionally.

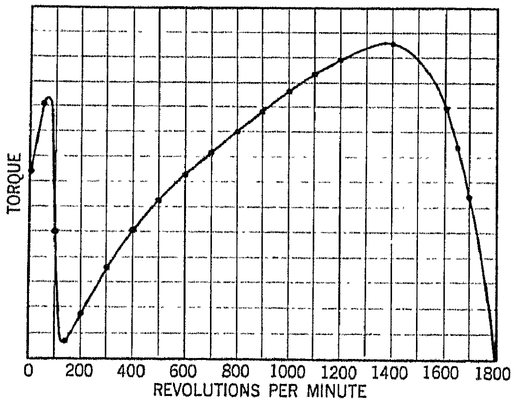

Fig. 12—Slot Ratio 36/62—4-Pole—Three-Phase

The greater number of hooks and crawlings which are observed in practise belong under this rule.

Experimental evidence

1. $G_1 = 24$, $G_2 = 42$, four-pole, unskewed. (Fig. 7.) A hook occurs at $3{,}600/(2 + 24) = 115$ r. p. m. Since the difference of slots is large and the motor is unskewed, the motor is useless.

2. $G_1 = 24$, $G_2 = 41$, four-pole, unskewed. Stiel mentions that this motor behaved as the 24/42 combination, and due to the hook it was also useless.

3. $G_1 = 36$, $G_2 = 62$, four-pole, unskewed. (Fig. 12.) The hook occurs at $3{,}600/(2 + 36) = 95$ r. p. m.

4. $G_1 = 36$, $G_2 = 58$, six-pole, unskewed. (Fig. 13.) The hook occurs at $3{,}600/(3 + 36) = 92$ r. p. m.

5. $G_1 = 12$, $G_2 = 19$, two-pole, 100-cycle. (Fig. 14.) The hook occurs at $6{,}000/(1 + 12) = 460$ r. p. m.

6. $G_1 = 48$, $G_2 = 60$, 12-pole. (Fig. 10.) The hook occurs at $3{,}600/(6 + 48) = 67$ r. p. m.

7. $G_1 = 36$, $G_2 = 57$, six-pole, two-phase. (Fig. 15.) The hook due to the forward revolving flux with $3 + 36 = 39$ pairs of poles occurs at $3{,}600/39 = 92$ r. p. m., while the hook due to the backward revolving flux with $36 - 3 = 33$ pairs of poles occurs at $3{,}600/33 = 109$ r. p. m.

8. $G_1 = 36$, $G_2 = 41$, six-pole, two-phase. (Fig. 16.) The two hooks occur again at 92 and 109 r. p. m.

9. In all motors tested where G_1 was less than $(G_1 + p)$ no trace of hook occurs at $S/(G_1 + p)$ r. p. m.

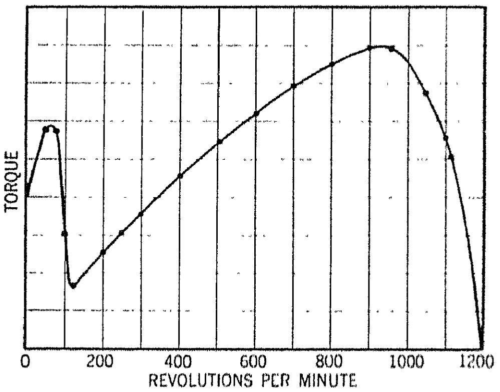

Fig. 13—Slot Ratio 36/38—6-Pole—Three-Phase

IX—Sixth Rule

When the rotor slots are divisible by the number of poles the flux due to the rotor slot opening causes a hook at S/G_2 r. p. m.

The fundamental m. m. f. and the rotor slot opening produce two fluxes with $(G_2 + p)$ and $(G_2 - p)$ pairs of poles revolving with a speed $S(v G_2 \pm 1)/(G_2 \pm p)$ with respect to the stator. These fluxes can induce e. m. fs. in the stator winding wound for p pairs of poles only if $(p \pm G_2)$ is an odd multiple of p. That is, when $p + G_2 = p(1 + 2k)$, or when $G_2/2p = k =$ integer. Since these fluxes stand still with respect to the stator at S/G_2, a hook occurs at that speed due to induction motor effect. Since the rotor slot openings are usually small, these hooks are small and do not cause crawling.

*Experimental evidence**

1. $G_1 = 24$, $G_2 = 20$, four-pole, two-phase. (Fig. 17.) A hook occurs at $3{,}600/20 = 180$ r. p. m.

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*See also the seventh rule.

2. $G_1 = 24$, $G_2 = 20$, four-pole, three-phase. (Fig. 4.) A hook occurs at 3,600/20 = 180 r. p. m.

3. $G_1 = 24$, $G_2 = 28$, four-pole, two-phase. (Fig. 9.) A hook occurs at 3,600/28 = 129 r. p. m.

4. $G_1 = 24$, $G_2 = 28$, four-pole, three-phase. (Figs. 6 and 8.) A hook occurs at 3,600/28 = 129 r. p. m.

5. All motors tested where G_2 is not divisible by p do not show any hook in the neighborhood of S/G_2 r. p. m.

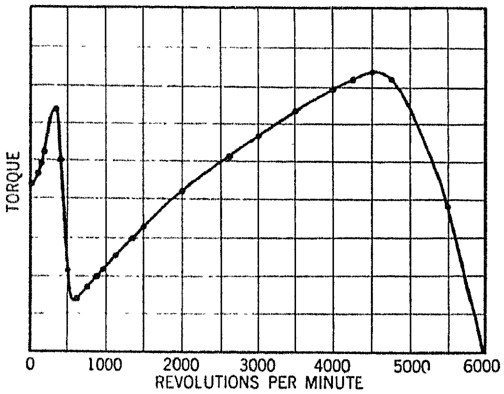

FIG. 14—SLOT RATIO 12/19—2-POLE—THREE-PHASE—100 CYCLE

6. In all self-starting synchronous motors $G_1 = 0$, $G_2 = 2\,p$. Here $p - 2\,p = -\,p$ is an odd multiple of p. Since the slot opening (the gap between the poles) is large, crawling may occur in the neighborhood of $S/2\,p$ = one-half synchronous speed.

II— Harmonic Fluxes due to Winding

X—HARMONIC M. M. FS.

So far it has been assumed that the air-gap is slotted and only the fundamental m. m. f. acts. Now it will be assumed that the air-gap is smooth and the harmonic m. m. fs. due to the distribution of winding also act.

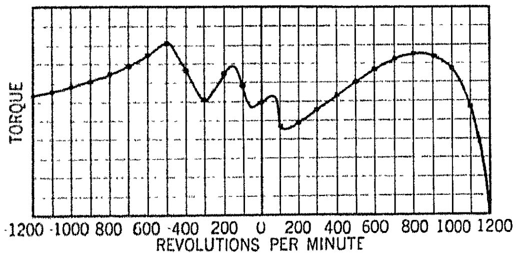

FIG. 15—SLOT RATIO 36/57—6-POLE—TWO-PHASE

It is well known that in a full-pitch stator winding with integer number of slots per pole phase group and with p pairs of poles the m. m. f. wave contains harmonics with $p \pm k\,\wp = n = p \pm k\,2\,p\,\psi$ pairs of poles (ψ = number of phases, $\wp$ = number of pole-phase groups) revolving with a speed S/n. It is also known that if a *squirrel-cage rotor* with G_2 slots is cut by a revolving flux with n pairs of poles the produced m. m. f. wave and flux have $(n \pm k\,G_2)$ pairs of poles revolving with a speed $S\,(1 \pm v\,k\,G_2)/(n \pm k\,G_2)$ with respect to the stator.

In a three-phase stator the *largest* forward revolving harmonic flux has $7\,p$ pairs of poles, in a two-phase motor it has $5\,p$ pairs of poles. In the rotor the largest harmonic m. m. fs. and fluxes have n, $(n + G_2)$ and $(n - G_2)$ pairs of poles and their magnitude is inversely proportional to the number of their poles.

XI—SEVENTH RULE

Whenever G_2 is divisible by the number of poles $2\,p$, a small hook occurs at $S/7\,p$ r. p. m. in a three-phase motor, at $S/5\,p$ in a two-phase motor and also at S/G_2 in both types, due to "internal concatenation" (cascade) effect of the harmonic fluxes.

If the harmonic flux due to the stator winding is small, say less than 8 per cent of the fundamental flux, the harmonic rotor current due to it is so small that the produced harmonic torque is negligible, provided the fluxes due to this harmonic rotor current can not induce an e. m. f. in the stator winding. If an e. m. f. can be induced in the stator winding, the stator acts as a short-circuited secondary to the harmonic rotor current, the rotor current is much larger and the resulting hook is noticeable.

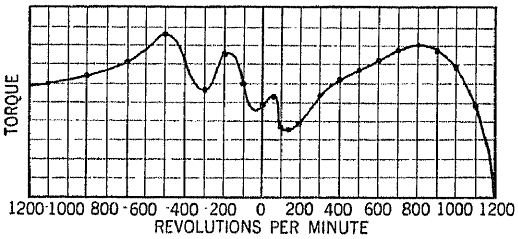

FIG. 16—SLOT RATIO 36/41—6-POLE—TWO-PHASE

Let the largest stator harmonic have $(p + 2\,p\,\psi)$ pairs of poles. When they cut the rotor, the rotor current produces fluxes with $(p + 2\,p\,\psi \pm G_2)$ pairs of poles. Now a flux can induce an e. m. f. in a winding wound for p pairs of poles only if the flux is an odd multiple of p, that is, if it has $p\,(1 + p\,k)$ pairs of poles. Hence, $(p + 2\,p\,\psi \pm G_2)$ is equal to $p\,(1 + p\,k)$ only if $G_2/2\,p = k$, that is if the number of rotor slots is divisible by the number of poles.

At $S/(p + 2\,p\,\psi)$ r. p. m. the stator harmonic flux stands still with respect to the rotor and the superimposed harmonic torque is zero. At S/G_2 r. p. m. all the rotor fluxes stand still with respect to the stator, the rotor current is small and the superimposed harmonic torque is again zero. At the neighborhood of these two speeds hooks occur in the speed-torque curves.

In three-phase motors the hook at S/G_2 is larger than the other as a consequence of the sixth rule. In two-phase motors however, the hook caused by the fifth harmonic is larger and under load it may even cause crawling.

Experimental evidence

1. $G_1 = 24$, $G_2 = 20$, four-pole, three-phase. (Fig. 4.) Small hooks occur at one-seventh and one-tenth synchronous speed.

2. $G_1 = 24$, $G_2 = 28$, four-pole, three-phase. (Figs. 6 and 8.) Hooks occur at one-seventh and one-fourteenth synchronous speeds.

3. $G_1 = 24$, $G_2 = 20$, two-pole, three-phase. (Fig. 19.) Hooks occur at 1/7 and 1/20 synchronous speed.

4. $G_1 = 24$, $G_2 = 18$, six-pole, three-phase. (Fig. 18.) The hooks at one-sixth and one-seventh synchronous speeds merge into one.

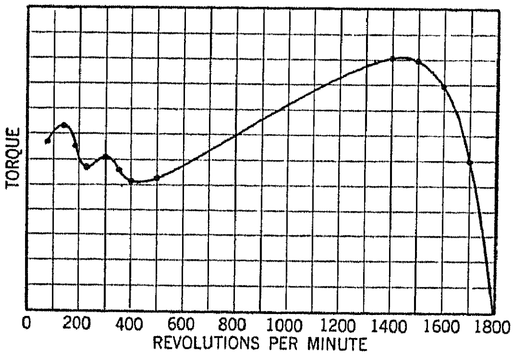

Fig. 17—Slot Ratio 24/20—4-Pole—Two-Phase—Skewed

5. $G_1 = 24$, $G_2 = 20$, four-pole, two-phase. (Fig. 17.) Hooks occur at one-fifth and one-tenth synchronous speeds.

6. $G_1 = 24$, $G_2 = 28$, four-pole, two-phase. (Fig. 9.) Hooks exist at one-fifth and one-fourteenth synchronous speeds.

7. All motors tested where G_2 was not divisible by the number of poles showed no sign of the presence of the harmonic m. m. f. waves.

XII—Eighth Rule

Whenever G_2 is divisible by the number of poles $2p$, a hook occurs at S/G_2 r. p. m. due to the internal cascade connection of the *fundamental* flux with the rotor harmonic fluxes. The smaller $G_2/2p$ is, the greater the hook, since the magnitude of the rotor harmonic fluxes is greater. When $G_2/2p = 1$ or $G_2 = 2p$, the produced harmonic flux $(p - G_2) = -p$ is as large as the fundamental flux and crawling occurs. Crawling may occur even if $G_2/2p$ is an integer close to unity.

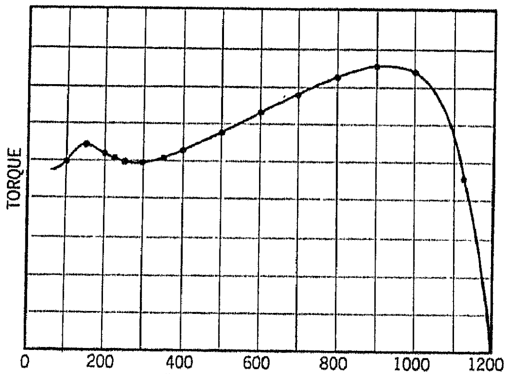

Fig. 18—Slot Ratio 24/18—6-Pole—Three-Phase

This phenomenon is known in the case of $G_2 = 2p$ as the "Görges effect" and it corresponds to a polyphase motor with single-phase rotor.

It should be noted that the hooks at S/G_2 r. p. m. are attributed to three different causes. They are attributed to the fluxes produced by (1) the fundamental m. m. f. and the rotor slot opening (this effect is noticeable especially in self-starting salient-pole synchronous motors), (2) the fundamental m. m. f. and the rotor currents (Görges effect), (3) the harmonic m. m. fs. and the rotor current (where the presence of a seventh or fifth harmonic is noticeable.)

XIII—Ninth Rule

It has been shown by Dreese that when the forward revolving stator m. m. fs. are the same as the backward revolving rotor m. m. fs. the fluxes produced by them lock at a certain speed. The locking occurs when $(p + k_1 2 p \psi) = -(p \pm G_2)$ or $G_2 = 2p(1 + k_1 \psi)$ and when the speeds (or frequencies) of the fluxes are the same, that is when $1 = -(1 \pm v k_2 G_2)$. From this $v = S(2/k_2 G_2)$.

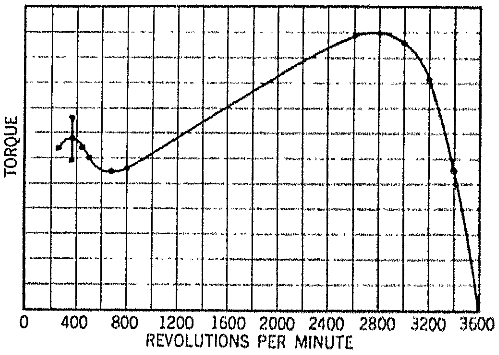

Fig. 19—Slot Ratio 24/20—2-Pole—Three-Phase Unskewed

It is shown in the fourth rule that locking occurs at $S(2/G_2)$ when $G_2 - G_1 = 2p$. In the case of a three-phase four-pole motor with 24 stator and 28 rotor slots (in the only example shown by Dreese) both of these rules apply. To find out whether the m. m. fs. or the permeances have the more powerful effect, the following tests were made.

A three-phase two-pole motor was tested with 24 stator and 20 rotor slots, unskewed, where the fourth rule does not apply, but only the ninth rule. By slowly approaching one-tenth speed a synchronous crawling was established after much trial and was watched through a stroboscope. But with slight variation of load (Fig. 19) the motor stepped out of synchronous speed, although it was not at all skewed. (In a skewed 24/20-two-phase four-pole motor this crawling could not be noticed. Fig. 17.)

Another three-phase 12-pole motor was tested with 48 stator and 60 rotor slots, where the ninth rule does not apply, but only the fourth rule. Although this motor has a full skew, it crawled at one-fifth synchronous speed if started under load and it crawled through a large variation of load as shown in Fig. 10.

XIV—TENTH RULE

Whenever a harmonic flux larger than say 8 per cent of the fundamental is produced by the winding, a hook occurs in the speed-torque curve by induction motor effect irrespective of the number of rotor slots.

In windings with integral number of slots per pole phase group, the only harmonic which causes a hook in the speed-torque curve irrespective of the number of rotor slots is the backward revolving third harmonic (with $3\,p$ pairs of poles) of the two-phase induction motor.

Experimental evidence

1. $G_1 = 36, G_2 = 57$, six-pole, two-phase. (Fig. 15.) The backward revolving third harmonic is pronounced, but no trace of the forward revolving fifth harmonic was found, since 57 is not divisible by 6.

2. $G_1 = 36, G_2 = 41$, six-pole, two-phase. (Fig. 16.) Here also the third harmonic is present, but the fifth harmonic is absent.

XV—SUMMARY OF RULES

A. Vibration and noise are liable to be present as follows:

1. When the slots differ by one or by the number of poles plus or minus one, transverse vibration and noise may occur.
2. When the slots differ by half the number of poles, torsional vibration and noise may occur.
3. When the slots differ by the number of poles, torsional vibration and noise may occur and also rumbling noises unaccompanied by critical vibrations.

The chances that the noise occurs in the working range are greater with smaller number of poles, higher speed and with smaller transverse or torsional critical speeds. When these noises do occur the motor is practically useless.

B. Crawling may be present as follows:

1. When the slots differ by the number of poles and the rotor has more slots than the stator, synchronous crawling (reaction machine effect) occurs at a speed $S\,(2/G_2)$. When the rotor has less slots the synchronous crawling occurs at a negative speed.
2. When the rotor slots are greater than $(G_1 + p)$ by, say, 80 per cent, crawling occurs at $S/(G_1 + p)$.
3. When the rotor slots are greater than $(G_1 - p)$ by, say, 80 per cent, crawling occurs at $S/(G_1 - p)$ if driven backward.
4. When the rotor slots are divisible by the number of poles, a two-phase motor may crawl at one-fifth synchronous speed it if is started under load.
5. A two-phase motor may crawl at one-third synchronous speed if it is driven backward under load.
6. When $G_2/2\,p$ is an integer equal to or close to unity, crawling may occur at S/G_2 r. p. m.
7. When the vibrations under A occur at low speeds, the motor crawls.
8. When the harmonic flux due to the winding is large, say larger than 10 per cent of the fundamental, the motor may crawl.

C. Smaller irregularities, hooks in the speed-torque curve, may appear as follows:

1. When the rotor slots are greater than $(G_1 + p)$, a hook occurs at $S\,(G_1 + p)$ r. p. m.
2. When the rotor slots are greater than $(G_1 - p)$, a hook occurs at $S/(G_1 - p)$ r. p. m. if it is driven backward.
3. When the rotor slots are divisible by the number of poles, a hook occurs at S/G_2 r. p. m.
4. When the number of rotor slots is divisible by the number of poles, a hook occurs at one-seventh synchronous speed in a three-phase motor and at one-fifth synchronous speed in a two-phase motor. In both cases the hook is accompanied by another hook at S/G_2 r. p. m.
5. When $G_2 = 2\,p\,(1 + k_1\,\psi)\,2\,p$, a small synchronous locking (synchronous motor effect) occurs at $S\,(2/k_2\,G_2)$ r. p. m.
6. Any two-phase motor has a hook at one-third synchronous speed if it is driven backward.
7. When a winding introduces a harmonic flux larger than, say, 8 per cent of the fundamental, it always produces a hook.
8. When the vibrations under A occur at higher speeds, proportionally smaller hooks occur at those speeds.

It should be emphasized that these rules are not necessarily complete. They only cover the cases known to the writer. The series of rules for smaller noises, which make the motor useless only for certain applications are not studied at all in this paper, neither are the variations of torque during starting and at very low speeds.

ACKNOWLEDGMENT

The writer acknowledges the helpful suggestions of Prof. E. E. Dreese during the investigation made in the Research Laboratories of the Lincoln Electric Co. The writer also wishes to thank Mr. J. F. Lincoln for permission to publish these rules.

NOMENCLATURE

p = number of pairs of poles of the stator winding
G_1 = number of stator slots
G_2 = number of rotor slots
D = difference of slots = $G_1 - G_2$
k = any positive integer
S = synchronous speed with two poles
v = ratio of rotor speed to two-pole synchronous speed
β = ratio of width of effective slot to tooth pitch
$\mathcal{P}_c$ = gap permeance without teeth

$\mathcal{P}_s$ = stationary permeance due to additional stator teeth

$\mathcal{P}_r$ = revolving permeance due to additional rotor teeth

$\mathcal{P}_o$ = Oscillating permeance due to coexistence of slots.

Appendix

PERMEANCE DUE TO REVOLVING TEETH

Calculations for the width and length of an effective tooth can be found in the writings of Carter, Lehmann and others.

Let the length of the whole air-gap be $2\,\pi$ radians, the number of slots be G and the ratio of the slot width to tooth pitch be β. The equation of the permeance curve of *one* revolving tooth (Fig. 20) is $f(x) = \mathcal{P}$ and the curve extends between the limits $2\,\pi/G + \omega\,t$ and $\pi\,(2-\beta)/G + \omega\,t$.

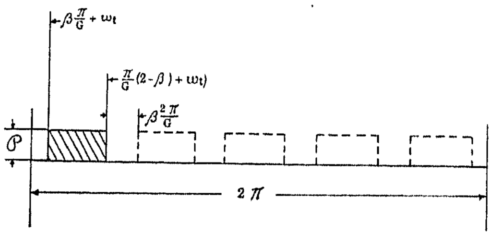
$\beta \frac{\pi}{G} + \omega_t$
$\frac{\pi}{G}(2-\beta) + \omega_t)$
$\beta \frac{2\pi}{G}$
$\mathcal{P}$
$2\,\pi$

FIG. 20—THE REVOLVING PERMEANCE

The coefficient of the cth harmonic *sine* term is

$$a_c = \frac{1}{\pi} \int_{\beta\pi/G+\omega t}^{\pi(2-\beta)/G+\omega t} \mathcal{P} \sin c\,x\,d\,x$$

$$= \frac{2\,\mathcal{P}}{c\,\pi} \sin \frac{c\,\pi}{G}\,(1-\beta)\,\sin\left(\frac{c\,\pi}{G} + c\,\omega\,t\right)$$

The coefficient of the cth harmonic *cosine* term is

$$b_c = \frac{1}{\pi} \int_{\beta\pi/G+\omega t}^{\pi(2-\beta)/G+\omega t} \mathcal{P} \cos c\,x\,d\,x$$

$$= \frac{2\,\mathcal{P}}{c\,\pi} \sin \frac{c\,\pi}{G}\,(1-\beta)\,\cos\left(\frac{c\,\pi}{G} + c\,\omega\,t\right)$$

The cth harmonic of the permeance curve is

$$\mathcal{P}_c = \frac{2\,\mathcal{P}}{c\,\pi} \sin \frac{c\,\pi}{G}\,(1-\beta)\,\cos\left(c\,x - c\,\omega\,t - \frac{c\,\pi}{G}\right)$$

There are G teeth displaced in space by $2\,\pi/G$ radians, hence the cth harmonic of the permeance curve is displaced by $2\,c\,\pi/G$ radians. When there are n sine waves displaced by α_1 the *magnitude* of the resultant is the magnitude of one component increased by $\sin n(\alpha/2)/\sin(\alpha/2)$, the *phase angle* of the resultant is $-\,(n-1)\,\alpha/2$.

Hence

The *phase angle* of the resultant is $(-\,c\,\pi + c\,\pi/G)$

The *magnitude* of the resultant is increased by $\sin(c\,\pi/\sin(c\,\pi/G)$.

The numerator is always zero, hence the expression is zero unless the denominator is also zero or $c/G = k$ = positive integer. (Fourier's series is valid only for positive harmonics.) Differentiating $\sin k\,G\,\pi/\sin k\,\pi = 0/0$, it gives $G \cos k\,G\,\pi/\cos k\,\pi$. Substituting,

$$\mathcal{P}_c = \pm \frac{2\,\mathcal{P}}{k\,\pi} \sin k\,\pi\,(1-\beta)\cos(k\,G\,x - k\,G\,\omega\,t)$$

$$+ A\cos(k\,G\,x - k\,G\,\omega\,t)$$

positive if k is even, negative if k is odd

a. The constant term is one-half of the found value if $k = 0$ is substituted. Constant permeance = $\mathcal{P}\,(1-\beta)$.

b. If $\omega = 0$ the *stationary* permeance is $\pm A \cos k\,G\,x$.

FLUX DUE TO CURRENT-DENSITY DISTRIBUTIONS

Let a sinusoidal current-density distribution with p pairs of poles be along the air-gap. Its equation is $i = A \sin(p_1\,x - \omega\,t)$.

The magnetomotive force acting along a closed path through the origin and a variable point x (Fig. 21).

$$\text{M. m. f.} = \int_0^x i\,d\,x = \frac{A}{p_1}\,[\cos \omega\,t - \cos(p_1\,x - \omega\,t)]$$

The magnetomotive force acting across the air-gap at any point x is

$$(\text{M. m. f.})_{xb} = (\text{M. m. f.})_{xbao} - (\text{M. m. f.})_{ao}$$
$$= (A/p_1)\,[\cos \omega\,t - \cos(p_1\,x - \omega\,t)] - M \quad (1)$$

Let the permeance at any point x be

$$\mathcal{P} = C + D \cos(p_2\,x + v\,G_2\,\omega\,t)$$

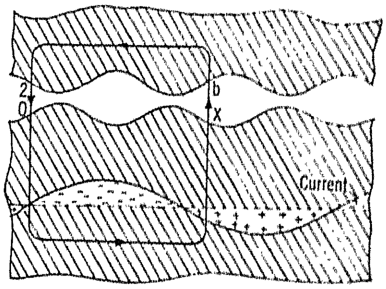

FIG. 21—M. M. F. DUE TO CURRENT

Then the flux density at any point k is

$$B = (A\,C/p_1)\cos \omega\,t - C\,M - (A\,D/2\,p_1)\cos[(p_1 - p_2)\,x +$$
$$+ (v\,G_2 + 1)\,\omega\,t] + (A\,D/2\,p_1)\cos[p_2\,x + (v\,G_2 + 1)\,\omega\,t] -$$
$$-\,(A\,D/2\,p_1)\cos[(p_1 + p_2)\,x + (v\,G_2 - 1)\,\omega\,t] -$$
$$-\,M\,D\cos(p_2\,x + v\,G_2\,\omega\,t) \quad (2)$$

But the space integral of the flux density from zero to $2\,\pi$ has to be equal to zero. This is possible only if in equation (2) the terms not containing x are zero. Two cases have to be distinguished.

1. When p_2 is different from p_1, the first two terms of equation (2) are zero, $(A\,C/p_1)\cos \omega\,t - C\,M = 0$. From this $M = (A/p_1)\cos \omega\,t$. If this value of M is substituted into equation (1)

$$(\text{M. m. f.})_{xb} = -\cos(p_1\,x - \omega\,t).$$

That is, the m. m. f. wave is the same as the current density wave.

2. When $p_2 = p_1$, $p_1 - p_2 = 0$ and the first three terms of equation (2) are zero. Finding the value for M and substituting into equation (1), it gives

$$(\text{M.m.f})_{xb} = (A\,D/2\,C\,p_1)\cos(v\,G_2+1)\ \omega\,t - \cos(p\,x - \omega\,t.)$$

That is, the m. m. f. wave has also an oscillating component besides the fundamental. If the value of M is substituted into equation (2) the resultant flux density is

$$\begin{aligned} B = (A\,D^2/4\,C\,p)\ \{ &\cos[\,p\,x + (2\,v\,G_2 + 1)\ \omega\,t] \\ &+ \cos(p\,x - \omega\,t)\ \} \\ &- (A\,C/p)\cos(p\,x - \omega\,t) \\ &- (A\,D/2\,p)\cos[2\,p\,x + (v\,G_2 - 1)\ \omega\,t] \end{aligned}$$

The first term represents a *single-phase* flux with p pairs of poles.

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Discussion

C. R. Boothby: Mr. Kron has given a very comprehensive set of rules by which irregularities in the speed-torque curves of induction motors may be predicted. He also gives a number of rules by which noise and vibration may be predicted.

The first inspection of these rules indicates that some difficulties would be encountered with almost any practicable slot combination. That the difficulties are not as great as one might interpret them, is amply illustrated by the speed-torque curves given in the paper. In many of the speed-torques shown, while indicating appreciable hooks, would not be unsatisfactory for most commercial applications. Combinations which are noisy at low speeds often are very quiet at normal operating speeds. In many of such cases the motor speeds up sufficiently fast that this noise is not objectionable.

It is, of course, desirable to avoid any combination which shows undesirable characteristics as to either noise or hooks. However, in some cases this is not possible due to other limitations imposed upon the designer. In such cases it is necessary to resort to other measures in order to reduce or eliminate the undesirable characteristic.

A hook in the speed-torque curve, when it occurs at a low speed, can be reduced to an inappreciable magnitude by skewing the rotor one pole pitch of the harmonic field causing the hook. In cases where two or more hooks exist at widely different speeds, it is obvious that they cannot both be eliminated by this method. A disadvantage resulting from skewing the rotor is the reduction in the break-down torque. This effect will be noticed in polyphase motors as well as single phase. It is due, in part at least, to an increase in leakage reactance. When the pull-off torque of a motor is low due to the presence of hooks it may be increased by making the rotor resistance larger. In the case of polyphase motors, torque at start and at low speed is increased, thus reducing the proportion of parasitic torque to fundamental field torque. The magnitude of the parasitic torque will not be reduced. In slip-ring motors that start with a high resistance in series with the rotor, hooks will, in general, be of such magnitude as to be negligible.

In the case of a single-phase induction motor started as a repulsion motor, the portion of the speed-torque curve below the switch operating speed is of no importance. In such cases slot combinations may be used which would be unsatisfactory for polyphase or split-phase motors. Such a case is the combination where the rotor and stator slots are divisible by the number of poles. Motors with this combination are often found to operate very satisfactorily.

It has been found that the synchronous motor effect described in rule VII can be reduced by skewing the rotor. However, it is seldom that it can be reduced to an extent as to make such a combination satisfactory.

H. C. Specht: The question of noise and cusps in the speed-torque curves of induction motors has been under discussion among designing engineers for many years and even today, we do not have a clear conception of all the factors involved. The writings of Dr. Stiel, Fritze, and Chapman, published about ten years ago, have added materially to the theory of this subject. Since that time, little has been added to the known theories.

Mr. Kron, in his paper, tries to present a clear picture by introducing a theory of revolving permeances. Whether or not this theory is more useful in simplifying this very complicated problem is somewhat doubtful. Moreover, the author considers the two-pole flux as the fundamental or first harmonic, regardless of the number of poles. This conveniently eliminates the use of sub-harmonics, *i. e.*, harmonics of a fractional order. However, on the other hand, this is apt to create some confusion in the minds of those accustomed to the conventional idea of harmonics.

When analyzing the phenomena of cusps in the speed-torque curve, a distinction should be made between induction motor action and synchronous motor action caused by the interlocking at certain speeds, including standstill. This latter mentioned action can occur only when the rotor slots become located symmetrically with respect to the stator slots, *i. e.*, when the resultant torque of all the rotor teeth under each pole is zero. By way of example may be mentioned the four-pole, 24-28 slot motor discussed in Prof. Dreese's paper. Mr. Kron in his paper, has given some more cases where such interlocking takes place at certain speeds, but does not discuss the case where interlocking at standstill occurs.

Mr. Kron gives ten rules for determining which of various slot combinations should be avoided. But when all these rules are applied, there seems to remain hardly any combinations which avoid undesirable cusps and noise. This is rather discouraging to a young designing engineer, leaving him wondering why someone does not tell him what combinations would be safe to use. As a matter of fact, there are hardly any slot combinations which do not show, at certain speeds, at least small cusps, although these may not be at all objectionable. The main problem, the one in which we are most interested today, is the building of motors which shall be as quiet as it is possible for them to be made, and which, moreover, are free from objectionable cusps.

In the earlier days, designers followed the practise of Tesla and Lamme in using a prime number of rotor slots. Application of this rule generally resulted in motors relatively free from dead points or positions of low torque. However, Dr. Stiel, Fritze and others have shown later that a prime number of rotor slots results in an unbalanced pull on the rotor and, therefore, a tendency towards noise and vibration. The modern art of design, therefore, avoids the use of prime numbers. If the number of rotor slots be even, especially if divisible by the number of poles, quieter operation will result than if the number were prime. However, the danger of obtaining cusps and low-torque points at starting is much greater. However, an increase of rotor slots, a proper skew, and sufficient rotor resistance will render the cusps and low-torque points harmless.

In addition to making the number of rotor slots divisible by the number of poles, it is advisable to make the difference between rotor and stator slots greater than the number of poles. For example, 36 stator slots and 48 rotor slots have proven quite satisfactory for four and six poles. But according to Mr. Kron's rules, this combination should be avoided. There are also other slot combinations which give better results than indicated by the author's rules. Possibly he bases his theories on motors which are not skewed. Normally, the rotors of all induction motors are, as they should be, skewed for the sake of quiet operation.

The author's rule number five seems too general. It does not hold for all cases where the number of rotor slots is more than the number of stator slots plus or minus the pole pairs. The slot combinations covered by rules 2, 4, 6, 7, and 8 are identical. Why not combine the cusps due to the various causes under one heading.

In the Appendix, the author gives some mathematical equations but he does not show how these can be applied to calculate crawling effects and the magnitude of cusps. Moreover, he does not show very clearly how his ten rules were derived from his mathematical work. If this were done, there is certainly no doubt but what the reader would profit a great deal more from the valuable data given by the author.

It is a well-known fact that the factor which has the most important bearing upon quiet operation is the slot combination. Nevertheless, there are many other factors which should be taken into consideration such as, flux density, length of air gap, skew, mechanical construction, etc. Before the question of cusps and noise is fully solved, much more work must be done along these lines.

L. A. Doggett: Mr. Kron deserves a great deal of credit for attacking this elusive problem. His analysis leads to a set of rules. It is hoped that Mr. Kron will go one step further and give us a few tables, recommending the number of slots for the more common cases. On trying to do this for the case of a two-pole three-phase machine, it was found that all possible values of rotor slot numbers were eliminated by one or more of Mr. Kron's ten rules. In the case of the four-pole three-phase machine, there were very few slot numbers that survived Mr. Kron's rules of elimination. Incidentally most of the rotor slot numbers, recommended by authors of design books, are eliminated by one or more of Mr. Kron's rules.

Although perfectly obvious and long well known, one more rule should be added for completeness. The number of rotor slots should not be equal to the number of stator slots, *i. e.*, $G_1 \neq G_2$.

Gabriel Kron: In this paper no attempt was made to recommend satisfactory slot combinations, only to determine what combinations are objectionable. The available experimental data are yet too meager to establish *all* the possible causes of irregular behavior, hence any rule recommending certain combinations may prove disastrous for some designer. The writer has in mind Stiel's recommendation of the 4-pole, 24-slot stator with a 22-slot rotor that Stiel found quiet and free from hooks. The writer also found a similar combination quiet, but a 4-pole motor with 36-stator and 34-rotor slots started up as a fire-syren. This experience resulted in rule number three and in the mathematical development in the second half of the Appendix. Later it was found that other companies had similar troubles with the 4-pole 24/22 combination itself. The rules should not be considered complete since there are other causes of noise and crawlings besides those considered.

As Mr. Boothby indicates, a large part of the irregularities, covered by rules VI-X, are insignificant, while the larger irregularities, covered by rules I-V, can be at least partially skewed out. However, when skewing is resorted to, the fundamental flux decreases and the pull-out torque is lowered, since the harmonic flux is not damped out.

Mr. Specht doubts the usefulness of introducing a new method of reasoning or assuming new units. The use of the "revolving permeances" the assumption of a two-pole flux as a first harmonic, the speed of the two-pole flux as unit speed, etc., can be appreciated, especially when one actually has to analyze a large number of seemingly contradictory experimental data for which no explanation has yet been given. Of course, after the explanation is found, any other reasoning or any other assumption may be used to get the same results. The writer acknowledges that it involves a certain amount of drudgery to set up the equations of the m. m. fs. permeances and fluxes for different slot-combinations, to follow the inter-relation of fluxes and to see the rules clearly, but there would be much more confusion if the cosine terms would involve fractions, or still worse, if a pictorial representation of the possible fluxes would be attempted for each m. m. f. wave with each possible slot combination and for each possible position of the rotor.

The rules have been worked out for unskewed motors. The first part of the Appendix gives the equation for the speed and the number of poles of the revolving permeances. The determination of the actual values of the fluxes and hooks is a subject which this paper does not touch at all. The writer was chiefly interested in the different *causes* of hooks, hence the rules are grouped according to causes and not according to slot combinations. Rules II and V refer to a difference of 2 p slots, while rules VI-VIII refer to rotors whose number of slots are divisible by 2 p, hence the possible slot combinations are not identical in these two cases, as assumed by Mr. Specht. Rule number five is general. In at least fifty different unskewed slot combinations this rule has been found to hold true without a single exception.

Prof. Doggett finds that few slot combinations survive the rules. Experimental data collected since the paper was submitted show that there are no slot combinations whose speed-torque curves do not have a considerable number of hooks. However, usually one has to worry about only the first five rules, covering ruinous irregularities caused by the fundamental flux. As far as the writer is aware, not one single rule recommended in standard design books is based on the analysis of the problem, only on experience. The problem of cogging, which is worse when $G_2 = G_1$ has not been mentioned in the paper due to the lack of experimental data.

It seems that before any definite rules can be arrived at for the selection of good slot combinations, more experimental data must be collected and published by manufacturing companies and engineering colleges.

The Pulling into Step of a Salient-Pole Synchronous Motor

BY HAROLD E. EDGERTON* and PAUL FOURMARIER†
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—A synchronous motor started as an induction motor must pass through an electro-mechanical transient after the field circuit is supplied with electrical excitation in order to change its action from that of an induction motor to that of a synchronous motor. The calculation of this problem has been difficult because the differential equation of motion is of a non-linear type of which no analytical solution has been made.

The integraph at the Massachusetts Institute of Technology has given a new and direct method of solving equations of the type encountered in these pulling-into-step transients. In this paper are compiled the results of many integraph solutions which were made after combining the factors that influence the performance of such machines into ratios and relative factors so that the results will apply to all practical cases. From the solutions several generalizations are made which can be directly applied to determine whether or not any type of salient-pole synchronous motor will synchronize properly.

These solutions take into account inertia, saliency, incompleteness of amortisseur windings, load, switching angle, and field current, and assume negligible armature resistance, negligible rotor leakage reactance, no saturation, constant load torque near synchronous speed, and negligible electrical transients. Oscillograph tests of a 160-hp. motor and a sample calculation are given.

I. INTRODUCTION

INDUCTION-MOTOR action is used to start synchronous motors and from the highest speed attainable from the induction action the mechanical and electrical torques, resulting from the application of field current, tend to accelerate and synchronize the rotor, that is, pull it into step. Steinmetz[1] in a discussion on this subject said, "....the least known or available in the literature are the phenomena of pulling into step..(of a) synchronous motor in that range of speed where it is not an induction motor anymore and where it is not yet a synchronous motor." Lamme,[2] about this same time, said of the problem, "It is difficult to 'see' just what is going on in the motor at the instant it pulls into synchronism."

Exact mathematical analysis of the pulling-into-step phenomena of even the simplest types of synchronous motors requires the solution of a non-linear differential equation. This difficult problem has been solved by approximate methods from a consideration of energy and by step-by-step analyses. Carr,[3] Cotton,[4] Dreyfus,[5] Böhm,[6] Fraenckel,[7] Lindstrom,[18] and others have published important papers treating this subject.

The advent of the integraph at the Massachusetts Institute of Technology several years ago made it possible for the first time to obtain rapidly solutions of non-linear differential equations of the type encountered in describing the motion of a synchronous motor during the pulling-into-step process. The results of investigations,[8,9] which utilized the integraph have recently appeared treating the round-rotor type of motor.

Solutions of the pulling-into-step transients of the salient-pole motor are given in this paper.

Before starting the mathematical solution of the pulling-into-step problem a discussion of the various factors that influence the phenomena will be given.

II. FACTORS INFLUENCING PULLING-INTO-STEP CHARACTERISTICS

Salient-pole synchronous motors are usually started as induction motors using amortisseur windings in the pole faces as the induction-motor rotor circuits. The field winding is commonly short-circuited through a resistance before the field current is applied.

Prior to the application of the field the speed of the motor will reach equilibrium at such a speed that the average induction-motor torque just balances the shaft load torque, and the windage and friction torques. An induction motor with a polyphase rotor having a uniform air gap will run at a constant speed if the load is constant. A salient-pole motor does not run at a constant speed for two reasons: first, it does not have a balanced polyphase amortisseur winding and secondly, the reluctance of the air gap is not uniform. Under these conditions the torque developed by the amortisseur winding due to induction motor or generator action does not depend upon the slip alone but also upon the angular displacement between the amortisseurs and the revolving stator field. Thus as the rotor slips the induction motor torque pulsates giving alternately an excess and a deficiency of torque which in turn causes variations in the slip. The salient-poles, due to their reluctance effect, always tend to place themselves so that the rotating magnetic field will be directly opposite them. As the rotor slips this reluctance torque tends alternately to accelerate and retard the rotor, causing slip variations. Summarizing—a salient-pole motor runs at such an average slip that its average torque equals the load torque. The degree of variation of the slip depends upon the closeness with which the amortisseur winding approaches a polyphase winding, upon the moment of inertia of the rotating masses, and upon the size of the reluctance torque.

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[1] For references see Bibliography.

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In case the shaft-load torque is relatively small, the motor may pull into synchronism and run as a "reluctance motor" with neither field current nor induced currents in the amortisseur bars. This interesting case is not considered in this paper since it occurs only when the shaft load is relatively small.

After a synchronous motor reaches its maximum induction-motor speed the field circuit is connected to an exciter or to a supply of direct current. The motor either pulls into synchronism or continues to slip but with violent current power and slip pulsations. The object of this paper is to demonstrate a means of computing these phenomena and to give a method of predicting the necessary conditions for satisfactory pulling into step.

A consideration of pulling-into-step problems in general for salient-pole motors shows that there are seven important factors which influence the phenomena. These will now be discussed separately.

1. The amount of load torque that is on the shaft is an important factor. When the machine is operating as an induction motor a considerable slip is required to supply a large shaft load. Consequently more kinetic energy is required to raise the speed of the rotor to synchronism than is required for smaller values of shaft load. The source of this energy is the excess of the sum of the induction and synchronous torques over that required to supply the load. The fact that a motor with a large shaft load may not pull into step is recognized by all.

2. Another factor is the maximum synchronizing torque available after the field is connected. The field current and the maximum synchronizing torque are nearly proportional for most motors. The greater this maximum torque the greater is the chance for pulling into step. The synchronizing torque also is indirectly effective through its effect on the natural frequency of mechanical oscillation. This effect is taken into account mathematically in a dimensionless equation of motion by the relative damping factor, k, which varies inversely as the square root of the maximum synchronizing torque (see equation (3) and the paragraph following it).

3. The amount of inertia of the rotor and the connected rotating masses determines the torque required to accelerate these parts from subsynchronous to synchronous speed during the synchronizing period. It also affects the damping in the same manner as does the maximum synchronizing torque by affecting the natural period of the angular oscillation.

4. A very important factor is the effectiveness of the amortisseur windings and the field circuit as an induction-motor rotor. For a given load torque the steepest slip-torque characteristic brings the rotor the closest to synchronous speed and it follows, for the same reason as stated in factor 1, the chance for pulling into step is more favorable. Besides making the slip smaller at the beginning of synchronizing period the greater induction-motor torque damps out the angular oscillations more quickly and thus tends to prevent these from becoming violent. These amortisseur requirements for good synchronizing qualities may conflict with those for high-starting torques.

5. Another factor is the angle between the terminal voltage and that induced by the d-c. field current which exists when the exciter is connected. If the exciter is connected when the angle is such as to cause generator action, the slip at first increases and the machine begins to depart further from synchronism, thus increasing the difficulty of subsequent synchronization.

6. The reluctance torque is another factor. This torque is a function of angular displacement as has already been discussed and causes the slip to vary before the field is connected. Afterwards it aids the motor to pull into step in some cases and influences somewhat the form of the angular transients.

7. The degree of unbalance in the induction motor torque due to the incompleteness of the amortisseur winding influences the pulling into step of a salient-pole synchronous motor by causing the initial slip to vary as previously discussed and by causing the damping to be a function of the angular displacement as well as of the slip during the transient.

It should be pointed out that factors 6 and 7 are not independent of each other. For instance the damping from a complete amortisseur winding is a variable with angle because of the variable air gap. The conclusions that are made in the paragraph preceding equation (4) state that neither of these factors greatly influences the criterion for pulling into step.

These seven major factors all appear in the terms of the differential equation of torques and solutions of this equation show their influence.

III. Differential Equation of Motion

The differential equation of motion of a salient-pole synchronous motor is generally given in approximately the following form, assuming negligible armature resistance, negligible rotor leakage reactance, no saturation of the magnetic materials, constant load torque near synchronous speed, and negligible electrical transients. The last assumption is justified by the high speed of the electrical transients in comparison with the relatively slow electro-mechanical transients.

$$P_i \frac{d^2 \theta}{d t^2} + P_d (1 - b \cos 2 \theta) \frac{d \theta}{d t} + P_R \sin 2 \theta + (P_m \sin \theta)\, 1 = P_L \qquad (1)$$

θ is the angle between the terminal voltage in one phase and the voltage induced in the same phase by the d-c. excitation. It is the mechanical angle (measured in electrical degrees) between the axis of a field pole and the axis of a corresponding imaginary field pole which would induce a voltage, with the armature open-circuited, at the terminals in phase with the applied voltage. θ would be zero in a lossless synchronous motor operating at no load. It can be taken as positive for either motor or generator action without altering the equation. In what follows θ is taken as positive for motor action. The units used by the authors are stated in part V, although any other consistent set of units can be used.

The first term, $P_j \dfrac{d^2\,\theta}{d\,t^2}$ represents the torque that results from acceleration of the rotor and the connected rotating masses of the load, where P_j is a coefficient proportional to the moment of inertia.

The second term is proportional to the slip since the slip is proportional to $\dfrac{d\,\theta}{d\,t}$. In contrast with the ordinary induction motor term for balanced polyphase rotor circuits the coefficient for this term is a function of angle. By methods such as employed by Park,[10] Linville,[11] or Putman[12] it can be shown that this function can be closely approximated by the form $(1 - b\cos 2\,\theta)$. This variation of the damping or induction motor effect is due to electrical unbalance of the rotor caused by incomplete amortisseur windings, by the field circuit, and by the non-uniform air gap. The double angle appears because of the symmetry about each pole instead of each pair of poles.

$P_R \sin 2\,\theta$ is the reluctance torque term. This torque is due to the tendency of the salient-poles to place themselves in alinement with the maximum magnetomotive force of the stator rotating magnetic field. The double angle appears here as in the damping term and for the same reason, namely, because the poles react individually and not in pairs as there is no d-c. excitation. The two-reaction theory[13] gives this term in units of synchronous torque as:

$$P_R = \frac{3\,(x_d - \overset{x_q}{x_d})}{(2\,x_d\,x\ )}\,\frac{V^2}{1000}\ \text{kw.}$$

where x_d is the synchronous reactance in the direct axis and x_q is the synchronous reactance in the quadrature axis, both per phase. V is the applied stator voltage per phase.

The synchronizing effect of the field current is expressed by $(P_m \sin\theta)\,\mathbf{1}$ in equation (**1**) where Heaviside's unit function, **1**, states the boundary conditions. It signifies the sudden appearance of the term $(P_m \sin\theta)$ at zero time. Before the exciter is connected there is no synchronizing torque except that due to reluctance. After the exciter is connected the synchronizing torque immediately appears neglecting a short electrical transient. The unit function is zero before and unity after $t = 0$, which is taken as the time of closing the exciter circuit. The numerical value of P_m is determined by $P_m = \dfrac{3\,E\,V}{x_d}\,\dfrac{\text{kw.}}{1000}$ (for a three-phase machine) which shows that it is proportional both to the terminal voltage and the field current. It is inversely proportional to the direct synchronous reactance. E is the induced electromotive force per phase in the armature due to the direct-current field excitation.

Equation (**1**) will be reduced to a more convenient form by dividing by P_m and by changing the time variable by the following relationship. Let $t = \lambda\,a$ where λ is a new variable and "a" is a constant to be determined. The derivatives with respect to the new variable λ in terms of those with respect to the original variable t are:

$$\frac{d}{d\,\lambda} = a\,\frac{d}{d\,t} \qquad \frac{d^2}{d\,\lambda^2} = a^2\,\frac{d^2}{d\,t^2}$$

Replacing t in equation (**1**) by λ gives:

$$\frac{P_j}{a^2\,P_m}\,\frac{d^2\,\theta}{d\,\lambda^2} + \frac{P_d}{a\,P_m}\,(1 - b\cos 2\,\theta)\,\frac{d\,\theta}{d\,\lambda} + \frac{P_R}{P_m}\sin 2\,\theta + (\sin\theta)\,\mathbf{1} = \frac{P_L}{P_m} \qquad (\mathbf{2})$$

The undetermined factor "a" is selected so that for any values of the coefficients in equation (**1**), the acceleration term of equation (**2**) has a unit coefficient. To do this equate the coefficient of this term to unity and solve for "a".

$$\frac{P_j}{a^2\,P_m} = 1 \qquad \therefore \qquad a = \sqrt{\frac{P_j}{P_m}}$$

This value of "a" substituted into equation (**2**) gives:

$$\frac{d^2\,\theta}{d\,\lambda^2} + k\,(1 - b\cos 2\,\theta)\,\frac{d\,\theta}{d\,\lambda} + \frac{P_R}{P_m}\sin 2\,\theta + (\sin\theta)\,\mathbf{1} = \frac{P_L}{P_m} \qquad (3)$$

where $k = P_d/\sqrt{P_j\,P_m}$; k is called the "relative damping coefficient" and is a very significant quantity. All machines having any given value of k respond with similar electro-magnetic transients when similar disturbances are imposed upon them, regardless of the size of the machines.

All motors, large or small, with large or small inertias, swing with the same period of oscillation in terms of the new variable, λ, which it will be noticed is dimensionless; λ is converted to time in seconds for any numerical case by multiplying by "a", *i. e.*, $t = \lambda\,a$.

In other words changing the variable from t to λ stretches or shrinks the time scale so that all motors having equal relative damping coefficients, values of b, and P_R/P_m have similar solutions of the same period. This transformation does not reduce the generality of the equation but does reduce the number of coefficients and thus aids materially in obtaining a solution covering all cases.

The M. I. T. integraph solves the differential equation of motion (3) for both the initial steady-state solution and the transient. The methods that it uses to achieve this result have been adequately described elsewhere.[8, 9, 11–17] However in this problem the integraph was used in a way slightly different from that previously described because of advantages that could be gained. Appendix I gives a brief description of the method of solving the pulling-into-step problem using the newer method. This method is also included in this paper so that those who are not familiar with this method of solving differential equations will be able to realize the directness and novelty of the integraph solutions.

IV. SOLUTIONS OF THE DIFFERENTIAL EQUATION OF MOTION AND THEIR CLASSIFICATION

The solutions assume that the motor is running in a steady-state condition as an induction motor at the time the exciter switch is closed. This means that the differential equation must be solved for a steady-state condition without the synchronizing term. As this induction-motor steady state is itself an oscillation due to the variable induction-motor torque and variable-reluctance torque, the initial conditions for the transient caused by applying the field current depend upon the part of the oscillation at which this switching occurs. As stated in the assumptions no allowance is made in the solutions for the time lag introduced by the electrical transients. These are usually over by the time the rotor has changed its angular position significantly because the angular transients are relatively slow compared to the electrical. However, the electrical transients in the stator and rotor certainly do influence the electrical torques and this effect is undoubtedly appreciable in large motors. Likewise no allowance is made for differences of the induction motor effects before and after applying the field current. The damping characteristic will be radically changed if the external impedance in the field circuit is changed very much during the switching operations. Usually the external impedance is considerably reduced when the exciter is connected, resulting in a beneficial effect.

The integraph made plots of angle against slip which satisfied the requirements of the differential equation of motion. Fig. 1 shows the form of these solutions. The curve *A B J* is the steady-state induction motor solution with no field excitation. The variation of the slip is due to the disturbing effects of unbalance in the rotor winding and the salient poles. Its magnitude

depends upon $k, \dfrac{P_R}{P_m}, b,$ and $\dfrac{P_L}{P_m}$.

Consider as an example the case where the field switch is closed at an angle of 120 degrees. The motor action of the synchronous torque is very large at that point and the slip decreases very rapidly as indicated by the steepness of the curve from *B* to *C*. At about the point *C* the sum of the synchronous and reluctance torques is equal to the load torque, the only remaining torque being that due to induction motor action which continues to accelerate the rotor until the point *D* is reached at which the sum of the slightly decreased synchronous and reluctance torques and the induction motor torque is just equal to the load torque. At *D* then the speed (and slip) are constant, but not at their synchronous values and θ therefore continues to increase, with a resulting decrease of net accelerating torque due to further loss of synchronizing and reluctance torques. Obviously the machine cannot synchronize at this minimum of slip and the rotor must slide along at least one pole of the armature field before again having an opportunity to pull into step.

The slip reaches a maximum in the region of zero angle. From this angle on, the motor action of the synchronous torque increases rapidly and the slip decreases correspondingly reaching zero at *F*. At that moment the motor is running at synchronous speed and is exactly in the situation of the same synchronous motor whose rotor is displaced from its steady-state normal position at *M*. The slip-angle curve from this point on is the curve of the free damped oscillation with constant load. This curve is a sort of spiral in the slip-angle plane turning around the final steady-state angle at *M*.

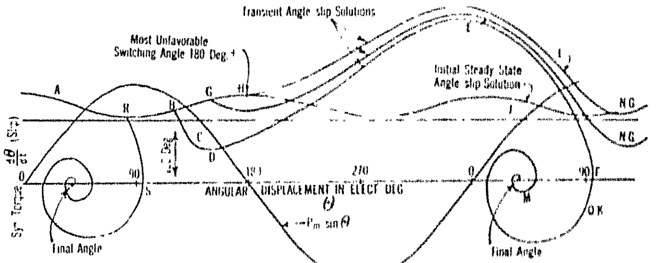

FIG. 1—TYPICAL INTEGRAPH ANGLE-SLIP SOLUTIONS OF THE EQUATION OF MOTION OF A SYNCHRONOUS MOTOR DURING PULLING-INTO-STEP TRANSIENTS

If the switching angle is more favorable, the slip will decrease to zero immediately and spiral about the steady-state operating angle, following a curve such as *R S*.

Should the relative damping coefficient be small, the switching angle unfavorable, and the initial slip large, the motor may not pull into step on the second possible pole but continue to slip many poles with violent variations in the slip. The motor may synchronize after several swings or possibly not at all. In either case the operation is not satisfactory.

For the classification of the integraph solutions for this paper the criterion of pulling into step was whether or not the motor synchronized within the first two opportunities as shown in the two characteristic swings, Fig. 1, that have been described. A solution such as curve *G-E* (Fig. 1) is not satisfactory although it may show synchronization within the next 360 deg.

When a machine is running as an induction motor, it is difficult to connect the exciter to the field at a given angle. For this analysis only the worst angle of switching will be considered, since synchronous motors are ordinarily designed so as to be capable of synchronizing when the field switch is closed independent of the slip and the angular position of the rotor. A series of solutions has definitely shown that the most unfavorable switching angle was always very close to 180 deg. At its exact value the slip-angle solution departs farthest from synchronous speed. (See solution *H E L*, Fig. 1). This curve is tangent to the initial steady-state induction-motor angle-slip solution and thus does not intersect the initial steady-state solution at two values of angle as does the solution marked *G E*.

All other factors being constant, the larger the value of *k*, the more favorable are the chances of pulling into step. This is because a large damping or induction motor effect, besides bringing the rotor to a higher initial speed, helps to damp out the violent angular oscillations and thus aids the synchronizing processes. In fact if *k* is sufficiently large a motor may synchronize even if the load ratio is unity.

The procedure for the integraph solutions was as follows. Numerical values of b, $\dfrac{P_R}{P_m}$, and $\dfrac{P_L}{P_m}$ were selected so that they were in the range of values that apply to practical cases. Then an integraph solution for the worst angle of switching (such as *H*, Fig. 1) was made with a numerical value of *k*. If the slip-angle curve showed that the motor did not pull into step (such as *H L* in Fig. 1) this fact was noted in a table such as Table I, column 5. A larger value of *k* was then used for the next integraph solution. In this way a series of solutions was made until a value of *k* was reached which showed that pulling into step just resulted. The first seven rows of Table I were made in this manner. The values of the constants were

$$P_L/P_m = 0.5$$
$$P_R/P_m = 0.25$$
$$b = 0.5$$

The critical value of *k* was in this way found to be between the values of 0.063 and 0.065.

The same procedure was then followed for a smaller value of load ratio and then for a larger value of load ratio.

Then other values of P_R/P_m and b were used in making integraph solutions. The most important results of these are given in Table I and are plotted in Fig. 2. It is important to notice the last column of this table which shows that the ratio of P_L/P_m to k for the critical value of k is *independent of* P_R/P_m and b within limitations on the solutions imposed by the errors of the integraph. This relation allows an important conclusion to be made, namely: *The motor represented by differential equation* (1) *will pull into step if the coefficients* (P_L/P_m and k) *satisfy the relationship*

$$P_L/P_m < 7.6\,k \qquad (4)$$

The number 7.6 is the slope of the line in Fig. 2, which has been arbitrarily placed so that it favors the points for the larger load ratios.

A limitation to the practical application of this relationship between coefficients necessary for satisfactory pulling-into-step characteristics will be noticed in the next section. This limitation is imposed by a linear slip-torque assumption. The results of this paper, therefore, are optimistic when they are applied to a motor which has at any time during the pulling-into-step transient a slip such that the slip-torque characteristic is curved. During the angular oscillations the maximum transient slip is about double the initial slip. Consequently, if double the initial slip has such a value that it corresponds to operation on the curved portion of the slip-torque curve a margin of safety should be included.

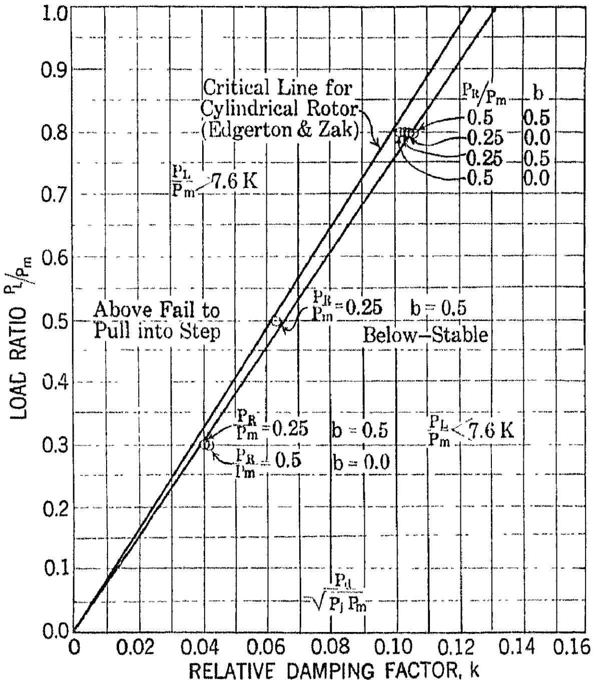

Fig. 2—Chart Showing Data of Table I

The region below the line represents the conditions for favorable pulling-into-step characteristics

The reluctance torque does not influence the value of the critical ratio to any considerable extent. This torque makes the initial slip more unfavorable than it would be if it were not present (as in round rotor machines) but a greater synchronizing effect is exerted by the joint action of the reluctance and synchronizing torques. These two opposite influences of the reluctance torque tend to cancel and its net effect, therefore, on the ability of the motor to synchronize is inappreciable.

The degree of unbalance of the damping winding apparently also has a rather minor effect on the critical ratio. This is undoubtedly due to the fact that the rotor swings through such a large range of values of angle that only the average damping is of importance.

TABLE I—TABLE OF RESULTS FROM INTEGRAPH SOLUTIONS

$\frac{P_L}{P_m}$	k	$\frac{P_R}{P_m}$	b	O N. G. X O. K.	Critical k_o	$\frac{P_L/P_m}{k_o}$
0.5	0.028	0.25	0.5	O		
0.5	0.038	0.25	0.5	O		
0.5	0.043	0 25	0.5	O		
0.5	0.055	0.25	0.5	O	0.64	7.80
0.5	0.063	0.25	0.5	O	0.064	
0.5	0.065	0.25	0.5	X		
0.5	0.082	0.25	0.5	X		
0.3	0.03	0.25	0.5	O		
0.3	0.04	0 25	0.5	O	0.41	7.3
0.3	0.042	0.25	0.5	X		
0.8	0.10	0.25	0.5	O		
0.8	0.105	0.25	0.5	X	1.04	7.7
0.8	0.11	0.25	0.5	X		
0.8	0.10	0.25	0	O		
0.8	0.105	0.25	0	X	1.05	7.6
0.8	0.11	0.25	0	X		
0.8	0.10	0.50	0	O		
0.8	0.105	0.50	0	X	1.03	7.75
0.8	0.11	0.50	0	X		
0.3	0.040	0.50	0	O		
0.3	0.043	0.50	0	X	0.42	7.15
0.8	0.105	0.50	−0.5	O		
0.8	0.11	0.50	−0.5	X	1 06	7.55
Edgerton and Zak—Cylindrical rotor motor						8 0

In Fig. 2 the corresponding values of load ratio and relative damping factor from Table I are plotted against each other. A curve from a similar study of a cylindrical-rotor synchronous motor also appears on Fig. 2. A comparison shows that the salient-pole synchronous machine does not pull into step under quite as unfavorable conditions as the cylindrical-rotor motor. However the difference is not great and no accurate comparison between the two types of motors can be made on the basis of these curves.

V. Practical Formula Giving Conditions for Pulling Into Step

The relation between the coefficients given in equation (4) which is necessary for satisfactory pulling-into-step characteristics is easily converted into a more usable form by substituting into it the values of the coefficients in terms of the $W R^2$ and the electrical constants as indicated in the following tabulation:

$$P_j = 18.5 \frac{(W R^2) f}{p^2} 10^{-6} \text{ kw/elect. deg./sec.}^2$$

(See Appendix II for derivation of P_j)

$(W R^2)$ = moment of inertia in lb. ft.^{2}
f = frequency in cycles per second
p = number of poles

$$P_d = \frac{P_L}{360 f s} \text{ kw/elect. deg./sec.}$$

P_L and s are taken from the near-synchronous straight-line slip-torque curve
P_L = load torque in synchronous kilowatts
s = the per unit slip

$$P_m = \frac{3 E V}{x_d (1000)} \text{ kw. (for a three-phase machine)}$$

where
V = applied stator voltage per phase
E = induced stator e. m. f. per phase due to the field current.
x_d = the direct-axis synchronous reactance per phase.
n = synchronous speed in r. p. m.

Substituting the values of P_j and P_d into equation (4) and solving for the slip gives the convenient relationship

$$s < \frac{590}{n} \sqrt{\frac{P_m}{(W R^2) f}} \qquad (5)$$

The amortisseur windings must furnish sufficient torque to raise the average speed of the rotor to such a value that the slip satisfies the inequality (5) if the motor is to pull into step.

Another convenient form for the criterion (5) is obtained by solving for the maximum synchronizing torque and thus determining the necessary field current to pull a motor into step.

$$P_m > 2.86 f (W R^2) n^2 s^2 \times 10^{-6} \qquad (5a)$$

The field current should be such that the maximum synchronous torque is greater than the indicated function of the frequency, moment of inertia, speed, and slip in order for synchronism to always result.

VI. Example

Pulling-Into-Step Tests on a 160-Hp. Salient-Pole Synchronous Motor

Oscillographic tests were made in the laboratory of the Massachusetts Institute of Technology of the pulling-into-step transients of a 160-hp. synchronous motor. This motor drives two synchronous generators, one rated at 50 kw., 60 cycles, 230 volts, the other at 50 kw., 24 cycles, 230 volts. An exciter is directly connected to the shaft. A photograph of the set is shown in Fig. 3.

The electrical connections as used in the tests are shown in the wiring diagram, Fig. 4. Auto-transformers supply the motor with about half voltage (1,150 volts) for starting. The 24-cycle generator was used as a shaft load with its armature connected to properly adjusted resistors. To measure angular displacements the 60-cycle generator was excited and its open-circuit terminal voltage was connected in series with a voltage from a potential transformer connected to the terminals of the motor. This potential transformer had such a ratio that the two voltages were equal in magnitude. The difference between these two voltages is a function of the angular displacement, since it is the vector difference between a voltage having a fixed relation with the excitation voltage, and the terminal voltage. The oscillograph recorded this voltage and it is termed "angle voltage" on the oscillograms. The poles of the 60-cycle generator rotor are 32 electrical deg. in advance of the poles of the motor rotor.

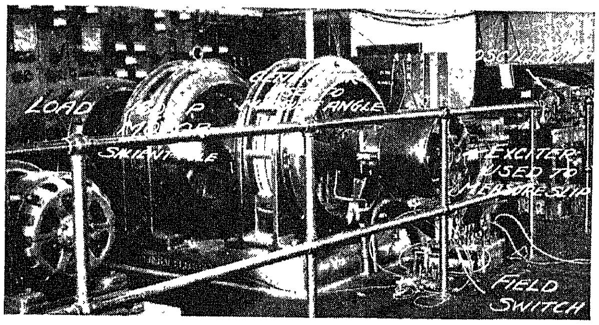

Fig. 3—160-Hp. Salient-Pole Synchronous Motor that was Used for Pulling-Into-Step Tests

The exciter was separately excited and its terminal voltage was opposed by a storage battery so that the resulting voltage at synchronous speed was equal to zero. In this way it was possible to record slip on the oscillograph. The commutator ripples were made insignificant by means of a choke coil. As a result the time constant of the circuit was about 1/10 of a second so that the current in the vibrator did not follow the slip exactly.

The polyphase power input to the stator of the motor was recorded by a Westinghouse instantaneous polyphase-power element. Armature current, field current and terminal voltage were also put on the oscillograms. The field current was held at a value which induced e. m. f. of about 1,150 volts (line to line) for all the oscillographic tests.

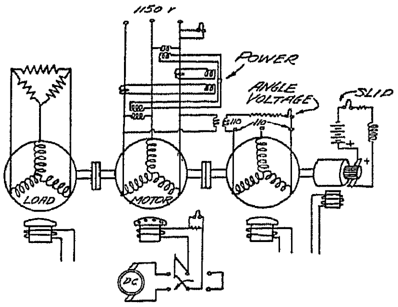

Fig. 4—Wiring Diagram of Connections Used for Oscillographic Tests

Oscillogram 1, Fig. 5, shows a satisfactory pulling-into-step transient. The initial slip has an average value of about 0.0149. The switching angle is about 40 deg. (generator). The oscillographic record of slip shows that the speed rises quickly to synchronous and then oscillates about that value.

Oscillogram 2, Fig. 6, gives a very interesting picture of the transients in a motor which is just on the verge of being satisfactory. The average initial slip for this transient is 0.0145 and the switching angle is 95 deg. (motor). The speed decreases, soon after the exciter is connected, then it increases to exact synchronous speed and runs there for nearly a second. The angular displacement at this point is such as to give unstable equilibrium. The rotor finally swings around again before reaching its steady-state operating angle.

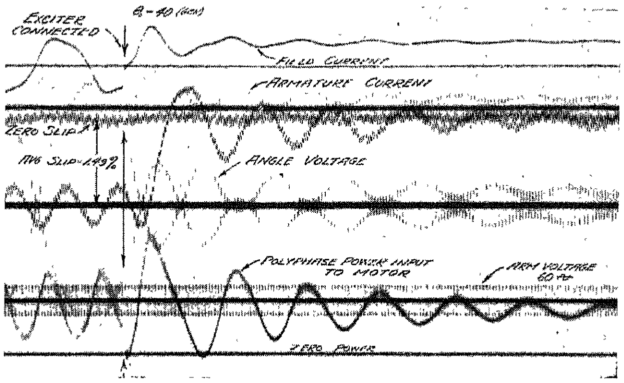

Fig. 5—Oscillogram No. 1

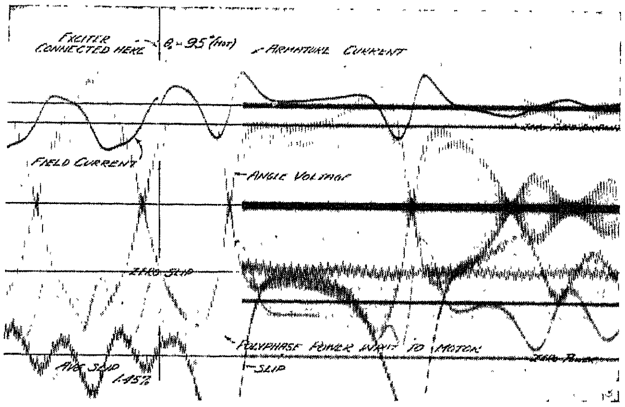

Fig. 6—Oscillogram No. 2

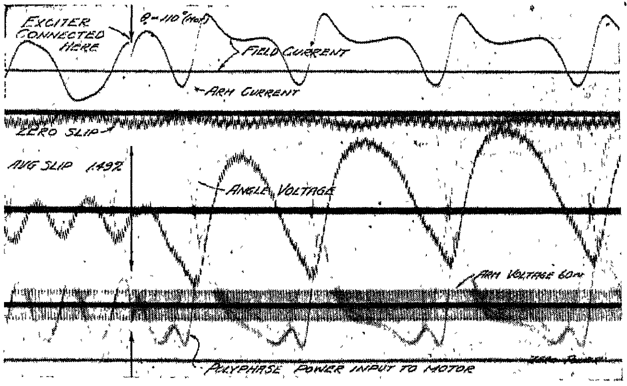

Fig. 7—Oscillogram No. 3

Oscillogram 3, Fig. 7 shows the transients in a motor which does not pull into step for seven or eight swings (only four shown on the oscillogram). The initial average slip is 0.0149 and the switching angle is 110 deg. (motor). During the oscillations following switching each oscillation brings the average speed closer to synchronous speed. The current and power oscillate violently at a low frequency.

A calculation of the limiting slip for this motor by means of expression (5) follows. The necessary factors are,

$$P_m = \frac{3\,V\,E}{1000\,x} = \frac{1150 \times 1150}{1000 \times 21.4} = 61.5 \text{ kw.}$$

since x = 21.4 ohms per phase
V = 1150 volts line to line
E = 1150 volts line to line
$W\,R^2$ = 2,762 lb. ft.^{2} (total of all three machines)
f = 60 cycles per sec.
n = 720 r. p. m.

$$s < \frac{590}{720}\sqrt{\frac{61.5}{2762\;\;60}}$$

$$s < 0.0143 \text{ per unit slip}$$

The three oscillograms that have been included in this paper are for values of slip slightly in excess of the critical value, but for switching angles more favorable than the worst condition which explains why oscillogram No. 1 showed satisfactory synchronization.

Preliminary tests to determine the critical value of slip for 100 per cent satisfactory pulling-into-step were made before the oscillograms were taken. The procedure was to switch the exciter at random for a certain value of shaft load. If the motor always synchronized then the load was increased slightly. As this was repeated, eventually a load was found where for some switching angles the motor did not pull into step the first swing. The per unit slip corresponding to this load for the motor that has been described was about 0.014 to 0.015 which is very close to the calculated value of 0.0143.

ACKNOWLEDGMENT

The authors wish to express their appreciation to those who have helped during the course of this investigation, especially to Professor Lyon, Dr. Bush, Professor Dahl and Mr. Hazen. Messrs. Alger and Terry of the General Electric Co. furnished data for the motor that was used for the experimental tests. Messrs. Kingsley and Levine assisted with the oscillograph testing and Messrs. Bower and Gillon aided in operating the integraph.

Appendix I

OUTLINE OF INTEGRAPH METHOD OF SOLVING DIFFERENTIAL EQUATION OF MOTION

Equation (3) is put into an integral form by transposing all terms except the acceleration term and then integrating both sides twice with respect to λ.

$$\theta = \int_0^\lambda\int_0^\lambda \frac{d^2\,\theta}{d\;\lambda^2}\,d\,\lambda\,d\,\tau = \int_0^\lambda\int_0^\lambda \left\{ \frac{P_{\mathrm{L}}}{P_m} - k\,(1 - b\cos 2\,\theta) \right.$$
$$\left. \frac{d\,\theta}{d\,\lambda} - \frac{P_{\mathrm{R}}}{P_m}\sin 2\,\theta - (\sin\,\theta)\,1 \right\} d\,\lambda \quad d\,\lambda \qquad (6)$$

Rewriting in a slightly different form by factoring out $(1 - b\cos 2\,\theta)$

$$\theta = \int_0^\lambda\int_0^\lambda \left\{ \frac{\dfrac{P_{\mathrm{L}}}{P_m} - \dfrac{P_{\mathrm{R}}}{P_m}\sin\,2\,\theta}{(1 - b\cos 2\,\theta)} \right.$$
$$\left. + \frac{-\sin\,\theta}{1 - b\cos 2\,\theta}\,1 - k\,\frac{d\,\theta}{d\,\lambda} \right\}(1 - b\cos 2\,\theta)\,d\,\lambda\;\;d\,\lambda \quad (7)$$

To solve equation (7) on the integraph it is necessary to have plots of the various terms (see Fig. 8).

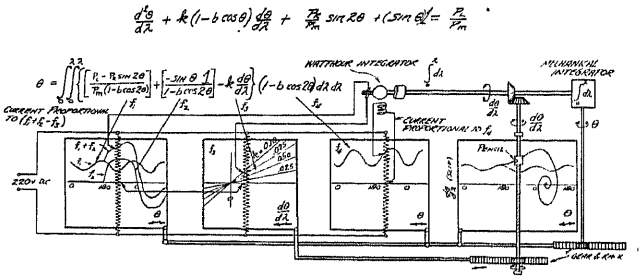
d²θ/dλ + k(1−b cos θ) dθ/dλ + P_R/P_m sin 2θ + (sin θ)1 = P_L/P_m
θ = ∫∫{[P_L − P_R sin 2θ / P_m(1−b cos 2θ)] + [−sin θ 1 / 1−b cos 2θ] − k dθ/dλ}(1−b cos 2θ) dλ dλ
f₁
f₂
f₃
f₄
WATTHOUR INTEGRATOR
∫dλ
MECHANICAL INTEGRATOR
CURRENT PROPORTIONAL TO (f₁+f₂−f₃)
CURRENT PROPORTIONAL TO f₄
dθ/dλ
θ
220V DC
f₁+f₂
k=.010
.025
.050
.075
Pencil
180
GEAR & RACK

FIG. 8.—SCHEMATIC DIAGRAM OF THE INTEGRAPH ILLUSTRATING METHODS EMPLOYED IN SOLVING THE CHARACTERISTIC EQUATION OF MOTION OF THE SALIENT-POLE SYNCHRONOUS MOTOR

Let f_1 be a plot of the following term as a function of angle, θ.

$$f_1 = \frac{\left(\dfrac{P_{\mathrm{L}}}{P_m} - \dfrac{P_{\mathrm{R}}}{P_m}\sin\;2\;\theta\right)}{(1 - b\cos 2\,\theta)}$$

Let f_2 be a plot of the term which is added at a time corresponding to switching of the exciter to the field circuit. This is also plotted as a function of angle.

$$f_2 = \frac{-\sin\,\theta}{1 - b\cos 2\,\theta}$$

Let f_3 be a plot of $-\,k\,\dfrac{d\,\theta}{d\,\lambda}$ against $\dfrac{d\,\theta}{d\,\lambda}$. This is a straight line having a slope corresponding to k.

Let f_4 be a plot of the term which is multiplied into the sum of the preceding functions. It is also plotted as a function of the angle.

$$f_4\,(1 - b\cos 2\,\theta)$$

Equation (7) now can be rewritten

$$\theta = \int_0^\lambda\int_0^\lambda (f_1 + f_2 + f_3)\,f_4\,d\,\lambda\;\;d\,\lambda.$$

$f_1 + f_2$ is put on a platen (see Fig. 8) on the integraph and this platen is back-coupled to the output of the second integrator. This output is the angle θ which is the variable against which f_1 and f_2 are plotted; f_3 is put on a platen which is back-coupled to the output of the first integrator. The output of this is equal to $d\,\theta/d\,\lambda$ as can be shown by differentiating equation (7) with respect to λ.

The potentiometers over these two platens are connected in series so that the resulting voltage is equal to $(f_1 + f_2 + f_3)$.

A third platen has a plot of f_4 upon it and since f_4 is a function of angle the platen is back-coupled to the output of the second integrator which is proportional to θ in the same manner as the platens upon which f_1 and f_2 are plotted.

Fig. 8 shows these plots upon the platens and the method of back-coupling them so that their displacements are proportional to the angular displacement and the slip.

The platen on the extreme right records the slip as a function of the angular displacement. Characteristic solutions are shown in Fig. 1 and are explained in the paper.

The procedure to find the initial steady-state induction motor solution was to start the integraph (with the synchronizing term (f_2) out) and let it run until the slip for each corresponding angle repeated itself every 360 electrical deg. This process is exactly the same as the operation of the motor inasmuch as an identical transient condition in both the motor and integraph must be run through before the final steady-state running condition is reached. The person operating the integraph soon learns to guess quite closely to the final value and this reduces the time necessary to grind out this preliminary steady state for the initial boundary conditions.

After the steady state is reached the pulling-into-step transient is obtained by following the curve $f_1 + f_2$ on the left-hand platen at a time corresponding to the switching angle that is under consideration. The sketch on this platen in Fig. 8 shows the approximate form of the curves at the most unfavorable angle.

Appendix II

DERIVATION OF P_j, THE INERTIA COEFFICIENT

Inertia torque $= J\,\alpha$ in lb. ft.

where $J = \dfrac{W\,R^2}{g}$ = the moment of inertia in poundals

α = the acceleration in mechanical radians per sec.

Inertia power $= \dfrac{2\,\pi\,n}{60}\,J\,\alpha$ in lb ft. per sec.

n = speed in r. p. m.

If the acceleration is to be expressed in electrical deg. per sec. ($d^2\,\theta/d\,t^2$) the conversion factors to give proper mechanical units are

$$\alpha = \frac{2\,\pi}{180\,p}\,\frac{d^2\,\theta}{d\,t^2} \text{ mechanical radians per sec.}^2$$

since $2\,\pi$ mechanical radians $= 180\,p$ electrical deg. The expression for inertia power is now written in terms of power in kilowatts, electrical deg. and sec. (at synchronous speed).

Inertia power in kw.

$$= \frac{2\,\pi\,n}{60}\left(\frac{W\,R^2}{g}\right)\frac{2\,\pi}{180\,p}\,\frac{0.746}{550}\,\frac{d^2\,\theta}{d\,t^2}$$

$$= P_j\,\frac{d^2\,\theta}{d\,t^2}$$

where

$$P_j = 0.154\,\frac{n\,(W\,R^2)}{p} \times 10^{-6} \text{ or } 18.5\,\frac{f\,(W\,R^2)}{p^2} \times 10^{-6}$$

(kw. per electrical deg. per sec.^{2})

since $n = \dfrac{f\,120}{p}$ r. p. m.

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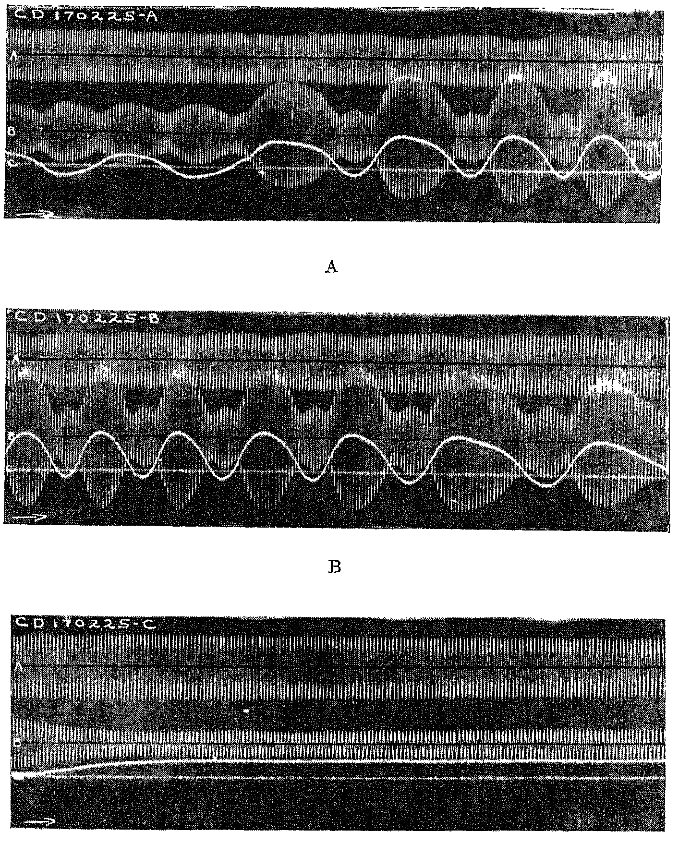
Discussion

D. W. McLenegan and I. A. Terry: This paper presents the solution of a problem of considerable complexity and is another illustration of the value of the M. I. T. integraph in solving problems which are very difficult to treat analytically. The practical value of the solution cannot be over emphasized. As the solution. In the differential equation (1) describing the rotor motion the synchronizing effect of the field current was expressed as ($P_m \sin \theta$) 1 which assumes that the normal torque-angle characteristic of the motor exists at the instant the excitation is applied to the motor. This is not true since there is a the field of application of synchronous motors is broadened, a clearer understanding of the complete theory must be available. For example, a synchronous motor driving a centrifugal pump must usually pull into step under full load but there is a relatively small inertia in the rotating system. On the other hand, a synchronous motor driving a sintering fan must pull into step under full load and in addition there is a large inertia in the rotating system. Since to synchronize, the motor must develop sufficient torque during one-half of a slip cycle to supply the load and in addition accelerate the inertia from the slip speed to full speed, it is evident that a synchronous motor which will develop normal torque at 95 per cent speed may be suitable for driving a centrifugal pump or other loads with a small inertia, but quite unsuitable for driving a sintering fan or other apparatus with a relatively large inertia. There are other cases where the torque of a motor at 95 per cent speed does not represent a true criterion for the ability to attain synchronism. As yet, there is not an Institute or even a commercial standard which truly represents the ability of a motor to synchronize under given conditions. This paper furnishes a theoretical basis for the ultimate formulation of such a standard.



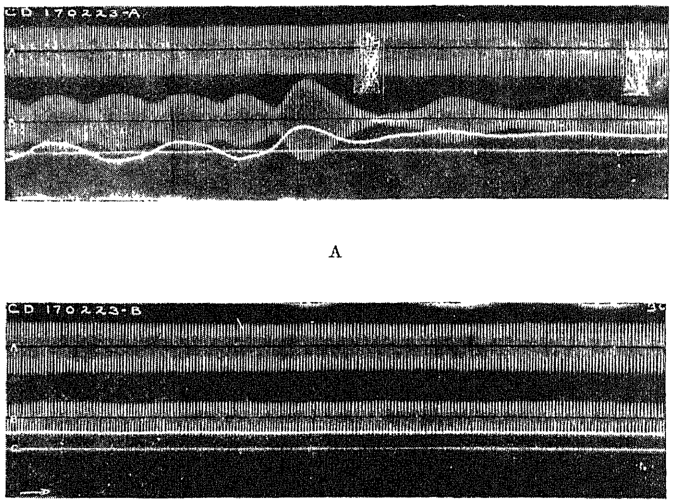
A

B

C

FIG. 1.—SYNCHRONIZING OF 75 HP., 900-R. P. M., SALIENT-POLE SYNCHRONOUS MOTOR WITH NORMAL FIELD TIME CONSTANT. LOAD 1.2 TIMES NORMAL

Curve *A*: Applied stator voltage
Curve *B*: Motor stator current
Curve *C*: Motor field current



A

B

FIG. 2.—SYNCHRONIZING OF 75-HP., 900-R. P. M. SALIENT-POLE SYNCHRONOUS MOTOR WITH HALF NORMAL FIELD TIME CONSTANT. INITIAL LOAD 1.2 TIMES NORMAL

Curve *A*: Applied stator voltage
Curve *B*: Motor stator current
Curve *C*: Motor field current

One of the most surprising results of the studies is the relatively unimportant part played by the reluctance torque in the synchronizing of a motor. The authors have shown that a salient-pole machine can synchronize from a slip which is less than

$$\frac{590}{n}\sqrt{\frac{P_m}{(W\,R^2)\,f}}$$

whereas it has been shown in bibliography 9 that a round rotor machine can synchronize from a slip less than

$$\frac{620}{n}\sqrt{\frac{P_m}{(W\,R^2)\,f}}$$

so that a salient-pole machine with a given average induction motor torque at 95 per cent speed and a given value of P_m has a slightly smaller ability to pull into synchronism than a round rotor machine with similar characteristics. Since a salient-pole machine has a greater synchronous pull-out torque with a given excitation and a greater synchronizing torque with very low excitations than a round rotor machine, it seems that it should also give a greater synchronizing or pull-in torque at normal excitation. We would like the authors to comment on this apparent inconsistency.

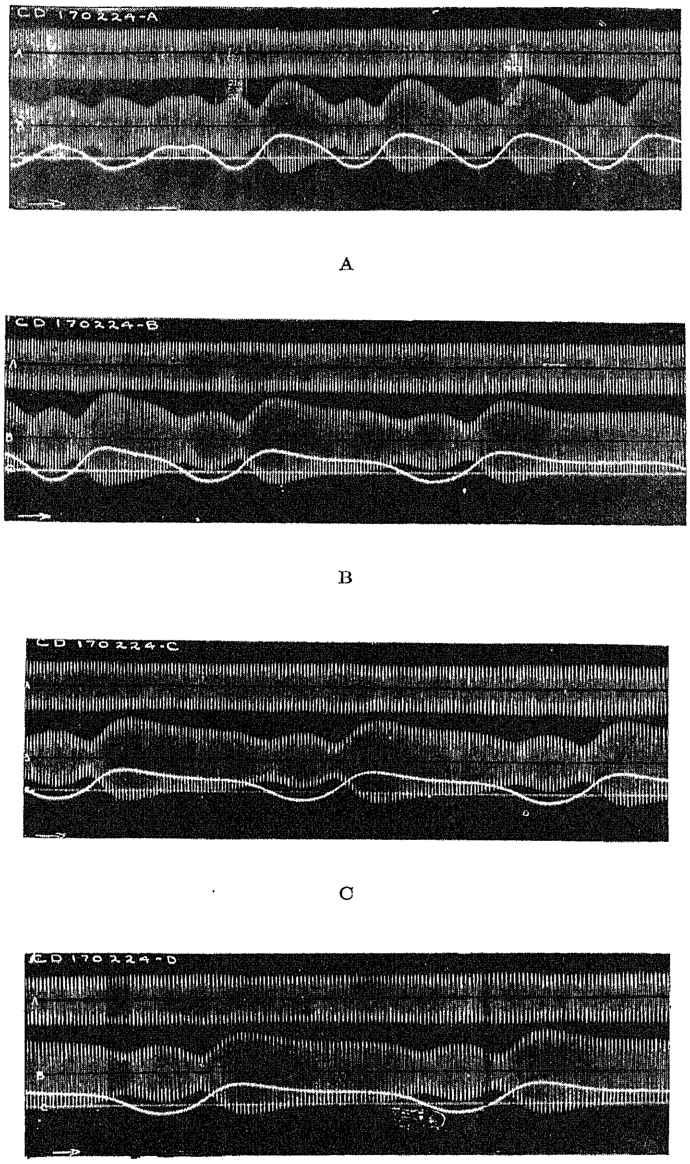


Fig. 3—Attempt to Synchronize 75-Hp., 900-R. P. M. Salient-Pole Synchronous Motor with Half Normal Field Time Constant

Curve A: Applied stator voltage
Curve B: Motor stator current
Curve C: Motor field current

In solving engineering problems certain assumptions must always be made to simplify the analysis. The accuracy with which the assumptions are realized determines the accuracy of definite time required to build up the current in the field winding. For present day industrial motors the short-circuit time constant of the field winding is frequently around a quarter to a half second which means that this time will have elapsed before 0.64 P_m will be available. The slip from which the motor must synchronize may represent a transient in the armature current of the same order. In the machine tested by the authors the short-circuit time constant of the field winding is much shorter than the time of the armature current transient.

To determine the effect of the field time constant upon the pull-in characteristics tests have been made upon a 75-hp., three-phase, 60-cycle, 900-r. p. m., 0.80-power factor synchronous motor direct-connected to a heavy fan. The total $W R$ of the rotating system was 3,870 lb. ft. sq. The motor was allowed to attain its maximum speed as an induction motor with various fan loads and then field current was applied under different conditions. It was found that for fan loads near the critical value (above which the motor could not be synchronized) the shorter the field time constant, the better the ability to synchronize although at lighter loads the data are not conclusive. The oscillogram shown in Fig. 1 was taken with the field winding excited at the time indicated by an abrupt change in the field current, from a constant potential of 125 volts with sufficient external resistance in the circuit to permit normal full load field current to flow under steady-state conditions. The short-circuit time constant of the field winding was 0.122 seconds. The initial slip was 0.021 with an observed load of 1.2 times normal. The motor slipped 10 poles before synchronism was attained. The oscillogram shown in Fig. 2 was taken with the field winding excited at the time indicated from a constant potential of 250 volts with sufficient external resistance in the circuit to permit normal full load field current to flow under steady-state conditions. The short-circuit time constant of the field winding was therefore half of the previous value. The initial slip was 0.02 and the observed load 1.2 times normal. Only two poles were slipped before synchronism was attained. The angle at which the field current was switched was slightly less favorable in the second case than in the first, but even then the shorter time constant permitted more rapid synchronization. These oscillograms were taken on a motor with a much shorter field time constant than armature current transient at the time application of field current. In case of a machine where the reverse is true then it appears that the effect of the time constant would be still greater. In general, there should be two results. One would be to shift forward (in time) the optimum angle of application of field voltage, and the other would be to make a slight reduction in the slip from which a motor may synchronize.

Another oscillogram, shown in Fig. 3, was taken on the same motor with the field excited from a constant potential source of 125 volts with sufficient external resistance to permit half of normal full load field current to flow under steady-state conditions. The switching angle was very unfavorable and the motor failed to synchronize. This oscillogram, together with the first one mentioned, therefore, afford a good means of checking the authors' equation (5). For the first oscillogram the observed slip from which the motor synchronized was 0.021 and the switch angle was quite favorable. From the authors' equation (5) the slip should be less than 0.0144 if the machine were to synchronize during the first two opportunities from an initial position of the worst switching angle. In the last mentioned oscillogram where the motor did not synchronize, the equation (5) gives the slip from which the motor would synchronize as 0.010. Table I shows a comparison between test values and values calculated from equation (5) for four different types of synchronous motors.

The conclusions this tabulation justifies are:

TABLE I

Motor rating	P_m	Total $W R^2$	Observed slip	Calculated slip	Comment
75 hp.-900 r. p. m.	112 kw.	3,870	0.021	0.0144	Normal-time constant synchronism attained
75 hp.-900 r. p. m	112 kw.	3,870	0.020	0.0144	Half-time constant synchronism attained
75 hp.-900 r. p. m	56 kw.	3,870	0.023	0.0102	Half-time constant (half excitation) motor failed to synchronize
1,700 hp.-720 r. p. m	2,600 kw	28,600	0.020	0.032	Normal-time constant synchronism attained
2,800 hp.-600 r. p. m.	4,240 kw	71,700	0.030	0.033	Normal-time constant synchronism attained
125 hp.-277 r p. m	150 kw.	6,375	0.040	0.0422	Normal-time constant synchronism attained

A motor may pull into step from a slip greater than that indicated by the formula, due to ideal timing of the instant at which field is applied; the formula will always give values such that if the slip calculated from it is attained, the motor will pull into step.

V. Bush: The excellent work presented in this paper appeals to me very strongly, especially as it was carried out by means of an integraph which is admittedly not particularly convenient nor accurate. This device at the Massachusetts Institute of Technology is really a first model, and we now have under construction a much better piece of equipment. This new apparatus will be capable of solving total differential equations of very considerable complexity and with much better precision than the present machine. I have every confidence that this new tool, put in the hands of Mr. Edgerton and the group which is interested in machine transients, will produce still further results along these lines, and may make it possible to include some of the effects which have necessarily been omitted in the present analysis.

The new machine for solving differential equations will shortly be described in some publication. I wish to emphasize again at this time that the Massachusetts Institute of Technology is anxious to cooperate in the use of these devices with any one having a serious research problem involving the solution of differential equations on which the instruments can be used to advantage.

Another very interesting matter in this paper is the new stroboscope, and I trust that this may have a separate presentation. It is interesting to note that the light intensities produced are very large indeed. Mr. Edgerton used a stroboscope which had a period of illumination of only about 10 microseconds occurring once each cycle, and yet was able to obtain sufficient light for photography. The instantaneous candle power of the source is hence enormous, as may readily be computed. This appears to have interesting possibilities for further development.

Ernst Weber: The paper deals generally with synchronous motors having damper windings so that an interaction of asynchronous and synchronous torques results when the field current is suddenly applied to the pole winding. It is known that the behavior of salient-pole machines is generally dependent on the materials used for the poles. Machines with solid iron poles and those with laminated poles show important differences especially with regard to mechanical oscillations. For this reason it would be pertinent to inquire as to the pole design features of the machine used in the tests described.

It is surprising to note that the typical integraph angle-slip solutions, given in Fig. 1, include only curves beginning with a decrease in slip, whereas oscillograms numbers 2 and 3 first show an important increase in slip, the transients being taken for switching angles of 95 and 110 degrees which are supposed to give good results. This might indicate some reaction between the establishment of the magnetic field in the poles and the damper winding. It is also possible (assuming the machine to have solid poles) that the eddy currents in the poles might have an important influence on the oscillations. In this case it would hardly be justifiable to neglect the rotor leakage reactance and all the electrical transients. When the oscillograms show a rather uncertain agreement with the integraph solution it would be necessary to include an additional term in the mechanical differential equation to account for the effect of the eddy currents. A second differential equation should include the effect of the electrical transients which have a relatively long duration compared with the variation of field current in the pole winding as shown in the oscillograms on page 775.

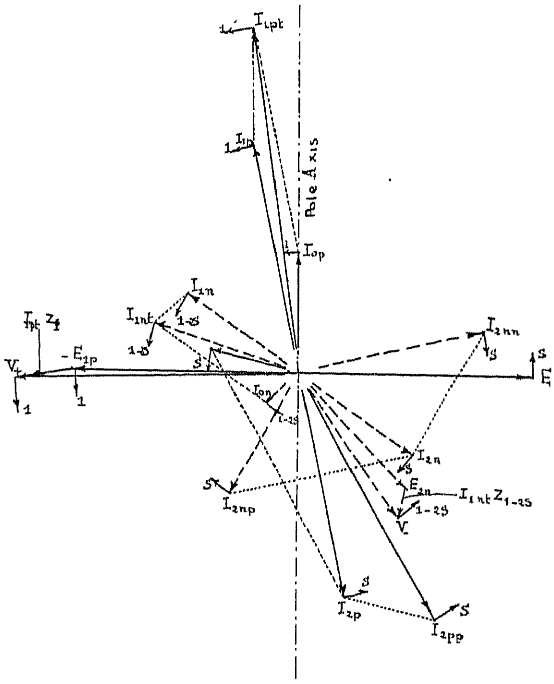


FIG 4

The work performed and presented in this paper is of the greatest value as a step forward in the graphical solution of nonlinear differential equations of complicated forms. In the special case of the application, however, the underlying theory of the phenomenon seems to be incomplete.

P. Fourmarier: Independently of a very exact calculation, it can be shown, by drawing the vector diagram of the synchronous motor running as an induction motor, that the induction motor power term is of the general form:

$$P_d\,[1 - b \cos (2\,\theta + \beta)]\,\frac{d\,\theta}{d\,t}$$

where β is a small angle.

Let us draw that diagram for a two-pole synchronous motor; in that case, the time diagram coincides with the space diagram.[1] All vectors are referred to the stator; the velocity of the rotor vectors is indicated with respect to the rotor; the velocity of the stator vectors is indicated with respect to the stator. The rotor vectors represent the values of the different variables in the middle bar of the pole.

The subscript 2 indicates the rotor vectors; the subscript 1, the stator vectors.

The notations in the diagram are listed below:

E_{2p} = e. m. f. induced by the positively rotating air-gap flux
E_{2n} = e. m. f. induced by the negatively rotating air-gap flux
I_{2pp} = positively rotating component of current due to E_{2p}
I_{2np} = negatively rotating component of current due to E_{2p}
I_{2pn} = positively rotating component of current due to E_{2n}
I_{2nn} = negatively rotating component of current due to E_{2n}
I_{2p} = total positively rotating rotor current
I_{2n} = total negatively rotating rotor current

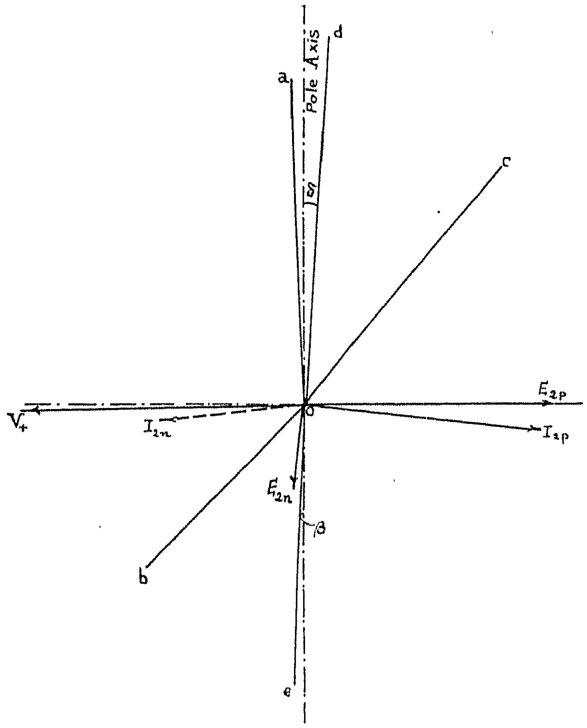


Fig. 5

I_{0p} = positively rotating magnetizing stator current
I_{0n} = negatively rotating magnetizing stator current
I^1_{pt} = total positively rotating stator current
I_{1nt} = total negatively rotating stator current
V_+ = total positively rotating terminal voltage
V_- = total negatively rotating terminal voltage

As the terminal voltage is supposed to be balanced, V_- must be equal to zero.

Power. The total power supplied to the shaft can be divided into two principal parts:

a. The constant powers including:
  1. The total positive power P_p [vectors E_p I_p]
  2. The total negative power P_n [vectors E_n I_n]

b. Two periodical powers P_{pn} and P_{np} of double frequency, caused by the voltages and currents rotating in opposite directions. Their average value is zero but their instantaneous value is given by the product of the vectors by the cosine of the angle between them. [vectors E_p I_n and E_n I_p]

Simplified Diagram. For small slips (pulling into step) the diagram can be shown to reduce to Fig. 5 as the rotor bars reactance and the stator winding resistance are neglected.

We can derive from that diagram the positions of E_{2p} corresponding to the maxima of unbalanced powers:

a Position of E_{2p} for maximum E_{2p} I_{2n}
b Position of E_{2n} for maximum E_{2n} I_{2p}
c Position of E_{2p} for maximum E_{2n} I_{2p}
d Position of E_{2p} for maximum total unbalanced power
e Position of V_+ for maximum total unbalanced power.

It must not be forgotten that the powers, being of double frequency, have to be added on a special double frequency diagram. The calculations show that β and δ are very small. We can thus represent this unbalanced power by

$$P_1 \cos (2\,\theta + \beta_1)\,\frac{d\,\theta}{d\,t}$$

where β_1 is a small angle.

Effect of the Field Winding. The field winding can be considered as a single-phase winding 90 deg. ahead of the bar winding; its unbalanced power can be represented by a similar expression

$$P_2 \cos (2\,\theta + \beta_2)\,\frac{d\,\theta}{d\,t} \qquad (2)$$

This unbalanced power is in general small as compared with the first one, which means that the sum of expressions (1) and (2) can be written in the form

$$P_d\, b\, (\cos 2\,\theta + \beta)\,\frac{d\,\theta}{d\,t}$$

where β is a small angle.

Harold E. Edgerton: It is a source of satisfaction to us to know that our efforts to solve the pulling-into-step problem of the salient-pole synchronous motor have resulted in a practical form which is of use to those who are concerned with the application of synchronous motors to drive difficult loads.

Messrs. Terry and McLenegan ask us to discuss the comparison that they show between our studies of the cylindrical and salient-pole synchronous machines. As we pointed out in the paper our accuracy is hardly sufficient to make a comparison of these equations, however, we are certain that the reluctance effect does not have a pronounced effect on the critical value of average initial slip for the worst switching angle. The reluctance torque has two main effects, one which is beneficial and the other detrimental. The beneficial effect of the reluctance torque is to help the motor pull into synchronism. The detrimental effect is the influence of the reluctance torque in causing speed variations so that the initial condition of slip is worse than the average for the unfavorable switching condition. For some favorable switching conditions the salient-pole synchronous motor will pull into step easier than the cylindrical-rotor type. The initial variations of slip that are caused by the reluctance torque are very noticeable in the oscillograms, Figs. 5, 6, and 7.

The discussion and the tests given by Messrs. Terry and McLenegan showing the influence of the electrical transients upon the pulling-into-step of a motor are interesting. We hope to obtain some integraph solutions of these transients in future studies. The tabulated comparisons show no disagreement. However, these tests are not conclusive since the switching conditions were uncertain.

There is a great variety of synchronous-machine dynamic problems that the integraph can solve. In some cases, such as in this problem, a general form of solution can be obtained by taking ratios of factors and by changing variables. In other problems the number of variables and factors are not easily simplified and a general solution for all cases is not possible. The integraph solutions are then only made for a specific set of parameters that fit some particular set of conditions that are to be investigated.

Soon a new mechanical integraph will be completed by Dr. Bush and those working with him. It has remarkable accuracy and will handle sixth order differential equations.

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[1] See Bibliography 12.

Steady Flow of Heat in Large Turbine-Generators

BY C. RICHARD SODERBERG[1]
Member, A. I. E. E.

Synopsis.—*The present paper represents an attempt at a reasonably complete solution of the heat flow in large turbine generators, where the temperature drops through the various iron paths are of consequence. The solution of two-dimensional problems of heat flow is obtained by means of "equivalent thermal circuits." The approximations involved in this method are discussed in two appendixes where exact solutions are developed. The theory has been applied to a number of large turbine generators with satisfactory agreement between measured and calculated temperatures. The application of the results to actual machines will be made the subject of a subsequent paper.*

* * * * *

INTRODUCTION

THE predetermination of the temperature in electrical machines involves three distinct problems:

1. The magnitude and distribution of the losses.
2. The flow of the cooling medium through the machine, and its cooling effect.
3. The flow of heat from the source of generation to the cooling medium.

All of these problems present great difficulties when accurate solutions are wanted, although the electrical industry has managed fairly well, as indicated by the comparatively close agreement between calculations and test results in modern machines. Agreement, however, between calculations and tests is not always a perfectly reliable criterion of the correctness of the underlying theory, because in machines designed on the basis of the performance of nearly similar units a very rough theory may give results in excellent agreement with tests. The same theory applied to a machine of different proportions or size may fail altogether.

The present paper is restricted to the third of the above problems and represents an attempt at a reasonably complete solution for the heat flow in large machines with deep slots, where the temperature drops through the various iron paths are of consequence. The analysis proceeds from a simple conception of the general aspect of the heat flow, by progressive stages of elaboration, to what is believed to be a fairly correct picture of the complicated phenomena involved. The more elaborate mathematical deductions are given in the appendixes. It is important to remember that the final accuracy of a temperature calculation depends primarily on the correctness of the loss distribution, and the thermal properties of the materials, upon which the calculation has been based. Several details of the following analysis have been carried to a degree of refinement which may be considered impractical for the ordinary types of machines. An approximation, however, is justified only when the degree of approximation is known, and this has not always been the case with the approximate methods commonly used.

Only the conditions of constant heat flow are considered. The problems encountered in predetermining the flow of heat before a stationary temperature is reached will be discussed in a later paper.

During the preparation of this paper the writer received many valuable suggestions from Dr. J. Wennerberg and others of the ASEA Company in Sweden, which suggestions are gratefully acknowledged.

SYSTEM OF UNITS AND NOMENCLATURE

In the following the watt-second will be selected as the unit of energy, making the *watt* the unit of power or of loss. The degree *centigrade* will be taken as the unit of temperature, and the *inch* as the unit of length.

Two fundamental phenomena are involved in the problem. The one is the flow of heat in solids, the law of which will be written in the same form as that for the flow of electricity. If there is a temperature variation of $d\,t$ over the length $d\,x$ of the path, the flow of energy $d\,P$ through the area $d\,a$ is

$$d\,P = -\frac{d\,t}{\rho\,\dfrac{d\,x}{d\,a}} = -\frac{1}{\rho}\,\frac{d\,t}{d\,x}\,d\,a \qquad (1)$$

Here ρ is the *specific thermal resistance* of the material in the direction of the flow. It will be expressed in deg. cent. inch per watt. The other fundamental phenomenon is the transfer of energy from a "cooled" surface by radiation and convection. This phenomenon is much more complicated, but for practical purposes it is permissible to assume that the flow of energy per unit area is proportional to the difference of temperature between the cooled surface and the cooling medium. The law will be written in the form

$$d\,P = -\frac{1}{C}\,t\,d\,a \qquad (2)$$

where C is defined as the *surface resistance* at the cooling surface, expressed in deg. cent. inch^2 per watt. *Emissivity* is the usual name given to the quantity $1/C$.

The conception of surface resistance may also be extended to the case of heat flow across comparatively thin layers of insulation, where the temperature gradient may be considered constant. Thus, the flow of heat across an insulation layer of thickness $\Delta\,x$ may be expressed by an equation of the same form as (2), where $C = \rho\,\Delta\,x$ may be defined as the surface resistance of the insulation layer.

The expression $d\,P/d\,a$ will be denoted by q and

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[1] Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

referred to as the *surface intensity* of the flow. When losses are generated in a solid, the loss per unit volume dP/dV will be denoted by p and referred to as the *volume intensity* of the loss.

The following list gives the general symbols, which are used in the paper, with the exception of occasional symbols which are always defined when used. Each symbol may be appended by one or several of the following indexes:

Index	Significance
1	Copper in slot
2	Iron in teeth
3	Airgap surface
a	Cooling medium
c	Iron in core back of teeth
i	Slot insulation
x	Axial direction
y (or r)	Radial direction

The sign * written thus, C^*, will be used to designate a "corrected" or "slightly different" value of the symbol in question.

Symbol	Units	Significance
a	inch^2	Area of cross section
b	inch	Width of copper in slot b_1, mean thickness of tooth b_2. Equivalent width of cooling surface b_a ("cooled width")
C	$\frac{\text{deg. cent. inch}^2}{\text{watt}}$	Surface resistance. Of insulation C_i, at cooling surface C_a, equivalent surface resistance for flow through tooth C
f (and k, m, n, o)		Numerical constants
h	inch	Depth of copper h_1, height of tooth h_2,
l		Half spacing of radial vent ducts l_1, half axial length of iron package l_2
P	watt	Loss
p	$\frac{\text{watt}}{\text{inch}^3}$	Volume intensity of loss
Q		Number of slots
q	$\frac{\text{watt}}{\text{inch}^2}$	Surface intensity of loss
R		Special cylinder function
r	inch	Radius, radial coordinate
t	deg. cent.	Temperature
x	inch	Axial coordinate
X and Y		Numerical constants (for expressions $m\alpha l$ and $n\alpha l$)
y	inch	Radial coordinate
α (and m, n)	$\frac{1}{\text{inch}}$	Argument constants (in expression $e^{\alpha x}$)
θ		Taper constant
κ		General numerical constant
ρ	$\frac{\text{deg. cent. inch}}{\text{watt}}$	Specific thermal resistance
τ	inch	Slot pitch

The significance of the various combinations of indexes will be self evident in most cases. b_{ax2} is the "cooled width" of the tooth in the x-direction (in the radial ducts), C_{ax2} is the corresponding surface resistance, ρ_{x1} is the specific thermal resistance of the copper in the axial direction, and so on. The order of the letters in the indexes just given will be retained throughout.

THE GENERAL PROBLEM

The present analysis applies to an arrangement of a general character, intended to embody the principal features of a rotor and a stator of a modern machine with radial ventilation. This arrangement is a slotted iron core with windings in the slots. In the core part the heat is dissipated from cooling surfaces in the airgap, at the back of the core, and in radial ducts. The latter divide the core into short, axial segments, which may be considered insulated from each other.

Thus, axial flow of heat takes place through the windings only. The general solution of the temperature problem involves a solution for unit or several of the core part (a vent package), a solution for the end windings, and a solution for the axial distribution of temperature in the windings. The present discussion will be restricted to the first and most important part of the problem. The dissipation of heat from the end-windings is not always easy to determine, but the difficulties are chiefly due to lack of data on the thermal properties of the insulation and the behaviour of the cooling medium. The axial distribution of the temperature of the windings is a comparatively simple one-dimensional problem, which has received considerable attention in the past.[2]

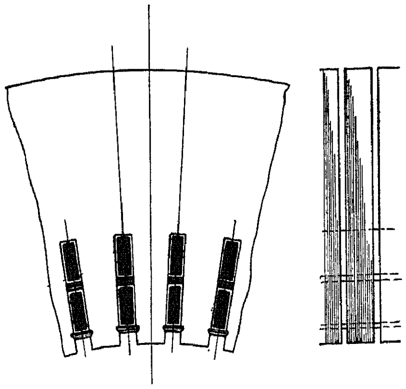

FIG. 1—TYPICAL ARRANGEMENT OF STATOR

The structure to be studied is thus a short section of the core, it may be either a stator section, Fig. 1, or a rotor section, Fig. 2 and Fig. 3. It is of comparatively short axial extension so that the temperature of the copper may be considered constant in the axial direction. In order to maintain the general nature of the problem the existance of the following losses will be assumed: 1. Copper Losses. These are generated inside the slot insulation and flow through the insulation layer into the tooth, whence they continue radially and axially to the cooling medium. 2. Iron Losses. These are generated in the core material itself whence they flow axially and radially to the cooling medium.

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[2] For an interesting discussion of the general subject of the flow of heat in electrical machines, and a particularly complete bibliography, see Edoard Roth: "Introduction a L'Étude Analytique de l'Echauffement des Machines Electriques."

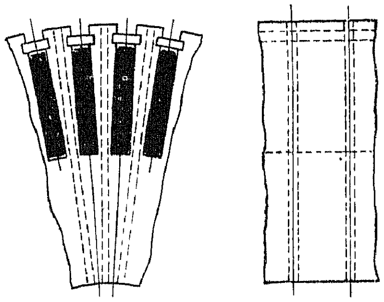


FIG. 2—TYPICAL ARRANGEMENT OF ROTOR AXIAL FLOW OF THE COOLING MEDIUM THROUGH CENTRAL HOLE IN ROTOR CORE

3. Surface Losses. These are assumed to be generated at the airgap surface of the core and are dissipated directly to the cooling medium.

Considering the structure as a whole there is no appreciable flow of heat in the tangential direction. A considerable flow takes place across the teeth, but since their dimensions are relatively small the resulting temperature variations may be neglected. The same results apply to the copper. The problems may thus be reduced to two-dimensional problems, the main directions of flow being axial and radial. Due to the symmetry of the arrangement, the axial flow of the iron package, and of the copper, divides along the central plane of the package. The division of the radial flow in the teeth and in the copper is somewhat more difficult to establish since it depends upon the relative intensity of the losses, and the cooling effects of the tooth region and at the back of the core.

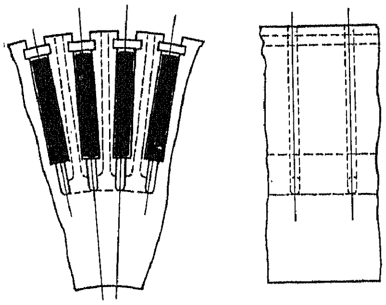


FIG. 3—TYPICAL ARRANGEMENT OF ROTOR AXIAL FLOW OF THE COOLING MEDIUM THROUGH VENT DUCTS AT THE BOTTOM OF SLOTS

Geometrically, the structure may be divided into the *tooth-region* and the *core,* bounded by a cylindrical surface through the bottom of the slots. In an arrangement of the type shown in Fig. 3 the cooling effect at the bottom of the slot is very pronounced, so that a considerable portion of the losses in the tooth-region will flow towards the back of the core. In the arrangement of Figs. 1 and 2 this tendency will be less pronounced, but in general, the radial flow will always divide in the tooth region. Before proceeding with the discussion of these problems the general method of analysis will be reviewed and applied to a few simple problems, the solutions of which will be used in the general solution of the problem.

THE THERMAL CIRCUIT

Consider a plate, Fig. 4, of thickness b, length l and height h, in which are generated the losses P watts with uniform intensity at all parts of the plate volume. These losses are carried to the cooling medium at the surfaces $x = l$ and $y = h$, all other surfaces being completely insulated. The effective cooling surfaces at these edges will be taken as $b_{ax}\, h$ and $b_{ay}\, l$, and the corresponding surface resistances as C_{ax} and C_{ay}, respectively.

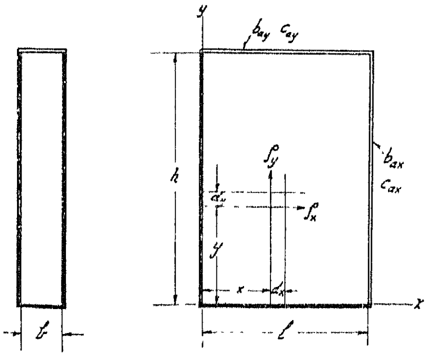


FIG. 4—SIMPLE TWO-DIMENSIONAL PROBLEM

Assuming at first that the surface $y = h$ also is heat insulated, a simple one-dimensional problem is obtained. The temperature of the plate is then easily found to be

$$t = P\left[\frac{C_{ax}}{b_{ax}\, h} + \frac{\rho_x}{b\, h}\,\frac{l}{2}\left(1 - \frac{x^2}{l^2}\right)\right] \tag{3}$$

The mean temperature is, consequently

$$t_m = P\left[\frac{C_{ax}}{b_{ax}\, h} + \frac{\rho_x}{b\, h}\,\frac{l}{3}\right] \tag{4}$$

This shows that the obstruction to the flow of heat from the mean temperature to the temperature of the cooling medium can be regarded as two thermal resistances in series $\dfrac{C_{ax}}{b_{ax}\; h}$ at the cooling surface, and $\dfrac{\rho_x}{b\; h}\;\dfrac{l}{3}$ in the plate. The latter is clearly the thermal resistance of a bar of area $b\,h$ and length $l/3$ (see Fig. 5). Similarly, the obstruction to a one-dimensional flow in the y-direction may be regarded as a combined thermal resistance of $\dfrac{C_{ay}}{b_{ay}\,l} + \dfrac{\rho\, y}{b\, l}\,\dfrac{h}{3}$.

The obstruction to the two-dimensional flow may now be regarded as the combination in parallel of these two groups of resistances. The *thermal circuit* of Fig. 6 may thus be used to illustrate the passage of the losses P from the mean temperature t_m to the temperature of the cooling medium. In solving for the mean temperature from this circuit introduce the quantities

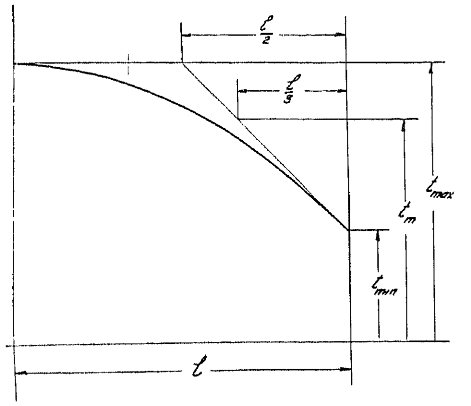
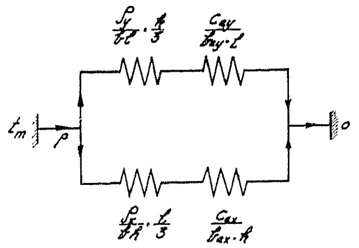


FIG. 5—TO ILLUSTRATE PARABOLIC TEMPERATURE DISTRIBUTION IN ONE-DIMENSIONAL FLOW OF DISTRIBUTED LOSSES



$\frac{\rho_y}{bl}\cdot\frac{h}{3}$
$\frac{C_{ay}}{b_{ay}\cdot l}$
t_m
P
0
$\frac{\rho_x}{bh}\cdot\frac{l}{3}$
$\frac{C_{ax}}{b_{ax}\cdot h}$

FIG. 6—THERMAL CIRCUIT FOR RECTANGULAR PLATE WITH DISTRIBUTED LOSSES

$$
\left.
\begin{aligned}
C_x &= \frac{2\,l\,C_{ax}}{b_{ax}} + \frac{2\,l^2\,\rho_x}{3\,b} \\
C_y &= \frac{2\,h\,C_{ay}}{b_{ay}} + \frac{2\,h^2\,\rho_y}{3\,b} \\
C &= \frac{C_x\,C_y}{C_x + C_y}
\end{aligned}
\right\} \quad (5)
$$

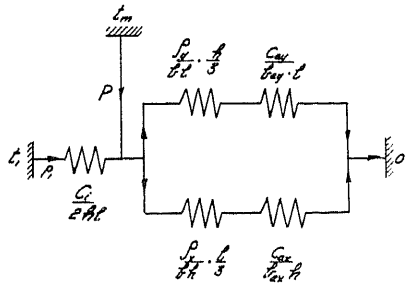
which all have the dimensions of surface resistance. Then the mean temperature becomes

$$t_m = \frac{P}{2\,h\,l}\,C \qquad (6)$$

The surface resistance C represents the insulating properties of an imaginary insulation layer which is placed on the two sides of the plate and which presents the same thermal resistance as the combined paths of the plate itself. C_x and C_y have the same significance for the one-dimensional flow in the x and y directions, respectively. The temperature at an arbitrary point (x, y) of the plate is easily obtained if the temperature variation is assumed to be parabolic. If it is found that

$$t = t_m\,\frac{\dfrac{2\,l\,C_{ax}}{b_{ax}} + \dfrac{l^2\,\rho_x}{b}\left(1 - \dfrac{x^2}{l^2}\right)}{C_x}\cdot\frac{\dfrac{2\,h\,C_{ay}}{b_{ay}} + \dfrac{h^2\,\rho_y}{b}\left(1 - \dfrac{y^2}{h^2}\right)}{C_y} \qquad (7)$$

It must be emphasized that this conception of the two-dimensional flow is a very rough approximation of the actual circumstances. As shown in Appendix I, however, it does give quantitative results which are sufficiently accurate for practical purposes.



t_m
P
$\frac{\rho_y}{bl}\cdot\frac{h}{3}$
$\frac{C_{ay}}{b_{ay}\cdot l}$
t_1
P_1
$\frac{C_i}{2hl}$
0
$\frac{\rho_x}{bh}\cdot\frac{l}{3}$
$\frac{C_{ax}}{b_{ax}\,h}$

FIG. 7—THERMAL CIRCUIT FOR RECTANGULAR PLATE WITH DISTRIBUTED LOSSES AND INCOMPLETELY INSULATED SIDES

Consider next the case where the sides of the plate are only partially insulated, the outside of the insulation being held at a constant temperature t_1, which is higher than the temperature of the plate itself. A certain amount of heat will now flow inward through the insulation layer where it will "mix" with the heat generated in the plate. The problem is now complicated by the fact that the intensity of flow across the insulation is not constant at all points. The total flow of heat across the insulation layer will be proportional to the average drop of temperature $t_1 - t_m$, but the contribution to the volume intensity of loss in the plate by the flow through the insulation layer will not be constant. The error in the mean temperature is small, however, as shown in Appendix I. Neglecting this error the problem may be represented by the thermal circuit shown in Fig. 7. P_1 is the loss flowing into the plate across the insulation, and C_i is the surface resistance of the insulation layer on the sides. With the notations given in (5) it is obvious from Fig. 7 that

$$t_m = \frac{P + P_1}{2\,h\,l}\,C \qquad (8)$$

and

$$t_1 = \frac{P_1}{2\,h\,l}\,C_i + \frac{P + P_1}{2\,h\,l}\,C \qquad (9)$$

In order to facilitate a comparison with the exact solution this result will be written in a different form by introducing the temperature

$$t_1' = t_1 + \frac{P}{2\,h\,l}\,C_i \qquad (10)$$

whence, for the mean temperature of the plate

$$t_m = t_1' \frac{C}{C + C_i} \qquad (11)$$

The temperature t_1' corresponds to the case when all the losses enter the plate from the outside through the insulation on the sides.

The thermal circuits shown so far cover cases of distributed losses. Similar circuits may be constructed for other methods of supplying the losses. Suppose, for example, that the losses P watts are introduced into the plate in Fig. 4 through the edge $y = o$. The exact solution for this case has not been investigated by the author, but it is evident that the flow in the y-direction will be essentially a flow with constant gradient of temperature; at any rate, this would certainly be the case if the edge $x = l$ were completely insulated. The flow in the x-direction, on the other hand, might be expected to approach a flow with distributed losses; this would, at any rate, be the case if the edge $y = h$ were completely insulated. The thermal circuit for the mean temperature of the plate would then take the form shown in Fig. 6 with the modification that the resistance $\frac{\rho_y}{b\,l}\,\frac{h}{3}$ is changed to $\frac{\rho_y}{b\,l}\,\frac{h}{2}$. Moreover, the temperature variation in the y-direction will be essentially linear so that the solution of the problem may be approximated by the equation

$$t = t_m \frac{\dfrac{2\,l\,C_{ax}}{b_{ax}} + \dfrac{2\,l^2\,\rho_x}{b}\left(1 - \dfrac{x^2}{l^2}\right)}{\dfrac{2\,l\,C_{ax}}{b_{ax}} + \dfrac{2\,l^2\,\rho_x}{3\,b}} \cdot \frac{\dfrac{2\,h\,C_{ay}}{b_{ay}} + \dfrac{2\,h^2\,\rho_y}{b}\left(1 - \dfrac{y}{h}\right)}{\dfrac{2\,h\,C_{ay}}{b_{ay}} + \dfrac{h^2\,\rho_y}{b}} \qquad (12)$$

where t_m is computed from the thermal circuit.

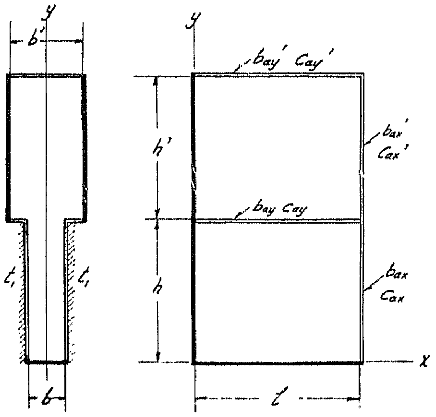


FIG. 8—COMPOUNDED RECTANGULAR PLATE

If, in addition to the surface losses along the edge $y = o$, there are uniformly distributed losses in the plate volume, the method of the thermal circuit suffers from a certain ambiguity. Of course, it is always possible to construct a circuit for each type of loss and superimpose the temperature fields thus obtained, but this method is somewhat too complicated for practical purposes. Whenever more than one type of loss is to be introduced into the same circuit it is usually necessary to make certain approximations for one or the other of the losses. The usual proportions of large turbo generators are such that the errors are negligible from a practical point of view.

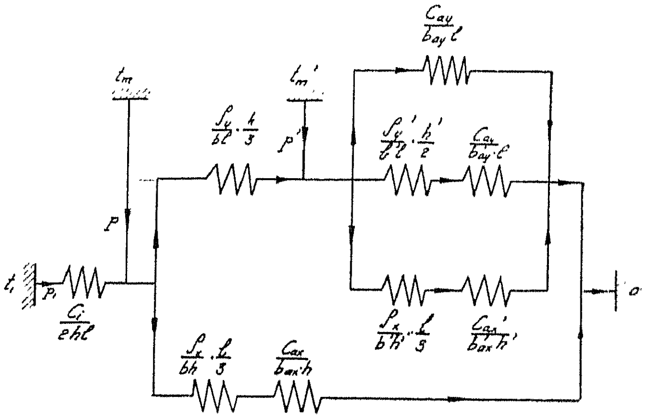


FIG. 9—THERMAL CIRCUIT FOR ARRANGEMENT IN FIG. 8

To illustrate this situation consider the problem shown in Fig. 8. The lower plate is identical in all respects with that shown in Fig. 4. The insulation on the sides has the surface resistance C_i and heat is flowing into this insulation from the temperature t_1. The sides of the upper plate are completely insulated. It is joined to the lower plate at the edge $y = h$ without affecting the cooling conditions for the lower plate at that edge. The losses in the lower and upper plates are P and P', respectively. The losses entering the lower plate through the insulation on the sides will be denoted by P_1.

If the losses P' may be considered concentrated along the edge $y = h$ of the upper plate, the thermal circuit of Fig. 9 will take care of the problem adequately. Obviously, the circuit is correct for the case that $P' = o$ as well. On the other hand, if the losses P' are evenly distributed throughout the volume, the circuit of Fig. 9 may be in error, depending on the importance of the paths of flow in the y-direction. If the conditions in the lower plate are such that only a small percentage of the flow is diverted in the y-direction, the thermal circuit of Fig. 9 will be more correct if the thermal resistance $\frac{\rho_y'}{b'\,l}\;\frac{h'}{2}$ be changed to $\frac{\rho_y'}{b'\,l}\;\frac{h'}{3}$ and the temperature variation in the upper plate be considered parabolic.

It is necessary, therefore, to exercise caution in applying the method of the equivalent thermal circuit. However, as shown in the following, the practical significance of this uncertainty is very small.

THE TEMPERATURE RISE CONSTANTS OF THE MACHINE

Returning now to the general problem presented by Figs. 1 to 3 it is assumed that the surface of division of the radial flow has been located. It is assumed, furthermore, since this always occurs in actual arrangements, that this surface is located in the tooth region, which it divides into two parts. The criterion for locating the surface of division is that the temperatures for the two regions "match" at this surface.

Considering first the inner part of the tooth-region (nearest the bottom of the slot) it is evident that the arrangement resembles that of Fig. 8. The lower plate corresponds to the tooth with the surface of division of the radial flow at the edge $y = o$. The upper part corresponds to the core. In constructing an equivalent thermal circuit for this system an uncertainty with regard to its construction will exist. This uncertainty is of very small practical consequence, however, as shown by the following reasoning. In the stator the radial component of the flow of heat in the tooth is usually very small so that the influence upon the tooth-region of the core is also small. Assuming the core losses to be uniformly distributed throughout the volume, the thermal circuit of Fig. 9, modified as suggested in the preceding paragraph, will be nearest to actual conditions. In the rotor, no core losses exist and the radial flow is large, so that the circuit of Fig. 9 is the correct one to use. It is evident, therefore, that a calculation of the conditions in the inner part of the tooth-region and the core is possible. The general process of analyzing the circuit thus decided upon will be obvious from the following discussion for the outer part.

It is a fortunate coincidence in the typical turbo generator that the surface of division of the radial flow in the tooth region is located near the bottom of the slots. Moreover, the error committed by placing it at the bottom of the slot is never very large and is on the side of higher temperatures. For this reason it is the outer part of the tooth region that is of greatest interest, and its solution leads to a set of quantities which may be referred to as the *temperature-rise constants of the machine*. Thus, the above discussion for the inner region merely serves as a check on the matching of the temperatures at the assumed position of the boundary.

The arrangement shown in Fig. 10 is a simplified tooth-region for a stator or a rotor of very large diameter. This may be referred to as two "coupled" plates of homogenous materials. The plate with index 1 represents the copper and has a thickness b_1 a length l_1 (one-half the vent-pitch) and a height h_1. Its specific thermal resistances are ρ_{x1} and ρ_{y1}, respectively, and the cooling conditions at the edges $x = l_1$ and $y = h_1$ are given by the cooling surfaces $b_{ax1}\,h_1$, and $b_{ay1}\,l_1$ and the surfaces resistances C_{ax1} and C_{ay1} respectively. The cooling surface at $x = l_1$ corresponds to the exposed part of the coil in the vent-duct, that at $y = h_1$ corresponds to the slot wedge. The plate with index 2 represents the iron, and the various quantities carry index 2; their significance is similar to that of the other plate. The insulation layer between the two plates is assumed to have a surface resistance of C_i. The following losses are considered:

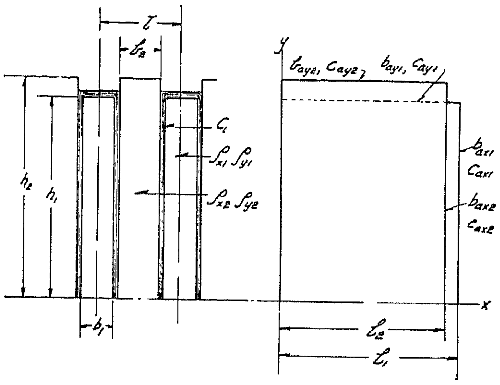


FIG. 10—SIMPLIFIED ARRANGEMENT OF SLOT REGION

1. Copper Losses. These are expressed either by the volume intensity p_1 in the copper volume $b_1\,h_1\,l_1$ or by the surface intensity q_1 on the air gap surface $\tau\,l_1$. The former is a function of x and y. If the mean value is denoted by $\bar{p}_1$ it is obvious that

$$q_1 = \frac{b_1\,h_1}{\tau}\,\bar{p}_1 \qquad (13)$$

2. Iron Losses. These are expressed either by the volume intensity p_2 in the iron volume $b_2\,h_2\,l_2$ or by the surface intensity q_2 on the air gap surface $\tau\,l_1$. The former is a function of x and y. If the mean value of p_2 is denoted by $\bar{p}_2$ it is obvious that

$$q_2 = \frac{b_2\,h_2\,l_2}{\tau\,l_1}\,\bar{p}_2 \qquad (14)$$

3. Surface Losses. (Pole-face losses.) These are assumed to be uniformly distributed over the tooth area $b_2\,l_2$ in the air gap, and are expressed by the equivalent surface intensity q_3 on the air-gap surface $\tau\,l_1$.

The principal object of the analysis is to determine the mean temperatures t_{1m} and t_{2m} of the copper and the iron. Assuming that the loss intensities p_1 and p_2

are uniform throughout the volumes the construction of the equivalent thermal circuit for the arrangement is comparatively simple. The results are shown in Fig. 11, which will thus give the solution of our problem with a degree of accuracy which will be discussed later. The solution will be put in the following form:

$$\begin{aligned} t_{1m} &= C_{11}\, q_1 + C_{12}\, q_2 + C_{13}\, q_3 \\ t_{2m} &= C_{21}\, q_1 + C_{22}\, q_2 + C_{23}\, q_3 \end{aligned} \tag{15}$$

The constants C_{11}, C_{12}, etc., expressed in deg. cent inch² per watt, will be defined as *the temperature rise constants of the machine.* The indexes are written in such a manner that the first number indicates the element (1 copper, 2 iron) to which the temperature refers, and the second indicates the kind of loss (1 copper losses, 2 iron losses, 3 surface losses) which cause the temperature rise. The thermal properties of plate 1 will be given by the surface resistances

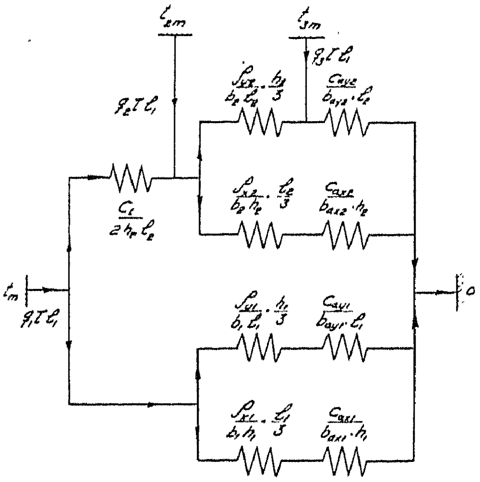

t_{2m}
t_{3m}
$q_3 \tau l_1$
$q_2 \tau l_1$
$\frac{\rho_{x2}}{b_2 l_2} \cdot \frac{h_2}{3}$
$\frac{C_{ay2}}{b_{ay2} \cdot l_2}$
$\frac{\rho_{x2}}{b_2 h_2} \cdot \frac{l_2}{3}$
$\frac{C_{ax2}}{b_{ax2} \cdot h_2}$
$\frac{C_i}{2 h_m l_2}$
t_m
$q_1 \tau l_1$
0
$\frac{\rho_{y1}}{b_1 l_1} \cdot \frac{h_1}{3}$
$\frac{C_{ay1}}{b_{ay1} \cdot l_1}$
$\frac{\rho_{x1}}{b_1 h_1} \cdot \frac{l_1}{3}$
$\frac{C_{ax1}}{b_{ax1} \cdot h_1}$

FIG. 11—THERMAL CIRCUIT FOR SLOT REGION

$$C_{x1} = \frac{2\, l_1\, C_{ax1}}{b_{ax1}} + \frac{2\, .\, l_1^2\, \rho_{x1}}{3\, b_1}$$

$$C_{y1} = \frac{2\, h_1\, C_{ay1}}{b_{ay1}} + \frac{2\, .\, h_1^2\, \rho_{y1}}{3\, b_1} \tag{16}$$

$$C_1 = \frac{C_{x1}\, C_{y1}}{C_{x1} + C_{y1}}$$

and those of the plate 2 by the surface resistances

$$C_{x2} = \frac{2\, l_2\, C_{ax2}}{b_{ax2}} + \frac{2\, l_2^2\, \rho_{x2}}{3\, b_2}$$

$$C_{y2} = \frac{2\, h_2\, C_{ay2}}{b_{ay2}} + \frac{2\, h_2^2\, \rho_{y2}}{3\, b_2} \tag{17}$$

$$C_2 = \frac{C_{x2}\, C_{y2}}{C_{x2} + C_{y2}}$$

The significance of these quantities is obvious from the above discussion of the two-dimensional problems. In addition, we introduce the dimensionless ratios

$$\left[\ \kappa_1 = \frac{C_1}{C_i + \dfrac{l_2}{l_1}\, C_1 + \dfrac{h_1}{h_2}\, C}\right.$$

$$\left.\kappa_2 = \frac{l_1}{l_2}\; \frac{C_i + \dfrac{l_2}{l_1}\, C_1}{C_i + \dfrac{l_2}{l_1}\, C_1 + \dfrac{h_1}{h_2}\, C_2}\ \right] \tag{18}$$

Then the temperature rise constants of the machine are given by

$$C_{11} = \frac{\tau}{2\, h_2}\; \kappa_1 \left[\frac{h_2}{h_1}\, C_i + C_2 \right]$$

$$C_{12} = C_{21} = \frac{\tau}{2\, h_2}\; \kappa_1\, C_2$$

$$C_{22} = \frac{\tau}{2\, h_2}\; \kappa_2\, C_2$$

$$C_{13} = C_{31} = \frac{\tau}{b_{ay2}}\; \frac{C_2}{C_{y2}}\; \kappa_1\, C_{ay2}$$

$$C_{23} = C_{32} = \frac{\tau}{b_{ay2}}\; \frac{C_2}{C_{y2}}\; \kappa_2\, C_{ay2} \tag{19}$$

The distribution of the copper and iron losses to the various paths is shown in Fig. 12 and Fig. 13. Of the copper losses in the slot, q_1, the portion $m\, q_1$ flows across the slot insulation to the tooth, the remainder $(1 - m)\, q_1$ flows directly through the copper to the cooling medium. Of the former portion $n\, m\, q_1$ flows in the radial direction of the tooth, and $(1 - n)\, m\, q_1$ flows in the axial direction. Of the latter portion $\sigma\, (1 - m)\, q_1$ flows in the radial direction. The iron losses in the tooth are divided into $n\, q_2$ flowing in the radial direction, and $(1 - n)\, q_2$ in the axial direction. The constants m, n and o are found to have the values

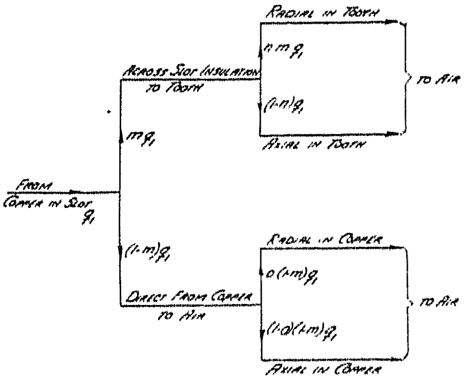


FIG. 12—DISTRIBUTION OF FLOW OF COPPER LOSSES

$$m = \kappa_1 \left[\frac{l_2}{l_1} - \frac{q_2}{q_1} \frac{h_1}{h_2} \frac{C_2}{C_1} - \frac{q_3}{q_1} \frac{2\,h_1}{b_{ay2}} \frac{C_2}{C_1} \frac{C_{ay2}}{C_{y2}} \right]$$

$$n = \frac{C_2}{C_{y2}} \left[1 - \frac{q_3}{q_1 m + q_2} \frac{2\,h_2}{b_{ay2}} \frac{C_{ay2}}{C_{x2}} \right]$$

$$o = \frac{C_1}{C_{y1}} \qquad (20)$$

ERRORS DUE TO THE APPROXIMATIONS

The solution given in the preceding paragraph is usually sufficiently accurate for practical problems, but it is of importance, nevertheless, to know the approximate errors that are committed by neglecting some of the features of the actual problem. Therefore, the present section will be devoted to a discussion of the influence upon the solution of the following factors: 1. The inaccuracy of the approximate solution of the two-dimensional problem with distributed losses; 2. The taper of the teeth; and 3. The variation of the volume intensity of the copper and iron losses.

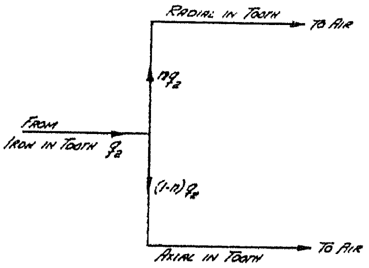


FIG. 13—DISTRIBUTION OF FLOW OF IRON LOSSES

The Inaccuracy of the Approximate Solution of the Two-Dimensional Problem

The solution given by the thermal circuit of Fig. 6 for the system shown in Fig. 4 gives too high a value of t_m, because the interference of the radial and axial flow has been neglected. It is obvious that the error is greatest when the flow is of the same intensity in the two directions. The error must also depend upon the relative magnitude of the temperature drops in the plate and at the cooling surfaces, because the approximate solution becomes exact when the former are zero. The result obtained in Appendix I-A agrees with this reasoning. It is shown there that a nearly exact value of t_m is obtained if the surface resistance C is replaced by a corrected value

$$C^* = \left[1 - \frac{1}{5} \frac{2\,\rho_x l^2}{3\,b\,C_x} \frac{2\,\rho_y h^2}{3\,b\,C_y} \right] C \qquad (21)$$

In extreme cases of very deep stator teeth the ratios $(2\rho_x l^2)/(3\,b\,C_x)$ and $(2\rho_y h)^2/(3\,b\,C_y)$ may have the value 0.65 and 0.45, respectively, so that an error in mean temperature of six per cent is possible from this source. This may mean 1 deg. cent. for the iron temperature and 1.5 deg. cent. on the copper temperature. The errors in maximum temperature may be twice as large.

This conclusion applies to cases in which the slot-insulation is very heavy (C_i large), so that the copper losses enter the tooth at uniform density. In actual cases, particularly for low-voltage machines and for the rotor, where this is not the case, there is an additional error. It is shown in Appendix I-B, that when this additional effect is included, the error in mean temperature may amount to 7 per cent. A "semi-exact" solution is also given, whereby it is possible to reduce this error to 1 or 2 per cent.

All of the above errors are on the "safe side," that is, the calculated temperatures are larger than the actual. We may conclude, therefore, that this group of errors may be neglected for the ordinary types of machines.

The Influence of the Taper of the Teeth

The taper of the teeth may influence the results in two ways. When, as in the rotor, there are no iron losses, the taper will modify the flow of heat in the tooth to some extent. When there are iron losses, as in the stator, the tapered tooth will have much higher flux density at the tip than at the root. The resulting concentration of the iron losses at the tip will be very pronounced, and this will in turn modify the radial (and to a less extent the axial) flow of heat. The resulting changes in the temperature are usually greater than those due directly to the taper, but both types of influences are comparatively small.

It has been found convenient to define the taper of the tooth by the ratio

$$\theta = \frac{\pi\,h}{b\,Q} \qquad (22)$$

where b and h are the mean thickness and the height of the tooth, respectively, and Q is the number of tooth pitches on the circumference. The ratio θ will be called the *taper constant.* This quantity represents the ratio of the depth of the tooth to the equivalent diameter[3] of the core at the middle of the tooth, and is found to appear in a simple manner in relations expressing the influence of tooth taper. θ is reckoned positive when the flow of heat is towards the wider part of the tooth (rotor), and negative when the flow is towards the narrow part of the tooth (stator). $\theta = 1$ represents a rotor tooth which goes to a point at the bottom, $\theta = 0$ a tooth with parallel sides, and $\theta = -1$ a stator tooth which goes to a point at the tip (see Fig. 15). Obviously, the conception of taper is easily extended to any cylinder problem.

In order to investigate the influence of the taper upon the flow of heat in the tooth when no readjustment of the losses takes place, consider the arrangement shown in Fig. 14 and assume that the thickness of the plate varies directly with y, with a mean value of b. Assume that the loss intensity in the plate varies inversely with the thickness, so that the loss per unit height of the plate is constant. This assumption will be the nearest to the actual case of the rotor, when the copper losses enter the tooth across the insulation with approximately constant intensity at all points. This problem is discussed at length in Appendix II-A. It is shown that for moderate values of the taper constant ($/ \theta / = 0.3$) the simple expression for the mean temperature can be retained if the specific thermal resistance ρ_y is given a corrected value, determined by the equation

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[3] The diameter for zero slot-width ($2r_m$ in Fig. 14.)

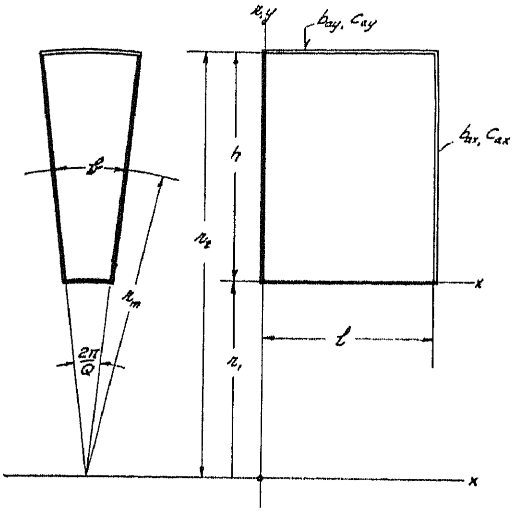


Fig. 14—Simple Cylinder Problem

$$\rho_y^* = \frac{1}{\sqrt{1 + \theta}}\, \rho_y \tag{23}$$

In this expression θ is to be given a positive sign for a rotor tooth and a negative sign for a stator tooth. This simple correction gives a remarkably close agreement with the accurate solution even when θ is greater than 0.3, although such cases are rare in actual practise. When $\theta = 0.3$, $\rho_y^* = 0.88\, \rho_y$ for the rotor, and $\rho_y^* = 1.2\, y$ for the stator.

To establish the influence of the taper upon the distribution of the iron losses, we assume that the same flux flows through each section of the tooth, and that the loss intensity varies with the nth power of the flux density. If the loss intensity at the mean flux density is denoted by p_{2m} it is easily seen that

$$p_2 = \frac{p_{2m}}{\left[1 - \theta\left(1 - \frac{2\,y}{h}\right)\right]^n} \tag{24}$$

From this expression, the mean loss intensity is found to be

$$\bar{p}_2 = k_n\, p_{2m} \tag{25}$$

where

$$k_n = -\frac{1}{2\,\theta\,(n-2)}\left[\frac{1}{(1+\theta)^{n-2}} - \frac{1}{(1-\theta)^{n-2}}\right] \tag{26}$$

For the present purpose it is more convenient to replace the function (**24**) by a parabola. It is found that a fair agreement is obtained by putting

$$p_2 = \frac{1 + 3\,k_2\left(\frac{y}{h_2}\right)^2}{1 + k_2} \tag{27}$$

where

$$k_2 = \frac{1}{3}\left[\left(\frac{1-\theta}{1+\theta}\right)^n - 1\right] \tag{28}$$

It should be noted that θ is to be entered in equations (**26**) and (**28**) with a negative sign. By these relations the iron losses have been divided into a constant part $\bar{p}_2/(1 + k_2)$ and a parabolically varying part $[\bar{p}_2/(1 + k_2)]$. $[1 + 3\,k_2\,(y/h_2)^2]$. The reason for selecting this particular type of division is found in the next paragraph where the influence of irregular distribution of the iron losses is discussed.

For the comparatively high inductions which are the rule in modern turbo-generators the losses vary with the 3rd or 4th power of the induction. Thus, if $n = 4$ we obtain, for a tooth-taper of $\theta = -0{,}20$, $k_n = 1{,}09$ and $k_2 = 1.35$. The minimum and maximum loss intensities, when figured from equation (**24**), are $0.44\,\bar{p}_2$ and $2.24\,\bar{p}_2$ respectively. The more approximate equation (**27**) gives for the same quantities $0.43\,\bar{p}_2$ and $2.15\,\bar{p}_2$, a slightly less sharp variation of the loss intensity than the one assumed for $n = 4$. In view of the uncertainty of the exponent n, however, and the uncertainty of the assumption of constant flux, there is no justification for a more accurate value of the constant k_2. The distribution thus obtained will be qualitatively correct, and the general magnitude of the influence upon the mean temperature will be sufficiently exact.

The Influence of the Variation of the Intensity of the Losses upon the Mean Temperature

The influence upon the temperature of irregular distribution of the losses is intimately connected with the specific thermal resistances of the copper and the tooth material. Therefore, this problem demands for its solution a discussion of the entire system of Fig. 10 as a whole, with particular emphasis given to the interaction of the two plates upon one another. In order to bring out the essential features, the investigation may be restricted to a straight tooth, and in addition, $h_1 = h_2 = h$ and $l_1 = l_2 = l$. These simplifications will not influence the main points of the results.

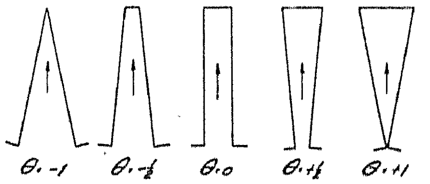


Fig. 15—To Illustrate Taper Constant θ

The copper losses are assumed to be made up of two parts, the losses due to the d-c. resistance, and the eddy current losses. The latter vary nearly parabolically with y, and their magnitude is a fraction k_1 of the d-c. losses, so that

$$p_1 = \frac{\bar{p}_1}{1 + k_1}\left[1 + 3\,k_1\left(\frac{y}{h}\right)^2\right] \qquad \textbf{(29)}$$

The quantity $1 + k_1$ is the ordinary eddy factor. Similarly, as shown in the preceding paragraph, the iron losses can be put

$$p_2 = \frac{\bar{p}_2}{1 + k_2}\left[1 + 3\,k_2\left(\frac{y}{h}\right)^2\right] \qquad \textbf{(30)}$$

The specific thermal resistance of the iron in the teeth does not vary a great deal from machine to machine. In general we may put for 0.014-in. thick enameled sheets $\rho_{2x} = 35$ and $\rho_{2y} = 1.25$ deg. cent. inch per watt. The specific thermal resistances of the copper are always the same in the axial direction, but in the radial direction are greatly dependent on the insulation between strands. For a typical 13,000-volt machine we may put $\rho_{1x} = 0.11$ and $\rho_{1y} = 60$ deg. cent. inch per watt. Thus, the essential features of the problem will be covered if we put $\rho_{1x} = 0$ and consider the effect of varying ρ_{1y} between zero and infinity.

$$\rho_{x1} = 0;\ \rho_{1y} = 0$$

In this case the copper temperature will remain practically constant, and the temperature drop across the insulation will vary with the iron temperature. Since the variation in intensity of the copper losses will not influence the temperatures at all in this case, only the variation of the iron losses need be considered. Since the losses in the teeth are concentrated towards the tip it is obvious that the mean temperature will be lowered. The amount will depend upon the relative magnitude of ρ_{2y} and C_i.

If the quantity $(2\,\rho_{y2}\,h^2)/(3\,b_2\,C_i)$ is small (< 0.5) the corrected mean iron temperature may be shown to be

$$t_{2m}^{*} = t_{2m} - 0.4\ \Delta\ t_2 \qquad \textbf{(31)}$$

where

$$\Delta\ t_2 \leqq \frac{\tau}{2\,h}\ \frac{2\,h^2\,\rho_{2y}}{3\,b_2}\ \frac{k_2}{1 + k_2}\ q_2 \qquad (32)$$

The decrease in the maximum iron temperature is found to be $0.75\ \Delta\ t_2$.

Thus the difference between the maximum and mean temperature will be changed by $(0.75 - 0.40)\ \Delta\ t_2 = 0.35\ \Delta\ t_2$. For a typical turbo-generator for 13,000 volts, the temperature $\Delta\ t_2$ is rarely above 5 deg. cent. Therefore, the correction in mean temperature of the iron is of the order of 2 deg. cent. If the quantity $(2\rho_{y2}\,h^2)/(3\,b_2 C_i)$ is large, the correction is a smaller part of $\Delta\ t_2$, and the temperatures must be computed by more complicated expressions.

It is worth noting that the direct influence of the tooth taper in obstructing the radial flow, and the indirect influence in redistributing the iron losses, compensate each other, so that the net effect for the stator is usually very small.

$$\rho_{x1} = 0;\ \rho_{y1} = \infty$$

This case is usually nearer to the actual conditions. The copper temperature will now remain constant in the axial direction, but it will vary in the radial direction, where it will tend to adjust itself to the iron temperature. The influence upon the mean temperature, beyond that due to the change in the mean iron temperature, is very small. The maximum copper temperature will be affected to a greater degree. If we put

$$\Delta\ t_1 = \frac{\tau}{2\,h}\ C_i\ \frac{k_1}{1 + k_1}\ q_1 \qquad \textbf{(33)}$$

it can be shown that the corrected maximum copper temperature may be put

$$t_1{}^{*}{}_{max} = t_{1m} + (t_{2\ max} - t_{2m}) - (0.35\ \Delta\ t_2 + \Delta\ t_1) \qquad \textbf{(34)}$$

Thus, the difference between maximum and minimum temperature of the copper will be the difference between the mean temperature of the iron at the root of the tooth and the mean iron temperature, less $(0.35\ \Delta\ t_2 + \Delta\ t_1)$. The former temperature difference may amount to 5 deg. cent. in a typical machine, the latter may be slightly larger. Thus, according to (**34**) the copper temperature may actually be lower at the bottom of the slot than at the top. The corrections in this equation are limiting values, however, which are usually not reached. We are justified, however, in concluding that the copper temperature in a typical turbo-generator is very nearly constant for the entire depth of the slot.

CONCLUSION

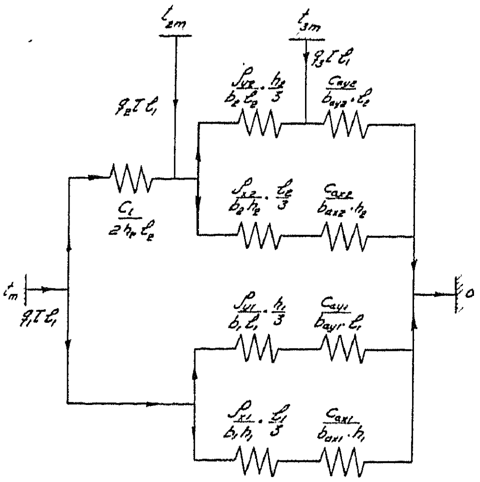
It is shown in the above that the conception of an equivalent thermal circuit is helpful in deriving approximate solutions for the complicated phenomena encountered in predetermining the flow of heat in large turbo-generators. The degree of complexity to which it is necessary to go will depend on the individual arrangement of the machines. In general, it is obvious that a simple circuit, emphasizing the essential features of the flow, is to be preferred to a more accurate, but more complicated circuit. It is believed that with this method the temperature can be predicted to within a few per cent of the iron temperature; the uncertainty of losses and thermal properties are usually so great that additional accuracy is chimerical.

The results obtained above have been applied to a number of turbine-generators of the general type of construction referred to. The agreement between measured and calculated temperatures is satisfactory in all cases, considering the unavoidable uncertainty in the thermal properties of the materials involved. The application of the results to actual machines will be made the subject of a separate paper in the future.

are uniform throughout the volumes the construction of the equivalent thermal circuit for the arrangement is comparatively simple. The results are shown in Fig. 11, which will thus give the solution of our problem with a degree of accuracy which will be discussed later. The solution will be put in the following form:

$$\begin{aligned} t_{1m} &= C_{11}\,q_1 + C_{12}\,q_2 + C_{13}\,q_3 \\ t_{2m} &= C_{21}\,q_1 + C_{22}\,q_2 + C_{23}\,q_3 \end{aligned} \qquad \textbf{(15)}$$

The constants C_{11}, C_{12}, etc., expressed in deg. cent inch^2 per watt, will be defined as *the temperature rise constants of the machine.* The indexes are written in such a manner that the first number indicates the element (1 copper, 2 iron) to which the temperature refers, and the second indicates the kind of loss (1 copper losses, 2 iron losses, 3 surface losses) which cause the temperature rise. The thermal properties of plate 1 will be given by the surface resistances



t_{2m}
t_{3m}
$q_3 \tau l_1$
$q_2 \tau l_1$
$\frac{\rho_{y2}}{b_2 l_2} \cdot \frac{h_2}{3}$
$\frac{C_{ay2}}{b_{ay2} \cdot l_2}$
$\frac{C_i}{2 h_1 l_2}$
$\frac{\rho_{x2}}{b_2 h_2} \cdot \frac{l_2}{3}$
$\frac{C_{ax2}}{b_{ax2} \cdot h_2}$
t_m
$q_1 \tau l_1$
0
$\frac{\rho_{y1}}{b_1 l_1} \cdot \frac{h_1}{3}$
$\frac{C_{ay1}}{b_{ay1} \cdot l_1}$
$\frac{\rho_{x1}}{b_1 h_1} \cdot \frac{l_1}{3}$
$\frac{C_{ax1}}{b_{ax1} \cdot h_1}$

FIG. 11—THERMAL CIRCUIT FOR SLOT REGION

$$\begin{aligned} C_{x1} &= \frac{2\,l_1\,C_{ax1}}{b_{ax1}} + \frac{2\,.\,l_1^2\,\rho_{x1}}{3\,b_1} \\ C_{y1} &= \frac{2\,h_1\,C_{ay1}}{b_{ay1}} + \frac{2\,.\,h_1^2\,\rho_{y1}}{3\,b_1} \qquad \textbf{(16)} \\ C_1 &= \frac{C_{x1}\,C_{y1}}{C_{x1} + C_{y1}} \end{aligned}$$

and those of the plate 2 by the surface resistances

$$\begin{aligned} C_{x2} &= \frac{2\,l_2\,C_{ax2}}{b_{ax2}} + \frac{2\,l_2^2\,\rho_{x2}}{3\,b_2} \\ C_{y2} &= \frac{2\,h_2\,C_{ay2}}{b_{ay2}} + \frac{2\,h_2^2\,\rho_{y2}}{3\,b_2} \qquad \textbf{(17)} \\ C_2 &= \frac{C_{x2}\,C_{y2}}{C_{x2} + C_{y2}} \end{aligned}$$

The significance of these quantities is obvious from the above discussion of the two-dimensional problems. In addition, we introduce the dimensionless ratios

$$\left[\begin{aligned} \kappa_1 &= \frac{C_1}{C_i + \dfrac{l_2}{l_1}\,C_1 + \dfrac{h_1}{h_2}\,C} \\ \kappa_2 &= \frac{l_1}{l_2}\;\frac{C_i + \dfrac{l_2}{l_1}\,C_1}{C_i + \dfrac{l_2}{l_1}\,C_1 + \dfrac{h_1}{h_2}\,C_2} \end{aligned}\right] \qquad \textbf{(18)}$$

Then the temperature rise constants of the machine are given by

$$\begin{aligned} C_{11} &= \frac{\tau}{2\,h_2}\;\kappa_1\left[\frac{h_2}{h_1}\,C_i + C_2\right] \\ C_{12} &= C_{21} = \frac{\tau}{2\,h_2}\;\kappa_1\,C_2 \\ C_{22} &= \frac{\tau}{2\,h_2}\;\kappa_2\,C_2 \\ C_{13} &= C_{31} = \frac{\tau}{b_{ay2}}\;\frac{C_2}{C_{y2}}\;\kappa_1\,C_{ay2} \\ C_{23} &= C_{32} = \frac{\tau}{b_{ay2}}\;\frac{C_2}{C_{y2}}\;\kappa_2\,C_{ay2} \end{aligned} \qquad \textbf{(19)}$$

The distribution of the copper and iron losses to the various paths is shown in Fig. 12 and Fig. 13. Of the copper losses in the slot, q_1, the portion $m\,q_1$ flows across the slot insulation to the tooth, the remainder $(1 - m)\,q_1$ flows directly through the copper to the cooling medium. Of the former portion $n\,m\,q_1$ flows in the radial direction of the tooth, and $(1 - n)\,m\,q_1$ flows in the axial direction. Of the latter portion $o\,(1 - m)\,q_1$ flows in the radial direction. The iron losses in the tooth are divided into $n\,q_2$ flowing in the radial direction, and $(1 - n)\,q_2$ in the axial direction. The constants m, n and o are found to have the values

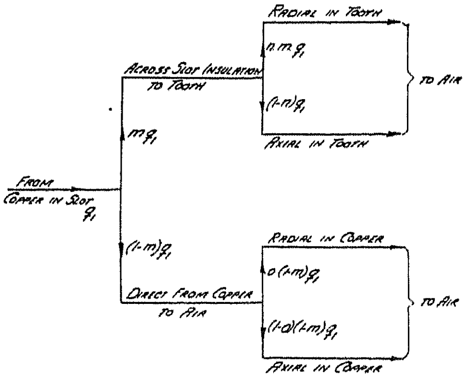


FIG. 12—DISTRIBUTION OF FLOW OF COPPER LOSSES

$$m = \kappa_1 \left[\frac{l_2}{l_1} - \frac{q_2}{q_1} \, \frac{h_1}{h_2} \, \frac{C_2}{C_1} - \frac{q_3}{q_1} \, \frac{2\,h_1}{b_{ay2}} \, \frac{C_2}{C_1} \, \frac{C_{ay2}}{C_{y2}} \right]$$

$$n = \frac{C_2}{C_{y2}} \left[1 - \frac{q_3}{q_1 m + q_2} \, \frac{2\,h_2}{b_{ay2}} \, \frac{C_{ay2}}{C_{x2}} \right]$$

$$o = \frac{C_1}{C_{y1}} \tag{20}$$

Errors Due to the Approximations

The solution given in the preceding paragraph is usually sufficiently accurate for practical problems, but it is of importance, nevertheless, to know the approximate errors that are committed by neglecting some of the features of the actual problem. Therefore, the present section will be devoted to a discussion of the influence upon the solution of the following factors: 1. The inaccuracy of the approximate solution of the two-dimensional problem with distributed losses; 2. The taper of the teeth; and 3. The variation of the volume intensity of the copper and iron losses.

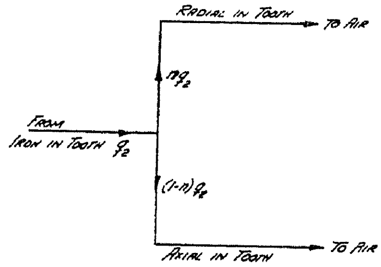


Fig. 13—Distribution of Flow of Iron Losses

The Inaccuracy of the Approximate Solution of the Two-Dimensional Problem

The solution given by the thermal circuit of Fig. 6 for the system shown in Fig. 4 gives too high a value of t_m, because the interference of the radial and axial flow has been neglected. It is obvious that the error is greatest when the flow is of the same intensity in the two directions. The error must also depend upon the relative magnitude of the temperature drops in the plate and at the cooling surfaces, because the approximate solution becomes exact when the former are zero. The result obtained in Appendix I-A agrees with this reasoning. It is shown there that a nearly exact value of t_m is obtained if the surface resistance C is replaced by a corrected value

$$C^* = \left[1 - \frac{1}{5} \, \frac{2\,\rho_x l^2}{3\,b\,C_x} \, \frac{2\,\rho_y h^2}{3\,b\,C_y} \right] C \tag{21}$$

In extreme cases of very deep stator teeth the ratios $(2\rho_x l^2)/(3\,b\,C_x)$ and $(2\rho_y h)^2/(3\,b\,C_y)$ may have the value 0.65 and 0.45, respectively, so that an error in mean temperature of six per cent is possible from this source. This may mean 1 deg. cent. for the iron temperature and 1.5 deg. cent. on the copper temperature. The errors in maximum temperature may be twice as large.

This conclusion applies to cases in which the slot-insulation is very heavy (C_i large), so that the copper losses enter the tooth at uniform density. In actual cases, particularly for low-voltage machines and for the rotor, where this is not the case, there is an additional error. It is shown in Appendix I-B, that when this additional effect is included, the error in mean temperature may amount to 7 per cent. A "semi-exact" solution is also given, whereby it is possible to reduce this error to 1 or 2 per cent.

All of the above errors are on the "safe side," that is, the calculated temperatures are larger than the actual. We may conclude, therefore, that this group of errors may be neglected for the ordinary types of machines.

The Influence of the Taper of the Teeth

The taper of the teeth may influence the results in two ways. When, as in the rotor, there are no iron losses, the taper will modify the flow of heat in the tooth to some extent. When there are iron losses, as in the stator, the tapered tooth will have much higher flux density at the tip than at the root. The resulting concentration of the iron losses at the tip will be very pronounced, and this will in turn modify the radial (and to a less extent the axial) flow of heat. The resulting changes in the temperature are usually greater than those due directly to the taper, but both types of influences are comparatively small.

It has been found convenient to define the taper of the tooth by the ratio

$$\theta = \frac{\pi\,h}{b\,Q} \tag{22}$$

where b and h are the mean thickness and the height of the tooth, respectively, and Q is the number of tooth pitches on the circumference. The ratio θ will be called the *taper constant.* This quantity represents the ratio of the depth of the tooth to the equivalent diameter[3] of the core at the middle of the tooth, and is found to appear in a simple manner in relations expressing the influence of tooth taper. θ is reckoned positive when the flow of heat is towards the wider part of the tooth (rotor), and negative when the flow is towards the narrow part of the tooth (stator). $\theta = 1$ represents a rotor tooth which goes to a point at the bottom, $\theta = 0$ a tooth with parallel sides, and $\theta = -1$ a stator tooth which goes to a point at the tip (see Fig. 15). Obviously, the conception of taper is easily extended to any cylinder problem.

In order to investigate the influence of the taper upon the flow of heat in the tooth when no readjustment of the losses takes place, consider the arrangement shown in Fig. 14 and assume that the thickness of the plate varies directly with y, with a mean value of b. Assume that the loss intensity in the plate varies inversely with the thickness, so that the loss per unit height of the plate is constant. This assumption will be the nearest to the actual case of the rotor, when the copper losses enter the tooth across the insulation with approximately constant intensity at all points. This problem is discussed at length in Appendix II-A. It is shown that for moderate values of the taper constant ($/\ \theta\ / = 0.3$) the simple expression for the mean temperature can be retained if the specific thermal resistance ρ_y is given a corrected value, determined by the equation

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[3] The diameter for zero slot-width ($2r_m$ in Fig. 14.)

$$\rho_y^* = \frac{1}{\sqrt{1 + \theta}}\, \rho_y \tag{23}$$

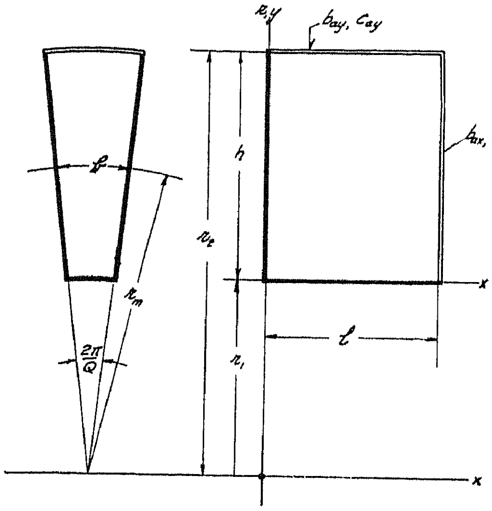

Fig. 14—Simple Cylinder Problem

In this expression θ is to be given a positive sign for a rotor tooth and a negative sign for a stator tooth. This simple correction gives a remarkably close agreement with the accurate solution even when θ is greater than 0.3, although such cases are rare in actual practise. When $\theta = 0.3$, $\rho_y^* = 0.88\ \rho_y$ for the rotor, and $\rho_y^* = 1.2\ y$ for the stator.

To establish the influence of the taper upon the distribution of the iron losses, we assume that the same flux flows through each section of the tooth, and that the loss intensity varies with the nth power of the flux density. If the loss intensity at the mean flux density is denoted by p_{2m} it is easily seen that

$$p_2 = \frac{p_{2m}}{\left[1 - \theta\left(1 - \frac{2\,y}{h}\right)\right]^n} \tag{24}$$

From this expression, the mean loss intensity is found to be

$$\bar{p}_2 = k_n\, p_{2m} \tag{25}$$

where

$$k_n = -\frac{1}{2\,\theta\,(n-2)}\left[\frac{1}{(1+\theta)^{n-2}} - \frac{1}{(1-\theta)^{n-2}}\right] \tag{26}$$

For the present purpose it is more convenient to replace the function (**24**) by a parabola. It is found that a fair agreement is obtained by putting

$$p_2 = \frac{1 + 3\,k_2\left(\frac{y}{h_2}\right)^2}{1 + k_2} \tag{27}$$

where

$$k_2 = \frac{1}{3}\left[\left(\frac{1-\theta}{1+\theta}\right)^n - 1\right] \tag{28}$$

It should be noted that θ is to be entered in equations (**26**) and (**28**) with a negative sign. By these relations the iron losses have been divided into a constant part $\bar{p}_2/(1 + k_2)$ and a parabolically varying part $[\bar{p}_2/(1 + k_2)]$. $[1 + 3\,k_2\,(y/h_2)^2]$. The reason for selecting this particular type of division is found in the next paragraph where the influence of irregular distribution of the iron losses is discussed.

For the comparatively high inductions which are the rule in modern turbo-generators the losses vary with the 3rd or 4th power of the induction. Thus, if $n = 4$ we obtain, for a tooth-taper of $\theta = -\ 0{,}20$, $k_n = 1{,}09$ and $k_2 = 1.35$. The minimum and maximum loss intensities, when figured from equation (24), are $0.44\ \bar{p}_2$ and $2.24\ \bar{p}_2$ respectively. The more approximate equation (27) gives for the same quantities $0.43\ \bar{p}_2$ and $2.15\ \bar{p}_2$, a slightly less sharp variation of the loss intensity than the one assumed for $n = 4$. In view of the uncertainty of the exponent n, however, and the uncertainty of the assumption of constant flux, there is no justification for a more accurate value of the constant k_2. The distribution thus obtained will be qualitatively correct, and the general magnitude of the influence upon the mean temperature will be sufficiently exact.

The Influence of the Variation of the Intensity of the Losses upon the Mean Temperature

The influence upon the temperature of irregular distribution of the losses is intimately connected with the specific thermal resistances of the copper and the tooth material. Therefore, this problem demands for its solution a discussion of the entire system of Fig. 10 as a whole, with particular emphasis given to the interaction of the two plates upon one another. In order to bring out the essential features, the investigation may be restricted to a straight tooth, and in addition, $h_1 = h_2 = h$ and $l_1 = l_2 = l$. These simplifications will not influence the main points of the results.

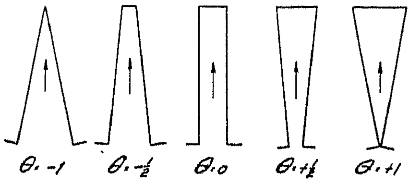

Fig. 15—To Illustrate Taper Constant θ

The copper losses are assumed to be made up of two parts, the losses due to the d-c. resistance, and the eddy current losses. The latter vary nearly parabolically with y, and their magnitude is a fraction k_1 of the d-c. losses, so that

$$p_1 = \frac{\bar{p}_1}{1 + k_1}\left[1 + 3\,k_1\left(\frac{y}{h}\right)^2\right] \qquad (29)$$

The quantity $1 + k_1$ is the ordinary eddy factor. Similarly, as shown in the preceding paragraph, the iron losses can be put

$$p_2 = \frac{\bar{p}_2}{1 + k_2}\left[1 + 3\,k_2\left(\frac{y}{h}\right)^2\right] \qquad (30)$$

The specific thermal resistance of the iron in the teeth does not vary a great deal from machine to machine. In general we may put for 0.014-in. thick enameled sheets $\rho_{2x} = 35$ and $\rho_{2y} = 1.25$ deg. cent. inch per watt. The specific thermal resistances of the copper are always the same in the axial direction, but in the radial direction are greatly dependent on the insulation between strands. For a typical 13,000-volt machine we may put $\rho_{1x} = 0.11$ and $\rho_{1y} = 60$ deg. cent. inch per watt. Thus, the essential features of the problem will be covered if we put $\rho_{1x} = 0$ and consider the effect of varying ρ_{1y} between zero and infinity.

$$\rho_{x1} = 0;\ \rho_{1y} = 0$$

In this case the copper temperature will remain practically constant, and the temperature drop across the insulation will vary with the iron temperature. Since the variation in intensity of the copper losses will not influence the temperatures at all in this case, only the variation of the iron losses need be considered. Since the losses in the teeth are concentrated towards the tip it is obvious that the mean temperature will be lowered. The amount will depend upon the relative magnitude of ρ_{2y} and C_i.

If the quantity $(2\,\rho_{y2}\,h^2)/(3\,b_2\,C_i)$ is small (< 0.5) the corrected mean iron temperature may be shown to be

$$t_{2m}{}^* = t_{2m} - 0.4\,\Delta\,t_2 \qquad (31)$$

where

$$\Delta\,t_2 \leqq \frac{\tau}{2\,h}\;\frac{2\,h^2\,\rho_{2y}}{3\,b_2}\;\frac{k_2}{1 + k_2}\,q_2 \qquad (32)$$

The decrease in the maximum iron temperature is found to be $0.75\,\Delta\,t_2$.

Thus the difference between the maximum and mean temperature will be changed by $(0.75 - 0.40)\,\Delta\,t_2 = 0.35\,\Delta\,t_2$. For a typical turbo-generator for 13,000 volts, the temperature $\Delta\,t_2$ is rarely above 5 deg. cent. Therefore, the correction in mean temperature of the iron is of the order of 2 deg. cent. If the quantity $(2\rho_{y2}\,h^2)/(3\,b_2 C_i)$ is large, the correction is a smaller part of $\Delta\,t_2$, and the temperatures must be computed by more complicated expressions.

It is worth noting that the direct influence of the tooth taper in obstructing the radial flow, and the indirect influence in redistributing the iron losses, compensate each other, so that the net effect for the stator is usually very small.

$$\rho_{x1} = 0;\ \rho_{y1} = \infty$$

This case is usually nearer to the actual conditions. The copper temperature will now remain constant in the axial direction, but it will vary in the radial direction, where it will tend to adjust itself to the iron temperature. The influence upon the mean temperature, beyond that due to the change in the mean iron temperature, is very small. The maximum copper temperature will be affected to a greater degree. If we put

$$\Delta\,t_1 = \frac{\tau}{2\,h}\,C_i\,\frac{k_1}{1 + k_1}\,q_1 \qquad (33)$$

it can be shown that the corrected maximum copper temperature may be put

$$t_1{}^*{}_{max} = t_{1m} + (t_{2\,max} - t_{2m}) - (0.35\,\Delta\,t_2 + \Delta\,t_1) \qquad (34)$$

Thus, the difference between maximum and minimum temperature of the copper will be the difference between the mean temperature of the iron at the root of the tooth and the mean iron temperature, less $(0.35\,\Delta\,t_2 + \Delta\,t_1)$. The former temperature difference may amount to 5 deg. cent. in a typical machine, the latter may be slightly larger. Thus, according to (**34**) the copper temperature may actually be lower at the bottom of the slot than at the top. The corrections in this equation are limiting values, however, which are usually not reached. We are justified, however, in concluding that the copper temperature in a typical turbo-generator is very nearly constant for the entire depth of the slot.

Conclusion

It is shown in the above that the conception of an equivalent thermal circuit is helpful in deriving approximate solutions for the complicated phenomena encountered in predetermining the flow of heat in large turbo-generators. The degree of complexity to which it is necessary to go will depend on the individual arrangement of the machines. In general, it is obvious that a simple circuit, emphasizing the essential features of the flow, is to be preferred to a more accurate, but more complicated circuit. It is believed that with this method the temperature can be predicted to within a few per cent of the iron temperature; the uncertainty of losses and thermal properties are usually so great that additional accuracy is chimerical.

The results obtained above have been applied to a number of turbine-generators of the general type of construction referred to. The agreement between measured and calculated temperatures is satisfactory in all cases, considering the unavoidable uncertainty in the thermal properties of the materials involved. The application of the results to actual machines will be made the subject of a separate paper in the future.

Appendix I

Two-Dimensional Problems

A.

This analysis applies to the system shown in Fig. 4 and has for its object a comparison of the approximate solution, given in the paper, and the exact solution. Considering the heat balance for an element $dx \,.\, dy \,.\, b$ of the plate, the following differential equation is easily obtained for the temperature t

$$\frac{1}{\rho_x}\,\frac{\partial^2 t}{\partial x^2} + \frac{1}{\rho_y}\,\frac{\partial^2 t}{\partial y^2} + p = 0 \qquad (35)$$

This partial differential equation is to be solved for the following boundary conditions

$$\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0 \quad \left(\frac{\partial t}{\partial x}\right)_{x=l} = -\,\frac{\rho_x\, b_{ax}}{b\, C_{ax}}\,(t)_{x=l}$$

$$\left(\frac{\partial t}{\partial y}\right)_{y=0} = 0 \quad \left(\frac{\partial t}{\partial y}\right)_{y=h} = -\,\frac{\rho_y\, b_{ay}}{b\, C_{ay}}\,(t)_{y=h} \qquad (36)$$

The loss intensity p is assumed to be constant throughout the volume.

It is obvious that the approximate solution becomes exact for two limiting conditions: *a*, when the problem is one-dimensional, and *b*, when the specific thermal resistances ρ_x and ρ_y are zero. It is easily seen that the essential aspects of the problem are embodied in the numerical constants

$$f_x = \frac{\rho_x\, b_{ax}\, l}{b\, C_{ax}} \text{ and } f_y = \frac{\rho_y\, b_{ay}\, h}{b\, C_{ay}} \qquad (37)$$

The approximate solution becomes exact if one or both of these constants are zero. One may be justified in concluding from this that the most unfavorable case for the approximate solution occurs when $f_x = f_y =$ the maximum value that may occur in practise.

The differential equation (35) may also be written

$$\frac{\partial^2 t}{\partial\left(x\sqrt{\dfrac{\rho_x}{\rho_y}}\right)^2} + \frac{\partial^2 t}{\partial y^2} + p\,\rho_y = 0 \qquad (38)$$

from which it is evident that the difference in specific thermal resistance may be obviated by changing the x-dimension of the plate in the ratio $\sqrt{\rho_x/\rho_y}$. Naturally, the cooling properties of the surface $y = h$ must be modified in a corresponding manner. This shows, however, that there is no loss of generality involved in putting $\rho_x = \rho_y = \rho$. Therefore, the following discussion will be restricted to the differential equation

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + p\,\rho = 0 \qquad (39)$$

with the boundary conditions

$$\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0 \qquad \left(\frac{\partial t}{\partial x}\right)_{x=l} = -\,\frac{f_x}{l}\,(t)_{x=l}$$

$$\left(\frac{\partial t}{\partial y}\right)_{y=0} = 0 \qquad \left(\frac{\partial t}{\partial y}\right)_{y=h} = -\,\frac{f_y}{h}\,(t)_{y=h} \qquad (40)$$

The classical problem solved by Fourier[4] corresponds to the case when $p = 0$. The solution

$$t = A \cos m\,x \cos n\,y \qquad (41)$$

satisfies the corresponding Laplaces' equation; it will also fit the boundary conditions if m and n are roots of the equations

$$m\,l \tan m\,l = f_x$$
$$n\,h \tan n\,h = f_y \qquad (42)$$

These equations have series of successive roots m_i $(i = 1, 2 \ldots)$ and n_j $(j = 1, 2 \ldots)$. The general solution for the case that $p = 0$ may thus be written

$$t = i\sum_{1}^{\infty} j\sum_{1}^{\infty} A_{ij} \cos m_i\,x \cos n_j\,y \qquad (43)$$

This solution may also be used for the case that $p \neq 0$. (Poissons' equation) by determining the constants A_{ij} appropriately.

Introducing (43) into (39) we obtain

$$p\,\rho = i\sum_{1}^{\infty} j\sum_{1}^{\infty} A_{ij}\,(m_i^2 + n_j^2) \cos m_i\,x \cos n_j\,y \qquad (44)$$

giving the relation which the constants A_{ij} must satisfy.

Multiplying this expression by $\int_0^l \cos m_i\,x\,d\,x$ and $\int_0^l \cos n_j\,y\,d\,y$, integrating, and using the expressions (42), it is found that[5]

$$A_{ij} = p\,\rho\,\frac{M_i\,N_j}{m_i^2 + n_j^2} \qquad (45)$$

where

$$M_i = \frac{4 \sin m_i\,l}{2\,m_i\,l + \sin 2\,m_i\,l} \qquad N_j = \frac{4 \sin n_j\,h}{2\,n_j\,h + \sin 2\,n_j\,h} \qquad (46)$$

For the present purpose the maximum and mean temperatures are of greatest interest. These are found to be

$$t_{max} = i\sum_{1}^{\infty} j\sum_{1}^{\infty} A_{ij} \qquad (47)$$

$$t_m = i\sum_{1}^{\infty} j\sum_{1}^{\infty} A_{ij}\,\frac{\sin m_i\,l}{m_i\,l}\,\frac{\sin n_j\,h}{n_j\,h} \qquad (48)$$

This result will be referred to as the "exact solution."

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[4] "The Analytical Theory of Heat," Freeman's translation, page 311.

[5] M. Roth's treatise, previously referred to, contains a useful diagram for determining the roots of the equations (42) and tables of the functions M_i and N_j.

The approximate solution given by (5) to (7) may now be written in such a form that a direct comparison with the exact solution is facilitated. The surface resistances C_x, C_y and C are found to be

$$C_x = \frac{2\,\rho_x\,l^2}{b}\left(\frac{1}{f_x} + \frac{1}{3}\right) \qquad C_y = \frac{2\,\rho_y\,h^2}{b}\left(\frac{1}{f_y} + \frac{1}{3}\right) \tag{49}$$

$$C = \frac{C_x\,C_y}{C_x + C_y}$$

When $\rho_x = \rho_y = \rho$, the maximum and minimum temperatures become

$$t_{max} = p\,\rho\,\frac{l^2\left(\frac{1}{f_x} + \frac{1}{2}\right)\;h^2\left(\frac{1}{f_y} + \frac{1}{2}\right)}{l^2\left(\frac{1}{f_x} + \frac{1}{3}\right) + h^2\left(\frac{1}{f_x} + \frac{1}{3}\right)} \tag{50}$$

$$t_m = p\,\rho\,\frac{l^2\left(\frac{1}{f_x} + \frac{1}{3}\right)\;h^2\left(\frac{1}{f_x} + \frac{1}{3}\right)}{l^2\left(\frac{1}{f_x} + \frac{1}{3}\right) + h^2\left(\frac{1}{f_x} + \frac{1}{3}\right)} \tag{51}$$

This result will be referred to as the "first approximation."

A somewhat closer approximation may be obtained by coordinating the solution (7) with the differential equation (35). For example, $\partial^2 t/\partial y^2$ may be evaluated from (7) and introduced into (35) which then becomes an ordinary differential equation of the form

$$\frac{d^2 t}{d\,x^2} = -\,p\,\rho\left[M + N\left(\frac{x}{l}\right)^2\right] \tag{52}$$

where M and N are two numerical constants determined by

$$M = \frac{C_y - \frac{\rho\,l^2}{3\,b}}{C_x + C_y} \text{ and } N = \frac{\rho\,l^2}{b\,(C_x + C_y)} \tag{53}$$

This equation is to be solved for the boundary conditions

$$\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0 \quad \left(\frac{\partial t}{\partial x}\right)_{x=l} = -\,\frac{f_x}{l}\,(t)_{x=l} \tag{54}$$

The result is

$$t = p\,\rho\left[M\,\frac{l^2 - x^2}{3} + N\,\frac{l^4 - x^4}{12\,l^2} + \frac{1}{f_x}\left(M + \frac{N}{3}\right)\right] \tag{55}$$

This gives for the corrected maximum and mean temperatures

$$t_{max}{}^* = t_{max}\left[1 - \frac{1}{6}\,\frac{\rho^2\,l^4}{b^2\,C_x\,C_y}\right]$$

$$t_m{}^* = t_m\left[1 - \frac{4}{45}\,\frac{\rho^2\,l^4}{b^2\,C_x\,C_y}\right] \tag{56}$$

where t_{max} and t_m are the values obtained from the solution given by (7). An alternative correction is obtained by starting out from the approximate value for $\partial^2 t/\partial x^2$. The result will differ from the above only to the extent that l will be replaced by h. The alternative corrections will be alike, therefore, when $l = h$. It is obvious on the other hand, that the approximate solution (7) is most inaccurate when $l = h$, because when $h = 0$ or $l = 0$, it becomes exact. This circumstance suggests as a reasonable correction the geometrical mean between the above and that which is obtained by solving a differential equation in y. This gives

$$\left[\begin{aligned} t_{max}{}^* &= t_{max}\left[1 - \frac{1}{6}\,\frac{\rho_x\,l^2}{b\,C_y}\cdot\frac{\rho_y\,h^2}{b\,C_y}\right] \\ &= t_{max}\left[1 - \frac{3}{8}\,\frac{f_x\,f_y}{(f_x + 3)\,(f_y + 3)}\right] \end{aligned}\right]$$

$$\left[\begin{aligned} t_m{}^* &= t_m\left[1 - \frac{4}{45}\,\frac{\rho_x\,l^2}{b\,C_x}\cdot\frac{\rho_y\,h^2}{b\,C_y}\right] \\ &= t_m\left[1 - \frac{1}{5}\,\frac{f_x\,f_y}{(f_x + 3)\,(f_y + 3)}\right] \end{aligned}\right] \tag{57}$$

This result will be referred to as the "second approximation."

Table I gives a comparison of these three solutions for the case that $p\,\rho\,l^2 = 1$, $h = l$, and the values 0.5, 1.5, 3, 5 and ∞ of the constants $f_x = f_y$.

In the problems encountered in turbo-generators the constants f_x and f_y rarely exceed 3 and 2, respectively. Thus, the first approximation gives an error in the mean temperature which does not exceed 4 per cent, and an error in maximum temperature which does not exceed 7.5 per cent. Usually the errors are much smaller, however. The second approximation will give both maximum and mean temperatures within one per cent.

B.

The two-dimensional problem, which is discussed in the preceding paragraphs, is a special case of the general tooth problem, Fig. 4, when the slot insulation is completely insulating ($C_i = \infty$). We now consider the case when the insulation of the sides is not complete, but restrict the discussion to the case when the external temperature of this insulation, the copper temperature t_{1m}, is constant. The differential equation is now found to be

$$\frac{1}{\rho_x}\,\frac{\partial^2 t}{\partial x^2} + \frac{1}{\rho_y}\,\frac{\partial^2 t}{\partial y^2} - \frac{2}{b\,C_i}\,(t - t_{1m}) + p = 0 \tag{58}$$

The boundary conditions are to remain the same as

TABLE I

	Maximum temp.					Mean temp.				
		1st approx.		2nd approx.			1st approx.		2nd approx.	
$f_x = f_y$	Exact t_{max}	t_{max}	Error per cent	t_{max}^*	Error per cent	Exact t_m	t_m	Error per cent	t_m^*	Error per cent
0,5	1.327	1.339	0.9	1.329	+ 0.2	1.161	1.167	0.5	1.162	+0.1
1,5	0.652	0.681	4.5	0.653	+ 0.2	0.490	0.500	2.0	0.489	− 0.2
3	0.479	0.521	8.8	0.472	− 1.5	0.319	0.333	4.1	0.317	− 0.6
5	0.408	0.459	12.5	0.392	− 3.9	0.249	0.267	7.2	0.246	1.2
∞	0.296	0.375	26.7	0.234	−21.0	0.140	0.167	19.3	0.133	− 5.0

for the simpler problem, (**40**). By putting

$$t_{1m}' = t_{1m} + \frac{p\,b}{2}\,C_i \tag{59}$$

and

$$\alpha = \sqrt{\frac{2\,\rho_y}{b\,C_i}} \tag{60}$$

the equation may be written

$$\frac{\partial^2 t}{\partial\left(x\sqrt{\frac{\rho_x}{\rho_y}}\right)^2} + \frac{\partial^2 t}{\partial_y^2} - \alpha^2\,(t - t_{1m}') = 0 \tag{61}$$

showing that: 1. The loss intensity of the plate p may be taken into account by adding $(p\,b/2)\cdot C_i$. to the temperature t_{1m}. The addition is clearly the mean drop through the insulation due to the losses p. 2. The specific thermal resistances may be made alike by stretching the x-dimension in the ratio $\sqrt{\rho_x/\rho_y}$. There will be no loss of generality, therefore, in making $\rho_x = \rho_y$, so that the following discussion will be applied to the differential equation

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} - \alpha^2\,(t - t_{1m}') = 0 \tag{62}$$

The exact solution will again be written in the form

$$t = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{ij}\cos m_i\,x\cos n_j\,y \tag{63}$$

where m_i and n_j are the series of roots of the equations (**42**). It is easily found by a reasoning almost identical with that used above that

$$A_{ij} = t_m'\,\frac{\alpha^2}{m_i^2 + n_j^2 + \alpha^2}\,M_i\,N_j \tag{64}$$

Where M_i and N_j are determined from (**46**). The exact solution thus gives the temperature as a function of the temperature t_{1m}'. When $C_i = \infty$ this solution becomes identical with that for the simple problem.

Another solution which is not exact, but which is very nearly exact for most cases, is obtained in the following manner. The differential equation (**62**) is easily shown to be satisfied by the solution.

$$t = t_{1m}' + A\cosh m\,\alpha\,x\cosh n\,\alpha\,y \tag{65}$$

where m and n ω now are two numerical constants satisfying the relation

$$m^2 + n^2 = 1 \tag{66}$$

This solution also satisfies the boundary conditions $(\partial t/\partial x)_{x=0} = 0$ and $(\partial t/\partial y)_{y=0} = 0$. The remaining boundary conditions $(\partial t/\partial x)_{x=l} = (-f_x/l)\,.\,(t)_{x=l}$ and $(\partial t/\partial y)_{y=h} = (-f_y/h)\,.\,(t)_{y=h}$ can not be fully satisfied, however, unless $t_{1m}' = 0$, which is a trivial solution. But it is possible to let the *mean* temperature at the cooling surfaces satisfy the boundary conditions. This should give fairly exact results, at least as far as the mean temperature is concerned. Mathematically, this means that the boundary conditions are written in the form.

$$\int_0^l\left(\frac{\partial t}{\partial y}\right)_{y=h} d\,x = -\int_0^l \frac{f_y}{h}\,(t)_{y=h}\,d\,x$$

and (**67**)

$$\int_0^h\left(\frac{\partial t}{\partial x}\right)_{x=l} d\,y = -\int_0^h \frac{f_x}{l}\,(t)_{x=l}\,d\,y$$

The integrated equations will not contain x or y, and will give the two equations required for determining m (or n) and A. Carrying out the integration, and writing for the arguments

$$X = m\,\alpha\,l \text{ and } Y = n\,\alpha\,h \tag{68}$$

it is found that

$$A = -\,t_m'\,\frac{1}{\sinh X\sinh Y}\,\frac{X\,Y}{F} \tag{69}$$

where F is the function

$$F = \frac{X^2}{f_x} + \frac{X}{\tanh X} = \frac{Y^2}{f_y} + \frac{Y}{\tanh Y} \tag{70}$$

The flow of heat will thus arrange itself in such a manner that the quantity F obtains the same value whether evaluated from X or from Y. Equations (**66**) and (**70**) may thus be used for determining the quantities m and n.

The curves of Fig. 16 may be used for determining F. These curves give the value of the quantity F as a function of z (= X or Y) and the parameter f (= f_x or f_y). m and n are first given values satisfying the equation (**66**). The corresponding values of X and Y give two values of F; if these are alike the assumed values of m and n are correct; if not the procedure is repeated with a new combination. Assume that this process has been carried out and write the solution

$$t = t_m' \left(1 - \frac{X\ Y}{F} \; \frac{\cosh X \dfrac{x}{l} \cosh Y \dfrac{y}{h}}{\sinh X \sinh Y} \right) \tag{71}$$

The mean temperature is given by a particularly simple expression

$$t_m = t_m' \frac{F - 1}{F} \tag{72}$$

Thus, to evaluate the mean temperature, it is sufficient to know the value of the quantity F. This solution will be referred to as the "semi-exact" solution.

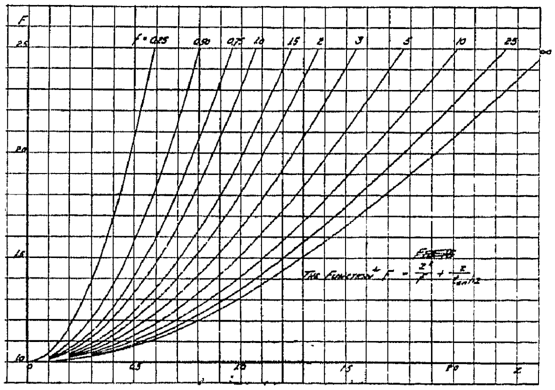

Fig. 16

When the arguments X and Y are small the function $z/\tanh z$ may be expanded into a series

$$\frac{z}{\tanh z} = 1 + \frac{z^2}{3} - \frac{z^4}{45} + \ldots \tag{73}$$

If terms of higher order of z than the second are neglected, it is seen that

$$F - 1 = \left[\frac{1}{f_x} + \frac{1}{3} \right] X^2 = \left[\frac{1}{f_y} + \frac{1}{3} \right] Y^2 \tag{74}$$

Now, introducing the values of f_x, f_y, it is found that

$$m^2 = \frac{C}{C_x} \tag{75}$$

$$n^2 = \frac{C}{C_y} \tag{76}$$

and

$$F - 1 = \frac{C}{C_i} \tag{77}$$

The mean temperature now becomes

$$t_m = t_m' \frac{C}{C + C_i} \tag{78}$$

This value is identical with that obtained by the approximate solution (**11**), and which has been referred to as the first approximation.

The semi-exact solution thus deteriorates into the ordinary approximate solution when C_i is very large. For this reason it is an improvement upon the approximate solution only when the quantities X and Y are large.

Table II has been calculated to illustrate this condition. This table gives the maximum and minimum temperature of the iron when $t_m' = 1$, $l = h$, $f_x = f_y = 3$, and for the value 0.5, 1.0, 2.0 and 3.0 for the argument $\alpha\, l = \alpha\, h$. For this symmetrical case it is evident from (**66**) that $m = n = 1/\sqrt{2}$. $f_x = f_y = 3$ may be regarded as a typical case for the stator; the value of $\alpha\, l = \alpha\, h$ does not usually exceed unity.

If the third term of the series (**73**) is retained it is found that the approximate solution for the mean temperature is still correct in form when the following corrected values for the quantities C_x and C_y are used

$$C_x{}^* = C_x \left[1 - \frac{1}{5} \frac{C}{C_i} \left(\frac{2\, \rho_x\, l^2}{3\, b\, C_x} \right)^2 \right];$$

$$C_y{}^* = C_y \left[1 - \frac{1}{5} \frac{C}{C_i} \left(\frac{2\, \rho_y\, h^2}{3\, b\, C_y} \right)^2 \right] \tag{79}$$

where C_x, C_y, and C are the ordinary surface resistances given by equations (**5**). This result also shows that when C_i is large compared with C the semi-exact solution becomes the ordinary approximate solution. It is an interesting fact that this correction is of the same form as that obtained for the simple two-dimensional problem (**57**).

Appendix II

Cylinder Problems

A.

Consider a two-dimensional problem of the same type as the one given in Fig. 4, but with varying thickness of the plate (Fig. 14). As before, it is assumed that

TABLE II

	Maximum temp.					Mean temp.				
		Semi-exact		1st approx.			Semi-exact		1st approx.	
$\alpha\, l = \alpha\, h$	Exact t_{max}	t_{max}	Error per cent	t_{max}	Error per cent	Exact t_m	t_m	Error per cent	t_m	Error per cent
0.5	0.110	0.111	0.9	0.120	9.1	00.73	0.077	5.5	0.077	5.5
1.0	0.347	0.360	3.8	0.391	12.7	02.37	0.246	3.8	0.250	5.5
2.0	0.744	0.763	2.6	0.893	20.0	05.38	0.558	3.7	0.572	6.3
3.0	0.917	0.927	1.1	1.171	27.7	07.10	0.728	2.5	0.750	5.6

the edges $x = l$ and $r = r_1$, and the two sides, are insulated. The ordinary cylinder problem with distributed losses is then obtained. The differential equation is now found to be, when $\rho_x = \rho_y = \rho$.

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\,\frac{\partial t}{\partial r} + \rho = 0 \ldots. \quad (80)$$

The difficulties in obtaining an exact solution of this problem are connected with the variation of p with r, and with the boundary conditions at the surface $x = l$. As mentioned before, the case when p varies inversely as r is particularly interesting but, unfortunately, a simple exact solution can be obtained only for the case when p is constant. The "cooled width," b_{ax}, varies inversely with the radius on stator problems, but on rotor problems it is usually constant. An exact solution is possible only in the former case. Therefore, the exact solution will be derived for the case where p is constant and b_{ax} varies inversely with the radius. The boundary conditions will then be

$$\left.\begin{matrix} \left(\dfrac{\partial t}{\partial x}\right)_{x=0} = 0 & \left(\dfrac{\partial t}{\partial r}\right)_{r=r_1} = 0 \\ \left(\dfrac{\partial t}{\partial x}\right)_{x=l} = -\dfrac{f_x}{l}\,(t)_{x=l} & \left(\dfrac{\partial t}{\partial r}\right)_{r=r_2} = -\dfrac{f_r}{h}\,(t)_{r=r_2} \end{matrix}\right\} \quad (81)$$

where $$f_x = \frac{\rho\, b_{ax}\, l}{b\, C_{ax}}$$

and $$f_r = \frac{r_m}{r_2}\;\frac{\rho\, b_{ay}\, h}{b\, C_{ay}} \quad (82)$$

The solution will contain products of the Bessel's functions $J_p\,(n\,r)$ and $K_p\,(n\,r)$[6] It will be found convenient to introduce the special functions

$$R_0\,(n\,r) = (n\,r_1)\begin{vmatrix} J_0\,(n\,r), & K_0\,(n\,r) \\ J_1\,(n\,r_1), & K_1\,(n\,r_1) \end{vmatrix}$$

and $$R_1\,(n\,r) = n\,r_1)\begin{vmatrix} J_1\,(n\,r), & K_1\,(n\,r) \\ J_1\,(n\,r_1), & K_1\,(n\,r_1) \end{vmatrix} \quad (83)$$

When $r = r_1$ it is found that $R_0 = 1$ and $R_1 = 0$, a property which these functions have in common with $\cos n\,(r - r_1)$ and $\sin n\,(r - r_1)$, respectively, and whose values they approach when r is very large compared with $r_2 - r_1 = h$.

The expression

$$t = A \cos m\,x\, R_0\,(n\,r) \quad (84)$$

satisfies the equation (**80**) when $p = 0$; the boundary conditions (**82**) will also be satisfied if m and n are roots of the equations

$$m\,l \tan m\,l = f_x \quad (85)$$

$$n\,h\,\frac{R_1(n\,r_2)}{R_0\,(n\,r_2)} = f_r \quad (86)$$

These equations have series of roots m_i ($i = 1, 2 \ldots$), and n_j ($j = 1, 2 \ldots$), so that the solution can be written as a series

$$t = i\sum_1^{\infty} j \sum_1^{\infty} A_{ij} \cos\,(m_i\,x)\, R_0\,(n_j\,r) \quad (87)$$

Introducing this series into (**80**), the following relation is obtained

$$i\sum_1^{\infty} j \sum_1^{\infty} A_{ij}\,(m_1^2 + n_j^2) \cos\,(m_i\,x)\, R_0\,(n_j\,r) = p \quad (88)$$

This enables us to determine the coefficients A_{ij} by multiplying by $\int_0^l \cos\,(m_i\,x)\,d\,x$ and $\int_{r_1}^{r_2} r.\; R_0\,(n_j\,r)\,d\,r$, integrating, and introducing the expression (**90**) and (**91**). It is found that

$$A_{ij} = p\,\frac{M_i\,N_j}{m_1^2 + n_j^2} \quad (89)$$

where

$$M_i = \frac{4 \sin m_i}{2\,m_i\,l + \sin 2\,m_i\,l} \quad (90)$$

$$N_j = \frac{2}{n_j\,r_2}\;\frac{R_0\,(n_j\,r_2)}{R_0^2\,(n_j\,r_2) + R_1^2\,(n_j\,r_2) - \dfrac{r_1^2}{r_2^2}} \quad (91)$$

Fig. 14, with which the above solution is associated, represents the conditions in a rotor tooth, or a stator core, where the heat flows towards the wider section. It is evident, however, that the solution will remain valid even for a stator tooth, where the flow is in the opposite direction. The notations of Fig. 14 may still be used if $r_2 < r_1$. Put in general

$$r = r_m - \frac{h}{2} + y = h\,\frac{1 - \theta\left(1 - \dfrac{2\,y}{h}\right)}{2\,\theta} \quad (92)$$

where

$$\theta = \frac{h}{2\,r_m} = \frac{\pi\,h}{b\,Q} \quad (93)$$

is the *taper constant*. The distinction between a rotor tooth and a stator tooth is that θ is positive for the former, and negative for the latter. The functions $R_0\,(n\,r)$ and $R_1\,(n\,r)$ may now be considered as functions of $n\,y$ and θ instead. The values of r_1 and r_2 in terms of θ and h are

$$r_1 = h\,\frac{1 - \theta}{2\,\theta} \quad \text{and} \quad r_2 = h\,\frac{1 + \theta}{2\,\theta} \quad (94)$$

---

[6] This notation is used by Jahnke and Emde in "Funktionentafeln," from which the numerical values have been taken.

Fig. 17 shows the function $R_0\left(n\,h\,\frac{1+\theta}{2\,\theta}\right)$ for different values of $n\,h$ and θ. Fig. 18 shows the corresponding values of $R_1\left(n\,h\,\frac{1+\theta}{2\,\theta}\right)$. It should be noted that when $\theta = 0$ the two functions become $\cos n\,h$ and $\sin n\,h$; when $\theta = 1$ they become $J_0\,(n\,h)$ and $J_1\,(n\,h)$. Fig. 19 shows the three lowest roots of the equation (**86**) for the same values of θ.

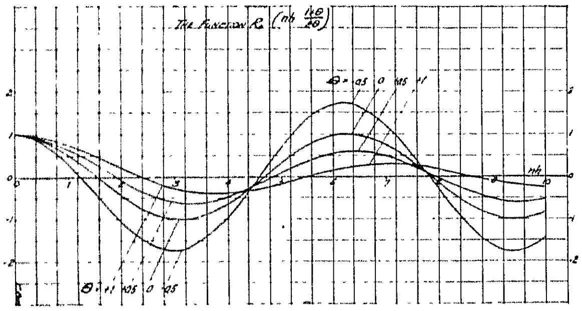

FIG. 17

The maximum and mean temperatures are of the greatest interest in making the comparisons with the more approximate solutions. In order to obtain a simple expression for the latter the definition

$$t_m = \frac{\int\limits_0^l \int\limits_{r_1}^{r_2} t\,.\,r\,.\,dx\,.\,dr}{\int\limits_0^l \int\limits_{r_1}^{r_2} r\,.\,dx\,.\,dr} \tag{95}$$

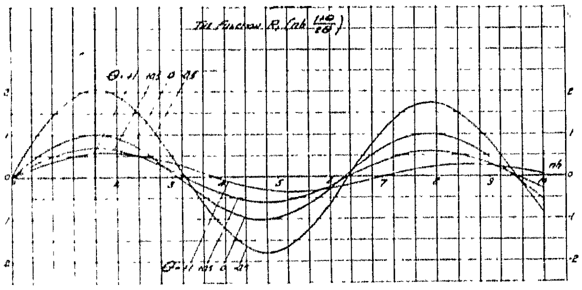

FIG. 18

will be used. It is found that

$$t_{max} = i\sum_1^{\infty} j\sum_1^{\infty} A_{ij} \tag{96}$$

$$t_m = i\sum_1^{\infty} j\sum_1^{\infty} A_{ij}\,\frac{\sin m_i\,l}{m_i\,l}\;\frac{R_1\,(n_j\,r_2)}{\frac{r_m}{r_2}\,.\,n_j\,h} \tag{97}$$

In establishing the approximate solution for the tapered tooth it is assumed that the radial and axial flow take place independently of each other, so that only the quantities entering in the radial flow will be corrected for the taper. Referring again to Fig. 14 assume that the edge $x = l$ is insulated and determine the solution for the corresponding one-dimensional problem. It is easily found that

$$t = p\,\rho\left[\frac{r_2^2 - r^2}{4} + \frac{r_1^2}{2}\log\frac{r_1}{r_2}\right] + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{98}$$

whence

$$t_{max} = p\,\rho\left[\frac{r_2^2 - r_1^2}{4} + \frac{r_1^2}{2}\log\frac{r_1}{r_2}\right] + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{99}$$

and

$$t_m = \frac{\int\limits_{r_1}^{r_2} t\,r\,.\,d\,r}{\int\limits_{r}^{r_2} r\,d\,r} = p\,\rho\left[\frac{r_2^2 - 3\,r_1^2}{8} + \frac{r_1^4}{2\,(r_2^2 - r_1^2)}\log\frac{r_2}{r_1}\right] + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{100}$$

Now replace r_1 and r_2 by their expressions in h and θ (**94**).

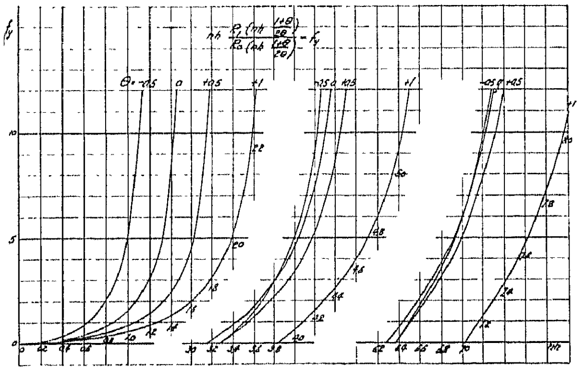

FIG. 19—THE THREE LOWEST ROOTS TO THE EQUATION

Assuming that θ is small ($\theta < 0.3$) we expand the logarithmical terms in series and neglect higher powers of θ than the second. Then

$$t_{max} = \frac{p\,\rho\,h^2}{2}\left(1 - \frac{2}{3}\,\theta\right) + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{101}$$

$$t_m = \frac{p\,\rho\,h^2}{3}\,(1 - \theta) + p\,\frac{b\,h\,C_{av}}{b_{av}} \tag{102}$$

The approximation will be somewhat closer for comparatively large values of θ if $1 - \frac{2}{3}\,\theta$ is replaced by $1/(1 + \frac{2}{3}\,\theta)$ and $1\text{-}\theta$ by $1/(1 + \theta)$. The result then becomes

$$t_{max} = \frac{p\,b}{2}\left[\frac{2\,hC_{ay}}{b_{ay}} + \frac{\rho\,h^2}{b}\,\frac{1}{1+\frac{2}{3}\theta}\right] \qquad (103)$$

$$t_m = \frac{p\,b}{2}\left[\frac{2\,h\,C_{ay}}{b_{ay}} + \frac{2\rho h^2}{3\,b}\,\frac{1}{1+\theta}\right] \qquad (104)$$

This shows that the simple solution for a straight tooth may be applied to the tapered tooth if the specific thermal resistance in the radial direction is multiplied by a factor, the value of which depends upon the taper constant. Table III gives a comparison of the solution thus obtained with the exact solution for an arrangement where $f_x = 3, f_r = \frac{r_m}{r_2} 3 = \frac{3}{1+\theta}, p\ \rho\ h^2 = 1$, $h = l$, and the values $-\,0.5$ and $+\,0.5$ for θ. It is obvious from this comparison that even for these extreme conditions the above method of correcting the specific thermal resistance gives results which are sufficiently accurate for practical purposes. The correction for tooth taper eliminates a part of the error due to the inaccuracy of the solution for the straight tooth, so that the errors are actually smaller than those which are obtained when $\theta = 0$.

TABLE III

	Max. temp.			Mean temp.		
		Approx.			Approx.	
θ	Exact t_{max}	t_{max}	Error per cent	Exact t_m	t_m	Error per cent
−0.5	0.581	0.590	+ 1.5	0.421	0.444	+ 5.5
+0.5	0.402	0.448	+ 11.4	0.248	0.267	+ 7.7

It is now necessary to derive a corresponding approximate solution for the case of greatest practical interest, namely, where the losses vary inversely with the tooth thickness. The differential equation is now found to be

$$\frac{1}{\rho_x}\,\frac{\partial^2 t}{\partial x^2} + \frac{1}{\rho_y}\,\frac{\partial^2 t}{\partial r^2} + \frac{1}{\rho_y}\,\frac{1}{r}\,\frac{\partial t}{\partial r} + \frac{r_m}{r}\,p = 0 \qquad (105)$$

The boundary conditions are to be identical with those given in (**82**). Assuming that the flow is entirely radial, so that $\partial^2 t/\partial x^2 = 0$, this equation yields the solution

$$t = p\,\rho_y\left[r_m\,h + r_1\,r_m \log\frac{r}{r_2}\right] + p\,\frac{b\,h\,C_{ay}}{b_{ay}} \qquad (106)$$

Replacing r_1 and r_m by their expressions in h and θ and neglecting powers above the second in θ,

$$t_{max} = \frac{p\,b}{2}\left[\frac{2\,h\,C_{ay}}{b_{ay}} + \frac{h^2\,\rho_y}{b}\left(1 - \frac{\theta}{3}\right)\right] \qquad (107)$$

In evaluating the mean temperature the expression $t_m = \frac{1}{h}\int_{r_2}^{r_1} t\,d\,r$ should be used rather than (**95**), which was introduced in order to simplify the exact solution. Then

$$t_m = \frac{p\,b}{2}\left[\frac{2\,h\,C_{ay}}{b_{ay}} + \frac{h^2\,\rho_y}{b}\left(1 - \frac{\theta}{2}\right)\right] \qquad (108)$$

In this case it is found that the accuracy for comparatively large values of θ is improved if the factor $1 - \theta/2$ is replaced by $1/\sqrt{1+\theta}$. The correction factor $1 - \theta/3$ in the expression for the maximum temperature should be changed to $1/\sqrt{1 + \frac{2}{3}\theta}$, but since the correction is always very small there will be no serious error involved in using the correction $1/\sqrt{1+\theta}$ for the maximum temperature as well.

It is concluded, therefore, that for machines of normal construction the taper of the teeth may be taken into account by using a corrected value for the specific thermal resistance in the radial direction. The value of this corrected resistance is

$$\rho_y^* = \rho_y\,\frac{1}{\sqrt{1+\theta}} \qquad (109)$$

B.

The above problem is only a special case of the general problem of the tooth region. This problem may also be solved by a semi-exact method similar to that developed in Appendix I-B, where cylinder function of the type (83) would appear. The results are too complicated to be of practical use however. Besides, the influence of ordinary tooth taper is so small that the results obtained in the preceding section are sufficiently accurate for all practical purposes.

Discussion

C. J. Fechheimer: Those who have attempted to obtain solutions of the thermal problems in electrical machines recognize the complexities of heat flow in more than one dimension, with different values of conductivities in the various paths, with irregular configurations, associated with which are varying rates of heat generation, etc. Most engineers have considered the problem unmanageable, if not confined to one dimension, and the necessary assumptions imposed by that limitation have usually introduced inaccuracies of such order that the calculated and test results were not in sufficiently close agreement. It should be regarded as an achievement for Mr. Soderberg to have succeeded in placing before us a solution of this intricate problem. By modifying his boundary conditions, he has simplified his solution, and then has put it into workable form by substituting equivalent electrical circuits for the heat circuits.

It is of interest to obtain a picture of the equations for the equivalent electrical circuit from another standpoint. The two equations which Mr. Soderberg has given the number 15 are:

$$t_1\,m = C_{11}\,q_1 + C_{12}\,q_2 + C_{13}\,q_3$$
$$t_2\,m = C_{21}\,q_1 + C_{22}\,q_2 + C_{23}\,q_3$$

$t_1\,m$ is the copper temperature; $t_2\,m$ the tooth temperature; q_1, q_2 and q_3 are respectively the loss intensities in the copper, teeth and surface. C_{11}, C_{12}, etc. are "influence factors." That is, C_{11} is the coefficient by which the copper loss intensity is multiplied to give the temperature rise of the copper due to the loss therein; C_{12} is the coefficient by which the tooth loss intensity is multiplied to give the temperature rise of the copper due to the tooth loss. Thus, C_{12} shows the influence of iron loss upon copper temperature, etc. In the second equation, the copper loss intensity q_1 is multiplied by C_{21} to give the temperature rise of the iron due to the loss in the copper. It is of interest to note that $C_{12} = C_{21}$ (equation **19**).

These equations will be recognized as embodying the principles of superposition and of reciprocity. According to the first, the temperature at any part of a system due to one source of heat, acting alone, may be added to the temperature in the same part due to another source of heat also acting alone, and the sum is the temperature at that point if both sources supplied heat simultaneously. The fact that $C_{12} = C_{21}$, involving the reciprocity principle, shows that the temperature rise in the first due to a given loss in the second equals the temperature rise in the second due to the same loss in the first. These are well-recognized principles.

Use of these principles may also be made in testing electrical machinery, particularly when the machine is too large to test at full load. Thus, the usual temperature measurements may be made, say on the armature, with the machine operating without load at normal voltage on open circuit, and a second run made at short circuit with normal current. The full load temperature rise is then the sum of the open- and short-circuit temperature rises. For example, if detectors are placed between coils in the slot, on open circuit they read (nearly) the temperature of the copper due to iron loss, and on short circuit the temperature of the copper due to the copper loss. The full load temperature rise is the sum of these two rises.

It is evident that the influence factors, C_{11}, C_{12}, etc. may be obtained from test. If run on open circuit, $q_1 = 0$, and if we neglect, q_3, (the surface losses)

$$C_{12} = \frac{t_1\,m}{q_2}.$$

The detector is then used to obtain the copper temperature rise, $t_1\ m$. Also from the open-circuit test,

$$C_{22} = \frac{t_2\,m}{q_2},$$

some suitable means being employed to measure the mean tooth temperature rise, $t_2\ m$. On short circuit, $q_2 = 0$, and

$$C_{11} = \frac{t_1\,m}{q_1};$$

also

$$C_{21} = \frac{t_2\,m}{q_1}.$$

The value of C_{21} thus obtained may then be compared with C_{12} from the open-circuit test.

The method was suggested by the writer in an article in the *Sibley Journal* in December, 1912, and has since been proposed by others.*

K. K. Palueff: It might be of interest to the Institute that a short-circuit table such as is used for the analysis of short-circuit characteristics of transmission systems can be very beneficially employed for solving some of the thermal problems similar to those mentioned by Mr. Soderberg. I had occasion to make use of such a table several years ago when it was necessary to determine temperature distribution between a number of bodies (see 1-2-3, etc. of Fig. 1), each generating a different amount of heat. It was necessary to solve this problem for a considerable number of numerical examples where the number of the bodies in the structure and the ratio A/B were varying through a wide range. It was found that an analytical solution was too complicated and laborious. In its place the following method was used:

Thermal resistance from each of the bodies to the external equi-potential surface S was determined on the assumption that the dotted vertical lines in Fig. 1 served as boundaries made up of perfect heat insulators. Then the thermal resistance between each adjacent pair of the bodies was determined, in which case the boundaries indicated by vertical lines were omitted. This permitted the formation of a network of resistances as shown in Fig. 1. This network was set up on the short-circuit table, then voltage was applied between S and each of the points, 1, 2, 3, etc. in turn. In each case the total current at the point of application (1, 2, 3, etc.) was made proportional to the heat dissipated by the respective body. The currents found in each branch (R_1, R_2, etc.) for all n readings were added together, giving quantities proportional to the total amount of heat flowing from each body directly to the equi-potential surface S. The product of this current and the corresponding resistance (R_1, R_2, etc.) gave quantities proportional to the temperature differences between the respective bodies and the surface S.

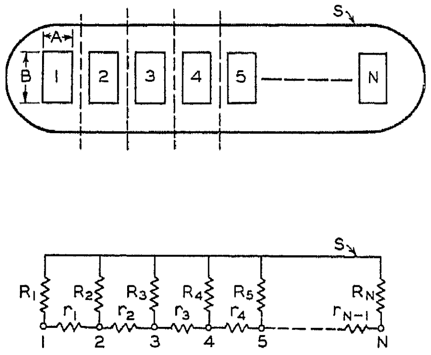

Fig. 1

D. S. Snell: The complexity of the solution given in this paper for the problem of stationary heat flow in a single tooth segment of a turbine-generator well illustrates the complex nature of the problem of temperature calculation in electrical machinery, and much credit is due the author for his unique and thorough treatment of this difficult problem. As pointed out in the paper, however, the correctness of any solution of this problem is, in the last analysis, wholly dependent upon the accuracy of the values of the constants used, and, it must be conceded, our present knowledge of several of these constants,—for example, the heat transfer coefficients for the air gap surfaces of the rotor and stator, is somewhat incomplete. Hence an approximate solution may, in actual practise, give as good results as a more involved one.

The assumption made by the author that the plane of division of radial heat flow in the tooth region is at the bottom of the slot, while probably justifiable for the types of turbine-generators considered, *i. e.*, with radial ventilating ducts in both stator and rotor, may give temperatures very much higher than the actual for turbine-generator rotors of the solid forged type, having ventilating channels below the coil slots. In Fig. 2 is shown the temperature distribution in a slot and tooth of a rotor of the latter type, as calculated by a method given by Dr. H. F. Shait in *Archiv fuer Elektrotechnik*, v., 22, p. 402, which shows the plane of division of radial heat flow to occur nearly half way up the tooth.

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*"The Temperature Rise on Electrical Machines," by Herbert Jehle, *E. T. Z.*, Aug. 14, 1930.

"Temperature Rise of Electrical Machine and Comparison between Single and Double Rating," by Jutaro Takedchi, *Journal of the Institute of Elec. Eng. of Japan*, Aug., 1926.

An application of the method just mentioned shows that in the rotor of a large turbine-generator a very small part of the total loss passes directly to the air gap or ventilating channel, hence negligible error would be introduced in calculating the rotor temperature if all of the loss were assumed to pass first to the tooth and thence to the gap and channel. By the use of this assumption and the further one that the loss is uniformly distributed in the tooth, an approximate method of calculating temperatures in the solid forged type of rotor has been developed which is illustrated in Fig. 3. In this method the position of the plane of division of radial heat flow (designated as the "neutral" plane) is first assumed and the various thermal resistances are calculated. The tooth temperatures on either side of this neutral plane are then determined, and if these are found to be equal, the position of the plane as assumed may be considered correct. If these temperatures are not in agreement, a new position of the neutral plane is assumed and the calculation repeated. Thus by successive approximations the neutral plane is finally located, and the temperature rise of the winding determined.

The equations from which the rotor temperature may be calculated by the above approximate method are given below, and an application of this method to three different turbine-generators is given in Table I.

TABLE 1

Kva. rating r. p. m.	67,000 1,500	43,750 1,800	62,500 1,800
f	0.655	0.565	0.615
T_g	9.6	20.	18.5
T_p	7.1	6.6	5.6
R_m	2.23	2.35	2.57
R_s	1.66	1.96	1.96
R_t	0.37	0.34	0.34
R_n	1.98	3.2	2.91
R_r	0.43	0.52	0.51
R_f	12.12	10.41	11.6
R_{im}	6.55	7.76	7.15
R_{in}	12.3	10.2	11.37
T_1	63.6	78.1	82.1
T_2	65.9	79.7	82.8
T_1'	36.0	47.2	49.5
T_2'	36.6	47.0	49.0
T_0 (calc.)	64.4	78.9	82.4
T_0 (test)	63.5	79.0	80.0

TEMPERATURE EQUATIONS

Let:

f = fraction of winding above neutral plane

N_1 = no. of field turns per slot

d = section area of conductor, sq. in.

I = current per conductor, amperes

u = resistance of a copper bar, 1 in. by 1 in. by 12 in., at temp. of ingoing air

k = temperature coefficient of copper at temp. of ingoing air

T_1 = mean temp. rise of winding, for section above neut. plane

T_2 = mean temp. rise of winding, for section below neut. plane

T_0 = mean temp. rise of winding, for entire slot

T_m = mean temp. rise of tooth, for section above neut. plane

T_n = mean temp. rise of tooth, for section below neut. plane

T_1' = tooth temp. rise at neutral plane, air gap side

T_2' = tooth temp. rise at neutral plane, channel gap side

T_g = mean temp. rise of air in air gap

T_p = mean temp. rise of rotor surface from pole-face losses

(All temperatures in deg. cent.)

Also, let:

$$Q = \frac{N_1 I^2 u}{24\, d};\; D = f\, Q;\; E = k\, f\, Q;\; F = (1-f)\, Q;\; G = (1-f)\, k\, Q$$

$$H = \frac{R_m}{3};\; J = \frac{R_n}{3};\; R_g = R_m + R_t + R_s;\; R_c = R_n + R_r + R_f$$

$$M = R_g + R_{im} - H;\; N = R_c + R_{in} - J$$

Then:

$$T_1 = \frac{T_g + T_p + D\,M}{1 - E\,M};\; T_2 = \frac{F\,N}{1 - G\,N};\; T_0 = f\,T_1 + (1-f)\,T_2$$

And

$$T_1' = T_g + T_p + R_g\,(D + E\,T_1);\; T_2' = R_c\,(F + G\,T_2)$$

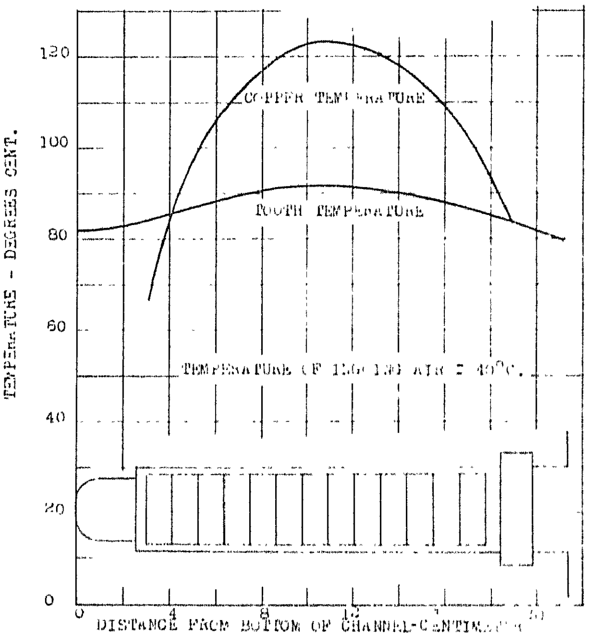

FIG. 2 -CALCULATED TEMPERATURE DISTRIBUTION IN ROTOR SLOT AND TOOTH FOR A FOUR-POLE 90,000-KVA. R. P. M. TURBINE-GENERATOR AT FULL LOAD

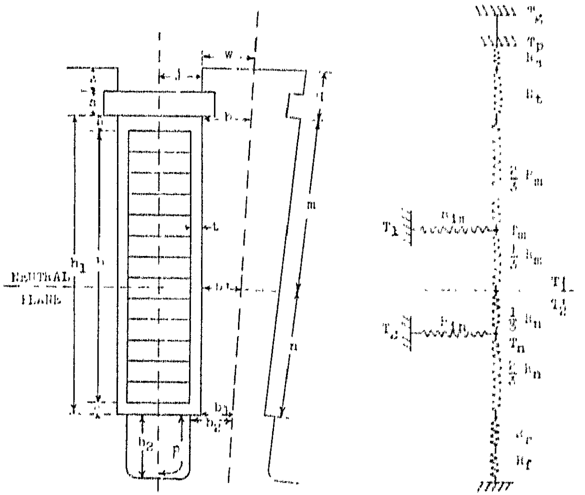

FIG. 3

THERMAL RESISTANCES

(All values in deg. per watt for 1 in. axial length, for ½ slot and tooth. Dimensions from Fig. 3 in inches)

Tooth:

Let λ = thermal conductivity of rotor steel, watt per cu. in. per deg. cent.

$$m = f\,h + a;\; n = (1 - f)\,h + c;\; r = \frac{b - b_1}{h_1}$$

Then

$$R_m = \frac{\theta}{\lambda\, m\,(b + b')};\; R_n = \frac{\Delta}{\lambda\, n\,(b_1 + b')}$$

$$\theta = \frac{m^2}{2} + \frac{b'\,m}{r} - \frac{(b')^2}{(r)^2}\log_\epsilon\left(1 + \frac{m\,r}{b'}\right)$$

$$\Delta = \frac{n^2}{2} - \frac{b'\,n}{r} - \frac{(b')^2}{(r)^2}\log_\epsilon\left(1 - \frac{n\,r}{b'}\right)$$

Insulation:

Let λ_i = thermal conductivity of slot insulation, watt per cu. in. per deg. cent.

$$m' = f\,h + .5\,a;\; n' = (1 - f)\,h + .5\,c$$

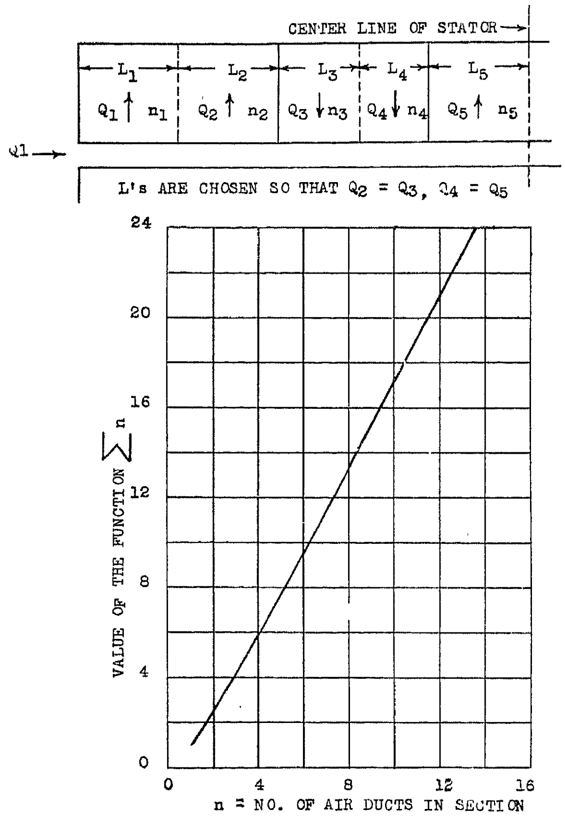

FIG. 4

Then

$$R_{im} = \frac{t}{m'\,\lambda_i};\; R_{in} = \frac{t}{n'\,\lambda_i}$$

Tooth Tip and Wedge:

$$W = 0.5\,(b + 2\,j + z + w);\; h' = \frac{s\,j + .5\,q\,(b + w)}{W};\; R_t = \frac{h'}{\lambda\,W}$$

Tooth Root:

Let $p = \frac{1}{2}$ perimeter of channel wall, in.

Then $R_r = \dfrac{h_2}{\lambda\,(b_2 + p)}$

Air gap surface:

Let

α_s = heat transfer coefficient for rotor surface, watt per in.^{2} per deg. cent.

S = no. of rotor coil slots

D_r = rotor diameter, in.

Then

$$R_s = \frac{2\,S}{\alpha_s\,\pi\,D_r}$$

Ventilating Channel Surface:

Let

α_m = mean heat transfer coefficient for channel, watt per in.^{2} per deg. cent.

ψ = a factor (usually between 0.7 and 0.8 for large generators) to eliminate channel air temperature from equations.

Then

$$R_f = \frac{1}{\alpha_m\,\psi\,p}$$

Surface temperature rise from pole-face losses:

$$T_p = \frac{\text{Pole-face loss in watts}}{\alpha_s\,\pi\,D_r\;x\;\text{rotor length}}$$

Mean temperature rise of air in air gap:

Referring to Fig. 4, the value of T_g for a typical turbine-generator may be approximated from the following equations:

Let:

Q = cu. ft. of air per min. to or from a given stator section

n = no. of air ducts in section of core of length L

F = kw. loss to stator air ducts in length L of inlet section

U = kw. loss to air gap from rotor and from tips of stator teeth, per in. length of air gap

V = kw. loss given to volume of air Q_1 before entering gap

$\Sigma\,n$ = a function given by the curve in Fig. 4.

Then

$$T_g = \frac{R_1\,L_1 + R_2\,L_2 + R_3\,L_3 + R_4\,L_4 + R_5\,L_5}{L_1 + L_2 + L_3 + L_4 + L_5}$$

Where:

$$R_1 = \frac{1852}{Q_1}\left(V + \frac{L_1\,U}{2\,n_1}\sum n_1\right)$$

$$R_2 = \frac{1852}{Q_2}\left(F_3 + L_3\,U + \frac{L_2\,U}{2\,n_2}\sum n_2\right)$$

$$R_3 = \frac{1852}{Q_3}\left(F_3 + \frac{L_3\,U}{2\,n_3}\sum n_3\right)$$

$$R_4 = \frac{1852}{Q_4}\left(F_4 + \frac{L_4\,U}{2\,n_4}\sum n_4\right)$$

$$R_5 = \frac{1852}{Q_5}\left(F_4 + L_4\,U + \frac{L_5\,U}{2\,n_5}\sum n_5\right)$$

C. Richard Soderberg: Referring at first to the discussion by Mr. Fechheimer, the principles of superposition and reciprocity are direct consequences of the simplification of the problems into thermal circuits. This fact was probably not emphasized sufficiently in my paper, and I fully agree with Mr. Fechheimer on the practical importance of these principles.

Mr. Palueff's reference to the use of short-circuit tables for the solution of problems of this type is very interesting. I am wondering, however, whether the type of problem to which Mr. Palueff is referring, involving a large number of similar bodies, would not be more simply treated by considering the aggregate as a homogeneous body. The errors by such a simplification are usually very small, and the numerical work is very greatly simplified.

Referring to the discussion by Mr. Snell, I wish to emphasize that, while most turbine-generators can be treated with sufficient accuracy by assuming the division of the radial flow to occur at the bottom of the slots, this restriction of the results obtained in the paper is not implied. The procedure outlined by Mr. Snell is mentioned in the paper; the methods outlined apply particularly to the case that the division of the radial flow occurs in the slot region, which is practically always the case. In an arrangement of the type shown in Fig. 3, (of the paper) where there are ventilating ducts at the bottom of the slot, the division of the radial flow occurs at about the middle of the tooth; when there are no axial ducts, the division occurs nearer the bottom of the slots. The formulas given by Mr. Snell will be found to give practically identical results with those obtained by the formulas given in the paper. The excellent agreement between calculated and tested values, given in Table I by Mr. Snell, is perhaps somewhat better than our usual experience, but it is my firm belief that even the temperature problem can be reduced to reliable theoretical treatment when more experience with rational methods of calculation has been obtained.

I fully agree with Mr. Snell that many niceties in the theoretical solutions may be over-shadowed by the uncertainties of the constants entering the solution, but it is in new, radical departure from standard practise that these may become important. For example, assume that it were possible to reduce the thermal resistance of the slot insulation of large turbo-rotors to one-half of the present usual value. Maintaining the rise of the copper the same, the mean rise of the teeth would increase by something of the order of 50 per cent above the present value. It would then become much more important to calculate this drop accurately and the error due to the neglection of tooth taper, for example, would begin to be serious. Thus, the justification for many of the details of the present treatment should be looked for in the treatment of new, abnormal constructions, and not in the application of the results to routine design practise, where much simpler methods may yield satisfactory results.

Effect of Transient Voltages on Power Transformer Design—III

Non-Resonating Auto-Transformer

BY K. K. PALUEFF*

Member, A. I. E. E.

Synopsis.—*Transient voltage phenomena within auto-transformers differ from those in transformers, because in the former there are two points of the winding connected to independent transmission lines. This allows independent variation of the potential of these points.*

When a surge comes to an auto-transformer from one line only, the other line acts practically as a "solid ground" for internal oscillations of the windings, provided it is long enough.

Transient voltage stresses between turns and between coils, therefore, are higher in auto-transformers than in transformers of the same voltage rating.

The non-resonating auto-transformer has been developed for commercial use. In mechanical construction it is essentially the same as the non-resonating transformer.

The presence of two independent lines connected to an auto-transformer permits several conditions of surge excitation. This allows corresponding variations in the design of shields.

* * * * *

INTRODUCTION

THE general principles controlling the high-frequency voltage phenomena within auto-transformer windings are the same as those governing these phenomena in ordinary transformer windings. These principles have been presented to the Institute in various communications mentioned in the bibliography of the present paper. On this account, the present paper should be looked upon as a continuation of the preceding papers of the author, and is written on the assumption that the reader is thoroughly familiar with the above mentioned papers.

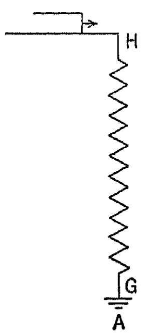

FIG. 1—A. TRANSFORMER WITH LINE CONNECTED AND NEUTRAL GROUNDED

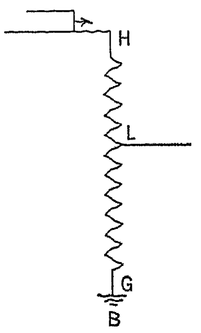

FIG. 1—B. AUTO-TRANSFORMER WITH TWO LINES CONNECTED AND NEUTRAL GROUNDED

The justification for writing this paper lies in the fact that in certain respects the high-frequency characteristics of an auto-transformer are essentially different from those of a straight transformer.

EFFECT OF TRANSMISSION LINE SURGE IMPEDANCE

In a grounded transformer one end of the high-voltage winding is connected to the transmission line, and the other end is grounded (see Fig. 1). In an auto-transformer, in addition to the above, some intermediate point of the high-voltage winding is connected to some other transmission line. It is this feature that makes the oscillation of an auto-transformer winding essentially different from that of a transformer winding.

In this paper the line that brings a given disturbance to an auto-transformer is called the "primary line," while the other line is called the "secondary line."

Furthermore, the transmission line connected to the highest point of the auto-transformer winding is termed the "high-voltage line," while the other line is termed the "low-voltage line."

The surge impedance of a transmission line generally does not exceed a few hundred ohms, while the impedance of a transformer to a surge is many times higher. On this account, the impedance of the primary line has practically no effect on the internal oscillation of windings, while the impedance of the secondary line dominates the phenomena.

As was pointed out before,[1] if a transformer neutral is connected to ground through a resistance of the order of a few hundred ohms, the transformer behaves under finite traveling wave excitation as produced by lightning, as if it were solidly grounded, because the surge impedance of the transformer is many times higher than the neutral resistance. Should the transformer neutral be connected to an infinite transmission line instead of being grounded through a resistance, the transformer will again act as if it were solidly grounded, provided the surge impedance of the line is of the same order of magnitude as the neutral resistance it replaces, *i. e.*, a few hundred ohms.

It is quite obvious that if the transmission line is connected not to the neutral point but to any other point of the winding, with the neutral left isolated,

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., January 26-30, 1931.

[1] See Bibliography 4.

the transformer will behave as if that were solidly grounded, provided there is a sufficient part of the high-voltage winding remaining between the primary and secondary transmission lines. If the secondary transmission line is connected only a few turns below the high-voltage line terminal, then the "surge impedance" of these few turns may be very small, and the transformer will act practically as a small inductance inserted between the two lines. For this reason, even a relatively short traveling wave may bring the potential of the secondary transmission line appreciably above ground. This potential depends on the surge impedance of the two lines, the value of the effective inductance of the part of the winding between the two lines, terminal conditions of any other winding on the same core, and the shape of the incident wave.

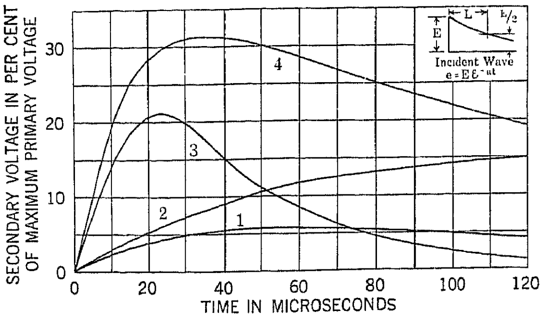

FIG. 2—VOLTAGE AT SECONDARY TERMINAL WITH INCIDENT WAVE APPLIED TO HIGH-VOLTAGE TERMINAL

1 and 3—Length *L* of incident wave 20 microseconds
2 and 4—Length *L* of incident wave 100 microseconds
1 and 2—High effective auto-transformer inductance
3 and 4—Low effective auto-transformer inductance
Long secondary line (see Fig. 4). Surge impedance of both lines 300 ohms auto-transformer ratio 2:1
High effective inductance covers range from 20,000 kva. 220,000 volts Y to 1,000 kva. 66,000 volts Y, and low effective inductance from 80,000 kva. 220,000 volts Y to 10,000 kva. 66,000 volts Y

The statement that the secondary line acts as a "solid ground" for internal oscillation is therefore only approximately correct. The potential of the secondary line rises in accordance with known law. In most cases this rise is very slow in comparison with the frequency of internal oscillations. It is for this reason that a long secondary line can be looked upon as a solid ground.

The curves of Figs. 2, 3 and 4 were obtained by use of the complete formula which is rather complicated, and will be published in the near future. Very nearly the same results may be obtained by assuming that the auto transformer acts as an air inductance coil connected between high- and low-voltage lines, the value of its inductance being equal to the short-circuit inductance between series and common windings. The effective surge impedance of primary line is its actual surge impedance, the effective surge impedance of the secondary line is its actual surge impedance multiplied by square of ratio of primary winding turns to secondary winding turns.

The complete phenomenon can, then, be divided into two parts: first, the gradual rise and fall of voltage on the secondary line, or the slow transient; and second, the internal oscillations within the windings, or the fast transient.

ORDINARY AUTO-TRANSFORMER

It can be seen that with the disturbances coming in from the high-voltage transmission line, with the low-voltage line connected and neutral grounded, the internal oscillations in an auto-transformer will be as if the end of the series winding were solidly grounded, and the common winding short-circuited.

It is also obvious that should the disturbance come in from the low-voltage line, with the high-voltage line connected, these oscillations would be as if both ends of the winding were solidly grounded.

The curves of Fig. 2 were prepared to determine more precisely the potential that is apt to appear on the secondary line in practise.

They show that under conditions commonly found, the potential of the secondary line remains a small fraction of the potential of the primary line.

There are four possible conditions of surge excitation of an auto-transformer when it is connected to both its lines:

A. Traveling wave arrives from high-voltage line.
B. Traveling wave arrives from low-voltage line.

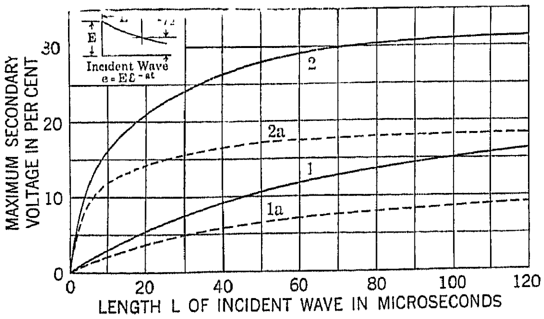

FIG. 3—MAXIMUM VOLTAGE AT SECONDARY TERMINAL WITH INCIDENT WAVE APPLIED TO HIGH-VOLTAGE TERMINAL

1 and 1A—High effective auto-transformer inductance
2 and 2A—Low effective auto-transformer inductance
1 and 2—Maximum secondary voltage expressed in per cent of maximum primary voltage
1A and 2A—Maximum possible secondary voltage expressed in per cent of primary line flashover voltage for very short wave. Obtained from curves 1 and 2 by taking impulse ratio of primary line insulation into account
Circuit constants the same as in Fig. 2

C. Two traveling waves of same polarity arrive simultaneously from both lines.

D. Two traveling waves of opposite polarity arrive simultaneously from both lines.

The first two cases are the most common. The last two are possible, practically speaking, in two cases only: first, when the lines are parallel, and second, when lightning storms take place directly over the auto-transformer.

It is believed that condition "D" can be satisfied only in case the same lightning discharge creates two traveling waves—one on one line by direct stroke, and another on the other line by induction.

For these reasons it appears that the probability of an auto-transformer being struck from both sides simultaneously, with waves of maximum possible amplitude, is very small.

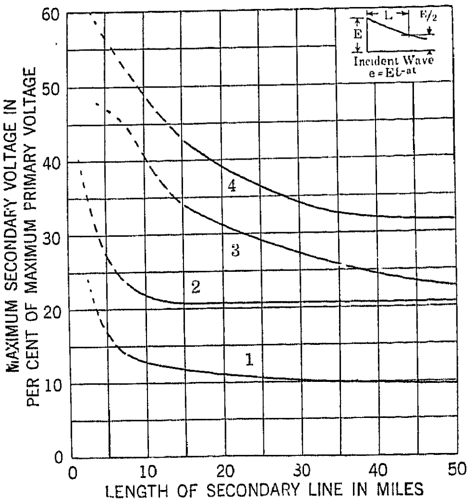

Fig. 4—Effect of Length of Secondary Line on Maximum Secondary Voltage

1 and 3—Length of incident wave 20 microseconds
2 and 4—Length of incident wave 100 microseconds
1 and 2—High effective auto-transformer inductance
3 and 4—Low effective auto-transformer inductance
Incident wave applied to high-voltage terminal
Circuit constants the same as in Fig. 2 and Fig. 3

Fig. 5 illustrates the initial and the maximum voltage to ground produced in an auto-transformer winding by a rectangular traveling wave under conditions A, B, C, and D. It is assumed that the wave is not shorter than half the duration of the natural period of the fundamental harmonics of the respective windings. This condition is generally fulfilled in service.

Condition "A."—The important conclusion that can be drawn from these diagrams is the fact that the insulation between coils and turns in the series part of the high-voltage winding is stressed to a much higher degree when the low-voltage line is connected than when it is disconnected.

It also follows that since the series winding contains only part of the total turns of the high-voltage winding, the stresses between turns are higher in the entire series winding than in the winding of a transformer of the same voltage rating. This occurs because the full voltage of the traveling wave appears across the series part of the high-voltage winding, not only during the initial voltage distribution but also during the oscillations that follow. The next important feature is that, owing to induction, relatively higher voltages are induced in the portions of the common and of the other windings adjacent to the series winding.

This increase in the voltage stresses between turns of an auto-transformer over those experienced in a straight transformer of the same voltage rating obviously becomes higher as the voltage ratio of the auto-transformer approaches unity. This is true because in case the rated voltages of an auto-transformer are in ratio of say 2:1, the series winding, which is one-half of the entire high-voltage winding, is subjected to the full voltage of the traveling wave on the high-voltage line, and its turn stresses are roughly twice the stresses that would be found in the high-voltage winding were the low-voltage line disconnected from the auto-transformer. However, in the case of an auto-transformer with the ratio of the two rated voltages equal to 1.1:1, the series winding is 9 per cent of the entire winding, and therefore the high-frequency voltage stresses between turns of the series winding will be very much higher than in an auto-transformer of 2:1 ratio. It should be noted however, that they will not be exactly $5\frac{1}{2}$ times as high, because the voltages of internal oscillations do not increase strictly in an inverse proportion to the turns.

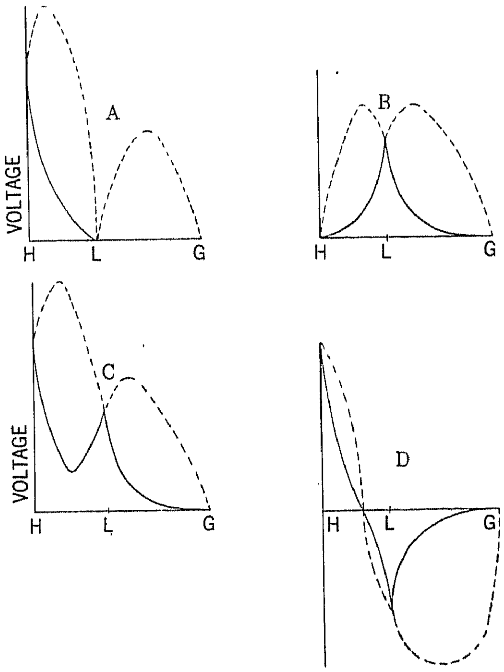

Fig. 5—Solid Lines Show Initial Voltage Distribution, and Dotted Lines Maximum Voltage to Ground Due to Subsequent Internal Oscillations

H, *L*, and *G* are high-voltage, low-voltage and ground terminals. Incident waves assumed to be proportional to rated terminal voltages.

Depending on the auto-transformer ratio the voltage to ground throughout the common winding under this mode of excitation may be substantially higher or lower than when the low-voltage transmission line is disconnected. The stresses between coils and turns in the common winding, adjacent to the series winding, may be of the same order or even greater.

Condition "B." The series winding is again subjected to the full voltage of the traveling wave, this time coming in from the low-voltage line. This means that should the number of turns of the series winding be substantially less than that of the common winding, the transient voltage stresses between turns of the series winding will be substantially greater than in the common winding.

In case the auto-transformer ratio approaches unity, it is obvious that exceedingly high stresses can be created in the series winding and, through induction, in the other windings adjacent to the series winding.

Voltage to ground at various points of the common winding may be higher under this type of excitation than under condition "A," but this depends on the ratio of the auto-transformer and the relative position of the windings.

Voltage to ground at various points of the series part of the winding will be either lower or equal to that caused by the previously described condition of excitation, depending again on the ratio of the auto-transformer. It becomes lower as the difference between the rated high and low voltages of the auto-transformer increases, provided the amplitudes of the two incident waves are proportional to the rated voltages of the high- and low-voltage transmission lines.

Condition "C." While it is quite possible to get simultaneous excitation on both lines due to lightning, it is considered quite improbable that the amplitude of both disturbances will be of sufficiently high magnitude to be of importance. However, to make the discussion general, the voltage distribution produced by such excitation is shown in Fig. 5. This figure illustrates the fact that out of all possible conditions of excitation, this condition of excitation creates the maximum voltages *to ground* in the series winding, as the series winding is excited from two ends simultaneously. However, voltages between coils and turns may not necessarily be the highest, as this would depend on what fraction of the total winding constitutes the series winding.

The voltages in the common winding, generally speaking, will not be materially higher than those found under condition B.

Condition "D." Under this condition, the voltages *across* the series winding are higher than under any other condition. While the voltages of various points of the series winding to ground may not be the highest, the voltages between coils and turns are the highest, and therefore the voltages induced in other windings, particularly parts adjacent to the series winding, are also the maximum. Voltages in the common winding are of the same order of magnitude as those found under condition B, except for the part of the common winding adjacent to the series winding, where they may be higher on account of induction, as above mentioned.

Fortunately, it appears most improbable that under this condition of excitation the amplitude of both traveling waves should approach extremely high values.

NON-RESONATING AUTO-TRANSFORMER

It was shown earlier in this paper that the internal transient voltages in auto-transformers may equal or considerably exceed those found in transformers.

The successful operation of all non-resonating transformers (totaling over 1,500,000 kva.) put into service during the last few years made it very desirable to extend the principle of electrostatic shielding to auto-transformers.

If perfect shielding is to be obtained, practical considerations make it essential that the potential of only *one* end of a winding be varied while the other end remains grounded either directly or through an impedor.

In the case of an auto-transformer, the neutral end of the winding is kept at ground potential, but there are *two* points in the winding instead of one, the potential of which can be varied independently. They are the line terminals of the series and common windings.

As has been shown, there are four possible conditions for transient voltage excitation of these terminals. This fact makes it impractical to obtain shielding which would be perfect for all four conditions of excitation. Fortunately, it is reasonable to consider only the first three conditions as important and probable. For these three conditions, the shielding methods described below give non-resonating characteristics to auto-transformer windings in a sufficient degree to make their use highly beneficial.

It was shown that under conditions experienced in service, the secondary transmission line connected to the auto-transformer may be considered as being at ground potential in regard to the internal voltage oscillations of the windings. Since traveling waves of very high magnitude are found to be relatively short, the point of the auto-transformer connected to the secondary transmission line may be considered as being solidly grounded during the period of internal oscillations.

Taking the above into consideration, it was found that a non-resonating auto-transformer can be produced without any radical change in the principal mechanical features from those used in a concentric winding core-type transformer.

It further was found that an auto-transformer can be made non-resonating to the various degrees of perfection that appear to be suitable for different service conditions.

In construction of windings and shields, the non-resonating auto-transformer is similar to the non-resonating transformer, (see Fig. 6).

The series and common windings are concentric with the tertiary winding. One of the most advantageous features of this construction is the absence of reentrant windings, which are undesirable but practically unavoidable in interleaved construction, (see Fig. 7). In addition to other reasons, reentrant windings are undesirable because of extremely non-uniform voltage distribution throughout the winding. This results in high local stresses, especially at the points of discontinuity, where the transient voltages of adjacent parts of different windings are generally in phase opposition.

There is a considerable number of methods which can be used in shielding auto-transformers. Six of them are shown in Fig. 8. On the left side of the figure are shown the connections of the shields to the windings; on the right side are shown the initial and maximum voltage distributions produced in service under four conditions. These conditions are:

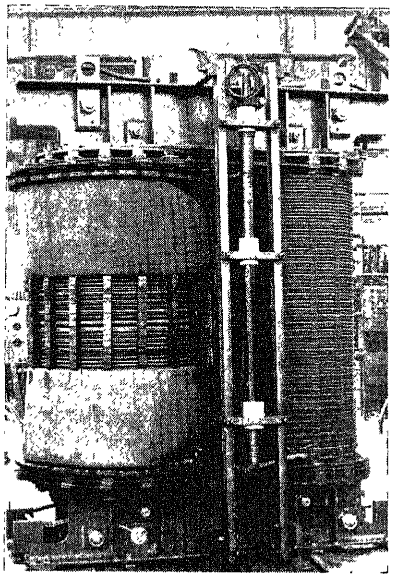

Fig. 6—20,000-Kva. (Output) 215,000- to 107,500- to 11,000-Volt Non-Resonating Auto-Transformer

W. Surge comes from the high-voltage line with the low-voltage line disconnected.

X. Surge comes from the high-voltage line with the low-voltage line connected.

Y. Surge comes from the low-voltage line with the high-voltage line connected.

Z. Surge comes from the low-voltage line with the high-voltage line disconnected.

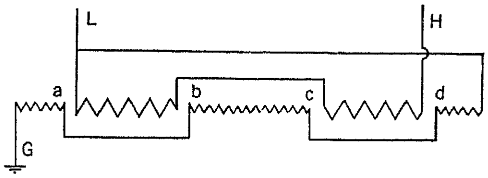

Fig. 7—Reentrant Winding Used in Auto-Transformer of Interleaved Construction. Points of Discontinuity at *a*, *b*, *c*, and *d*

The dotted lines in all of these diagrams indicate the maximum voltage to ground produced by transient voltages. The solid lines indicate the initial or electrostatic voltage distribution. Where two lines coincide only the solid line is shown.

It should be noted that under most of the methods of shielding, the internal voltage distribution created by transient voltages coincides with that of the standard acceptance induced voltage test. This is a very important feature, as it makes the standard induced voltage test a real test of ability of the insulation to withstand the stresses to which it can be subjected in service. As was pointed out before, this is not true for either transformers or auto-transformers of conventional design.

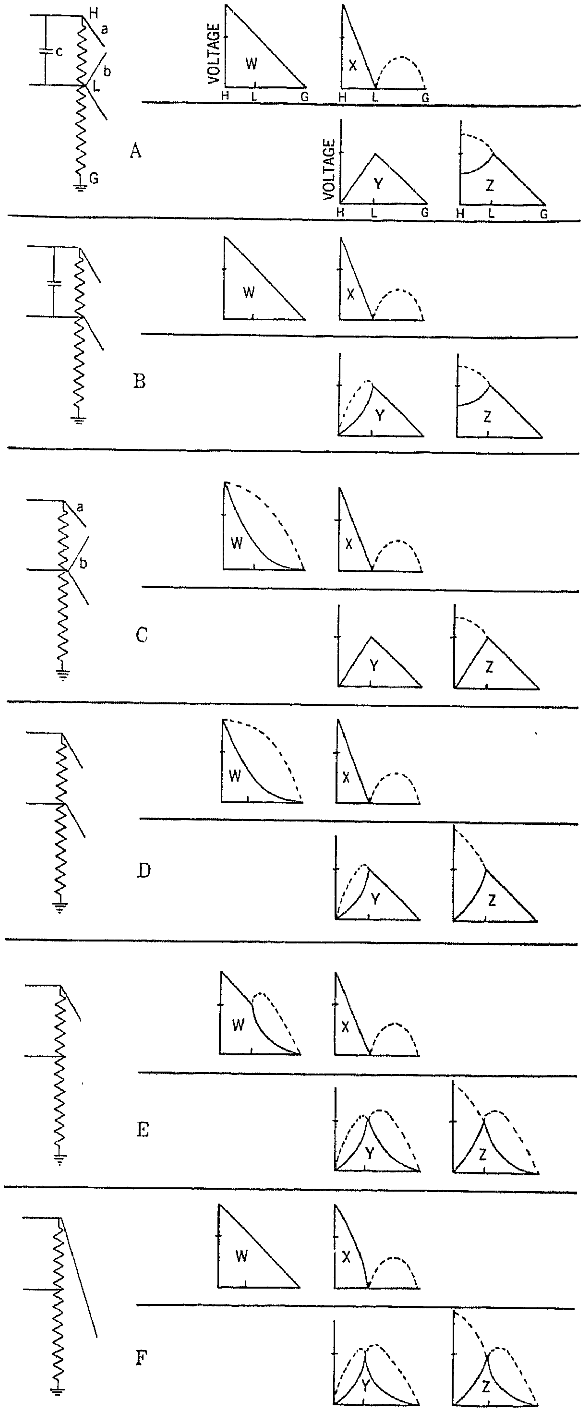

Fig. 8—Effect of Various Methods of Shielding on Voltage Distribution in Auto-Transformers

In some of the cases shown, where only partial shielding is employed, the voltages produced by oscillation exceed those produced by the standard induced voltage test (see *C, D,* and *E* of Fig. 8), to a lesser degree however than that found in ordinary auto-transformers. The insulation strength in this part of the winding should therefore correspondingly exceed that required for the standard induced voltage test.

Method A. The shielding method illustrated by diagram *A* of Fig. 8 is the most perfect shielding. It gives uniform voltage distribution throughout the entire winding for all conditions of operation except one. The condition under which the series part of the winding is not free from oscillation is when the high-voltage line is disconnected from the auto-transformer and the surge enters from the low-voltage line. Even under this condition, however, the internal voltages to ground do not exceed the equivalent values of voltage produced during the standard induced voltage test.

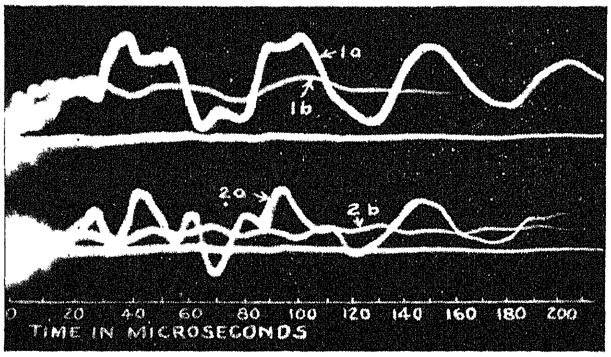

FIG. 9—INTERNAL VOLTAGES IN AUTO-TRANSFORMER

1—32 per cent from neutral
2—12 per cent from neutral
a—Without shields
b—Series and common windings shielded (see Fig. 8 *B* W)
Long rectangular wave applied to high-voltage terminal

Low-voltage line disconnected. Tests made in air, thereby creating some electrostatic unbalance

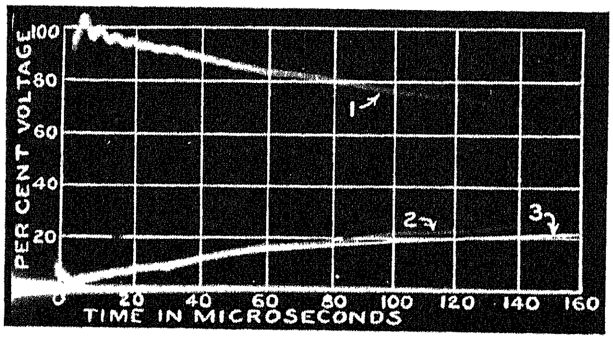

FIG. 10—SECONDARY VOLTAGE FOR AUTO-TRANSFORMER OF FIG. 6. LONG SECONDARY LINE

1—Wave applied to high-voltage terminal
2—Measured secondary voltage
3—Calculated secondary voltage

Since this condition of operation is very unusual, the shielding, as illustrated in this diagram, can be considered as being complete for all practical purposes.

Capacitor *c* consists of two concentric electrodes with a suitable number of herkolite cylinders and oil ducts between them.

Method B. This method is similar to method A, except that shield *b*, as shown in diagram *A*, is omitted. When this method is used, under conditions W and X the windings are non-resonating; while under conditions Y and Z the series winding oscillates but within the limits of stresses obtained during the induced voltage test.

Method C. This method is similar to method A, except for the omission of capacitor *c*.

The capacitor is desirable only when the low-voltage line is either disconnected or too short to act as an effective ground to the low-voltage terminal for disturbances entering from the high-voltage line (condition W).

The omission of the capacitor results in oscillations in the series and common windings under condition W, and some oscillation in the series winding under condition Z. However, under conditions X and Y, which are the most important and in which the series and common windings as a whole are subjected to the highest stresses (condition X for series and Y for common winding), the respective windings are non-resonating.

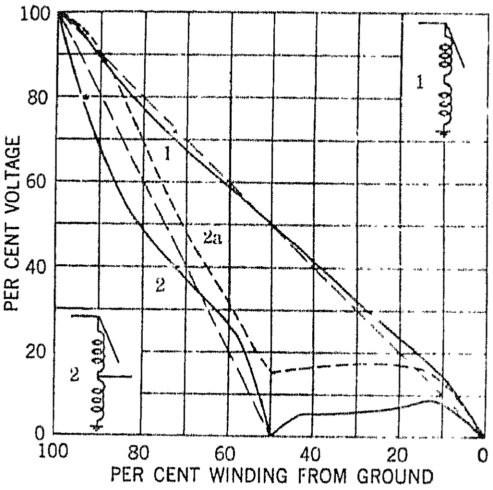

FIG. 11 MEASURED VOLTAGE DISTRIBUTOIN IN AUTO-TRANSFORMER OF FIG. 6

1 Initial voltage distribution and maximum voltage to ground, secondary line disconnected
2—Initial voltage distribution, secondary line connected
3—Maximum voltage to ground, secondary line connected

Method D. Without much increase in local stresses, method C can be simplified by the omission of shield *b*. This results in oscillation of the series winding when a surge comes from the low-voltage line. Otherwise it gives results as satisfactory as are obtained by the previous method.

Method E. When it is desired to make only the series winding non-resonating, which is the most common case since disturbances in the series winding are higher than those in the common winding, shielding method illustrated in diagram *E* is employed.

Method F. When the condition of operation shown nder W dominates, a longer shield than that employed method E may be used. This modifies the results s shown in diagram *F*.

Figs. 9, 10, and 11 show oscillograms and voltage stribution obtained in a non-resonating auto-transrmer built for commercial use and shown in Fig. 6.

CONCLUSIONS

rdinary Auto-Transformer

a. With only one transmission line connected to an to-transformer, a traveling wave produces internal cillations similar to those produced in a transformer.

b. With both transmission lines connected to their spective terminals, a traveling wave impressed from e line creates two kinds of voltage transients in the indings. The first is relatively slow, and is responble for the rise of potential on the secondary line. he second is relatively fast, and consists of internal ee oscillations in the windings.

c. For the internal free oscillation, a secondary line sufficient length acts as a "solid ground."

d. Internal stresses, particularly those between rns, are higher in an auto-transformer than in a ansformer of the same voltage rating. This becomes ore so as the auto-transformer voltage ratio approaches unity.

on-Resonating Auto-Transformer.

a. Auto-transformers can be built which are nonsonating for usual conditions of operation.

b. The windings of non-resonating auto-transrmers are uniform and free of breaks.

c. Non-resonating auto-transformers are similar to n-resonating transformers in general appearance and nstruction.

ACKNOWLEDGMENT

The credit for establishing some of the rigorous athematical relations used in this paper is due to Ir. J. H. Hagenguth. The general assistance of Ir. W. A. McMorris in the preparation of this paper also acknowledged.

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3. *Lightning Studies of Transformers by the Cathode Ray cillograph*, F. F. Brand and K. K. Paluoff, A. I. E. E. TRANS., l. 48, July 1929, p. 998.

4. *Effect of Transient Voltages on Power Transformer Design*, K. Palueff, A. I. E. E. TRANS., Vol. 49, July 1930, p. 1179.

Discussion

F. J. Vogel: Mr. Palueff's use of the term "non-resonating" licates that it should be defined as a transformer in which ans have been provided to reduce oscillations rather than a nsformer in which there are no oscillations. It is obviously orrect to speak of varying degrees of non-resonance unless the rm is defined as above.

In the description of the ordinary auto-transformer, condition *A*, the important conclusion is drawn that the insulation between coils and turns in the series part of the high-voltage winding is stressed to a much higher degree when the low-voltage line is connected. Further, this is true regardless of ratio, true whether due to initial distribution or oscillation, and as the author explains, will be roughly stressed in the order of the ratio of the transformer. Designers of auto-transformers have been familiar with these facts. Experience has always indicated that practically the entire surge voltage is across the series part of the winding whether the low-voltage line is connected or not and good insulation practise has been established on this basis.

In connection with Mr. Palueff's statements regarding interleaved windings, Fig. 1 (of this discussion) shows the initial distribution and oscillations within the type of winding most commonly employed. Note that all the stress is distributed over the series part of the winding. According to Mr. Palueff, point *b c* is a point of discontinuity, where high voltages prevail. This is not the case. The relation between the voltage stress at point *a c* and the insulation at that point is obviously the same regardless of whether or not the winding is of interleaved type or not, assuming, of course, that the same factors of safety are used. The normal graded insulation at point *b c* is approximately 50 per cent of that at point *a c*, while the corresponding stress is a much smaller proportion, so that instead of the insulation being very highly stressed at this point, there is a very large factor of safety.

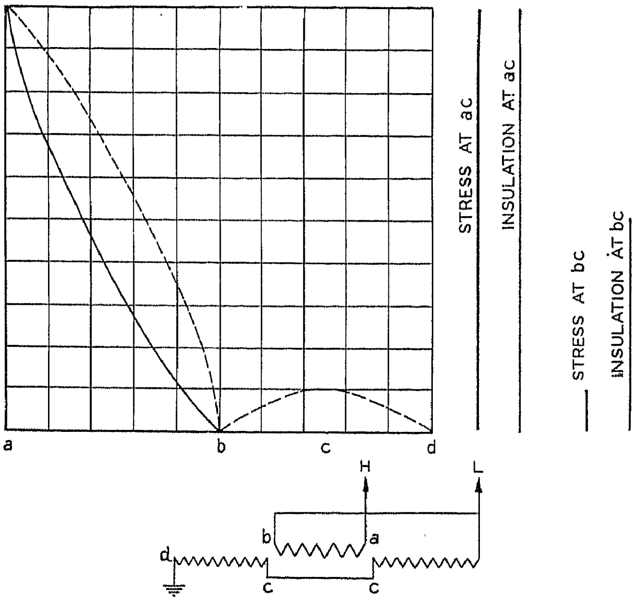

FIG. 1

In Mr. Palueff's Fig. 8A, an arrangement of shielding is shown which purports to show a uniform distribution of potential in the series part of the winding under all conditions of excitation. It is doubtful if the method of shielding shown will bring about this desired condition. It could be approximately obtained if excited from the high-voltage line by a single shield *a* connected to the high-voltage line lead, as he has shown in Fig. 8B (*X*). However, if shield *b* is used as shown in Figs. 8A and 8C, this shield will be at practically ground potential, when the surge enters from the high-voltage line, and when the low-voltage line is connected, for reasons previously explained. The part of the winding influenced by this shield will then have a large capacity to ground, and will approach ground potential very rapidly. The result of this will be to distort the distribution as I have shown from curve Fig. 2A to Fig. 2B and permit oscillations as indicated. Similarly, if a surge comes in on the low-voltage line, shield *a* will be at practically ground potential, and the part of the winding influenced by this shield will approach ground potential very rapidly. The result of this will be to distort the distribution as I have shown from curve Fig. 2C to Fig. 2D.

In Fig. 8E, covering the case of the single high-voltage shield, sketch *W* indicates a much lower voltage gradient than sketch *X*, sketch *W* being the case for the low-voltage line disconnected and sketch *X* with the low-voltage line connected. Since the slope of the curve of initial distribution is a function of the capacity between elements of the winding adjacent to the high-voltage terminal and the capacities between these elements and the ground and shield, it is obvious that the initial slope of this curve cannot be affected by the connection of the low-voltage lead. Mr. Palueff's sketch *W*, Fig. 8E, which I have shown as Fig. 2E, should be modified as indicated in Fig. 2F.

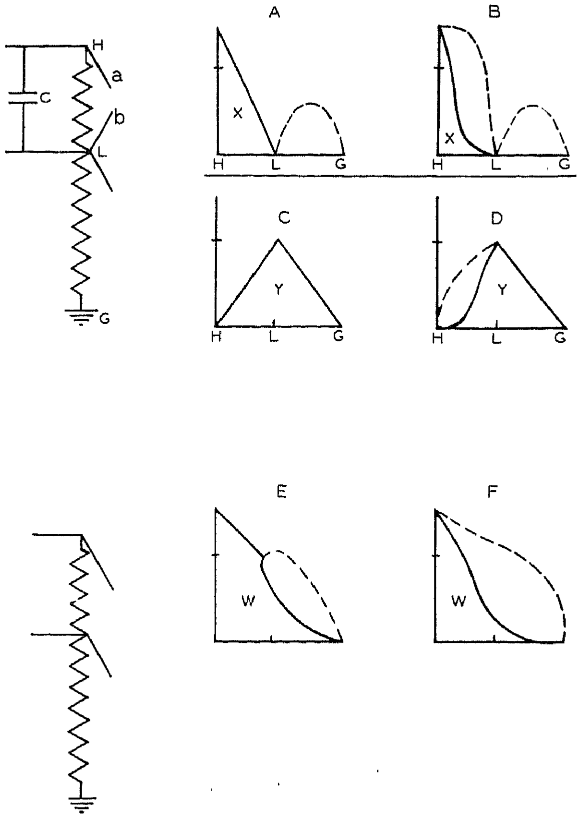

FIG. 2

K. K. Palueff: I shall attempt to answer Mr. Vogel's criticisms in the order he presented them.

The term 'non-resonating" is proper for the transformers bearing that name, as these transformers do not oscillate. In case of auto-transformers, a variety of shielding methods (Fig. 8) is possible which reduces oscillations very materially. Some of these methods (for example *W*, *X*, and *Y* of Fig. 8A) eliminate oscillations under operating conditions in the winding subjected to the surge, and in such cases the auto-transformers are non-resonating.

I believe that Mr. Vogel's remark regarding effect of connecting secondary line is referred to initial distribution only and he neglects to consider the subsequent oscillation.

Regarding points of discontinuity, Mr. Vogel informs us that the distribution of windings shown in his Fig. 1 is most commonly used, rather than that shown in Fig. 7. Apparently, he has in mind small units, for this arrangement of windings is not practical for large auto-transformers.

The insulation allowances shown in Fig. 1 of the discussion may be sufficient when operating with the low-voltage line connected, but with the low-voltage line being short-circuited or disconnected the oscillations are similar to those in a transformer, and the stresses at b_C will be much higher than shown.[2,3,4] The insulation must be sufficient for both conditions of operation.

The initial distribution shown by the solid line in Fig. 1 of Mr. Vogel's discussion is much better than the curve he found to be representative of his transformers, and published in Fig. 3 on page 62 of the A. I. E. E. TRANS., Vol. 50, March 1931. Correspondingly, the maximum voltage to ground shown by the dotted line is also too optimistic. Fig. 3 herewith compares his present data with that shown in his paper referred to above.

The voltage distribution shown in Fig. 8A is correct. If Mr. Vogel would make a calculation for the shielding method shown, he would find that it is an easy task to balance the effects of shields *a* and *b* to give the desired results.

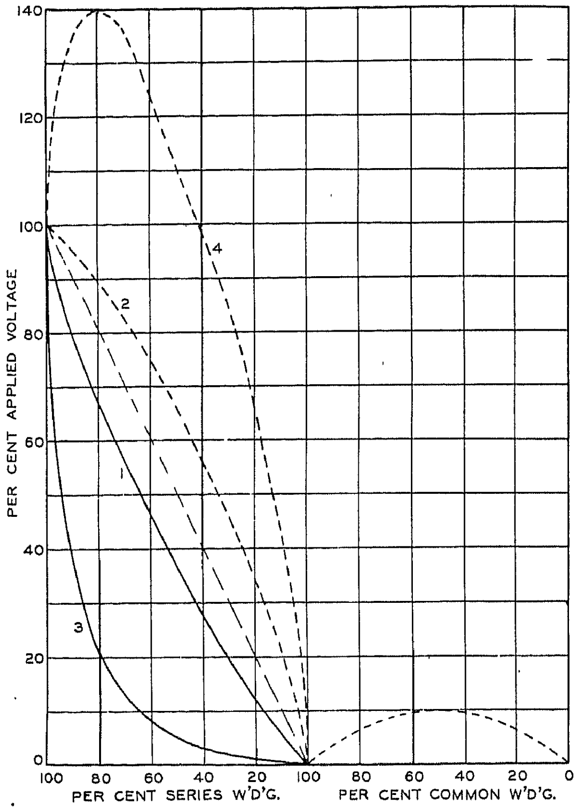

FIG. 3—1 AND 2—INITIAL DISTRIBUTION AND MAXIMUM VOLTAGE TO GROUND RESPECTIVELY, AS SHOWN IN FIG. 1 OF MR. VOGEL'S DISCUSSION

3—INITIAL DISTRIBUTION CURVE PUBLISHED BY MR. VOGEL (SEE REFERENCE IN DISCUSSION)

4—MAXIMUM VOLTAGE TO GROUND, ESTIMATED FROM CURVE 3

Sketch *X* under Fig. 8E is correct as shown. This condition is more important than that shown under *W*, since in the former case the entire impulse voltage appears across the series winding. Mr. Vogel is right in his correction of sketch *W*. I agree that the corrected distribution is better than that originally shown, since it does not have such high concentration of voltage near point *L*.

Harmonic Generation by Means of Grid-Circuit Distortion

BY F. E. TERMAN,[1] D. E. CHAMBERS,[2] and E. H. FISHER[3]
Associate, A. I. E. E. Associate, A. I. E. E. Non-member

Synopsis.—*The usual vacuum tube harmonic generators are fundamentally high distortion amplifiers operated under conditions of grid bias and plate voltage which, to distort the input voltage wave, utilize the non-linear relation between plate voltage and plate current. This paper gives the result of an investigation into the possibilities of using the non-linear grid voltage-current characteristic to produce harmonics. Grid distortion can be utilized by operating the tube so that the grid goes positive for a part of each fundamental frequency cycle and then inserting in series with the grid lead an impedance across which the grid current impulses produce a voltage drop rich in harmonics, which is then amplified in the plate circuit.*

The grid circuit distortion harmonic generator has the advantages of producing the harmonic in one circuit and developing the power in another, combined with the very high distortion properties of the grid circuit. The result is that the grid-distortion harmonic generator develops from 100 to over 1,000 times as much power on the higher harmonics as does the usual plate distortion generator. Thus, when using a suitable parallel resonant circuit as the distorting grid impedance, a steady output of 135 volts effective has been obtained on the 22nd harmonic from a radio receiving tube. When the grid impedance consists of a large inductance it is possible to generate simultaneously at least 1/20th milliwatts of power on every harmonic up to the 50th. The grid circuit input power required to give these results is several hundred milliwatts in the case of the parallel resonant circuit, and considerably less when a grid inductance is used.

The paper gives the mechanism of operation, illustrated with oscillograms and harmonic spectra, and includes a discussion of the effect of circuit conditions on the operation of the grid-distortion generator.

INTRODUCTION

FREQUENCY multiplication by the use of harmonic generators has an important place in communication. Thus in the type *B* carrier-current equipment of the Bell System the carrier frequencies of the different channels are harmonics of a single master frequency. Again, it is common practise to control the frequency of a radio transmitter by means of a piezoelectric crystal, and where the transmitted frequency is too high to be obtained directly from such a crystal it can be derived indirectly from a lower frequency by making use of harmonic generators. Another example of the practical value of the harmonic generator in communication technique is furnished by the broadcasting of the same chain program from a number of stations all operating on a common carrier frequency, as is being done by the British Broadcasting Company. Synchronization is maintained between the carrier waves of the stations in this English chain by deriving each carrier from a 1015.6 cycle current transmitted over wires and run through a series of 10-frequency doublers.

The vacuum-tube harmonic generators of present practise are fundamentally amplifiers operated under conditions of input voltage and grid bias such that the output voltage is a distorted replica of the input wave, and accordingly contains harmonics of the a-c. voltage that is applied to the grid. Harmonic generators of this type have the disadvantage of using the plate circuit of the tube to produce both the distortion and the power output. As a consequence it is impossible to adjust for a maximum percentage of harmonics and at the same time to obtain optimum conditions for large harmonic power. The two do not go together in a high-distortion amplifier operated in the usual way.

An improvement can be made in harmonic generators by producing the distortion in the grid circuit, and selectively amplifying the grid distortion in the plate circuit. In this way the grid conditions can be adjusted to give a high percentage of distortion without regard to the amount of power developed, and the plate current can be adjusted for optimum power output. A grid distortion method of harmonic generation has the additional advantage that the grid circuit is inherently capable of producing greater distortion than is the plate circuit. That is to say, the voltage-current characteristic of the grid circuit is more curved and contains a sharper bend than does the corresponding plate voltage-current relation.

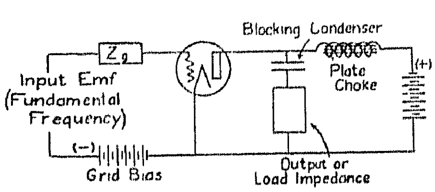

FIG. 1—BASIC CIRCUIT OF THE GRID-DISTORTION HARMONIC GENERATOR

Showing the distorting grid impedance Z_g

THE GRID-DISTORTION HARMONIC GENERATOR

Grid distortion can be utilized to produce harmonics by means of the arrangement shown in Fig. 1, in which an impedance Z_g has been placed in series with the grid. Operation of the tube so that the grid goes somewhat positive at each positive crest of the input voltage

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produces impulses of grid current which flow through the grid impedance Z_g and develop a voltage drop across it. Since the impulses of grid current represent a highly distorted current wave rich in harmonics, the voltage drop across Z_g will also be distorted, particularly if the added grid impedance Z_g offers a high impedance to the higher harmonics contained in the grid current impulses. These harmonic voltages developed across the grid impedance Z_g are amplified in the plate circuit of the tube, and under favorable conditions can develop a surprising amount of harmonic power in the output impedance shown in Fig. 1.

ANALYSIS OF HARMONIC GENERATOR OUTPUTS

Analysis of the output voltage of the harmonic generators investigated was made possible by the "power circuit harmonic analyzer" which is part of the equipment of the Ryan High Voltage Laboratory. This piece of apparatus, which was developed by the American Telephone and Telegraph Company for inductive interference investigations, operates by selecting the desired harmonic component with suitable tuned circuits and measuring its potential with a vacuum-tube voltmeter. This analyzer has a frequency range up to the 50th harmonic of 60 cycles, and measures potentials down to 0.5 millivolt with good accuracy.

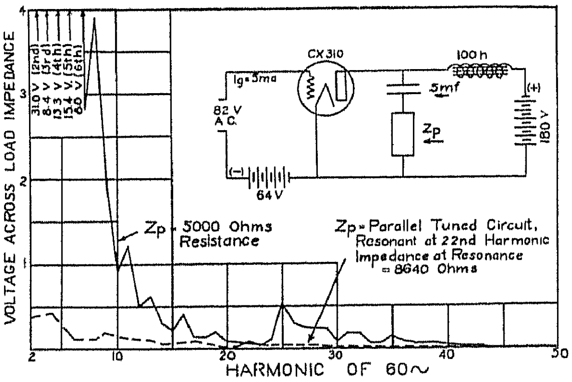

Fig. 2—Harmonic Spectrum Produced by Plate-Distortion Harmonic Generator with 60-Cycle Input

Harmonic analysis of the voltage developed across a resistance load by a plate-distortion type of harmonic generator gives frequency spectra of which Fig. 2 is typical. In the special case shown the grid bias and input voltage were adjusted to give a maximum output on the 22nd harmonic. For this particular curve the grid was allowed to become positive for a portion of each cycle. If the input voltage had been reduced sufficiently to keep the grid negative at all times the result would have been a harmonic spectrum similar to that of Fig. 2, but with all amplitudes somewhat reduced.

It is characteristic of all plate-distortion harmonic generators to develop a relatively large power output on the low order harmonics, particularly the second, while at the same time producing practically nothing on the high harmonics. This is clearly evident in Fig. 2. Replacing the load resistance by a circuit tuned to a high harmonic does not appreciably increase the output even on the resonant frequency, as is made apparent by the dotted line in Fig. 2, which is for a resonant circuit load tuned to the 22nd harmonic.

The possibilities of harmonic generation by grid distortion are shown by comparing Fig. 3 with Fig. 2. The results of Fig. 3 were obtained by using a distorting-grid impedance consisting of a large inductance, as shown by the circuit diagram on this figure, and by adjusting grid bias and input voltage to develop maximum voltage on a high order harmonic. For purposes of comparison, the harmonic spectrum developed by the plate-distortion generator with resistance load is replotted on this figure in dotted lines. The effect of the grid-circuit distortion introduced by the added grid inductance is primarily to increase, without producing a significant change in the lower order output, the voltage output on the high order harmonics to many times the value given by the plate-distortion method. Interpreted in terms of power, the distorting grid inductance produces an improvement in power output

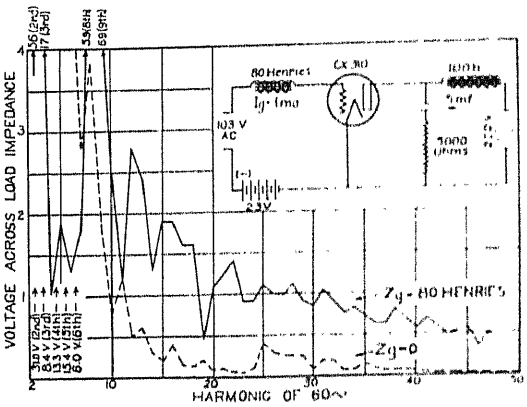

Fig. 3—Harmonic Spectrum Produced by Grid-Distortion Harmonic Generator

When the distorting grid impedance is an 80 henry inductance and the fundamental frequency is 60 cycles

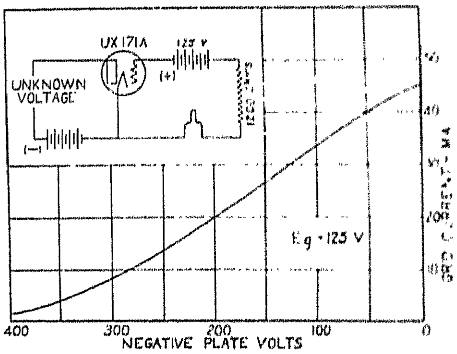

Fig. 4—Circuit and Typical Calibration Curve for Oscillograph Work Using the Inverted Vacuum Tube

on harmonics above the 20th ranging from 100 to 1,000 times, and develops a workable output (exceeding 1/20 milliwatt) on every harmonic up to the 50th.

By means of a parallel resonant distorting-grid impedance it is possible to accentuate the power output on a given harmonic. Replacing the resistance load by a suitable tuned circuit serves to increase the selective effect even more. An idea of what can be done by means of a tuned grid-harmonic generator is given by the oscillograms of Fig. 6. Here the 22nd harmonic voltage developed across the tuned circuit is 135 volts (which is the plate-filament voltage stepped up 5.6 times), and the 22nd harmonic circulating current in the tuned load circuit is 25 milliamperes. This represents several thousand times as much power as can be developed on this harmonic by plate distortion under comparable conditions.

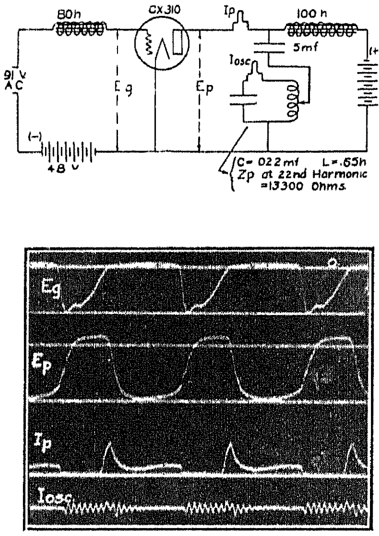

FIG. 5—CIRCUIT, TEST CONDITIONS, AND SERIES OF OSCILLOGRAMS

Showing performance of grid-distortion harmonic generator when using a distorting grid impedance consisting of an inductance

The use of a grid-distorting impedance consisting of a resistance was found to give little if any more harmonic power than no grid distortion at all. The principal effect of a grid resistance is to eliminate to a large extent the power loss in the grid circuit when the grid goes positive, by limiting the grid current.

The results presented in Figs. 2, 3, and 6 are typical examples selected from a large number of data. These figures serve as an index of what can be expected from grid- and plate-distortion harmonic generators under practical conditions.

THE MECHANISM OF THE GRID-DISTORTION HARMONIC GENERATOR

The rather complex action taking place in grid-distortion harmonic generators was investigated by means of oscillograms. In order to do this it was necessary to take oscillograms of rather high potentials, such as grid-filament and plate-filament voltages, without drawing any current, which was very satisfactorily accomplished by using an inverted vacuum tube.[4]

The inverted vacuum tube is essentially an ordinary triode with the functions of the grid and plate interchanged. That is, the grid is the positive anode, and the plate is kept at a negative potential and is the control electrode. Operated in this inverted manner, the tube acts as a voltage-reducing power amplifier. Any voltage applied to the negative plate causes a proportional variation in the grid current, and, since the plate of a small tube is capable of handling hundreds of volts, the inverted tube is ideally suited to oscillographic work in high-voltage low-current circuits. Fig. 4 gives the connection used for taking oscillograms with the inverted vacuum tube, and shows a typical calibration curve.

The series of oscillograms given in Fig. 5 shows the action taking place in the grid-distortion harmonic generator when the distorting grid impedance is an inductance, and the plate load is an impedance tuned to the 22nd harmonic. The effect of the grid inductance is to cause the grid-filament voltage to be very irregular even though the input voltage is a pure sine wave. The distorted grid voltage is characterized by a sudden change of grid potential as the grid goes negative, accompanied by an oscillation which is probably due to the distributed capacity of the grid inductance. The plate current wave is seen to be highly distorted, with sudden rises and falls which indicate the presence

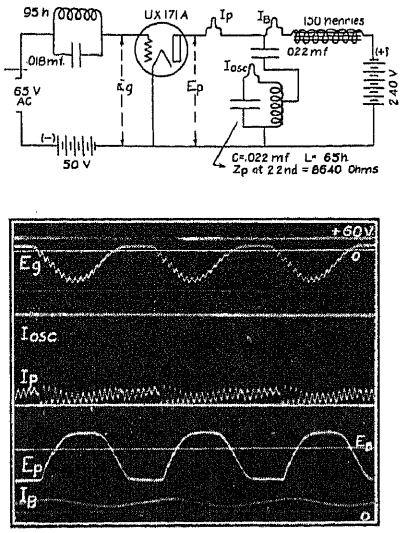

FIG. 6—CIRCUIT, TEST CONDITIONS, AND SERIES OF OSCILLOGRAMS

Showing performance of grid-distortion harmonic generator when using a distorting grid impedance tuned to the 22nd harmonic. The 22nd harmonic component of I_{osc} amounts to 25 milliamperes of high frequency components. The response obtained from this current in the circuit tuned to the 22nd harmonic is seen to be quite appreciable.

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[4] "The Inverted Vacuum Tube, a Voltage Reducing Power Amplifier," by F. E. Terman, *Proceedings I. R. E.*, **16** p. 447, April 1928.

The series of oscillograms in Figs. 6, 7, 8A, and 8B shows the action taking place in the harmonic generator when the distorting-grid impedance is a resonant circuit tuned to a harmonic. The results obtained with this type of distortion depend very considerably upon the circuit adjustments. In particular, best results are obtained when the plate current flows throughout the cycle, but at the same time the plate voltage must approach zero when the grid is positive.

The oscillograms in Fig. 6 show the operation of the tuned grid impedance under the most favorable conditions. It is seen that when the grid potential goes negative an oscillation is set up in the grid tuned circuit. This oscillation is amplified in the plate circuit, producing a very considerable 22nd harmonic circulating current in the tuned load impedance. This circulating current is a true harmonic of the fundamental frequency even when the resonant circuits are slightly detuned, because the action taking place repeats exactly at each cycle of fundamental frequency. The circulating current is not a pure wave of 22nd harmonic, but rather is modulated somewhat at 60 cycles and harmonics thereof, giving sideband components on the harmonics near the 22nd.

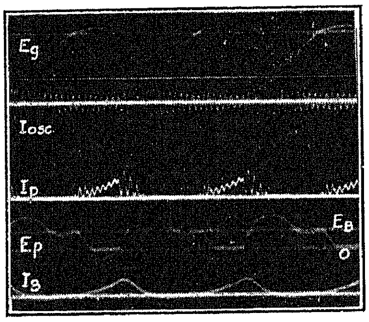

Fig. 7—Series of Oscillograms Similar to Fig. 6

Except that the circuit conditions are such as to allow the plate current to become zero for part of the cycle

Adjusting the tuned grid-harmonic generator to give a continuous flow of plate current throughout the entire period when the grid is negative makes it possible for the amplified grid oscillation to supply energy to the load during the entire existence of the oscillation. The effect of allowing the plate current to go to zero for part of the cycle is shown in Fig. 7, and is seen to cause a very great reduction in the harmonic output.

Conditions most favorable for a large harmonic power output are obtained by using a small plate blocking condenser, a very high impedance in the plate choke, and a tube with an amplification constant such that satisfactory grid distortion can be secured with a C bias that does not entirely cut off the plate current. The poor results of Fig. 7 are caused by a negative grid bias that is unnecessarily great, complicated by insufficient inductance in the plate reactor. Both of these factors combine to reduce the portion of the time the plate current flows.

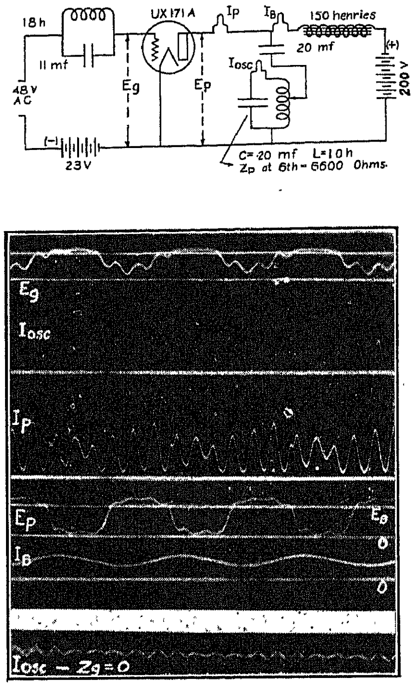

Fig. 8A—Circuit, Test Conditions, and Oscillograms

Showing performance of harmonic generator with and without a distorting-grid impedance tuned to the 6th harmonic. In the former case the 6th harmonic component of I_{osc} is 54 milliamperes

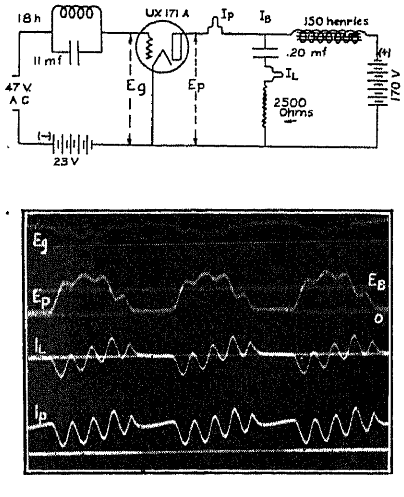

Fig. 8B—6th Harmonic Generation

Under conditions similar to those in Fig. 8A except that a resistance load is used

The use of a small blocking condenser is necessary in order to make the plate voltage drop to a very low value when the grid goes positive, and to cause the plate potential to rise rapidly when the grid again goes negative. The small blocking condenser can do this because it charges and discharges rapidly due to its small capacity. The blocking condenser capacity giving optimum results with a given set of circuit conditions is rather critical. Values approximately the same as the plate circuit tuning condenser were found to be the best. A large condenser keeps the plate voltage high even when the grid is considerably positive. Under these conditions, with grid and plate both at positive potentials of considerable magnitude, secondary electron emission takes place at the grid, which results in a negative grid-filament tube resistance, and consequent self-oscillation at a frequency determined by the circuit constants. This condition can ordinarily be detected by the existence of an abnormally low or even negative grid current, a consequence of secondary electron emission at the grid.

Characteristics of the Tuned Grid-Harmonic Generator

Harmonic generation by the use of a suitable parallel resonant circuit for a distorting grid impedance will generate low order harmonics with great success, as is evident from the series of oscillograms in Fig. 8A, which shows sixth harmonic generation. The sixth harmonic circulating current in the tuned plate circuit in this case amounts to 54 milliamperes, while the harmonic voltage across the tuned circuit is 117 volts, including a step-up ratio of 2 to 1, representing 530 milliwatts. It would be a relatively simple matter to combine low loss tuned circuits with a ratio of transformation of approximately ten, thus giving 585 volts effective from the generator instead of 117, and this output could be obtained from a radio receiving tube!

The advantages of using grid distortion instead of plate distortion in the production of harmonics are very vividly shown by the two oscillograms in Fig. 8A which contrast the current in the tuned plate circuit ($I_{os\,c.}$) when the tuned grid impedance is present, and when it is removed. With grid distortion the 6th harmonic current circulating in the output impedance is strong and steady, enduring from cycle to cycle with little change of amplitude, while plate distortion gives an output that, in addition to being weak, is of irregular amplitude. The current ratio in the two cases is about 10 to 1, representing a power ratio of 100 to 1.

The effect of replacing the tuned plate impedance by a load resistance is seen in Fig. 8B. It is apparent that this change does not alter the harmonic output very greatly, as is to be expected because the harmonic is actually generated by the grid in both cases.

The amount of harmonic voltage produced in the grid circuit between grid and filament is roughly proportional to the grid current and to the inductance used in the tuned grid circuit. Whatever harmonic voltage is produced in this way at the grid is then amplified in the plate circuit and produces a harmonic output that depends upon the extent to which the load impedance is able to absorb the available power. The way these relations work out in a practical case is shown in Fig. 9. In particular, it is to be noted that when the plate load impedance is varied by changing the circuit resistance, there is a very definite condition for maximum power output, as is always the case in amplifier operation.

The resistance of the tuned grid circuit should be low enough to prevent the harmonic currents in the circuit from dying out before the cycle of fundamental voltage is completed. In Fig. 6 it is evident that the grid oscillation is almost damped out before it is extinguished by the grid going positive. A lower tuned circuit resistance would have caused the grid oscillation to maintain itself better, and would have resulted in a higher harmonic output, while higher tuned circuit resistances would have reduced the output very considerably. Contrasting with this is the situation presented in Fig. 8, where the oscillation in the tuned grid impedance is substantially unchanged in amplitude for the fundamental cycle. Here the tuned circuit resistance could be increased considerably before the damping of the grid oscillation would be sufficient to alter extensively the harmonic output.

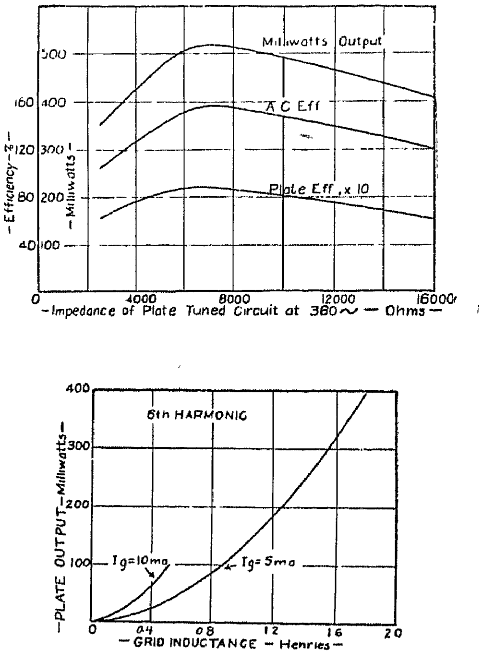

Fig. 9—Performance of Tuned Grid-Harmonic Generator Under Varying Circuit Conditions

In view of the large harmonic power developed by the tuned grid generator it is interesting to compute the efficiencies involved. This has been done in Fig. 9 for a number of conditions involving the sixth harmonic. The plate efficiency is the ratio of harmonic power to the plate battery power, and is seen to be about the same as when the tube is used as a distortionless audio frequency amplifier. The "a-c. efficiency" is the ratio of the harmonic power output to the fundamental frequency power supplied to the grid circuit of the harmonic generator, and can of course exceed 100 per cent. When the "a-c. efficiency" is 100 per cent it means that the harmonic power output is equal to the input power required by a stage of harmonic generator. It is accordingly apparent from Fig. 9 that a grid distorting generator will put out an amount of power on the sixth harmonic that is easily sufficient to operate directly the next stage of generation.

MISCELLANEOUS COMMENTS

The only disadvantage of the grid-distortion method of harmonic generation compared with the plate-distortion method lies in the fundamental frequency power required as a result of the grid current that must flow to produce grid distortion. With tubes of the size used in this study the input power was in the order of a few hundred milliwatts for a tuned grid impedance, and much less for a grid impedance consisting of a large inductance. Since the grid current flows only when the fundamental frequency input voltage is at or near its crest value, the grid power is approximately equal to the product of the d-c. grid current and crest input voltage.

The output on all except the very lowest order harmonics is very irregular with the plate-distortion generator. This is because the presence of the higher harmonics depends upon minor operating conditions which change from moment to moment sufficiently to vary the output on a high harmonic as much as 50 per cent.

In the case of grid-distortion harmonic generators this unsteadiness is very much less, particularly in the tuned grid case, where the 22nd harmonic under the conditions shown in Fig. 6 was perfectly steady. The amount of power on a given harmonic and the steadiness of this power are roughly proportional to each other, irrespective of how the harmonic is generated.

The experimental results given in this paper are all for a fundamental frequency of 60 cycles, but they can be applied to other frequencies by keeping all circuit reactances and resistances constant as the frequency is changed. Thus the results obtained with an 80-henry distorting grid inductance at a fundamental frequency of 60 cycles are the same as would be obtained at a 1,200-cycle fundamental frequency with 4 henrys distorting inductance. At radio frequencies certain factors such as tube capacities and plate to grid feedback must be taken into account in making such transformations. The use of a neutralized grid-distorting harmonic generator is very desirable when handling high frequencies.

CONCLUSIONS

The results of this study show that the grid-distortion method of harmonic generation is very definitely superior to the usual plate-distortion method in respect to: (1) power that can be obtained on a desired harmonic, particularly when this is a high order harmonic; (2) power that can be obtained simultaneously on a large number of harmonics; and (3) steadiness of output on a particular harmonic frequency. The power developed by the grid-distortion generator at a given harmonic ordinarily is at least 10 times the maximum value obtainable from the plate-distortion method and in some cases will be over 1,000 times as great.

These advantages of the grid-distortion harmonic generators are gained at the cost of a small power loss in the grid circuit of the generator. This power consumption will ordinarily be several hundred milliwatts or less, and must be supplied by the fundamental frequency input to the generator.

The authors wish to take this opportunity to express their appreciation of the cooperation and assistance rendered by Professor J. S. Carroll of the Ryan Laboratory.

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1931

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its forty-seventh Annual Report, for the fiscal year ending April 30, 1931. A general balance sheet showing the condition of the Institute's finances on April 30, 1931, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year, more detailed information having been published from month to month in the Institute JOURNAL and its successor, ELECTRICAL ENGINEERING.

Director's Meetings. The Board of Directors held seven meetings during the year, four in New York, one at Toronto, Ont., Canada, one at Philadelphia, Pa., and one at Pittsburgh, Pa. The Executive Committee acted upon various matters during intervals between Board meetings.

Information regarding the more important activities of the Institute which have been under consideration by the Board of Directors, the committees, and the various officers is published each month in the section of ELECTRICAL ENGINEERING devoted to "News of Institute and Related Activities."

President's Visits. President Lee attended the two national conventions and four District meetings held since the beginning of his administration, and visited a number of the Sections.

The following is a list of places visited in addition to many trips to Institute headquarters in New York: Regina, Sask., Canada; Vancouver, B. C., Canada; Portland, Ore. (Pacific Coast Convention); San Francisco, Calif.; Los Angeles, Calif.; Salt Lake City, Utah; Denver, Colo.; Omaha, Nebr.; Washington, D. C. (A. E. C. Meeting); Pittsburgh, Pa. (District Meeting); New York, N. Y. (including Winter Convention); Louisville, Ky. (District Meeting); Philadelphia, Pa. (District Meeting); Rochester, N. Y. (District Meeting); and Toronto, Ont., Canada.

In May and June, President Lee will visit other Sections and attend the Summer Convention in Asheville, N. C.

Meetings. Three national conventions and five District meetings were held during the year, and a brief report on each is given below.

Annual Meeting. The Annual Business Meeting was held at the Royal York Hotel, Toronto, Ont., Canada, on Monday morning, June 23, 1930, during the annual Summer Convention. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1930, was presented, and the Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1930.

Summer Convention.—The forty-sixth annual Summer Convention was held at Toronto, Ont., Canada, June 23-27, 1930. Forty-five papers were presented at eight technical sessions, and printed copies of fifteen technical committee reports were available. The Annual Conference of Officers, Delegates, and Members, under the auspices of the Sections Committee and the Committee on Student Branches, was held on Monday, June 23, and Tuesday, June 24, and fifty-one Section delegates, eight District Secretaries, and eight Counselor delegates were present. The entertainment features included golf and tennis tournaments, a dinner, and an all-day trip to the Welland Canal, with optional additional trips to Queenston and Niagara Falls. The Lamme Medal, awarded by the Institute, was presented to Rudolf E. Hellmund at the dinner. More than 1,100 members and guests attended the convention.

Pacific Coast Convention.—The nineteenth Pacific Coast Convention was held in Portland, Oregon, September 2-5, 1930. At five technical sessions, seventeen papers were presented. Ten technical papers by students were presented at two additional sessions. The program included various social events and trips. The attendance was more than 300.

Winter Convention.—The nineteenth Winter Convention was held in New York, January 26-30, 1931. Fifty papers were presented at ten sessions. At an evening session, the Edison Medal was presented to Dr. Frank Conrad, and a lecture on "Your Nimble Servant—The Electron," was given by Dr. Karl Taylor Compton, President of Massachusetts Institute of Technology. Numerous inspection trips, a smoker, and a dinner-dance were held. Attendance about 2,000.

District Meetings.—

District No.	Location	Dates	Papers	Attendance
1	Springfield, Mass.	May 7-10	24	360
2	Philadelphia, Pa.	Oct. 13-15	22	500
4	Louisville, Ky.	Nov. 19-22	18	300
2	Pittsburgh, Pa.	Mar. 11-13	16	500
1	Rochester, N. Y.	Apr. 29-May 2	22	370

Papers were presented at District meetings by students as follows: Springfield 6, Louisville 8, and

Rochester 8. A Conference on Student Activities was held at each of the five meetings listed above.

Sections.—Practically all Sections have been active, and many of the meetings have been especially effective. The programs have included a wide variety of subjects, and much attention has been given to the desirability of increasing the participation of Section members in the meetings.

The Memphis and San Antonio Sections, organized early in the fiscal year, have been very active. The Florida Section was organized March 27, 1931.

The four technical groups of the New York Section and the Power Group of the Chicago Section have been highly successful in their activities.

The New York Section organized three classes in effective speaking, with a total enrolment of more than 90, which met weekly for three months, and the results were highly satisfactory to those participating.

The two lecture courses, "Engineering Economics" at Armour Institute of Technology, and "Recent Developments in Electron Physics and Chemistry" at the University of Chicago, conducted through the coöperation of the Chicago Section and the Western Society of Engineers, have aroused so much interest that the Chicago Section has requested its members to indicate their preferences in a list of seven lecture courses and four courses in class instruction which are under consideration for next year.

President Lee visited a considerable number of Sections (see names under heading "President's Visits"), and will visit others later.

Under the plan for Institute Lecturers to visit Sections, approved by the Board of Directors in June, 1930, Past-President William McClellan visited the Atlanta, Cincinnati, Dallas, Houston, Louisville, Memphis, Oklahoma City, and San Antonio Sections in February and March, and Past-President A. E. Kennelly visited the Akron, Erie, Niagara Frontier, and Schenectady Sections in March.

Some of the Vice-Presidents visited all or nearly all of the Sections within their Districts.

The addresses of S. P. Grace, Assistant Vice-President, Bell Telephone Laboratories, Inc., have continued to be very popular, the average attendance per meeting at the ten meetings reported to headquarters being more than 3,200.

Many of the Sections have continued their close coöperation with neighboring Student Branches, with excellent results.

Student Activities.—In many respects, the student activities have shown remarkable improvement in each of the last two years. Nearly all Branches have been active during the past year, and the number of those depending to a considerable extent upon students for their programs was greater than ever before.

New Branches were organized at the University of British Columbia, Harvard University, University of Illinois, and Rice Institute.

The keen interest of Counselors and Branch Chairmen in the District Conferences on Student Activities has been continued.

Reports on such conferences were given at many Branch meetings. Many student conventions have been held, some sponsored by Sections and some by Districts.

Students gave talks at Branch meetings on a wide variety of subjects, including many of the latest technical developments, summer experiences, current events, and biographies. Many Branches held general discussions of their activities.

The suggestions made by Professor W. H. Timbie, Chairman, Committee on Student Branches, that the Branches have talks by students upon the life and work of Faraday, and that they purchase and present to their institutions pictures of Faraday, have already been carried out by about one-third of them.

A considerable number of the Branches were visited by the Vice-Presidents in their Districts.

The Constitution and By-laws were amended to change the provisions for Student enrolment and to provide for admitting Enrolled Students to the Institute as Associates, upon the expiration of their term of enrolment, without the payment of the usual entrance fee.

As a result of a recommendation of the Counselor Delegates at the Summer Convention in Toronto that the Institute make available to high school students information regarding the field of electrical engineering, and later more detailed recommendations of a subcommittee of the Committee on Student Branches, the Board of Directors authorized the publication of a pamphlet for this purpose, and the material is being prepared.

Section and Branch Statistics.—

	For fiscal year ending			
	April 30 1925	April 30 1927	April 30 1929	April 30 1931
SECTIONS				
Number of Sections	49	52	54	59
Number of Section meetings held	386	431	460	491
Total attendance	49,029	60,708	73,254	108,523
BRANCHES				
Number of Branches	82	95	100	109
Number of Branch meetings held	548	842	940	1,137
Total attendance	27,603	42,650	47,408	51,807

Meetings and Papers Committee.—The arrangement of technical programs for national conventions, coöperation with District meeting program committees, and the reviewing of technical papers have comprised the more important activities of the Meetings and Papers Committee for the year 1930-1931. During the year, the committee has reviewed 277 papers; of these 175 have been recommended for publication in the TRANSACTIONS, others are awaiting scheduling for future meetings, a number were recommended for ELECTRICAL

ENGINEERING publication only, and a few were returned to the authors. The committee members have, furthermore, assisted the Committee on Award of Institute Prizes in the selection of papers deserving of final consideration for national Institute paper prize awards.

Three national conventions and five District meetings were held during the year. 229 papers were presented, including 15 annual reports of technical committees. This represented an increase of 83 papers over the number which was presented during the previous year or an increase of more than 50 per cent. Many excellent contributions to the theory and practise of electrical engineering in its various fields are contained in these papers. Details regarding attendance and the number of papers presented at each meeting are given above in the reports on the various conventions and meetings.

The policy of requesting discussors to submit their discussion in writing was inaugurated as a trial at the Summer Convention, 1930. Discussors were requested to submit their remarks in writing within two weeks after the close of the meeting. Results attained indicate that discussion prepared in writing is more comprehensive than discussion given extemporaneously. The adoption of this policy also saves the expense of engaging a reporter, and results in a saving of time in the handling of discussion. The plan met with favor and, as a result, it was continued for all of the subsequent meetings throughout the year.

There is at all times an abundance of material for meetings and publication. To a considerable extent the selection of this material should be based on the breadth of the interest in the subjects on the part of the whole membership as well as on the intensity of interest by special groups in papers of limited appeal. In order that the committee may in the future better balance the meeting programs from this point of view, the committee adopted a resolution, April 10, 1931, requesting the Board of Directors to have a survey made of the membership for the purpose of ascertaining the principal fields of electrical interest of each member.

The technical program of the 1931 Summer Convention is completed; the program for the Pacific Coast Convention has taken on definite form; and consideration has also been given to appropriate subjects for technical sessions at the 1932 Winter Convention.

Publication Committee.—The country-wide survey referred to in last year's report having been completed and a careful analysis made of the various opinions expressed in the numerous replies received, the Publication Committee prepared a report which reviewed the situation, and in which recommendations were made that the present policy of publishing the TRANSACTIONS be continued but that certain important changes be made in the JOURNAL.

This report was presented to the Board of Directors at their May, 1930, meeting, but action was deferred pending its consideration at the Conference of Officers, Delegates, and Members at the Summer Convention, held in Toronto, June 25, 1930. At this conference the recommendations of the Committee were discussed with great interest and the report was endorsed with a recommendation that the Board of Directors adopt it as a statement of the Institute's policy regarding publications.

The general policy as recommended by the Publication Committee in its report may be briefly summarized as follows:

TRANSACTIONS.—Continue the present policy of publishing the TRANSACTIONS quarterly and printing papers in full together with discussion thereon.

MONTHLY JOURNAL.—

1. Publish interpretive abstracts of all papers presented at conventions and District meetings, and papers selected from those presented at Section and Branch meetings; and print in full only a limited number of outstanding papers which are of general interest to the engineering profession.
2. Print all annual reports of technical committees in full in both the JOURNAL and TRANSACTIONS.
3. Print in full special articles on timely scientific subjects of particular interest to engineers.
4. Print articles and reviews on recent developments in electrical engineering and related fields, constituting a review of the world-wide achievements in engineering and allied arts and sciences.
5. Record more adequately the outstanding activities of Sections and Branches.
6. Publish important electrical items from the Engineering Index to the extent of possibly four pages of the JOURNAL per month.

The report, including these recommendations, was formally approved by the Board of Directors at the meeting held in Toronto, June 25, 1930, with the understanding that the recommended changes become effective January 1, 1931. At a subsequent meeting, the Board approved the recommendation of the Publication Committee that the title of the monthly publication be changed from JOURNAL to ELECTRICAL ENGINEERING.

The Publication Committee and the National Secretary, after giving careful consideration to the necessary rearrangement and additions to the editorial staff with a view to building up an organization fully capable of successfully executing the recommendations embodied in its report to the Board of Directors, published the first issue of ELECTRICAL ENGINEERING in January, 1931.

This monthly official organ of the Institute which has a much broader scope than the JOURNAL previously published has been favorably received by the membership at large. This is indicated by the many comments received commending the Publication Committee and the editorial staff on their work.

Because of the many questions arising in connection with the new publication policy and because of many other matters coming up from time to time relative to all of the Institute's publications, the Committee has been meeting with the editorial staff on an average of once a month during the past year for the purpose of giving careful consideration to the various problems as they present themselves.

Coordination Committee.—The Committee corresponded with the officers of all Districts and Sections, and obtained their views regarding national and District meetings during the year 1932, resulting in recommendations to and adoption by the Board of Directors, in January and March, 1931, of a complete schedule of national and District meetings to be held during 1932.

The Committee considered and made recommendations upon various matters referred to it by the Board of Directors and the National Secretary.

Standards Committee.—Three new Standards have been added to the A. I. E. E. series during the past year, as follows: Constant Current Transformers, Air Circuit Breakers, and Switchboards and Switching Equipment for Power and Light. Four other Standards have been revised, *i. e.*, Transformers, Automatic Stations, Insulators, and Recommended Practise for Electrical Installations on Shipboard. Three reports on proposed Standards were likewise issued. Recognizing the economic importance and the demand on the part of users to obtain in service, under appropriate conditions, outputs greater than the test rating, as defined in the A. I. E. E. Standards, there was approved and issued a pamphlet, dealing with "Recommendations for the Operation of Transformers." Progress reports of further development of Standards through the agency of some of the Institute's technical committees indicated that work on two proposed Standards is practically completed and that the development of a series of test codes is likewise well under way.

Working under the procedure of the American Standards Association, the Institute has continued its sole or joint sponsorship in some twenty projects and is, in addition, represented on thirty additional undertakings going forward under the sponsorship of other organizations. A number of the Sectional Committees have submitted reports on proposed American Standards which the Institute has approved as a sponsor, notably the Sectional Committee on Insulated Wires and Cables and that on Scientific and Engineering Symbols and Abbreviations. The Sectional Committee on Electric Welding has been organized and there have been consolidations and realignments of other Sectional Committees in the interest of progress and economy of time. The Sectional Committee on Rotating Electrical Machinery reported recently that the task of revising and coördinating the five Institute Standards before them is now nearing completion and as a result of their work the five Standards will be consolidated and eventually issued in a single pamphlet. The Sectional Committee on Electrical Definitions, of which the Institute is sole sponsor, and whose undertaking is of such proportions as to require the organization of fifteen subcommittees, including three regularly organized Sectional Committees, has been making rapid progress and expects to issue its first report, containing from 2,500 to 3,000 definitions in the fall of 1931.

Committee on Safety Codes.—The most important subject within the Committee's field this year was the recommendation for changes in the National Electrical Code, made to the Electrical Committee of National Fire Protection Association, which Committee met on February 17, 18, and 19, in New York City. The recommendations which had previously had the consideration of Articles Committees were available in printed form, and copies were forwarded to all members of the Committee on Safety Codes, and opinions obtained as to whether the contained matter warranted a meeting for its consideration. The Chairman, and Vice-Chairman as alternate, attended the meeting of the Electrical Committee as representatives of the Committee on Safety Codes.

Committee on the Engineering Profession.—In August, 1929, the President of the Institute was authorized to appoint a "Committee on the Engineering Profession" to consider various matters affecting the status of the engineering profession, to coöperate with the American Engineering Council Committee on Engineering and the Allied Technical Professions, and to assume the duties of the A. I. E. E. Committee on Licensing of Engineers, which was at that time discontinued.

The Board of Directors at the October, 1931, meeting authorized the President to enlarge the Committee to include members located in many parts of the United States, and the personnel was promptly increased to fourteen. A list of the members may be found on page 53 of the January, 1931, issue of ELECTRICAL ENGINEERING.

This Committee has several very important fields of activity, but has, during the past year, devoted its principal efforts to coöperation with other organizations in the development of suitable provisions for the licensing of engineers, as proposed laws and amendments on this subject in various states have demanded a large amount of attention.

U. S. National Committee of the I. E. C.—The Seventh Plenary Meeting of the International Electrotechnical Commission was held June 27-July 9, 1930, with thirty-two delegates representing the United States National Committee, and about 300 representing other countries. The opening plenary meeting was held in Copenhagen, the meetings of the advisory (working) committees were held in Stockholm, and the final plenary meeting to confirm the actions taken by the advisory committees was held in Oslo.

At meetings of seventeen of the advisory committees, during the week beginning June 30, the following important actions were taken:

1. The official adoption by the I. E. C. of names for magnetic C. G. S. units, as follows: magnetic flux, maxwell; flux density, gauss; magnetic field intensity, oersted; magnetomotive force, gilbert.

2. Approval of a revised standard specification for rating of electrical machinery, containing new material relating to tolerances and revised values of temperature rise.

3. Approval of revision of Publication 35, dealing with graphical symbols for heavy-current systems, and of graphical symbols for radio communication.

4. Approval of document on Steam Turbines comprising two parts:

   I. Specifications for Condensing Steam Turbines.
   II. Rules for Acceptance Tests of Condensing Steam Turbines.

5. The methods of sampling of insulating oils proposed by the U. S. National Committee were accepted for adoption under the six months' rule.

6. Specifications for two classes of a-c. watthour meters and two classes of current and voltage transformers where especially intended for use with watthour meters were adopted.

7. Bases for computing and units for expressing waterpower resources for statistical use and for the purpose of making comparisons between different rivers, river basins, regions, and countries were adopted.

8. In connection with oil switches and circuit breakers, definitions or rules for the following items were agreed upon: operating duty, series of operations, standard series of interrupting operations, standard series of circuit making tests, recovery voltages, making and breaking current (for determining the performance of the circuit breaker), short-time current, power factor, and breaking power and breaking capacity.

In addition to the above, much progress was made in the work of the Commission which does not lend itself to being summarized in any brief way. The vitality, practical usefulness, and predominating influence of the I. E. C. in its field were clearly demonstrated anew.

Professor Enstrom of Sweden was elected President of the I. E. C. in succession to Professor Feldman.

It was decided to hold the next I. E. C. Plenary Meeting in Czechoslovakia in 1934.

Since the I. E. C. meeting, the U. S. National Committee has held several meetings, and is going forward diligently in preparation for meetings of various advisory committees, two of which will be held in the fall: Magnetic Units, in London, September 1931, of which Dr. A. E. Kennelly is Chairman; and Electric Traction Equipment, Brussels, October 1931. It is also proposed to convene meetings of several of the advisory committees during the year 1932.

In accordance with the statutes, the U. S. N. C. was reorganized following the I. E. C. Meeting. It is lending coöperation to the plans which are being developed by the Electrical Advisory Committee for the establishment of an electrical standards committee in the United States.

U. S. National Committee of the International Commission on Illumination.—The primary aim of the Committee during the past year was to arrange for creditable representation of American interests, developments, and opinions at the sessions of the International Commission which will be held in Great Britain, September 2-19, 1931.

The U. S. Committee is particularly charged with four important divisions of the Commission's work: (1) motor-vehicle lighting, (2) factory and school lighting, (3) aviation lighting, and (4) applied lighting practise in fields not otherwise specifically covered. The last two are new assignments, and in these there has been the keenest interest.

A special coördinating committee was formed to serve as a general clearing house for American contributions to the congress and commission sessions outside the scope of the secretariat committees.

Other international organizations sought the assistance of the commission on two important questions of standardization: (1) lamp bases and sockets, and (2) problems of photometry.

Technical Committees.—Reports of technical committees embracing an outline of the year's work and a summary of progress in the industry will be presented at the annual Summer Convention and printed in ELECTRICAL ENGINEERING and the TRANSACTIONS.

Membership.—The work of the Membership Committee has been carried on this year in much the same manner as in previous years, except that the practise of sending names of candidates furnished by the membership at large to headquarters was restored. This practise seems to be favored by the majority of the Section Membership Committee Chairmen.

A number of the Section Membership Committees have improved their organizations for membership work during the year.

The change in the By-laws admitting Enrolled Students to Associate membership without admission fee has had a good effect in increasing the number of such students applying for admission. In accordance with the established policy of the Institute, emphasis has continued to be placed on the qualifications of candidates.

In the absence of specific assignment to any one of the duty of promoting transfers to the higher grades, the Chairman of the Membership Committee has assumed the initiative in this activity by suggesting to the District Vice-Chairmen of the Committee that they undertake this work.

The additions to and deductions from the membership are shown in the following table. The total membership as of April 30, is 18,334.

	Honorary	Fellow	Member	Associate	Total
Membership on April 30, 1930	10	702	3,557	13,734	18,003
Additions:					
Transferred		56	334		
New members qualified		4	126	1,552	
Reinstated			13	113	
Total	10	762	4,030	15,399	20,201
Deductions:					
Died		6	34	61	
Resigned		3	43	422	
Transferred			46	344	
Dropped		2	59	847	
Membership on April 30, 1931	10	751	3,848	13,725	18,334

Deaths.—The following deaths have occurred during the year.

Fellows: Percy E. Hart, Charles D. Knight, C. Wellman Parks, George W. Patterson, William N. Ryerson, Robert J. Scott.

Members: Daniel Adamson, Harry B. Alverson, Arthur N. Anderson, Charles K. Badger, Seymour D. Benedict, Miles W. Birkett, William H. Bristol, Peter A. Coghlin, Edward G. Connette, Edwin F. Creager, Huibert Doyer, Frederick P. Fish, Frank R. Ford, Charles H. Gaffeney, Theodore I. Jones, John H. Lendi, Stanley L. B. Lines, John J. Lyng, Fred D. Lyon, Andrew S. MacDonald, Elwood F. McLaughlin, Paul H. Reinholdt, Andrew L. Riker, Edward P. Roberts, John F. Schoemaker, Elmer A. Sperry, Thomas Sproule, Edward J. Stanton, Ivan H. Summers, Harold L. Turner, Harold M. Walmsley, William F. Willmann, B. C. Wolverton, Beverly L. Worden.

Associates: Nathan B. Ambler, Koh-ichi Arai, Elwood W. Beck, Hyla H. Betts, Alexander L. Black, Edmund H. Bru, Waldo C. Bryant, Edgar E. Campbell, Louis M. Campfield, Voris O. Chewning, William B. Clarkson, Howard R. Connell, G. Stanley Covey, Donald K. Crawford, Alfred H. Crossman, William E. Davis, Samuel W. Dean, Eugene L. Delafield, William K. Detlor, Cyrus L. Doub, McKee Duncan, Leonard W. Egan, Joseph S. Fogerty, Howard H. Fuller, Harold W. Geis, Harold R. Goss, Hans Grabow, Eugene A. Hart, W. L. Henry, David C. Hershberger, Thornton L. High, Charles F. Hill, Benjamin C. Howard, Otto E. Huebner, Harry J. Hunsicker, Oscar C. Larson, J. H. Livsey, John Lundie, Francis MacLehose, Albert B. Mathis, Edmund D. McCarthy, William F. McKnight, Alvin Meyers, William H. Moses, Charles A. Mudge, Irvine M. Noble, Joseph C. Norton, Fenton L. Osgerby, Frederick Platt, Edward B. Plenge, Edward D. Priest, Joseph B. Rawlings, William A. Rosenbaum, Walter H. Seaver, Erik G. Sohlberg, Edward J. Stertman, Ralph E. Thurston, Harold W. Wikle, Robert M. Wilson, Thor Wollebak, Thorsten Zweigbergk.

Board of Examiners.—The Board of Examiners held nine meetings during the past year, averaging about three and one-quarter hours. A total of 3,773 cases were considered, divided as shown in the following table:

APPLICATIONS FOR ADMISSIONS		
Recommended for grade of Associate	1,129	
Not recommended	32	1,161
Recommended for grade of Member	130	
Not recommended	33	166
Recommended for grade of Fellow	3	
Not recommended	1	4
Recommended for enrollment as Students		2,042
APPLICATIONS FOR TRANSFER		
Recommended for grade of Member	322	
Not recommended	31	353
Recommended for grade of Fellow	44	
Not recommended	3	47
		3,773

Institute Prizes.—Four National Prizes and fifteen District Prizes for papers were awarded during the year, as announced through the Institute JOURNAL. These prizes were for papers presented at Institute meetings during 1929. The National Prizes were presented at the 1930 Summer Convention and the District Prizes at various meetings in the respective Districts. The National Prizes consist of $100.00 each and a certificate, while the District Prizes consist of $25.00 cash and a certificate.

Scholarships.—The governing bodies of Columbia University have placed at the disposal of the Institute each year a scholarship in electrical engineering for each class. The awards are made annually by an Institute committee. Each scholarship pays $350.00 toward annual tuition, with provision for reappointment.

Complete details governing prizes and scholarships may be obtained by applying to the National Secretary of the Institute.

Edison Medal.—The Edison Medal, founded by associates and friends of Thomas A. Edison, is awarded annually by a committee consisting of twenty-four members of the Institute "for meritorious achievement in electrical science, electrical engineering, or the electrical arts." The medal for 1930 was awarded to Dr. Frank Conrad, "for his contributions to radio broadcasting and short-wave radio transmission." The medal was presented at the Winter Convention of the Institute, January 28, 1931.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of civil, mining, mechanical, and electrical engineers, awarded the twenty-seventh medal to Admiral David Watson Taylor.

Lamme Medal.—The Lamme Medal was founded as a result of a bequest of the late Benjamin G. Lamme, Chief Engineer of the Westinghouse Electric and Manufacturing Company, who died on July 8, 1924. The bequest provides for the award by the Institute of a gold medal (together with a bronze replica thereof) annually to a member of the A. I. E. E. "who has shown meritorious achievement in the development of electrical apparatus or machinery" and for the award of two such medals in some years if the accumulation from the funds warrants.

The Lamme Medal Committee of the Institute awarded the third (1930) Lamme Medal to Dr. William James Foster, "for his contributions to the design of rotating alternating current machinery." Arrangements are being made for the presentation of the medal at the annual Summer Convention at Asheville, N. C., June 22-26, 1931.

Commission of Washington Award.—The Washington Award for 1931 was made to Dr. Ralph Modjeski, "for preëminent service in advancing human progress."

The award is made annually "to an engineer whose work in some special instance, or whose services in general have been noteworthy for their merit in promoting the public good," by a committee composed of nine representatives of the Western Society of Engineers and two each from the A. S. C. E., the A. I. M. E., the A. S. M. E., and the A. I. E. E.

Employment Service.—The Institute coöperates with the national societies of civil, mechanical, and mining engineers in the operation of the Engineering Societies Employment Service with its main office in the Engineering Societies Building, New York. Offices are operated in Chicago and San Francisco also, with the coöperation of the Western Society of Engineers in the former city, and the California Section of the American Chemical Society and the Engineers Club of San Francisco in the latter.

The service is supported by the joint contributions of the societies and their individual members who are benefited. As in the past it consists principally in acting as a medium for bringing together the employer and the employee. In addition to the publication of the Employment Service announcements monthly in ELECTRICAL ENGINEERING, weekly subscription bulletins are issued for those seeking positions.

American Engineering Council.—Since its organization about ten years ago, American Engineering Council has experienced a steady, normal growth. It includes in its membership twenty-six national, state, and local engineering societies which have a total membership of about 60,000.

The Council represents its constituents at conferences, conventions, and congressional hearings upon matters involving engineering and on important national committees. It also extends valuable individual service to its member organizations. Several notable surveys on matters of concern to the entire engineering profession have been conducted.

Through a standing Committee on Engineering and Allied Technical Professions, the Council endeavors to secure information upon and to improve the general status of the engineering profession.

Other standing committees are Constitution and By-laws, Finance, Membership and Representation, Public Affairs, Publicity and Publications, and Regional Activities.

The Council has many special committees, including those on: Airports; Airport Drainage and Surfacing; Air Transport Service in Foreign Commerce; Bridges; Communications; Competition of Governmental Agencies with Engineers in Private Practise; Dams; Flood Control; Government Reorganization; Man-Hour Information; Oil Pollution of Streams; Patents; Power; Program for Research; Reforestation; Representation; Street and Highway Safety; Street Signs, Signals, and Markings; Water Resources and Control; Weather Bureau; and Who's Who in Engineering.

In recent months, the Council, through its headquarter's staff and the A. S. C. E., A. S. M. E., and A. I. E. E., rendered assistance to the President's Emergency Committee for Employment. Some of the other important problems which have recently been considered are: the use of engineering works as means of alleviating the effects of the economic depression; methods of balancing the forces of production, distribution, and consumption; government coöperation with engineers; methods of making the work of the U. S. Weather Bureau of more use to engineers; flood control; studies of earthquakes; prompt publication of water supply and other data of the U. S. Geological Survey; government regulation of communication systems; pollution of navigable waters; single court of patent appeals; aircraft equipment tests; Federal relations to education; bridge legislation; Muscle Shoals; Federal Department of Public Engineering Works; and Ship model towing basin.

The Council publishes the American Engineering Council Bulletin monthly except July and August.

United Engineering Trustees, Inc.—This organization, formerly United Engineering Society, was set up by the four national societies of civil, mining, mechanical, and electrical engineers to hold in trust and to administer for them the Engineering Societies Building, in which their headquarters are located, certain funds, and the Library. Its charter gives it broad powers for the advancement of the engineering arts and sciences.

Extracts from the annual report of United Engineering Trustees, Inc., were published in the April, 1931, issue of ELECTRICAL ENGINEERING.

Engineering Foundation.—A department of United Engineering Trustees, Inc., was established in 1914 by the national societies of civil, mining, mechanical, and electrical engineers "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." It was conceived by Ambrose Swasey, of Cleveland, Ohio, and he has made three gifts towards its endowment. The fund has been generously increased through the gifts of Edward D. Adams and others, and also through a bequest of the late Henry R. Towne.

Appropriations have been made for various research projects and coöperation has been extended in others.

Engineering Societies Library.—The Library is administered as a free public library under the direction of the Library Board of United Engineering Trustees, Inc., this Board being composed of representatives of the national societies of civil, mining, mechanical, and electrical engineers.

The Library contains nearly 200,000 books and pamphlets. It receives regularly about 1,200 technical periodicals in many languages, and about 800 additional publications issued irregularly.

A staff of technically trained searchers and translators is maintained. The staff is prepared to furnish the following types of service: photoprints, abstracts, translations, bibliographies, searches, etc. Special arrangements have been made for lending books.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years. A complete list of representatives is published in the September and January issues of ELECTRICAL ENGINEERING.

Finance Committee.—During the year the Committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins & Sells, certified public accountants, have audited the books, and their report follows.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON,

National Secretary.

May 19, 1931.

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS

OFFICES IN THE PRINCIPAL CITIES OF
THE UNITED STATES OF AMERICA
—AND IN—
LONDON, PARIS, BERLIN, SHANGHAI,
MANILA, MONTREAL, HAVANA,
MEXICO CITY

CABLE ADDRESS "HASKSELLS"

NEW YORK CENTRAL BUILDING
75 EAST 45TH STREET
NEW YORK

May 16, 1931.

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

We have audited your accounts for the year ended April 30, 1931, and submit the following exhibits and schedule:

Exhibit
- A—Balance Sheet, April 30, 1931.
  Schedule
  - 1—Reserve Capital Fund—Securities.
- B—Summary of Income and Surplus for the Year Ended April 30, 1931.

WE HEREBY CERTIFY that in our opinion Exhibits A and B set forth the financial condition of the Institute at April 30, 1931, and the results of its operations for the year ended that date.

Yours truly,
HASKINS & SELLS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

EXHIBIT A.

BALANCE SHEET, APRIL 30, 1931

ASSETS

REAL ESTATE:			
One-fourth interest in Land, Building, and Equipment of United Engineering Trustees, Inc.,—25 to 33 West 39th Street (depreciation carried on books of the United Engineering Trustees, Inc.)			$496,948.48
EQUIPMENT:			
Library—Volumes and fixtures		$37,796.37	
Works of art, paintings, etc		3,001.35	
Office furniture and fixtures	$31,226.33		
Less reserve for depreciation (including $10,431.79 funded)	15,510.78	15,715.55	
Total equipment			56,513.27
WORKING ASSETS:			
"Transactions," etc		$ 5,354.90	
Text and cover paper		1,904.89	
Badges		1,734.31	
Total working assets			8,994.10
CURRENT ASSETS:			
Cash		$ 9,151.39	
Accounts receivable:			
Members—For dues		27,317.95	
Advertisers		1,632.84	
Miscellaneous		2,064.30	
Accrued interest on investments		3,349.96	
Total current assets			43,516.44
FUNDS:			
Reserve Capital Fund:			
Cash	$ 5,621.75		
Securities—Schedule 1	214,010.50	$219,632.25	
Life Membership Fund:			
Cash	$ 6,559.99		
Chicago, Burlington & Quincy Railroad 4% bonds, 1958, registered, face value, $5,000.00	4,868.75		
Accrued interest	33.33	11,462 07	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 1,295.61		
New York City 4½% corporate stock, 1957, par value, $2,000.00	2,204.05		
New York Telephone Company 4½% bond, 1939, registered, face value, $1,000.00	878.75		
Accrued interest	67.50	4,445.91	
Mailloux Fund:			
Cash	$ .17		
New York Telephone Company 4½% bond, 1939, registered, face value, $1,000.00	1,000.00		
Accrued interest	22.50	1,022.67	
Lamme Medal Fund:			
Cash	$ 280.28		
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series C bond, 1995, face value, $4,000.00	4,330.00		
Accrued interest	100.00	4,710.28	
Depreciation of Furniture and Fixtures Fund:			
Cash	$ 366.79		
Cleveland Union Terminals Company 5% sinking fund series B gold bonds, 1973, registered, face value, $4,000.00	4,010.00		
Fidelity Union Title and Mortgage Guaranty Company first mortgage certificates (on property, Nos. 75-79 Prospect Street, East Orange, N. J.) 5½%, due 1933	1,000.00		
United Gas Improvement Company $5.00 preferred stock, 20 shares	1,995.00		
Consolidated Gas Company of New York $5.00 cumulative preferred stock, 30 shares	3,060.00	10,431.79	
Total funds			251,704.97
TOTAL			$857,677.26

LIABILITIES

CURRENT LIABILITIES:		
Accounts payable	$11,556.34	
Dues received in advance	2,489.70	
Entrance fees and dues advanced by applicants for membership	275 00	
Subscriptions for "Transactions" received in advance	303.00	
Total current liabilities		$ 14,624.01
FUND RESERVES (not including depreciation reserve):		
Reserve Capital Fund	$219,632.25	
Life Membership Fund	11,462.07	
International Electrical Congress of St. Louis—Library Fund	4,445.91	
Mailloux Fund	1,022.67	
Lamme Medal Fund	4,710.28	
Total fund reserves (not including depreciation reserve)		241,273.18
SURPLUS, Per Exhibit "B"		601,780.04
TOTAL		857,677.26

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

SUMMARY OF INCOME AND SURPLUS FOR THE YEAR ENDED APRIL 30, 1931

EXHIBIT B

INCOME:			
Dues		*$233,169.04	
Students' fees		10,423.50	
Entrance fees		12,229.00	
Transfer fees		1,975.00	
Advertising		75,439.78	
"Electrical Engineering" subscriptions		8,863.34	
"Transactions" subscriptions		12,340.10	
Sales—Miscellaneous publications		10,553.96	
Sales—Badges	$ 3,233.75		
Less cost	2,895.13	338.62	
Interest on securities in Reserve Capital Fund		11,130.66	
Interest on securities in Depreciation Reserve Fund		210.84	
Interest on bank balances		951.20	
Total			$377,625.04
EXPENSES:			
Publications:			
"Electrical Engineering"	$93,761.05		
"Transactions"	33,428.73		
Technical papers	16,320.94		
Year book	9,700.86		
Miscellaneous	4,556.13	$157,767.71	
Institute meetings		19,239.40	
Administrative expenses		63,363.08	
President's special appropriation		639.89	
Board of Directors, traveling expenses		4,422.15	
National nominating committee, traveling expenses		948.97	
Institute representatives, traveling expenses		13.25	
Sections		35,371.30	
Geographical districts' expenses:			
Traveling expense:			
Executive committees	$ 2,873.88		
Vice-Presidents	1,250.82		
District best paper prizes	175.00		
District prizes for initial paper	150.00		
Institute lecturers	464.25	4,913.95	
Branches		10,493.86	
Membership		8,903.67	
FORWARD		$306,077.23	$377,625.04

*Includes $90,015.00 allocated to "Electrical Engineering" subscriptions.

TOTAL INCOME (FORWARD)		$377,625.04
EXPENSES (FORWARD)	$306,077.23	
Finance	419.74	
Code Committee	60.00	
Technical Committee	205.88	
Headquarters Committee	240.48	
Standards	7,602.13	
American Standards Association	1,500.00	
International Electrotechnical Commission	1,207.25	
Engineering Societies Library, maintenance	10,297.48	
Engineering Societies Employment Service	204.37	
United Engineering Trustees, Inc., assessment	5,713.56	
American Engineering Council	17,607.50	
Institute prizes	418.00	
Edison Medal Committee	63.92	
Edison Statue	500.00	
United States National Committee of International Commission on Illumination	300.00	
Total		352,417.54
NET INCOME		$ 25,207.50
SURPLUS CREDITS:		
Increase in value of equity in building	$ 3,595.88	
Adjustment of Life Membership Fund Reserve—Excess of fund over required amount as determined by the Institute in accordance with the by-laws	995.65	
Total		4,591.53
GROSS SURPLUS FOR THE YEAR		$ 29,799.03
SURPLUS CHARGES:		
Uncollectible dues and members' charges written off	$ 9,159.32	
Provision for depreciation of furniture and fixtures	2,260.87	
Loss on disposal of office fittings	94.36	
Decrease in value of library	3,765.86	
Total		15,280.41
SURPLUS FOR THE YEAR		$ 14,518.62
SURPLUS, MAY 1, 1930	$595,882.42	
Less transferred to Reserve Capital Fund in accordance with resolution of the Board of Directors	8,621.00	587,261.42
SURPLUS, APRIL 30, 1931		$601,780.04

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

RESERVE CAPITAL FUND—SECURITIES

APRIL 30, 1931

EXHIBIT A.
SCHEDULE No. 1.

	Face Value or Number of Shares	Book Value
RAILROAD BONDS:		
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series C bonds, due 1995, registered	$ 8,000.00	$ 8,910.00
Central of Georgia Railway Company 5½% refunding and general mortgage series B bonds, due 1959, registered	5,000.00	5,283.75
Chicago and Erie Railroad Company 5% first mortgage gold bond, due 1982, registered	1,000.00	1,105.00
Chicago, Burlington & Quincy Railroad Company 5% first and refunding mortgage series A gold bond, due 1971, registered	1,000.00	1,010.00
Chicago, Terre Haute & Southeastern Railway Company 5% first and refunding mortgage gold bonds, due 1960, registered	8,000.00	7,910.00
Florida East Coast Railway Company 5% first and refunding mortgage series A gold bonds, due 1974, registered	10,000.00	9,818.75
Great Northern Railroad Company 5½% general mortgage series B gold bonds, due 1952, registered	10,000.00	9,817.50
New York Central Railroad Company 5% refunding and improvement mortgage series C bonds, due 2013, registered	6,000.00	5,742.50
Pennsylvania Railroad Company 4½% general mortgage series A gold bonds, due 1965, registered	5,000.00	5,130.00
St. Louis, San Francisco Railway Company 5% prior lien mortgage series B bonds, due 1950, registered	6,000.00	5,497.50
Southern Railway Company 5% first consolidated mortgage gold bonds, due 1994, registered	1,000.00	980.00
Total		$ 61,295.00
PUBLIC UTILITY BONDS:		
American Telephone and Telegraph 5% sinking fund gold debentures, due 1960, registered	$15,000.00	$ 14,625.00
Consolidated Gas Company of New York 5½% gold debentures, due 1945, registered	5,000.00	5,187.50
Duquesne Light Company 4½% first mortgage series A gold bonds, due 1967, registered	3,000.00	2,970.00
New York Telephone Company 6% gold debenture bonds, due 1949, registered	5,000.00	5,525.00
Northern States Power Company 5½% first lien and general mortgage gold bonds series B, due 1950, registered	10,000.00	10,400.00
Ontario Power Service Corporation, Limited, 5½% first closed mortgage sinking fund gold bonds, due 1950, registered	5,000.00	4,711.00
Pacific Gas & Electric Company 5½% first and refunding mortgage series C gold bonds, due 1952, registered	5,000.00	5,137.50
Philadelphia Company secured 5% series A gold bonds, due 1967, registered	10,000.00	10,000.00
Shawinigan Water and Power Company 4½% first mortgage and collateral trust sinking fund series A gold bonds, due 1967	5,000.00	4,581.25
Texas Electric Service Company 5% first mortgage gold bonds, due 1960, registered	4,000.00	3,910.00
United Light and Power Company 5½% first lien and consolidated mortgage gold bonds, due 1959	5,000.00	4,975.00
Total		$ 72,022.25
Total Railroad Bonds (Forward)		$ 61,295.00
Total Public Utility Bonds (Forward)		72,022.25
INDUSTRIAL BONDS:		
American Smelting and Refining Company 5% first mortgage series A gold bonds, due 1947, registered	$ 9,000.00	$ 9,085.00
Bethlehem Steel Company 5% purchase money and improvement mortgage sinking fund gold bonds, due 1936, registered	5,000.00	5,033.75
Fidelity Union Title and Mortgage Guaranty Company first mortgage certificates (on property Nos. 75-79 Prospect Street, East Orange, N. J.) 5½%, due 1933	14,000.00	14,000.00
International Match Corporation 5% convertible gold debentures, due 1941	3,000.00	2,880.00
New York Steam Corporation 6% first mortgage gold bonds, due 1947, registered	10,000.00	10,837.50
United States Rubber Company 5% first and refunding mortgage series A bonds, due 1947, registered	2,000.00	1,915.00
Western Electric Company 5% debentures, due 1944, registered	10,000.00	9,818.75
Youngstown Sheet and Tube Company 5% first mortgage sinking fund series A gold bonds, due 1978, registered	20,000.00	20,275.00
Total		$ 73,845.00
STOCKS:		
Commonwealth Edison Company	12 shares	$ 2,892.00
Public Service Corporation of New Jersey, $5.00 preferred	30 "	2,958.75
United Gas Improvement Company, $5.00 preferred	10 "	997.50
Total		$ 6,848.25
TOTAL		$214,010.50

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